

Stochastic Modeling of Repairable Redundant System Comprising One Big Unit and Three Small Dissimilar Units

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Abstract This paper deals with the stochastic modeling of system comprising two subsystems A and B in series. Subsystem A consists three active parallel units. Failure time and repair time are assumed exponential. We developed explicit expressions for mean time to system failure (MTSF), system availability, busy period and profit function using Kolmogorov's forward equations method and perform graphical analysis to see the behavior of failure rates and repair rates on measures of system effectiveness such MTSF, system availability and profit function.

Keywords MTSF, System Availability, Profit Function, Active Parallel

1. Introduction

Stochastic models of redundant systems as well as methods of evaluating system reliability indices such as mean time to system failure (MTSF), system availability, busy period of repairman, profit analysis, etc have been researched in order to improve the system effectiveness.

There are systems of three units in which two units are sufficient to perform the entire function of the system. Such systems are called 2-out-of-3 redundant systems. These systems have wide application in the real world. The communication system with three transmitters can be sited as a good example of 2-out-of-3 redundant system. Many research results have been reported on reliability of 2-out-of-3 redundant systems. For example, Chander and Bhardwaj[1], analyzed reliability models for 2-out-of-3 redundant system subject to conditional arrival time of the server. Chander and Bhardwaj[2] present reliability and economic analysis of 2-out-of-3 redundant system with priority to repair. Bhardwaj and Malik[3] studied MTSF and cost effectiveness of 2-out-of-3 cold standby system with probability of repair and inspection. Taneja et al[4] deals with the reliability and cost benefit analysis of a system consisting of a big unit and two identical small units. A single repair facility appears and disappears from the system randomly with constant rates, Malik et al[5] analyzed two reliability models for a system of non

identical units original and duplicate using regenerative point technique., Mahmoud and Moshrefa[6] deal with the study of the stochastic analysis of a two unit cold standby system considering hardware failure, human error failure and preventive maintenance, Yusuf and Bala[7], studied stochastic two models of two unit parallel system. In model I, the system can be normal, deterioration (slow, mild or fast deterioration), failure whereas in model II, the system can either be in normal or failure modes. Using linear first order linear differential equations, various measures of system effectiveness such as mean time to system failure (MTSF) and availability are obtained to see the effect of deterioration on such measures, Kumar and Kadyan[8] deal with profit analysis of two unit non identical system with degradation and replacement while Surera et al[9] studied cost benefit analysis of a computer system with priority to software replacement over hardware repair, Bhardwaj and Malik[15] developed two models for 2-out-of-3 system to study cost benefit analysis using semi-Markov and regenerative process.

1.1. Objective

In this paper, we study a system comprising of two subsystems A and B in series. Subsystem A consists of three active parallel units while subsystem B is a single unit. The system is attended by four repairmen and considered up when: (1) all the units of subsystem A and subsystem B are working (2) two units of subsystem A and subsystem B are working. The system is down when two units of subsystem A failed or at the failure of subsystem B. We analyzed the system behavior using kolmogorov's forward equation methods. Explicit expression for measures of system

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effectiveness like mean time to system failure (MTSF), system availability, busy period of repairman, and profit analysis have been developed. The objective is to study the effect of failure and repair rates parameters with respect to subsystems A and B on reliability indices such as MTSF, availability and profit. Graphs were plotted to see the behavior of failure and repair rates on system performance.

Notations

A_{iO} Unit i in subsystem A is operational $i = 1, 2, 3$

A_{Ri} Failed unit in subsystem A under type i repair

A_{iG} Unit i in subsystem A is good

B_O Subsystem B is operational

B_{R4} Subsystem B is failed and under type 4 repair

β_i Type i failure rate of unit A_i in subsystem A

α_i Type i repair rate of unit A_i in subsystem A

λ Failure rate of subsystem B

μ Repair rate of subsystem B

1.2. Model Description and Assumptions

1. The system consist of two non identical subsystems A and B
2. Subsystem A consist three active parallel units
3. Units in subsystem A and subsystem B can have two modes: operation and failure
4. The system is attended by four repairmen
5. The system is down when two units of subsystem A failed or at the failure of subsystem B
6. The system is up when all the units of subsystem A

and subsystem B are operational or two units of subsystem A and subsystem B are operational

7. Units in subsystem A suffer three types of failures while subsystem B suffer one type of failure

8. Failure rates and repair rates are constant

1.3. State of the System

Up states:

$S_0(A_{1O}, A_{2O}, A_{3O}, B_O)$

$S_1(A_{R1}, A_{2O}, A_{3O}, B_O)$

$S_2(A_{1O}, A_{R2}, A_{3O}, B_O)$,

$S_3(A_{1O}, A_{2O}, A_{R3}, B_O)$

Failed states:

$S_4(A_{1O}, A_{2O}, A_{3O}, B_{R4})$

$S_5(A_{R1}, A_{R2}, A_{3G}, B_G)$

$S_6(A_{R1}, A_{2G}, A_{3G}, B_{R4})$,

$S_7(A_{1G}, A_{R2}, A_{R3}, B_G)$

$S_8(A_{1G}, A_{R2}, A_{3G}, B_{R4})$

$S_9(A_{R1}, A_{2G}, A_{R3}, B_G)$

$S_{10}(A_{1G}, A_{2G}, A_{R3}, B_{R4})$

2. Model Formulation

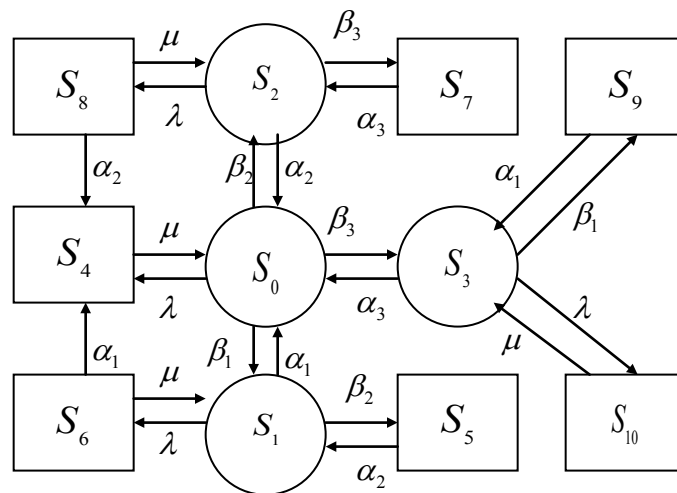


Figure 1. schematic diagram of the System

2.1. Mean Time to System Failure for System

Let $P(t)$ be the probability row vector at time t , then the initial conditions for this problem are as follows:

$$P(0) = [P_0(0), P_1(0), P_2(0), P_3(0), P_4(0), P_5(0), P_6(0), P_7(0), P_8(0), P_9(0), P_{10}(0)] = [1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]$$

we obtain the following system of differential equations from Figure 1 above:

$$\begin{aligned}
 \frac{dP_0(t)}{dt} &= -(\lambda + \beta_1 + \beta_2 + \beta_3)P_0(t) + \alpha_1 P_1(t) + \alpha_2 P_2(t) + \alpha_3 P_3(t) + \mu P_4(t) \\
 \frac{dP_1(t)}{dt} &= -(\lambda + \alpha_1 + \beta_2)P_1(t) + \beta_1 P_0(t) + \alpha_2 P_5(t) + \mu P_6(t) \\
 \frac{dP_2(t)}{dt} &= -(\lambda + \alpha_2 + \beta_3)P_2(t) + \beta_2 P_0(t) + \alpha_3 P_7(t) + \mu P_8(t) \\
 \frac{dP_3(t)}{dt} &= -(\lambda + \alpha_3 + \beta_1)P_3(t) + \beta_3 P_0(t) + \alpha_1 P_9(t) + \mu P_{10}(t) \\
 \frac{dP_4(t)}{dt} &= -\mu P_4(t) + \lambda P_0(t) + \alpha_1 P_6(t) + \alpha_2 P_8(t) \\
 \frac{dP_5(t)}{dt} &= -\alpha_2 P_5(t) + \beta_2 P_1(t) \\
 \frac{dP_6(t)}{dt} &= -(\mu + \alpha_1)P_6(t) + \lambda P_1(t) \\
 \frac{dP_7(t)}{dt} &= -\alpha_3 P_7(t) + \beta_2 P_2(t) \\
 \frac{dP_8(t)}{dt} &= -(\mu + \alpha_2)P_8(t) + \lambda P_2(t) \\
 \frac{dP_9(t)}{dt} &= -\alpha_1 P_9(t) + \beta_1 P_3(t) \\
 \frac{dP_{10}(t)}{dt} &= -\mu P_{10}(t) + \lambda P_3(t)
 \end{aligned} \tag{1}$$

