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Dynamic Responses of Composite Plates on the Pasternak Foundation Subjected to a Moving Mass by a Cell-based Smoothed Discrete Shear Gap (CS-FEM-DSG3) Method

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Abstract A cell-based smoothed discrete shear gap method (CS-FEM-DSG3) using triangular elements was recently proposed to improve the performance of the discrete shear gap method (DSG3) for static and dynamics analyses of Mindlin plates. In this paper, the CS-FEM-DSG3 is incorporated with spring systems for dynamic analyses of composite plates on the Pasternak foundation subjected to a moving mass. The composite plate-foundation system is modeled as a discretization of triangular plate elements supported by discrete springs at the nodal points representing the Pasternak foundation. The position of the moving mass with specified velocity on triangular elements at any time is defined, and then the moving mass is transformed into loads at nodes of elements. The accuracy and reliability of the proposed method is verified by comparing its numerical solutions with those of other available numerical results. A parametric examination is also conducted to determine the effects of various parameters on the dynamic response of the composite plates on the Pasternak foundation subjected to the moving mass.

Keywords Smoothed Finite Element Methods (S-FEM), Composite Plate, Cell-based Smoothed Discrete Shear Gap Technique (CS-FEM-DSG3), Pasternak Foundation, Moving Mass

1. Introduction

Dynamic response of plates on foundations subjected to a moving mass can be found in various types of engineering structures and real life applications such as basement foundations of building, traffic highways, airport runways, raft foundations, etc.

For numerical analysis of plates on elastic foundations subjected to a moving mass, Thompson[1] first carried out an analysis of dynamic behavior of roads subjected to longitudinally moving loads by assuming the pavement as an infinitely long thin plate resting on elastic foundations. This analysis however cannot be used effectively for pavements of finite dimensions. Gbadeyan and Oni[2] contributed a closed form solution by using double Fourier sine integral transformation to analyse a simply supported rectangular plate resting on elastic Pasternak foundation traversed by an arbitrary number of moving concentrated masses. Kim and Roesset[3] have studied an infinite plate resting on elastic Winkler foundation subjected to moving loads with transformed field domain analyses using Fourier transform.

Further, the elastic foundation is also represented as a Pasternak model and characterized by two moduli, one is the vertical spring modulus of foundation and the other is the shear modulus of foundation. In vibrations of continuous systems, types of support conditions have direct effect on the natural frequencies. Cheng and Kitipornchai[4] proposed a membrane analogy to derive an exact explicit eigenvalues for vibration and buckling of simply supported FG plates resting on elastic foundation using the first order shear deformation theory (FSDT). Chiena and Chen[5] studied the effect of Pasternak foundation on non-linear vibration of laminated plates. Aiello and Ombres[6] used a Rayleigh – Ritz method to evaluate the vibrations of laminates resting on a Pasternak foundation. Omurtag et al.[7] investigated the vibration of Kirchoff plates on Winkler and Pasternak foundations. Malekzadeh et al.[8] used ANSYS software to analyze the vibration of non-ideal simply supported laminated plate on an elastic foundation subjected to in-plane stresses.

In the other frontier of developing advanced finite element technologies, Liu and Nguyen Thoi Trung[9] have applied a strain smoothing technique[10] into the conventional FEM

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using linear interpolations to formulate a series of smoothed finite element methods (S-FEM) including the cell-based smoothed FEM (CS-FEM)[11-14] a node-based smoothed FEM (NS-FEM)[15-17], an edge-based smoothed FEM (ES-FEM)[18,19] and a face-based smoothed FEM (FS-FEM)[20]. Each of these smoothed FEM has different properties and has been used to produce desired solutions for a wide class of benchmark and practical mechanics problems. The S-FEM models have also been further investigated and applied to various problems such as plates and shells[21-26], piezoelectricity[27,28], visco-elastoplasticity[29,30], limit and shakedown analysis for solids[31], and some other applications[32-34], etc. Extending the idea of the CS-FEM to plate structures, Nguyen-Thoi et al.[35] have recently formulated a cell-based smoothed stabilized discrete shear gap element (CS–FEM-DSG3) for static, and free vibration analyses of isotropic Mindlin plates by incorporating the CS-FEM with the original DSG3 element[36]. In the CS-FEM-DSG3, each triangular element will be divided into three sub-triangles, and in each sub-triangle, the stabilized DSG3 is used to compute the strains. Then the strain smoothing technique on whole the triangular element is used to smooth the strains on these three sub-triangles. The numerical results showed that the CS–FEM-DSG3 is free of shear locking and achieves the high accuracy compared to the exact solutions and others existing elements in the literature.

This paper hence extends the CS-FEM-DSG3 to dynamic responses of composite plates on the Pasternak foundation subjected to a moving mass. The composite plate-foundation system is modeled as a discretization of triangular plate elements supported by discrete springs at the nodal points representing the Pasternak foundation. The position of the moving mass with specified velocity on triangular elements at any time is defined, and then the moving mass is transformed into loads at nodes of elements. The accuracy and reliability of the proposed method is verified by comparing its numerical solutions with those of others available numerical results. A parametric examination is conducted to determine the effects of various parameters on the dynamic response of the plates on the Pasternak foundation subjected to the moving mass.