The differential equations above can be put in matrix form as $\dot{P} = AP$ where

$$A = \begin{bmatrix}
 -(\lambda + \beta_1 + \beta_2 + \beta_3) & \alpha_1 & \alpha_2 & \alpha_3 & \mu & 0 & 0 & 0 & 0 & 0 & 0 \\
 \beta_1 & -(\lambda + \alpha_1 + \beta_2) & 0 & 0 & 0 & \alpha_2 & \mu & 0 & 0 & 0 & 0 \\
 \beta_1 & 0 & -(\lambda + \alpha_2 + \beta_3) & 0 & 0 & 0 & 0 & \alpha_3 & \mu & 0 & 0 \\
 \beta_3 & 0 & 0 & -(\lambda + \alpha_3 + \beta_1) & 0 & 0 & 0 & 0 & 0 & \alpha_1 & \mu \\
 \lambda & 0 & 0 & 0 & -\mu & 0 & \alpha_1 & 0 & \alpha_2 & 0 & 0 \\
 0 & \beta_2 & 0 & 0 & 0 & -\alpha_2 & 0 & 0 & 0 & 0 & 0 \\
 0 & \lambda & 0 & 0 & 0 & 0 & -(\mu + \alpha_1) & 0 & 0 & 0 & 0 \\
 0 & 0 & \beta_3 & 0 & 0 & 0 & 0 & -\alpha_3 & 0 & 0 & 0 \\
 0 & 0 & \lambda & 0 & 0 & 0 & 0 & 0 & -(\mu + \alpha_2) & 0 & 0 \\
 0 & 0 & 0 & \beta_1 & 0 & 0 & 0 & 0 & 0 & -\alpha_1 & 0 \\
 0 & 0 & 0 & \lambda & 0 & 0 & 0 & 0 & 0 & 0 & -\mu
 \end{bmatrix}$$

It is difficult to evaluate the transient solutions hence following El-Said[10], Haggag[11], El-Said and Shrbeny[12], and Wang et al[14], we delete the rows and columns of absorbing state of matrix A and take the transpose to produce a new matrix, say Q .

The expected time to reach an absorbing state is obtained from

$$E[T_{P(0) \rightarrow P(\text{absorbing})}] = P(0)(-Q^{-1}) \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

$$\text{Where } Q = \begin{bmatrix}
 -(\lambda + \beta_1 + \beta_2 + \beta_3) & \beta_1 & \beta_2 & \beta_3 \\
 \alpha_1 & -(\lambda + \alpha_1 + \beta_2) & 0 & 0 \\
 \alpha_2 & 0 & -(\lambda + \alpha_2 + \beta_3) & 0 \\
 \alpha_3 & 0 & 0 & -(\lambda + \alpha_3 + \beta_1)
 \end{bmatrix}$$

This method is successful of the following relations:

$$E\left[T_{P(0) \rightarrow P(\text{absorbing})}\right] = P(0) \int_0^{\infty} e^{At} dt \int_0^{\infty} e^{At} dt = -A^{-1}, \text{ for } A^{-1} < 0$$

Expression for MTSF can therefore be obtain from

$$E\left[T_{P(0) \rightarrow P(\text{absorbing})}\right] = MTSF = \frac{N_1}{D_1} \quad (2)$$

Where

$$N_1 = (\lambda + \alpha_1 + \beta_2)(\lambda + \alpha_2 + \beta_3)(\lambda + \alpha_3 + \beta_1) + \beta_1(\lambda + \alpha_2 + \beta_3)(\lambda + \alpha_3 + \beta_1) + \beta_1(\lambda + \alpha_1 + \beta_2)(\lambda + \alpha_3 + \beta_1)$$

$$\beta_3(\lambda + \alpha_1 + \beta_2)(\lambda + \alpha_2 + \beta_3)$$

$$\begin{aligned} D_1 = & \alpha_1\beta_3^2\lambda + \beta_1\beta_2^2\lambda + \alpha_1\beta_1\beta_3^2 + \alpha_2\beta_1\lambda^2 + \alpha_3\beta_2^2\lambda + \alpha_2\beta_2^2\lambda + \beta_1\beta_2\beta_3^2 + \alpha_1\alpha_2\beta_2\lambda + \alpha_1\alpha_2\alpha_3\beta_2 + \alpha_1\alpha_2\beta_1\beta_2 + \\ & \alpha_1\alpha_2\alpha_3\lambda + \alpha_1\alpha_3\beta_3\lambda + 2\alpha_1\beta_1\beta_3\lambda + 2\alpha_2\alpha_3\beta_2\lambda + 2\alpha_2\beta_1\beta_2\lambda + 2\alpha_3\beta_2\beta_3\lambda + 4\beta_1\beta_2\beta_3\lambda + \alpha_3\beta_2\beta_3\lambda - \alpha_1\alpha_2\alpha_3\beta_1 + \\ & \alpha_3\beta_1\beta_2\lambda + \alpha_3\beta_1\beta_2\beta_3 + \alpha_1\alpha_3\beta_2\lambda + \alpha_1\beta_1\beta_2\lambda + \alpha_2\alpha_3\lambda^2 + \alpha_1\alpha_2\lambda^2 + \lambda^4 + \alpha_3\lambda^3 + 2\beta_1\lambda^3 + \alpha_2\lambda^3 + 2\beta_3\lambda^3 + \alpha_1\lambda^3 + \\ & 2\beta_2\lambda^3 + \beta_1^2\lambda^2 + \beta_2^2\lambda^2 + \beta_3^2\lambda^2 + 2\alpha_1\beta_3\lambda^2 + 2\alpha_2\beta_2\lambda^2 + 3\beta_2\beta_3\lambda^2 + \beta_1^2\beta_3\lambda - \alpha_1\alpha_2\beta_1^2 + \beta_1^2\beta_2\lambda + \beta_1^2\beta_2\beta_3 + \\ & \alpha_1\beta_2\lambda^2 + \alpha_2\beta_3\lambda^2 + \alpha_3\beta_3\lambda^2 + 3\beta_1\beta_3\lambda^2 + \alpha_1\alpha_3\lambda^2 + \alpha_1\beta_1\lambda^2 + 2\alpha_3\beta_2\lambda^2 + 3\beta_1\beta_2\lambda^2 + \alpha_3\beta_1\lambda^2 + \alpha_2\alpha_3\beta_2^2 + \alpha_2\beta_1\beta_2^2 \\ & + \beta_2^2\beta_3\lambda + \alpha_3\beta_2^2\beta_3 + \beta_1\beta_2^2\beta_3 + \beta_1\beta_3^2\lambda + \beta_2\beta_3^2\lambda + \alpha_1\beta_2\beta_3\lambda + \alpha_1\alpha_3\beta_2\beta_3 + \alpha_1\beta_1\beta_2\beta_3 + \alpha_2\beta_1\beta_3\lambda + \alpha_1\alpha_2\beta_3\lambda + \\ & \alpha_1\alpha_2\beta_1\beta_3 + \alpha_2\beta_2\beta_3\lambda + \alpha_2\beta_1\beta_2\beta_3 \end{aligned}$$

2.2. Steady state availability Analysis for System

For the availability case of Figure 1 following El-Said[10], Haggag[11], El-Said and Shrbeny[12], and Wang et al[14], the initial conditions for this system are:

$$P(0) = [P_0(0), P_1(0), P_2(0), P_3(0), P_4(0), P_5(0), P_6(0), P_7(0), P_8(0), P_9(0), P_{10}(0)] = [1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]$$

The system of differential equations in for System 1 above can be expressed as:

$$\begin{bmatrix} \dot{P}_0(t) \\ \dot{P}_1(t) \\ \dot{P}_2(t) \\ \dot{P}_3(t) \\ \dot{P}_4(t) \\ \dot{P}_5(t) \\ \dot{P}_6(t) \\ \dot{P}_7(t) \\ \dot{P}_8(t) \\ \dot{P}_9(t) \\ \dot{P}_{10}(t) \end{bmatrix} = \begin{bmatrix} -(\lambda + \beta_1 + \beta_2 + \beta_3) & \alpha_1 & \alpha_2 & \alpha_3 & \mu & 0 & 0 & 0 & 0 & 0 & 0 \\ \beta_1 & -(\lambda + \alpha_1 + \beta_2) & 0 & 0 & 0 & \alpha_2 & \mu & 0 & 0 & 0 & 0 \\ \beta_1 & 0 & -(\lambda + \alpha_2 + \beta_3) & 0 & 0 & 0 & 0 & \alpha_3 & \mu & 0 & 0 \\ \beta_3 & 0 & 0 & -(\lambda + \alpha_3 + \beta_1) & 0 & 0 & 0 & 0 & 0 & \alpha_1 & \mu \\ \lambda & 0 & 0 & 0 & -\mu & 0 & \alpha_1 & 0 & \alpha_2 & 0 & 0 \\ 0 & \beta_2 & 0 & 0 & 0 & -\alpha_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & \lambda & 0 & 0 & 0 & 0 & -(\mu + \alpha_1) & 0 & 0 & 0 & 0 \\ 0 & 0 & \beta_3 & 0 & 0 & 0 & 0 & -\alpha_3 & 0 & 0 & 0 \\ 0 & 0 & \lambda & 0 & 0 & 0 & 0 & 0 & -(\mu + \alpha_2) & 0 & 0 \\ 0 & 0 & 0 & \beta_1 & 0 & 0 & 0 & 0 & 0 & -\alpha_1 & 0 \\ 0 & 0 & 0 & \lambda & 0 & 0 & 0 & 0 & 0 & 0 & -\mu \end{bmatrix} \begin{bmatrix} P_0(t) \\ P_1(t) \\ P_2(t) \\ P_3(t) \\ P_4(t) \\ P_5(t) \\ P_6(t) \\ P_7(t) \\ P_8(t) \\ P_9(t) \\ P_{10}(t) \end{bmatrix}$$

The steady-state availability is given by

$$A_v(\infty) = P_0(\infty) + P_1(\infty) + P_2(\infty) + P_4(\infty) + P_7(\infty) \quad (3)$$

In the steady state, the derivatives of the state probabilities become zero so that

$$AP = 0 \quad (4)$$

which in matrix form

$$\begin{bmatrix} -(\lambda + \beta_1 + \beta_2 + \beta_3) & \alpha_1 & \alpha_2 & \alpha_3 & \mu & 0 & 0 & 0 & 0 & 0 & 0 \\ \beta_1 & -(\lambda + \alpha_1 + \beta_2) & 0 & 0 & 0 & \alpha_2 & \mu & 0 & 0 & 0 & 0 \\ \beta_1 & 0 & -(\lambda + \alpha_2 + \beta_3) & 0 & 0 & 0 & 0 & \alpha_3 & \mu & 0 & 0 \\ \beta_3 & 0 & 0 & -(\lambda + \alpha_3 + \beta_1) & 0 & 0 & 0 & 0 & 0 & \alpha_1 & \mu \\ \lambda & 0 & 0 & 0 & -\mu & 0 & \alpha_1 & 0 & \alpha_2 & 0 & 0 \\ 0 & \beta_2 & 0 & 0 & 0 & -\alpha_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & \lambda & 0 & 0 & 0 & 0 & -(\mu + \alpha_1) & 0 & 0 & 0 & 0 \\ 0 & 0 & \beta_3 & 0 & 0 & 0 & 0 & -\alpha_3 & 0 & 0 & 0 \\ 0 & 0 & \lambda & 0 & 0 & 0 & 0 & 0 & -(\mu + \alpha_2) & 0 & 0 \\ 0 & 0 & 0 & \beta_1 & 0 & 0 & 0 & 0 & 0 & -\alpha_1 & 0 \\ 0 & 0 & 0 & \lambda & 0 & 0 & 0 & 0 & 0 & 0 & -\mu \end{bmatrix} \begin{bmatrix} P_0(t) \\ P_1(t) \\ P_2(t) \\ P_3(t) \\ P_4(t) \\ P_5(t) \\ P_6(t) \\ P_7(t) \\ P_8(t) \\ P_9(t) \\ P_{10}(t) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Using the following normalizing condition

$$P_0(\infty) + P_1(\infty) + P_2(\infty) + P_3(\infty) + P_4(\infty) + P_5(\infty) + P_6(\infty) + P_7(\infty) + P_8(\infty) + P_9(\infty) + P_{10}(\infty) = 1 \quad (5)$$

We substitute (5) in any of the redundant rows in (4) to give

$$\begin{bmatrix} -(\lambda + \beta_1 + \beta_2 + \beta_3) & \alpha_1 & \alpha_2 & \alpha_3 & \mu & 0 & 0 & 0 & 0 & 0 & 0 \\ \beta_1 & -(\lambda + \alpha_1 + \beta_2) & 0 & 0 & 0 & \alpha_2 & \mu & 0 & 0 & 0 & 0 \\ \beta_1 & 0 & -(\lambda + \alpha_2 + \beta_3) & 0 & 0 & 0 & 0 & \alpha_3 & \mu & 0 & 0 \\ \beta_3 & 0 & 0 & -(\lambda + \alpha_3 + \beta_1) & 0 & 0 & 0 & 0 & 0 & \alpha_1 & \mu \\ \lambda & 0 & 0 & 0 & -\mu & 0 & \alpha_1 & 0 & \alpha_2 & 0 & 0 \\ 0 & \beta_2 & 0 & 0 & 0 & -\alpha_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & \lambda & 0 & 0 & 0 & 0 & -(\mu + \alpha_1) & 0 & 0 & 0 & 0 \\ 0 & 0 & \beta_3 & 0 & 0 & 0 & 0 & -\alpha_3 & 0 & 0 & 0 \\ 0 & 0 & \lambda & 0 & 0 & 0 & 0 & 0 & -(\mu + \alpha_2) & 0 & 0 \\ 0 & 0 & 0 & \beta_1 & 0 & 0 & 0 & 0 & 0 & -\alpha_1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} P_0(\infty) \\ P_1(\infty) \\ P_2(\infty) \\ P_3(\infty) \\ P_4(\infty) \\ P_5(\infty) \\ P_6(\infty) \\ P_7(\infty) \\ P_8(\infty) \\ P_9(\infty) \\ P_{10}(\infty) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

We solve for the system of equations in the matrix above to obtain the steady-state probabilities

$$P_0(\infty), P_1(\infty), P_2(\infty), P_3(\infty)$$

$$A_v = \frac{N_2}{D_2}$$

Where

$$\begin{aligned} N_2 = & \alpha_1 \alpha_2 \alpha_3 \mu (\mu^2 + 2\mu\lambda + \alpha_1 \mu + \alpha_2 \mu + \lambda^2 + \alpha_1 \lambda + \alpha_2 \lambda + \alpha_1 \alpha_2) + \alpha_2 \alpha_3 \beta_1 \mu (\mu^2 + \mu\lambda + \alpha_1 \mu + \alpha_2 \mu + \alpha_1 \lambda + \alpha_1 \alpha_2) + \\ & \alpha_1 \alpha_3 \beta_1 \mu (\mu^2 + \mu\lambda + \alpha_1 \mu + \alpha_2 \mu + \alpha_2 \lambda + \alpha_1 \alpha_2) - \alpha_1 \alpha_2 \mu (-\beta_2 \mu + \beta_1 \mu^2 - \beta_3 \mu^2 + 2\beta_1 \mu \lambda - 2\beta_2 \mu \lambda - 2\beta_3 \mu \lambda - \alpha_1 \beta_2 \mu + \\ & \alpha_1 \beta_1 \mu - \alpha_1 \beta_3 \mu - \alpha_2 \beta_2 \mu + \alpha_2 \beta_1 \mu - \alpha_2 \beta_3 \mu - \beta_2 \lambda^2 + \beta_1 \lambda^2 - \beta_3 \lambda^2 - \alpha_1 \beta_2 \lambda + \alpha_1 \beta_1 \lambda - \alpha_1 \beta_3 \lambda - \alpha_2 \beta_2 \lambda + \alpha_2 \beta_1 \lambda - \\ & \alpha_2 \beta_3 \lambda - \alpha_1 \alpha_2 \beta_2 + \alpha_1 \alpha_2 \beta_1 - \alpha_1 \alpha_2 \beta_3) \\ D_2 = & \alpha_1^2 \alpha_2^2 \alpha_3 \lambda + 2\alpha_1 \alpha_3 \beta_1 \mu^2 \lambda + \alpha_1 \alpha_3 \beta_1 \mu \lambda^2 - 2\alpha_2 \beta_1^2 \mu^2 \lambda - \alpha_2 \beta_1^2 \mu \lambda^2 - \alpha_1 \alpha_2 \beta_1^2 \mu \lambda + \alpha_1^2 \alpha_3 \beta_1 \mu \lambda + \alpha_3 \beta_1 \beta_2 \mu^2 \lambda + \\ & \alpha_1 \alpha_3 \beta_1 \beta_2 \mu \lambda + \alpha_1^2 \alpha_2^2 \alpha_3 \beta_2 + \alpha_1 \alpha_2^2 \alpha_3 \beta_2 \lambda - \alpha_1^2 \alpha_2^2 \alpha_3 \beta_1 + \alpha_1 \alpha_2 \alpha_3 \mu^3 + \alpha_1^2 \alpha_2 \alpha_3 \mu^2 + \alpha_1 \alpha_2^2 \alpha_3 \mu^2 + \alpha_1^2 \alpha_2^2 \alpha_3 \mu + \\ & \alpha_1 \alpha_2 \beta_3 \mu^3 + \alpha_1 \alpha_2 \beta_2 \mu^3 + \alpha_1 \alpha_3 \beta_1 \mu^3 + \alpha_1 \alpha_2 \alpha_3 \beta_1 \beta_2 \mu + \alpha_2^2 \alpha_3 \beta_1 \mu \lambda + 2\alpha_2 \alpha_3 \beta_1 \mu^2 \lambda + 3\alpha_1 \alpha_2 \beta_3 \mu^2 \lambda + \alpha_2^2 \beta_1 \beta_3 \mu^2 + \\ & \alpha_1 \alpha_2 \beta_1 \beta_2 \mu^2 + \alpha_1 \alpha_2^2 \beta_3 \mu^2 + 3\alpha_1 \alpha_2 \beta_2 \mu \lambda^2 + 2\alpha_1 \alpha_2 \alpha_3 \beta_2 \mu \lambda + 2\alpha_1^2 \alpha_2 \alpha_3 \mu \lambda + \alpha_1^2 \alpha_2^2 \beta_3 \mu + 2\alpha_1^2 \alpha_2 \beta_3 \mu \lambda + \alpha_1^2 \alpha_2 \beta_1 \beta_3 \mu - \end{aligned}$$