2. Weak Form for the Laminate Composite Plate

Consider a laminate composite plate under bending deformation as shown in Figure 1. The middle (neutral) surface of plate is chosen as the reference plane that occupies a domain $\Omega \subset R^2$. The displacement field according to Reissner–Mindlin model which based on the first-order shear deformation theory[37] can be expressed by

$$\begin{aligned} u(x,y,z) &= u_0(x,y) + z\beta_x(x,y) \\ v(x,y,z) &= v_0(x,y) + z\beta_y(x,y) \\ w(x,y,z) &= w(x,y) \end{aligned} \tag{1}$$

where u_0, v_0, w are the displacements of the mid-plan of plate; β_x, β_y are the rotations of the middle plane around y-axis and x–axis, respectively, with the positive directions defined in Figure 1.

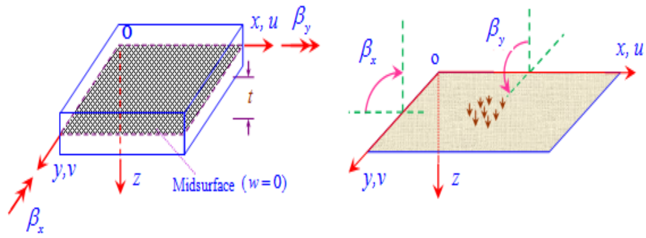


Figure 1. Reissner-Mindlin thick plate and positive directions of the displacement u, v, w and β_x, β_y

The linear strain can be given as

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} u_{0,x} \\ v_{0,y} \\ u_{0,x} + v_{0,y} \end{Bmatrix} + z\begin{Bmatrix} \beta_{x,x} \\ \beta_{y,y} \\ \beta_{x,y} + \beta_{y,x} \end{Bmatrix} \tag{2}$$

$$= \boldsymbol{\varepsilon}_0 + z\boldsymbol{\kappa}$$

$$\begin{Bmatrix} \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix} = \begin{Bmatrix} w_{,x} + \beta_x \\ w_{,y} + \beta_y \end{Bmatrix} = \boldsymbol{\gamma} \tag{3}$$

In the laminate composite plate, the constitutive equation of a k^{th} orthotropic layer in local coordinate is derived from Hook's law for plane stress as

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \\ \tau_{xz} \\ \tau_{yz} \end{Bmatrix}^{(k)} = \begin{bmatrix} Q_{11} & Q_{12} & Q_{16} & 0 & 0 \\ Q_{21} & Q_{22} & Q_{26} & 0 & 0 \\ Q_{61} & Q_{62} & Q_{66} & 0 & 0 \\ 0 & 0 & 0 & Q_{55} & Q_{54} \\ 0 & 0 & 0 & Q_{45} & Q_{44} \end{bmatrix}^{(k)} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \\ \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix}^{(k)} \tag{4}$$

where material constants are given by

$$Q_{11} = \frac{E_1}{1-\nu_{12}\nu_{21}}, Q_{12} = \frac{\nu_{12}E_2}{1-\nu_{12}\nu_{21}}, \; Q_{22} = \frac{E_2}{1-\nu_{12}\nu_{21}} \tag{5}$$

$$Q_{66} = G_{12}, \; Q_{55} = G_{13}, \; Q_{44} = G_{23}$$

in which E_1, E_2 are the Young modulus in the 1 and 2 directions, respectively, and G_{12}, G_{23}, G_{13} are the shear modulus in the 1-2, 2-3, 3-1 planes, respectively, and ν_{12} are Poisson's ratios.

The laminate is usually made of several orthotropic layers in which the stress-strain relation for the kth orthotropic lamina (with the arbitrary fiber orientation compared to the reference axes) is computed by

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \\ \tau_{xz} \\ \tau_{yz} \end{Bmatrix}^{(k)} = \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} & 0 & 0 \\ \overline{Q}_{21} & \overline{Q}_{22} & \overline{Q}_{26} & 0 & 0 \\ \overline{Q}_{61} & \overline{Q}_{62} & \overline{Q}_{66} & 0 & 0 \\ 0 & 0 & 0 & \overline{Q}_{55} & \overline{Q}_{54} \\ 0 & 0 & 0 & \overline{Q}_{45} & \overline{Q}_{44} \end{bmatrix}^{(k)} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \\ \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix}^{(k)} \quad (6)$$

where $\overline{Q}_{ij}$ are transformed material constants of the kth lamina as in Ref[37].