$$\begin{aligned}
& 3\alpha_1\alpha_2\beta_1\mu\lambda^2 + \alpha_2\beta_1\beta_3\mu\lambda^2 + \alpha_2\beta_1\beta_3\mu\lambda^2 - 2\alpha_1\alpha_2^2\beta_1\mu\lambda + \alpha_2^2\beta_1\beta_3\mu\lambda + \alpha_2^2\beta_1\beta_2\mu\lambda - \alpha_1^2\alpha_2^2\beta_1\mu + \alpha_1^2\alpha_2^2\beta_2\mu + \\
& \alpha_1^2\alpha_2\alpha_3\beta_2\mu - \alpha_1\alpha_2^2\beta_1^2\mu + \alpha_1\alpha_2^2\alpha_3\beta_2\mu + \alpha_1\alpha_2^2\beta_1\beta_3\mu + 2\alpha_1\alpha_2^2\alpha_3\mu\lambda + 2\alpha_1\alpha_2\alpha_3\beta_1\mu\lambda + 2\alpha_1\alpha_2\beta_1\beta_3\mu\lambda + \\
& 2\alpha_1\alpha_2^2\beta_3\mu\lambda + 2\alpha_1\alpha_2^2\beta_2\mu\lambda + \alpha_2\alpha_3\beta_1\mu\lambda^2 + 2\alpha_1^2\alpha_2\beta_2\mu\lambda + \alpha_1\alpha_2^2\beta_1\beta_2\mu - \alpha_1^2\alpha_2^2\beta_1\lambda + \alpha_1^2\alpha_2^2\beta_3\lambda + \alpha_1^2\alpha_2^2\beta_2\lambda + \\
& \alpha_1^2\alpha_2\alpha_3\beta_2\lambda + \alpha_1^2\alpha_2\alpha_3\lambda^2 - \alpha_1^2\alpha_2\beta_1\lambda^2 + \alpha_1^2\alpha_2\beta_3\lambda^2 + \alpha_1^2\alpha_2\beta_2\lambda^2 + \alpha_1\alpha_2^2\alpha_3\lambda^2 - \alpha_1\alpha_2^2\beta_1\lambda^2 + \alpha_1\alpha_2^2\beta_3\lambda^2 + \\
& \alpha_1\alpha_2^2\beta_2\lambda^2 + \alpha_1\alpha_2\alpha_3\beta_1\lambda^2 + \alpha_1\alpha_2\alpha_3\beta_2\lambda^2 + \alpha_1\alpha_2\alpha_3\lambda^3 - \alpha_1\alpha_2\beta_1\lambda^3 + \alpha_1\alpha_2\beta_3\lambda^3 + \alpha_1\alpha_2\beta_2\lambda^3 + \alpha_1\alpha_3\beta_1\beta_2\mu^2 + \\
& \alpha_1\alpha_2\alpha_3\beta_1\mu^2 + \alpha_1\beta_1\beta_3\mu^2\lambda + \alpha_1^2\alpha_3\beta_1\mu^2 + \alpha_2^2\beta_1\beta_2\mu^2 + \alpha_1^2\alpha_3\beta_1\beta_2\mu^2 - 3\alpha_1\alpha_2\beta_1\mu^2\lambda + 2\alpha_2\beta_1\beta_3\mu^2\lambda + 2\alpha_2\beta_1\beta_2\mu^2\lambda + \\
& \alpha_1^2\alpha_2\beta_2\mu^2 - \alpha_1\alpha_2^2\beta_1\mu^2 - \alpha_1\alpha_2\beta_1^2\mu^2 + \alpha_2^2\alpha_3\beta_1\mu^2 - \alpha_2^2\beta_1^2\mu^2 + \alpha_1^2\alpha_2\beta_3\mu^2 + 3\alpha_1\alpha_2\beta_2\mu^2\lambda - \alpha_1^2\alpha_2\beta_1\mu^2 + \\
& \alpha_1\alpha_2^2\beta_2\mu^2 + \alpha_1^2\beta_1\beta_3\mu^2 + \alpha_1\alpha_2\beta_1\beta_2\mu\lambda - \alpha_2^2\beta_1^2\mu\lambda - 2\alpha_1^2\alpha_2\beta_1\mu\lambda + 3\alpha_1\alpha_2\beta_3\mu\lambda^2 + 3\alpha_1\alpha_2\alpha_3\mu\lambda^2 + \alpha_1\beta_1\beta_3\mu^3 + \\
& \alpha_2\alpha_3\beta_1\mu^3 - \alpha_2\beta_1^2\mu^3 + \alpha_2\beta_1\beta_3\mu^3 + \alpha_2\beta_1\beta_2\mu^3 + \alpha_3\beta_1\beta_2\mu^3 + 2\alpha_1\alpha_2\beta_1\beta_3\mu^2 + \alpha_1\alpha_2\alpha_3\beta_2\mu^2 + 3\alpha_1\alpha_2\alpha_3\mu^2\lambda - \alpha_1\alpha_2\beta_1\mu^3
\end{aligned}$$

2.3. Busy Period Analysis

Using the same initial conditions as for the reliability case:

$$\begin{aligned}
P(0) &= [P_0(0), P_1(0), P_2(0), P_3(0), P_4(0), P_5(0), P_6(0), P_7(0), P_8(0), P_9(0), P_{10}(0)] \\
&= [1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]
\end{aligned}$$

The differential equations can be expressed as

$$\begin{bmatrix} \dot{P}_0(t) \\ \dot{P}_1(t) \\ \dot{P}_2(t) \\ \dot{P}_3(t) \\ \dot{P}_4(t) \\ \dot{P}_5(t) \\ \dot{P}_6(t) \\ \dot{P}_7(t) \\ \dot{P}_8(t) \\ \dot{P}_9(t) \\ \dot{P}_{10}(t) \end{bmatrix} = \begin{bmatrix} -(\lambda + \beta_1 + \beta_2 + \beta_3) & \alpha_1 & \alpha_2 & \alpha_3 & \mu & 0 & 0 & 0 & 0 & 0 & 0 \\ \beta_1 & -(\lambda + \alpha_1 + \beta_2) & 0 & 0 & 0 & \alpha_2 & \mu & 0 & 0 & 0 & 0 \\ \beta_1 & 0 & -(\lambda + \alpha_2 + \beta_3) & 0 & 0 & 0 & 0 & \alpha_3 & \mu & 0 & 0 \\ \beta_3 & 0 & 0 & -(\lambda + \alpha_3 + \beta_1) & 0 & 0 & 0 & 0 & 0 & \alpha_1 & \mu \\ \lambda & 0 & 0 & 0 & -\mu & 0 & \alpha_1 & 0 & \alpha_2 & 0 & 0 \\ 0 & \beta_2 & 0 & 0 & 0 & -\alpha_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & \lambda & 0 & 0 & 0 & 0 & -(\mu + \alpha_1) & 0 & 0 & 0 & 0 \\ 0 & 0 & \beta_3 & 0 & 0 & 0 & 0 & -\alpha_3 & 0 & 0 & 0 \\ 0 & 0 & \lambda & 0 & 0 & 0 & 0 & 0 & -(\mu + \alpha_2) & 0 & 0 \\ 0 & 0 & 0 & \beta_1 & 0 & 0 & 0 & 0 & 0 & -\alpha_1 & 0 \\ 0 & 0 & 0 & \lambda & 0 & 0 & 0 & 0 & 0 & 0 & -\mu \end{bmatrix} \begin{bmatrix} P_0(t) \\ P_1(t) \\ P_2(t) \\ P_3(t) \\ P_4(t) \\ P_5(t) \\ P_6(t) \\ P_7(t) \\ P_8(t) \\ P_9(t) \\ P_{10}(t) \end{bmatrix}$$

In the steady state, the derivatives of the state probabilities become zero this will enable us to compute steady state busy :

$$B(\infty) = 1 - P_0(\infty) \quad (6)$$

$$AP = 0$$

$$\begin{bmatrix} -(\lambda + \beta_1 + \beta_2 + \beta_3) & \alpha_1 & \alpha_2 & \alpha_3 & \mu & 0 & 0 & 0 & 0 & 0 & 0 \\ \beta_1 & -(\lambda + \alpha_1 + \beta_2) & 0 & 0 & 0 & \alpha_2 & \mu & 0 & 0 & 0 & 0 \\ \beta_1 & 0 & -(\lambda + \alpha_2 + \beta_3) & 0 & 0 & 0 & 0 & \alpha_3 & \mu & 0 & 0 \\ \beta_3 & 0 & 0 & -(\lambda + \alpha_3 + \beta_1) & 0 & 0 & 0 & 0 & 0 & \alpha_1 & \mu \\ \lambda & 0 & 0 & 0 & -\mu & 0 & \alpha_1 & 0 & \alpha_2 & 0 & 0 \\ 0 & \beta_2 & 0 & 0 & 0 & -\alpha_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & \lambda & 0 & 0 & 0 & 0 & -(\mu + \alpha_1) & 0 & 0 & 0 & 0 \\ 0 & 0 & \beta_3 & 0 & 0 & 0 & 0 & -\alpha_3 & 0 & 0 & 0 \\ 0 & 0 & \lambda & 0 & 0 & 0 & 0 & 0 & -(\mu + \alpha_2) & 0 & 0 \\ 0 & 0 & 0 & \beta_1 & 0 & 0 & 0 & 0 & 0 & -\alpha_1 & 0 \\ 0 & 0 & 0 & \lambda & 0 & 0 & 0 & 0 & 0 & 0 & -\mu \end{bmatrix} \begin{bmatrix} P_0(t) \\ P_1(t) \\ P_2(t) \\ P_3(t) \\ P_4(t) \\ P_5(t) \\ P_6(t) \\ P_7(t) \\ P_8(t) \\ P_9(t) \\ P_{10}(t) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