The Galerkin weakform of transient analysis of composite plates on Pasternak foundation can be written as[8]:

$$\int_\Omega \delta\boldsymbol{\varepsilon}_p{}^T \overline{\mathbf{D}} \boldsymbol{\varepsilon}_p \,\mathrm{d}\Omega + \int_\Omega \delta\boldsymbol{\gamma}^T \overline{\mathbf{D}}_s \boldsymbol{\gamma} \,\mathrm{d}\Omega + \int_\Omega \delta\mathbf{u}^T \mathbf{m}\, \ddot{\mathbf{u}} \mathrm{d}\Omega$$
$$+ \int_\Omega \delta w^T k_w w \,\mathrm{d}\Omega - \int_\Omega \delta w^T k_g \nabla^2 w \mathrm{d}\Omega = \int_\Omega \delta\mathbf{u}^T \mathbf{b}\, \mathrm{d}\Omega \quad (7)$$

where k_w and k_g contains the elastic moduli of Pasternak foundation; $\mathbf{m}$ is the matrix containing the mass density of the material; $\boldsymbol{\varepsilon}_p = [\boldsymbol{\varepsilon}_0 \quad \boldsymbol{\kappa}]^T$; $\overline{\mathbf{D}}$ and $\overline{\mathbf{D}}_s$ are material constant matrices given in the form of

$$\overline{\mathbf{D}} = \begin{bmatrix} \mathbf{D}_m & \mathbf{B} \\ \mathbf{B} & \mathbf{D}_b \end{bmatrix} \quad \overline{\mathbf{D}}_s = k\int_{-t/2}^{t/2} \overline{Q}_{ij} \mathrm{d}z \,; i, j = 4, 5 \quad \left(k = 5/6\right) \quad (8)$$

in which

$$\mathbf{D}_{mij} = \int_{-t/2}^{t/2} \overline{Q}_{ij} \mathrm{d}z \,; \quad \mathbf{B}_{ij} = \int_{-t/2}^{t/2} z\overline{Q}_{ij} \mathrm{d}z;$$
$$\mathbf{D}_{bij} = \int_{-t/2}^{t/2} z^2 \overline{Q}_{ij} \mathrm{d}z \quad (i, j = 1, 2, 6) \quad (9)$$

3. FEM Formulation for Composite Plates on Pasternak Foundation

Now, by discretizing the bounded domain Ω of the composite plate into N_e finite elements such that $\Omega = \bigcup_{e=1}^{N_e} \Omega_e$ and $\Omega_i \cap \Omega_j = \varnothing$, $i \neq j$, then the finite element solution $\mathbf{u}^h = [u \quad v \quad w \quad \beta_x \quad \beta_y]^T$ of a displacement model for the composite plate is expressed as

$$\mathbf{u}^h = \sum_{i=1}^{N_n} \begin{bmatrix} N_i & 0 & 0 & 0 & 0 \\ 0 & N_i & 0 & 0 & 0 \\ 0 & 0 & N_i & 0 & 0 \\ 0 & 0 & 0 & N_i & 0 \\ 0 & 0 & 0 & 0 & N_i \end{bmatrix} \mathbf{d}_i = \mathbf{N}\mathbf{d} \quad (10)$$

where N_n is the total number of nodes of problem domain discretized; N_i is shape function at ith node; $\mathbf{d}_i = [u_i \ v_i \ w_i \ \beta_{xi} \ \beta_{yi}]^T$ is the displacement vector of the nodal degrees of freedom of $\mathbf{u}^h$ associated to ith node, respectively.

The membrane, bending and shear strains can be then expressed in the matrix forms as

$$\boldsymbol{\varepsilon}_0 = \sum_i \mathbf{B}_i^m \mathbf{d}_i \ ; \ \boldsymbol{\kappa} = \sum_i \mathbf{B}_i^b \mathbf{d}_i \ ; \ \boldsymbol{\gamma} = \sum_i \mathbf{B}_i^s \mathbf{d}_i \quad (11)$$

where

$$\mathbf{B}_i^m = \begin{bmatrix} N_{i,x} & 0 & 0 & 0 & 0 \\ 0 & N_{i,y} & 0 & 0 & 0 \\ N_{i,y} & N_{i,x} & 0 & 0 & 0 \end{bmatrix}; \ \mathbf{B}_i^b = \begin{bmatrix} 0 & 0 & 0 & N_{i,x} & 0 \\ 0 & 0 & 0 & 0 & N_{i,y} \\ 0 & 0 & 0 & N_{i,y} & N_{i,x} \end{bmatrix}; \quad (12)$$
$$\mathbf{B}_i^s = \begin{bmatrix} 0 & 0 & N_{i,x} & N_i & 0 \\ 0 & 0 & N_{i,y} & 0 & N_i \end{bmatrix}$$

in which $N_{i,x}$ and $N_{i,y}$ are the derivatives of the shape functions in x-direction and y-direction, respectively

The discretized system of equations of composite plates on Pastarnak foundation using the FEM for transient analysis then can be expressed as

$$\mathbf{M}\ddot{\mathbf{d}} + \mathbf{K}\mathbf{d} = \mathbf{F} \quad (13)$$

where $\mathbf{K}$ is the global stiffness matrix given by

$$\mathbf{K} = \int_\Omega \mathbf{B}^T \overline{\mathbf{D}} \mathbf{B} \,\mathrm{d}\Omega + \int_\Omega \mathbf{B}^{sT} \overline{\mathbf{D}}_s \mathbf{B}^s \mathrm{d}\Omega + \int_\Omega \mathbf{N}_w{}^T k_w \mathbf{N}_w \,\mathrm{d}\Omega$$
$$- \int_\Omega \mathbf{N}_{w,x}{}^T k_g \mathbf{N}_{w,x} \,\mathrm{d}\Omega - \int_\Omega \mathbf{N}_{w,y}{}^T k_g \mathbf{N}_{w,y} \,\mathrm{d}\Omega \quad (14)$$

in which $\mathbf{N}_w = [0 \ 0 \ N_1 \ 0 \ 0 \ 0 \ 0 \ N_2 \ 0 \ 0 \ 0 \ 0 \ N_3 \ 0 \ 0]$; $\mathbf{B}$ =[$\mathbf{B}^m$ $\mathbf{B}^b$] and $\mathbf{F}$, $\mathbf{M}$ are the load vector defined by

$$\mathbf{F} = \int_\Omega p\mathbf{N} \,\mathrm{d}\Omega + \mathbf{f}^b \ ; \ \mathbf{M} = \int_\Omega \mathbf{N}^T \mathbf{m} \mathbf{N} \,\mathrm{d}\Omega \quad (15)$$

4. Formulation of the CS-FEM-DSG3 for Composite Plates on Pasternak Foundation

In the CS-FEM-DSG3[35], the domain discretization is the same as that of the DSG3[36] using N_n nodes and N_e triangular elements. However in the formulation of the CS-FEM-DSG3, each triangular element is divided into three sub-triangles by connecting the central point O of the element to three field nodes as shown in Figure 2. Using the DSG3 formulation[35] for each sub-triangle, the membrane, bending and shear strains in 3 sub-triangles are then obtained, respectively, by

$$\boldsymbol{\varepsilon}_0^{e\Delta_j} = \mathbf{B}^{m\Delta_j} \mathbf{d}^e \quad , j = 1, 2, 3 \quad (16)$$

$$\boldsymbol{\kappa}^{e\Delta_j} = \mathbf{B}^{b\Delta_j} \mathbf{d}^e \quad , j = 1, 2, 3 \quad (17)$$

$$\boldsymbol{\gamma}^{e\Delta_j} = \mathbf{B}^{s\Delta_j} \mathbf{d}^e \quad , j = 1, 2, 3 \quad (18)$$

where $\mathbf{d}^e$ is the vector containing the nodal degrees of freedom of the element; $\mathbf{B}^{m\Delta_j}$, $\mathbf{B}^{b\Delta_j}$, $\mathbf{B}^{s\Delta_j}$, j=1,2,3, are membrane, bending and shearing gradient matrices by the DSG3[36] of j^{th} sub-triangle, respectively.

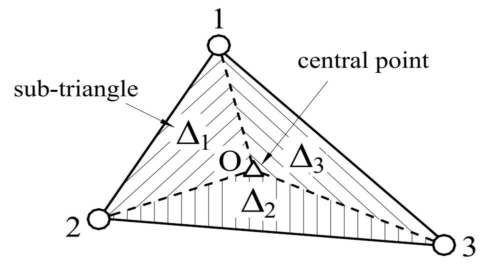


Figure 2. Three sub-triangles (Δ_1 , Δ_2 and Δ_3) created from the triangle 1-2-3 in the CS-MIN3 by connecting the central point O with three field nodes 1, 2 and 3

Now, applying the cell-based strain smoothing operation in the CS-FEM[11], the constant membrane, bending and shear strains $\boldsymbol{\varepsilon}_0^{e\Delta_j}$, $\boldsymbol{\kappa}^{e\Delta_j}$, $\boldsymbol{\gamma}^{e\Delta_j}$, $j = 1,2,3$ are, respectively, used to create element smoothed strains $\tilde{\boldsymbol{\varepsilon}}_0^e$, $\tilde{\boldsymbol{\kappa}}^e$ and $\tilde{\boldsymbol{\gamma}}^e$ on the triangular element Ω_e, such as:

$$\tilde{\boldsymbol{\varepsilon}}_0^e = \tilde{\mathbf{B}}^m \mathbf{d}^e \quad ; \quad \tilde{\boldsymbol{\kappa}}^e = \tilde{\mathbf{B}}^b \mathbf{d}^e \quad ; \quad \tilde{\boldsymbol{\gamma}}^e = \tilde{\mathbf{B}}^s \mathbf{d}^e \qquad (19)$$

where $\tilde{\mathbf{B}}^m$, $\tilde{\mathbf{B}}^b$ and $\tilde{\mathbf{B}}^s$ are the smoothed strain gradient matrices, respectively, given by

$$\tilde{\mathbf{B}}^m = \frac{1}{A_e}\sum_{j=1}^{3} A_{\Delta_j}\mathbf{B}^{m\Delta_j} ; \tilde{\mathbf{B}}^b = \frac{1}{A_e}\sum_{j=1}^{3} A_{\Delta_j}\mathbf{B}^{b\Delta_j} ;$$
$$\tilde{\mathbf{B}}^s = \frac{1}{A_e}\sum_{j=1}^{3} A_{\Delta_j}\mathbf{B}^{s\Delta_j} \qquad (20)$$