We solve for $P_0(\infty)$

Using the following normalizing condition

$$P_0(\infty) + P_1(\infty) + P_2(\infty) + P_3(\infty) + P_4(\infty) + P_5(\infty) + P_6(\infty) + P_7(\infty) + P_8(\infty) + P_9(\infty) + P_{10}(\infty) = 1$$

We substitute (5) in any of the redundant rows in (4) to give

$$\begin{bmatrix} -(\lambda + \beta_1 + \beta_2 + \beta_3) & \alpha_1 & \alpha_2 & \alpha_3 & \mu & 0 & 0 & 0 & 0 & 0 & 0 \\ \beta_1 & -(\lambda + \alpha_1 + \beta_2) & 0 & 0 & 0 & \alpha_2 & \mu & 0 & 0 & 0 & 0 \\ \beta_1 & 0 & -(\lambda + \alpha_2 + \beta_3) & 0 & 0 & 0 & 0 & \alpha_3 & \mu & 0 & 0 \\ \beta_3 & 0 & 0 & -(\lambda + \alpha_3 + \beta_1) & 0 & 0 & 0 & 0 & 0 & \alpha_1 & \mu \\ \lambda & 0 & 0 & 0 & -\mu & 0 & \alpha_1 & 0 & \alpha_2 & 0 & 0 \\ 0 & \beta_2 & 0 & 0 & 0 & -\alpha_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & \lambda & 0 & 0 & 0 & 0 & -(\mu + \alpha_1) & 0 & 0 & 0 & 0 \\ 0 & 0 & \beta_3 & 0 & 0 & 0 & 0 & -\alpha_3 & 0 & 0 & 0 \\ 0 & 0 & \lambda & 0 & 0 & 0 & 0 & 0 & -(\mu + \alpha_2) & 0 & 0 \\ 0 & 0 & 0 & \beta_1 & 0 & 0 & 0 & 0 & 0 & -\alpha_1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} P_0(\infty) \\ P_1(\infty) \\ P_2(\infty) \\ P_3(\infty) \\ P_4(\infty) \\ P_5(\infty) \\ P_6(\infty) \\ P_7(\infty) \\ P_8(\infty) \\ P_9(\infty) \\ P_{10}(\infty) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

The steady state busy period $B(\infty)$ is therefore

$$B(\infty) = \frac{N_3}{D_2}$$

$$\begin{aligned} N_3 = & \alpha_1 \alpha_2 \beta_3 \mu^3 + \alpha_1 \alpha_2 \beta_2 \mu^3 + \alpha_1 \alpha_3 \beta_1 \mu^3 + \alpha_1 \alpha_2 \alpha_3 \beta_1 \beta_2 \mu + \alpha_2^2 \alpha_3 \beta_1 \mu \lambda + 2 \alpha_2 \alpha_3 \beta_1 \mu^2 \lambda + 3 \alpha_1 \alpha_2 \beta_3 \mu^2 \lambda + \alpha_2^2 \beta_1 \beta_3 \mu^2 + \\ & \alpha_1 \alpha_2 \beta_1 \beta_2 \mu^2 + \alpha_1 \alpha_2^2 \beta_3 \mu^2 + 3 \alpha_1 \alpha_2 \beta_2 \mu \lambda^2 + 2 \alpha_1 \alpha_2 \alpha_3 \beta_2 \mu \lambda + \alpha_1^2 \alpha_2 \alpha_3 \mu \lambda + \alpha_1^2 \alpha_2^2 \beta_3 \mu + 2 \alpha_1^2 \alpha_2 \beta_3 \mu \lambda + \alpha_1^2 \alpha_2 \beta_1 \beta_3 \mu - \\ & 3 \alpha_1 \alpha_2 \beta_1 \mu \lambda^2 + \alpha_2 \beta_1 \beta_3 \mu \lambda^2 + \alpha_2 \beta_1 \beta_3 \mu \lambda^2 - 2 \alpha_1 \alpha_2^2 \beta_1 \mu \lambda + \alpha_2^2 \beta_1 \beta_3 \mu \lambda + \alpha_2^2 \beta_1 \beta_2 \mu \lambda - \alpha_1^2 \alpha_2^2 \beta_1 \mu + \alpha_1^2 \alpha_2^2 \beta_2 \mu + \\ & \alpha_1^2 \alpha_2 \alpha_3 \beta_2 \mu - \alpha_1 \alpha_2^2 \beta_1^2 \mu + \alpha_1 \alpha_2^2 \alpha_3 \beta_2 \mu + \alpha_1 \alpha_2^2 \beta_1 \beta_3 \mu + \alpha_1 \alpha_2^2 \alpha_3 \mu \lambda + 2 \alpha_1 \alpha_2 \alpha_3 \beta_1 \mu \lambda + 2 \alpha_1 \alpha_2 \beta_1 \beta_3 \mu \lambda + \\ & 2 \alpha_1 \alpha_2^2 \beta_3 \mu \lambda + 2 \alpha_1 \alpha_2^2 \beta_2 \mu \lambda + \alpha_2 \alpha_3 \beta_1 \mu \lambda^2 + 2 \alpha_1^2 \alpha_2 \beta_2 \mu \lambda + \alpha_1 \alpha_2^2 \beta_1 \beta_2 \mu - \alpha_1^2 \alpha_2^2 \beta_1 \lambda + \alpha_1^2 \alpha_2^2 \beta_3 \lambda + \alpha_1^2 \alpha_2^2 \beta_2 \lambda + \\ & \alpha_1^2 \alpha_2 \alpha_3 \beta_2 \lambda + \alpha_1^2 \alpha_2 \alpha_3 \lambda^2 - \alpha_1^2 \alpha_2 \beta_1 \lambda^2 + \alpha_1^2 \alpha_2 \beta_3 \lambda^2 + \alpha_1^2 \alpha_2 \beta_2 \lambda^2 + \alpha_1 \alpha_2^2 \alpha_3 \lambda^2 - \alpha_1 \alpha_2^2 \beta_1 \lambda^2 + \alpha_1 \alpha_2^2 \beta_3 \lambda^2 + \\ & \alpha_1 \alpha_2^2 \beta_2 \lambda^2 + \alpha_1 \alpha_2 \alpha_3 \beta_1 \lambda^2 + \alpha_1 \alpha_2 \alpha_3 \beta_2 \lambda^2 + \alpha_1 \alpha_2 \alpha_3 \lambda^3 - \alpha_1 \alpha_2 \beta_1 \lambda^3 + \alpha_1 \alpha_2 \beta_3 \lambda^3 + \alpha_1 \alpha_2 \beta_2 \lambda^3 + \alpha_1 \alpha_3 \beta_1 \beta_2 \mu^2 + \\ & \alpha_1 \alpha_2 \alpha_3 \beta_1 \mu^2 + \alpha_1 \beta_1 \beta_3 \mu^2 \lambda + \alpha_1^2 \alpha_3 \beta_1 \mu^2 + \alpha_2^2 \beta_1 \beta_2 \mu^2 + \alpha_1^2 \alpha_3 \beta_1 \beta_2 \mu^2 - 3 \alpha_1 \alpha_2 \beta_1 \mu^2 \lambda + 2 \alpha_2 \beta_1 \beta_3 \mu^2 \lambda + 2 \alpha_2 \beta_1 \beta_2 \mu^2 \lambda + \end{aligned}$$