Therefore the global stiffness matrix of the CS-FEM-DSG3[35] is computed by

$$\tilde{\mathbf{K}} = \int_{\Omega} \tilde{\mathbf{B}}^T \mathbf{D} \tilde{\mathbf{B}} \, d\Omega + \int_{\Omega} \tilde{\mathbf{B}}^{sT} \mathbf{D}_s \tilde{\mathbf{B}}^s \, d\Omega + \int_{\Omega} \mathbf{N}_w{}^T k_w \mathbf{N}_w \, d\Omega$$
$$- \int_{\Omega} \mathbf{N}_{w,x}{}^T k_g \mathbf{N}_{w,x} \, d\Omega - \int_{\Omega} \mathbf{N}_{w,y}{}^T k_g \mathbf{N}_{w,y} \, d\Omega \qquad (21)$$

where $\tilde{\mathbf{B}} = \begin{bmatrix} \tilde{\mathbf{B}}^m & \tilde{\mathbf{B}}^b \end{bmatrix}$

5. Transformation of Moving Mass into the Load at Nodes of Elements

In the moving mass problem, the mass M is considered as a concentrated load which has magnitude P=Mg, where g is the acceleration of gravity. The discretization of problem domain into triangular elements is arbitrary, and hence when a concentrated mass (or a concentrated load) moves with velocity v on a line along the longitudinal direction of the plate, this concentrated load will cross triangular elements arbitrarily. We hence need to define the position of the moving mass crossing triangular elements and to transform the moving mass into the load at nodes of elements at any time t.

Figure 3 shows a model of a moving mass crossing triangular elements. In this model, the mass moves along the line inclined an angle θ compared with x axis. Suppose that at the time point t, the position of the moving mass is (a,b) in the Cartesian coordinate system Oxy. Then, the position of the moving mass $(\bar{x}, \bar{y})$ at the time $\bar{t} = t + \Delta t$ are defined as

$$\bar{x} = v\Delta t \cos\theta + a \; ; \quad \bar{y} = v\Delta t \sin\theta + b \qquad (22)$$

where v is velocity of the moving mass and Δt is step time.

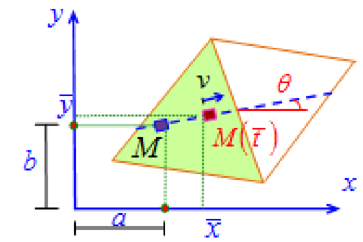


Figure 3. Position of a moving mass crossing triangular elements

The force vector $\overline{\mathbf{F}}$ is transformed from the moving mass at the position $(\bar{x}, \bar{y})$ into the load at nodes of elements is defined by

$$\overline{\mathbf{F}} = Mg\mathbf{N}_w \qquad (23)$$

Note that in the moving mass problem, it is necessary to add a numerical scheme for defining which elements containing the moving mass.

6. Numerical Results

In this section, various numerical examples are performed to show the accuracy and stability of the CS-FEM-DSG3 compared to the others existing numerical solutions. The section will include three parts. The first two-part aims to verify the accuracy of the CS-FEM-DSG3 by comparing its numerical solutions with those of others available numerical results for the static and free vibration analyses of composite plates on the Pasternak foundation. The third part aims to illustrate the performance of the present method for the dynamic analysis of composite plates on Pasternak foundation subjected to a moving mass.

Table 1. Non-dimensional defections of composite plate under SSL load with a/t = 10, 20, 100

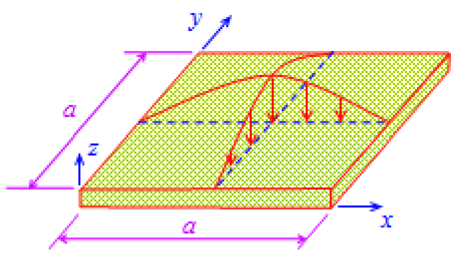
Ply	Method	SSL		
		10	20	100
[0/90/0]	FEM-T3[38]	0.6281	0.4516	0.3714
	FEM-Q4[38]	0.6458	0.4666	0.4073
	CS-FEM-DSG3	0.6604	0.4909	0.4425
	Reddy[39]	0.6693	0.4921	0.4337
[0/90/90/0]	FEM-T3[38]	0.6211	0.4503	0.3675
	FEM-Q4[38]	0.6387	0.4655	0.4073
	CS-FEM-DSG3	0.653	0.4898	0.4423
	Reddy[39]	0.6627	0.4912	0.4337
[0/90/0/90/0]	FEM-T3[38]	0.5868	0.4406	0.3661
	FEM-Q4[38]	0.6034	0.4556	0.4069
	CS-FEM-DSG3	0.6164	0.4792	0.4394
	Reddy[39]	0.6277	0.4814	0.4333

6.1. Static and Free Vibration Analysis of Composite Plates

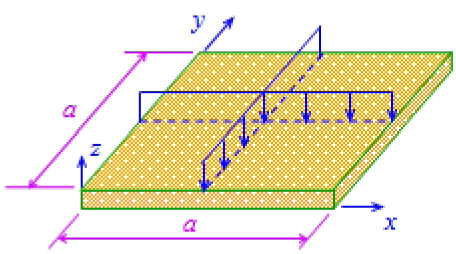
6.1.1. Static Analysis

We now consider a simply supported square laminate plate (length a, thickness t) subjected to sinusoidally distributed load (SSL) and uniform distributed load (UDL) shown in Figure 4.