$$\begin{aligned}
& \alpha_1^2 \alpha_2 \beta_2 \mu^2 - \alpha_1 \alpha_2^2 \beta_1 \mu^2 - \alpha_1 \alpha_2 \beta_1^2 \mu^2 + \alpha_2^2 \alpha_3 \beta_1 \mu^2 - \alpha_2^2 \beta_1^2 \mu^2 + \alpha_1^2 \alpha_2 \beta_3 \mu^2 + 3\alpha_1 \alpha_2 \beta_2 \mu^2 \lambda - \alpha_1^2 \alpha_2 \beta_1 \mu^2 + \\
& \alpha_1 \alpha_2^2 \beta_2 \mu^2 + \alpha_1^2 \beta_1 \beta_3 \mu^2 + \alpha_1 \alpha_2 \beta_1 \beta_2 \mu \lambda - \alpha_2^2 \beta_1^2 \mu \lambda - 2\alpha_1^2 \alpha_2 \beta_1 \mu \lambda + 3\alpha_1 \alpha_2 \beta_3 \mu \lambda^2 + 2\alpha_1 \alpha_2 \alpha_3 \mu \lambda^2 + \alpha_1 \beta_1 \beta_3 \mu^3 + \\
& \alpha_2 \alpha_3 \beta_1 \mu^3 - \alpha_2 \beta_1^2 \mu^3 + \alpha_2 \beta_1 \beta_3 \mu^3 + \alpha_2 \beta_1 \beta_2 \mu^3 + \alpha_3 \beta_1 \beta_2 \mu^3 + 2\alpha_1 \alpha_2 \beta_1 \beta_3 \mu^2 + \alpha_1 \alpha_2 \alpha_3 \beta_2 \mu^2 + \alpha_1 \alpha_2 \alpha_3 \mu^2 \lambda - \alpha_1 \alpha_2 \beta_1 \mu^3 + \\
& \alpha_1^2 \alpha_2^2 \alpha_3 \lambda + 2\alpha_1 \alpha_3 \beta_1 \mu^2 \lambda + \alpha_1 \alpha_3 \beta_1 \mu \lambda^2 - 2\alpha_2 \beta_1^2 \mu^2 \lambda - \alpha_2 \beta_1^2 \mu \lambda^2 - \alpha_1 \alpha_2 \beta_1^2 \mu \lambda + \alpha_1^2 \alpha_3 \beta_1 \mu \lambda + \alpha_3 \beta_1 \beta_2 \mu^2 \lambda + \\
& \alpha_1 \alpha_3 \beta_1 \beta_2 \mu \lambda + \alpha_1^2 \alpha_2^2 \alpha_3 \beta_2 + \alpha_1 \alpha_2^2 \alpha_3 \beta_2 \lambda - \alpha_1^2 \alpha_2^2 \alpha_3 \beta_1
\end{aligned}$$

2.4. Profit Analysis

Following El-Said[10], Haggag[11], El-said and sherbeny[13] and Wang et al[14], the expected profit per unit time incurred to the system in the steady-state is given by:

Profit = total revenue generated from system using - total cost due to repair of failed unit or subsystem B

$$PF = C_0 A_{v2}(\infty) - C_1 B_2(\infty) \quad (6)$$

Where PF : is the profit incurred to the system

C_0 : is the revenue per unit up time of the system

C_1 : is the cost per unit time which the system is under repair

3. Results

The following particular cases are considered:

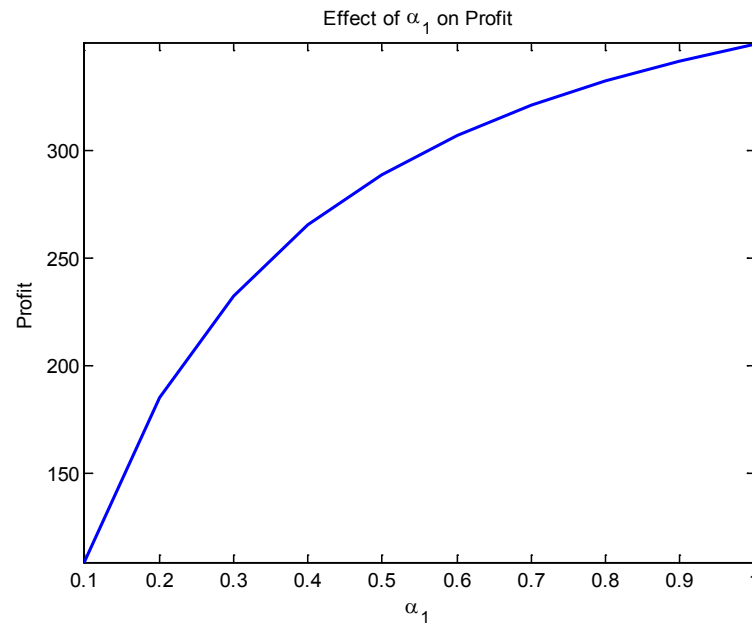


Figure 2. effect of α_1 on Profit

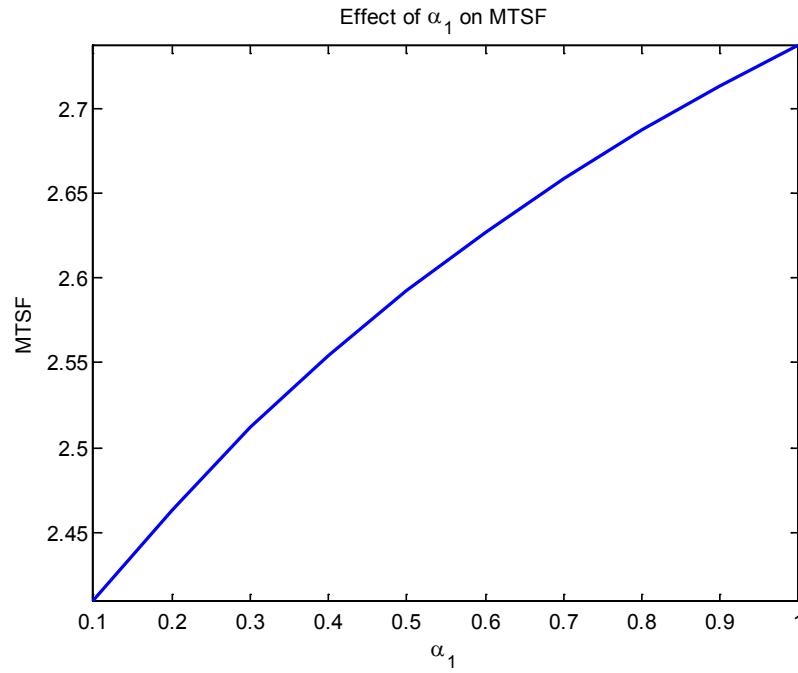


Figure 3. effect of α_1 on MT SF

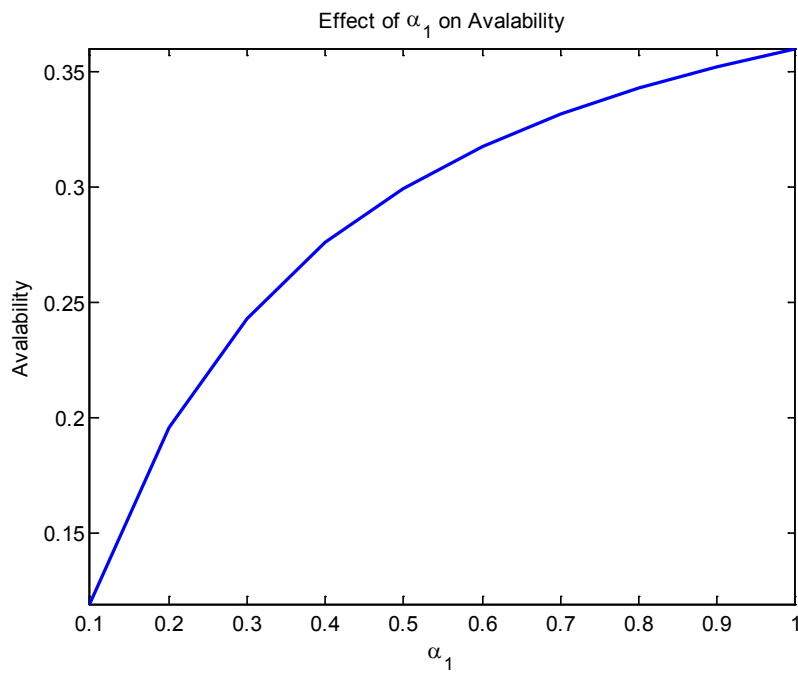


Figure 4. effect of α_1 on system availability

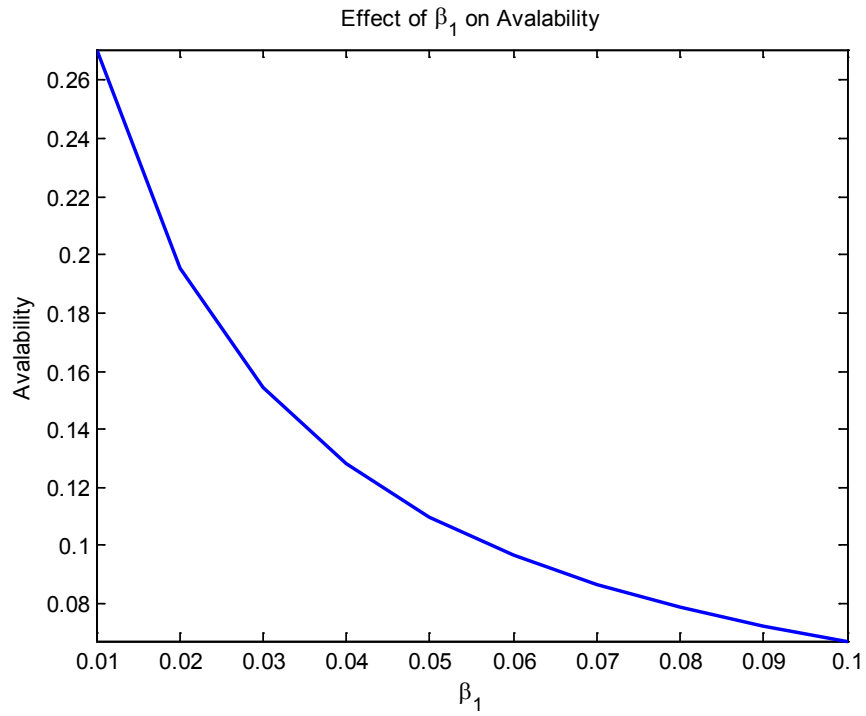


Figure 5. effect of β_1 on system availability

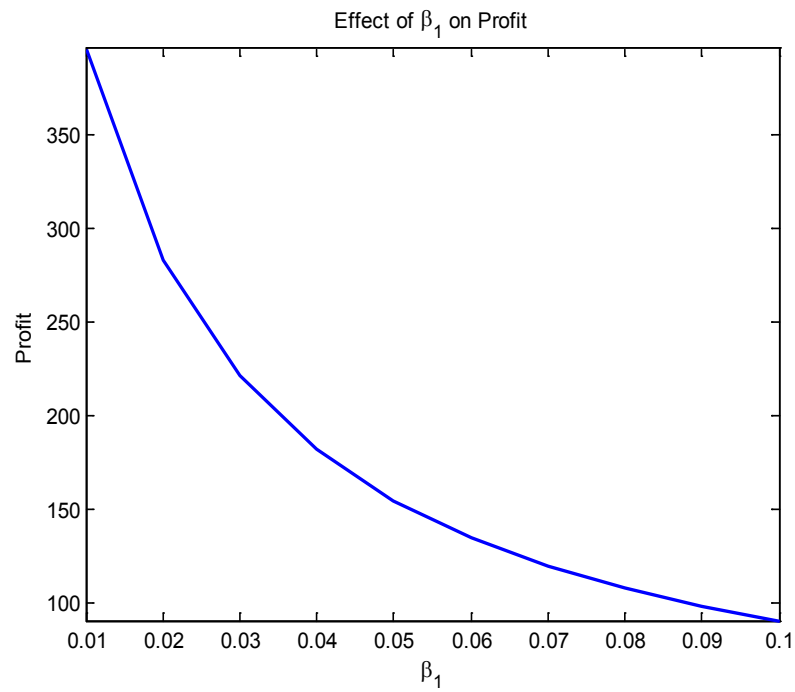


Figure 6. effect of β_1 on Profit

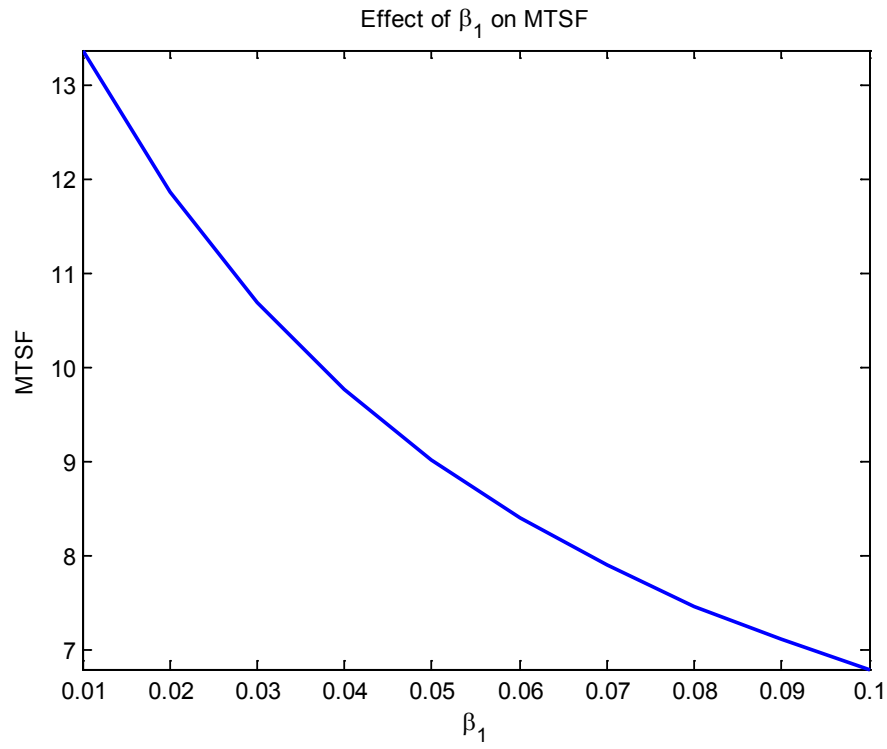


Figure 7. effect of β_1 on MTSF

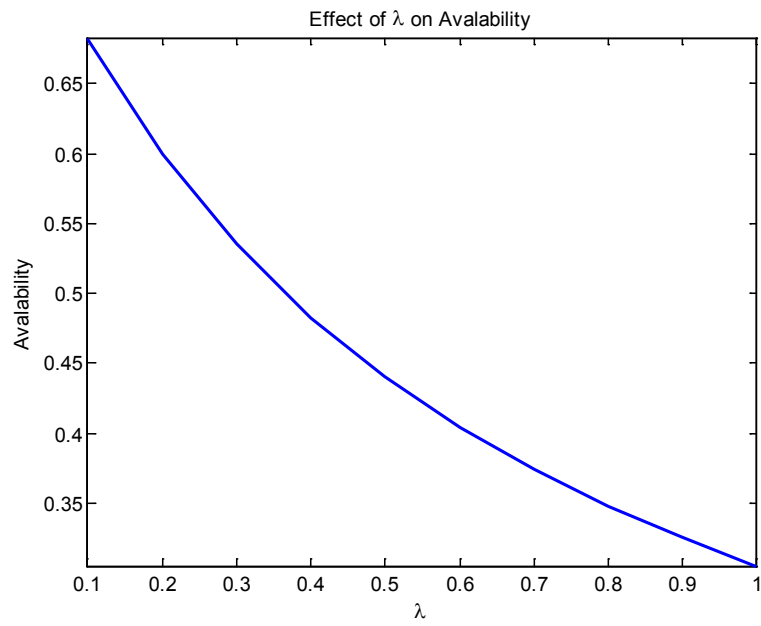


Figure 8. effect of λ on system availability

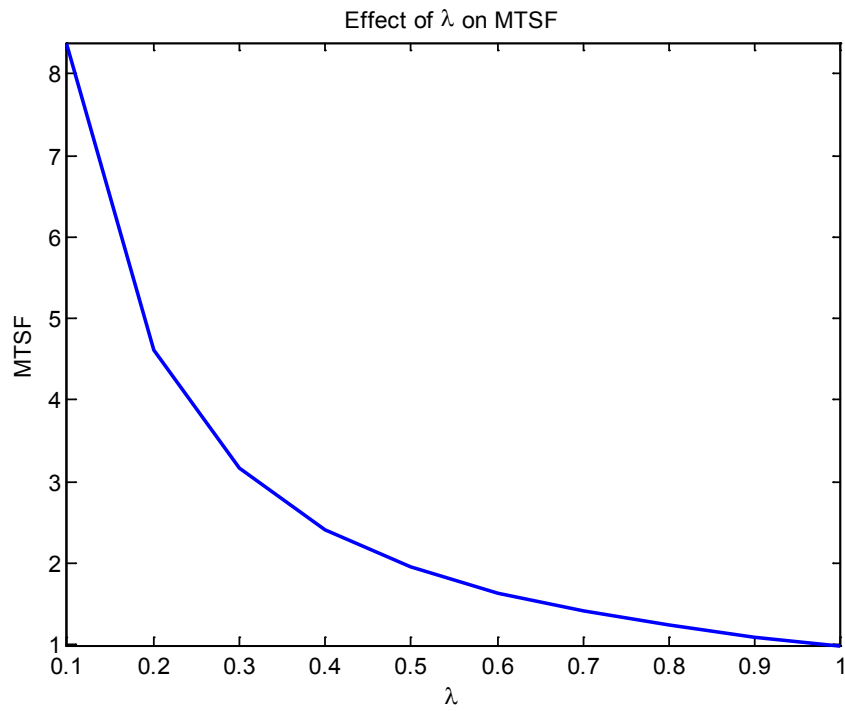


Figure 9. effect of λ on MTSF

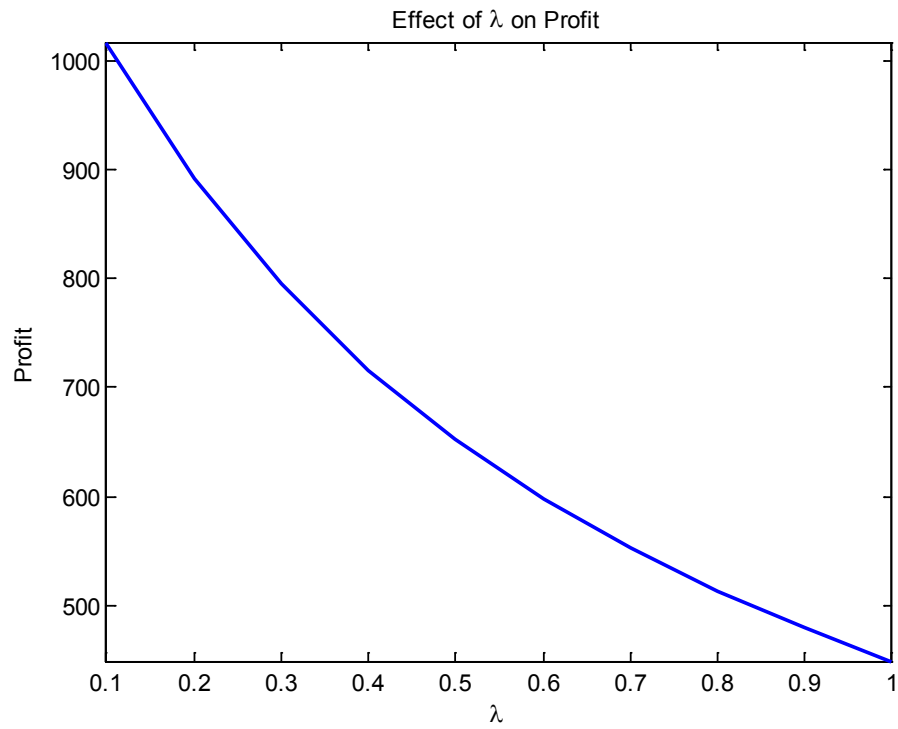


Figure 10. effect of λ on Profit

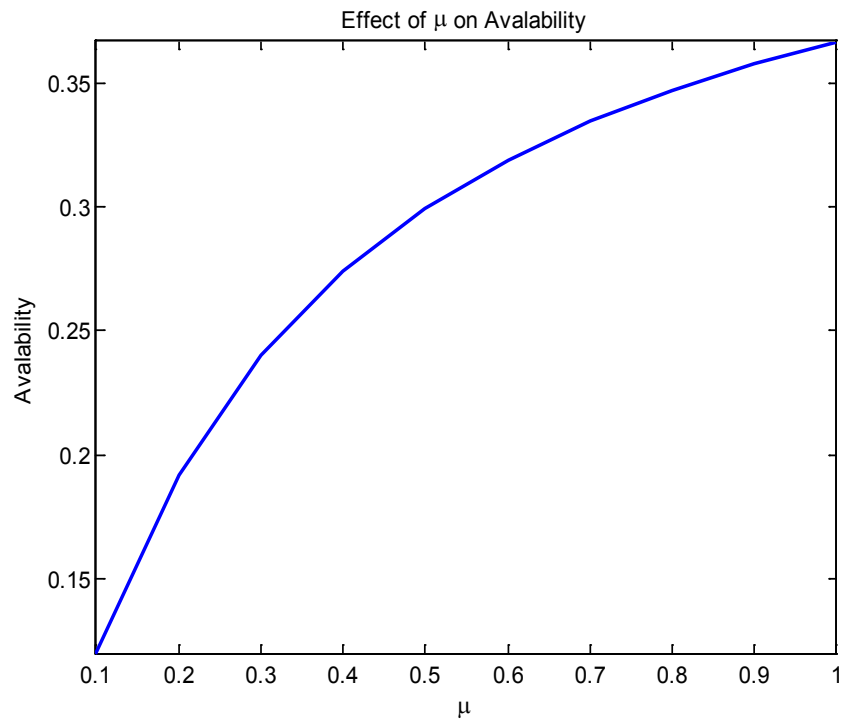


Figure 11. effect of μ on system availability

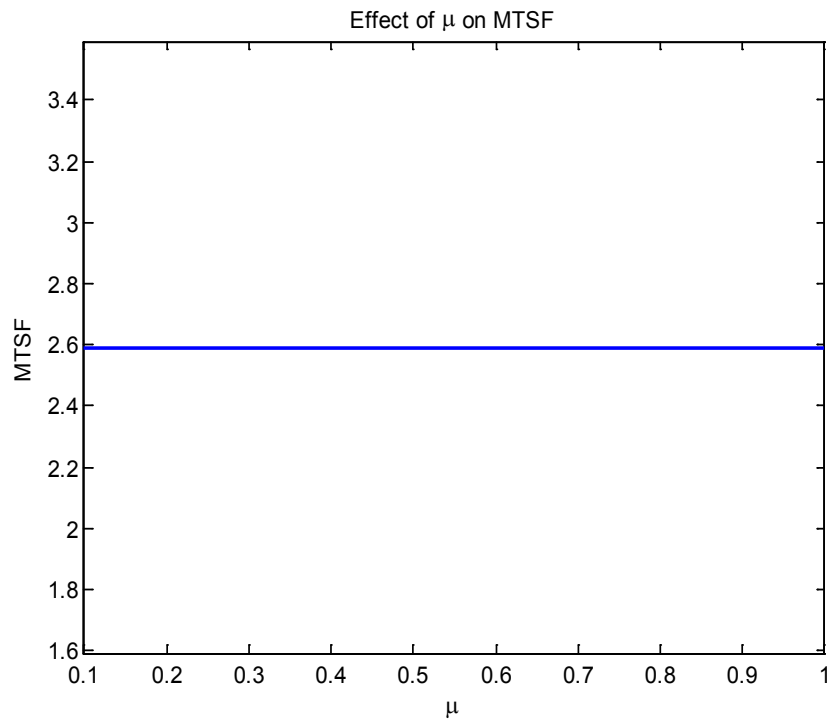


Figure 12. effect of μ on MT SF

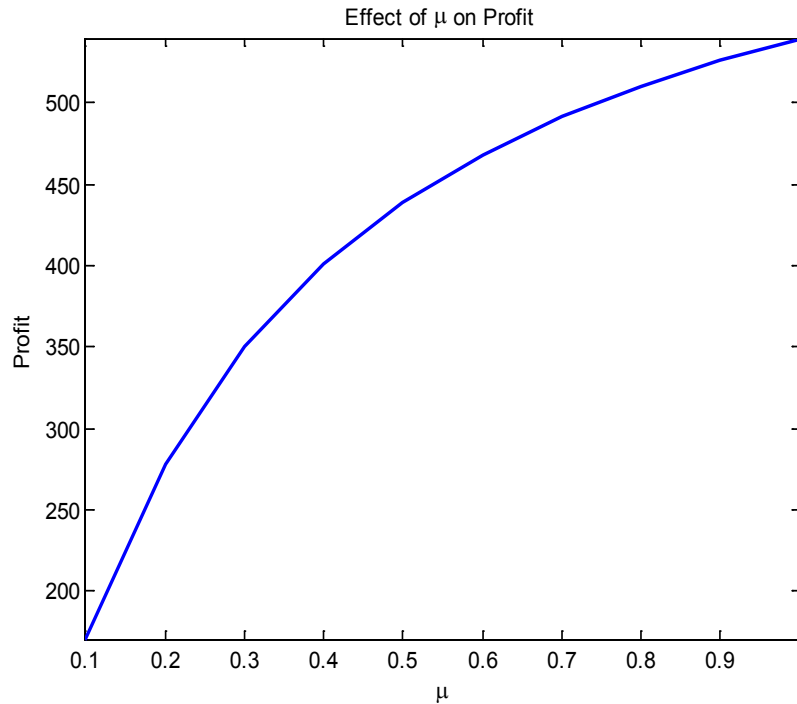


Figure 13. effect of μ on Profit

4. Discussion

Case I:

$\alpha_2 = 0.5$, $\alpha_3 = 0.02$, $\beta_1 = 0.5$, $\beta_2 = 0.06$, $\beta_3 = 0.01$, $\lambda = 0.6$, $\mu = 0.5$, $C_0 = 1000$, $C_1 = 10$ and vary α_1 for Figure 2 to 4.

Case II:

$\alpha_2 = 0.01$, $\alpha_3 = 0.02$, $\alpha_1 = 0.01$, $\beta_2 = 0.06$, $\beta_3 = 0.9$, $\lambda = 0.06$, $\mu = 0.05$, $C_0 = 1500$, $C_1 = 10$ and vary β_1 for Figure 5 to 7.

Case III:

$\alpha_1 = 0.09$, $\alpha_2 = 0.5$, $\alpha_3 = 0.02$, $\beta_1 = 0.05$, $\beta_2 = 0.06$, $\beta_3 = 0.01$, $\mu = 0.5$, $C_0 = 1500$, $C_1 = 10$ and vary λ for Figure 8 to 10.