Material properties are given by $E_2=1$, $E_1=25E_2$, $G_{23}=0.2E_2$, $G_{12}=G_{13}=0.5E_2$, $\nu_{12}=0.25$. A non-dimensional $\overline{w}=100E_2wt^3/(qa^4)$ is used. Table 1 and Table 2 display the non-dimensional central node deflection of the simply supported composite plate subjected to SSL and UDL load with ratios length-to-thickness a/t = 10, 20, 100. It is seen that the results by the CS-FEM-DSG3 agree well with those by the Reddy[39] and are better than those of FEM-T3[38] and FEM-Q4[38].



(a) Sinusoidally distributed load (SSL)



(b) Uniform distributed load (SSL)

Figure 4. Model of a simply supported square laminate plate subjected to sinusoidally distributed load (SSL) and uniform distributed load (UDL)

Table 2. Non-dimensional defections of composite plate under UDL load with a/t = 10, 20, 100

Ply	Method	UDL		
		10	20	100
[0/90/0]	FEM-T3[38]	0.9639	0.6989	0.5744
	FEM-Q4[38]	0.9874	0.7195	0.6307
	CS-FEM-DSG3	1.0127	0.7565	0.6787
	Reddy[39]	1.0219	0.7572	0.6697
[0/90/90/0]	FEM-T3[38]	0.9641	0.7085	0.5795
	FEM-Q4[38]	0.9883	0.7302	0.643
	CS-FEM-DSG3	1.0111	0.7676	0.6965
	Reddy[39]	1.025	0.7694	0.6833
[0/90/0/90/0]	FEM-T3[38]	0.912	0.6966	0.5812
	FEM-Q4[38]	0.935	0.7182	0.6465
	CS-FEM-DSG3	0.9553	0.7547	0.6961
	Reddy[39]	0.9727	0.7581	0.6874

6.1.2. Free Vibration Analysis

We analyze a clamped square plates (CCCC) (length a, thickness t) shown in Figure 5 with the material properties are $E_1 = 40E_2$, $G_{12} = G_{13} = 0.6E_2$, $G_{23} = 0.5E_2$, $\nu_{12} = 0.25$. A non-dimensional frequency parameter $\varpi = \left(\omega a^2/\pi^2\right)\sqrt{\rho t/D_0}$ is also used, where $D_0 = E_2t^3/(12(1-\nu_{12}\nu_{21}))$ is the flexural rigidity of the plate.

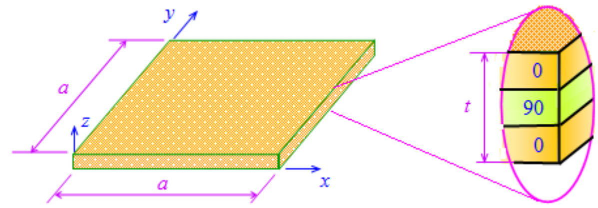


Figure 5. A three layers (0/90/0) square composite laminated plate model

Table 3 shows five lowest non-dimensional frequency parameters of a CCCC composite plate. Again, it is seen that the results by the CS-FEM-DSG3 agree well with those of[40-42]. In addition, Figure 6 plots the shape of six lowest eigenmodes of composite plate using the CS-FEM-DSG3. It is seen that the shapes of eigen-modes reveal the real physical modes.

Table 3. Five lowest non-dimensional frequency parameters of a CCCC composite plate

a/t	Method	Mode				
		1	2	3	4	5
5	Liew[40]	4.44	6.64	7.70	9.18	9.74
	Zhen[41]	4.54	6.52	8.17	9.47	9.49
	Ferreira[42]	4.44	6.64	7.69	9.18	9.74
	CS-FEM-DSG3	4.68	7.00	8.00	9.62	10.11
10	Liew[40]	7.41	10.39	13.91	15.43	15.81
	Zhen[41]	7.48	10.21	14.34	14.86	16.07
	Ferreira[42]	7.41	10.39	13.91	15.43	15.81
	CS-FEM-DSG3	7.31	10.53	13.41	15.55	15.83
20	Liew[40]	10.95	14.03	20.39	23.20	24.99
	Zhen[41]	11.00	14.06	20.32	23.50	25.35
	Ferreira[42]	10.95	14.03	20.39	23.20	24.98
	CS-FEM-DSG3	11.03	14.18	20.73	23.36	25.25
100	Liew[40]	14.66	17.61	24.51	35.53	39.15
	Zhen[41]	14.60	17.81	25.23	37.16	38.52
	Ferreira[42]	14.43	17.39	24.31	35.40	37.78
	CS-FEM-DSG3	13.71	16.83	23.93	33.78	35.06

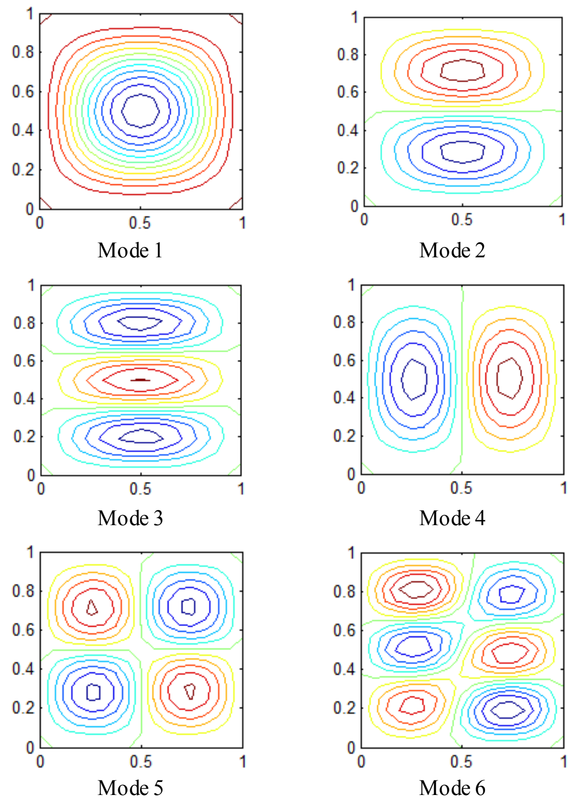


Figure 6. Shape of six lowest eigenmodes of composite plate[0/90/0] by the CS-FEM-DSG3.

6.2. Free Vibration Analysis of Composite Plates on Pasternak Foundation

We now analyze a simply supported composite plate on Pasternak foundation (length a, thickness t) shown in Figure 7 with the material properties are E_2=10.3e9; E_1=40E_2; G_{23}=0.5E_2; G_{13}=0.6E_2; $\nu_{12}=0.25$; G_{12}=0.6E_2; a/t = 100. A non-dimensional frequency parameter $\varpi=\omega\left(a^2/\mathrm{t}\right)\sqrt{\rho/E_2}$ is also used.

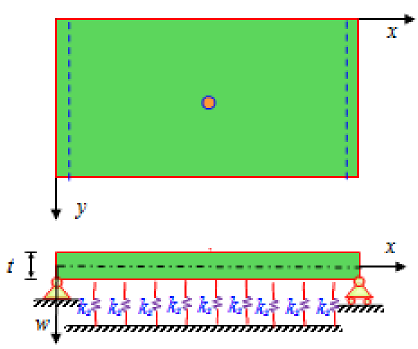


Figure 7. Model of composite laminated plate on Pasternak foundation

Table 4 shows three lowest frequencies of composite plate on Pasternak foundation. It is observed that the results of CS-FEM-DSG3 agree well with the reference solutions in[43,8].

We next study the deflection of free vibration modes of the plate on the Pasternak foundation corresponding to three sets of various foundation coefficients. Figure 8, Figure 9 and Figure 10 plot the deflection of the first free vibration modes of the plate on the Pasternak foundation at the middle line along the longitudinal direction x. It can be seen that when the stiffness of foundation becomes stiffer, the deflections of modeshape of the plate on the elastic foundation change significantly comparing with those of the plate without foundation.

Table 4. Three lowest non-dimensional frequency parameters of a simply supported composite plate on Pasternak foundation

Foundation	Method	Mode 1	Mode 2	Mode 3
$k_1=100,\ k_2=0$	Ref[43]	21.60	-	-
	Ref[8]	21.38	25.27	66.83
	CS-FEM-DSG3	21.64	26.23	61.70
$k_1=100,\ k_2=10$	Ref[43]	25.76	-	-
	Ref[8]	25.57	41.68	72.83
	CS-FEM-DSG3	25.20	41.21	70.74

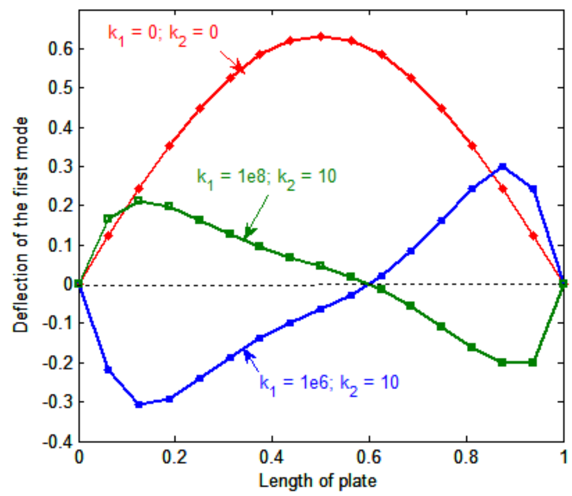


Figure 8. Deflection of the first mode of the plate on the Pasternak foundation at middle line along the longitudinal direction x