Case IV:

$\alpha_1 = 0.5$, $\alpha_2 = 0.5$, $\alpha_3 = 0.02$, $\beta_1 = 0.5$, $\beta_2 = 0.06$, $\beta_3 = 0.01$, $\lambda = 0.6$, $C_0 = 1500$, $C_1 = 10$ and vary μ for Figure 11 to 13.

Figure 2 to 4 provides description of profit function, MTSF and system availability with respect to α_1 . From these figures, it is clear that both profit function, MTSF and system availability increase as α_1 increases. In Figure 5 to 7, the behavior of system availability, profit function and MTSF are shown with respect to β_1 . It is observed that system availability decrease as β_1 increases. In Figure 8 to 10, the behavior of system availability, MTSF and profit

function with respect to λ . The results in these figures have shown that system availability, MTSF and profit function decrease as λ increases. Figure 11 to 13 provides description of system availability, MTSF and profit function with respect to μ . System availability and profit in Figure 11 and 13 increase with increase in μ while MTSF in Figure 12 is constant with respect to μ .

5. Conclusions

In this paper, we developed the explicit expressions for the mean time to system failure (MTSF), system availability, busy period and profit function for the system and performed graphical study to see the behavior of failure rates and repair rates parameters on system performance. It is observed that from graphical study system performance increase with repair rates and decrease with failure rates.

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