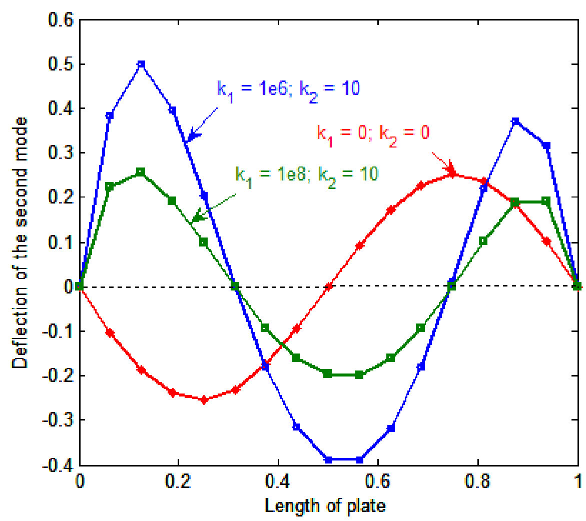


Figure 9. Deflection of the second mode of the plate on the Pasternak foundation at middle line along the longitudinal direction x

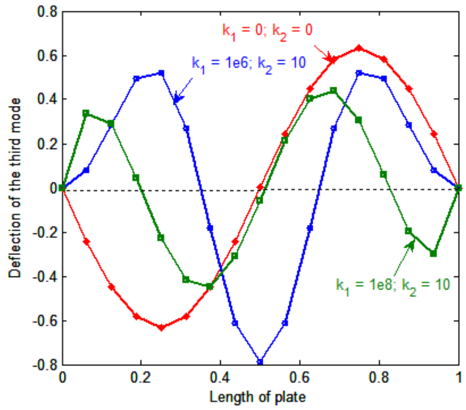


Figure 10. Deflection of the third mode of the plate on the Pasternak foundation at middle line along the longitudinal direction x

6.3. Dynamic Analysis of Composite Plates on Pasternak Foundation Subjected to a Moving Mass

In this section, model of composite plate is similar to section 6.2. We consider a concentrated mass M=1000kg moving with velocity v=20m/s on the middle line along the longitudinal direction x of a composite plate with the simply supported boundary.

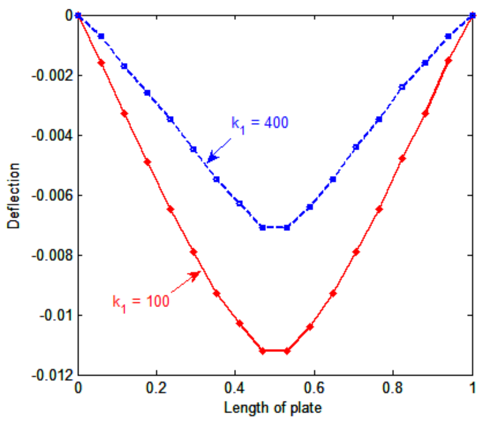


Figure 11. Effect of k_1 to deflection of middle line of the composite plate on Pasternak foundation when the mass moves to the middle position of the composite plate

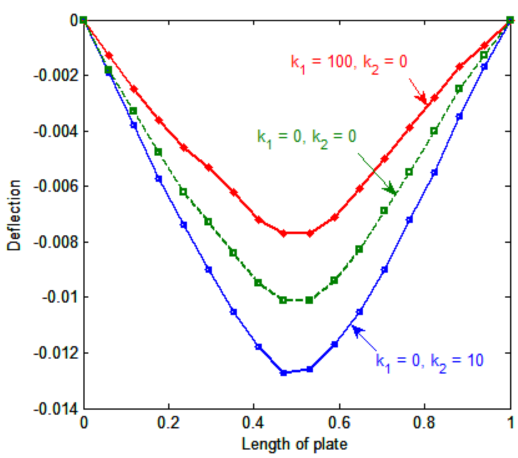


Figure 12. Effect of various foundation coefficients to deflection of middle line of the composite plate on Pasternak foundation when the mass moves to the middle position of the composite plate

Now, a parametric examination by the CS-FEM-DSG3 is conducted to determine the effects of various parameters on the dynamic response of the composite plates on the Pasternak foundation subjected to a moving mass. The variation of the deflection of middle line along the length of plate by the CS-FEM-DSG3 with various foundation coefficients, is shown in Figure 11 and Figure 12. The results show that when the stiffness of foundation becomes stiffer, the deflection of the plate becomes smaller, as expected.

7. Conclusions

The paper presents an incorporation of the original CS-FEM-DSG3 with spring systems for dynamic analyses of composite plates on the Pasternak foundation subjected to a moving mass. The composite plate-foundation system is modeled as a discretization of triangular plate elements supported by discrete springs at the nodal points representing the Pasternak foundation. The position of the moving mass with specified horizontal velocity on triangular elements at any time is defined and transformed into loads at nodes of elements. The accuracy and reliability of the proposed method is verified by comparing its numerical solutions with those of others available numerical results. A examination of effects of various parameters on the dynamic response of the composite plates on the Pasternak foundation subjected to a moving mass is conducted and gives the expected results.

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