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A Robust Adaptive Carrier Frequency Offset Estimation Algorithm for OFDM

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Abstract The carrier frequency offset (CFO) severely degrades the bit error rate (BER) performance of orthogonal frequency division multiple (OFDM) systems. In this paper, our aim is to develop a robust adaptive CFO estimation algorithm for OFDM system which is able to provide accurate estimate of CFO even in the presence of impulsive noise. To this end, we formulate the CFO estimation in terms of the maximum correntropy criterion (MCC) which is a robust optimality criterion for impulsive noise. Then, we utilize the gradient-ascent method and also apply instantaneous approximation to derive the robust adaptive algorithm. The proposed algorithm has computational complexity similar to the popular least mean-square (LMS), while it is robust against the impulsive signal because of using higher order moments beyond just second order moments. The performance of the proposed algorithm is evaluated under different conditions, including the Gaussian noise, impulsive noise, and time-varying CFO, where simulation results reveal the effectiveness of the proposed algorithm.

Keywords Adaptive algorithm, CFO, maximum correntropy criterion, OFDM

1. Introduction

The orthogonal frequency division multiplexing (OFDM) enables high data rate transmissions in wireless communication systems. It has been chosen as the standards for the European digital and video broadcasting, IEEE802.11a and HIPERLAN/2 [1]. However, the OFDM system is very sensitive to the carrier frequency offset (CFO). CFO is mainly caused by two sources [2]. The first one is the mismatch of carrier frequencies between oscillators in transmitter and receiver, and the other one is due to the Doppler shift, which may change from time to time. Unfortunately CFO destroys the orthogonality among subcarriers, which in turn causes inter-carrier interference (ICI) and degrades the bit error rate (BER) performance severely [2]. This issue motivated the development of CFO estimation algorithms.

So far several schemes have been proposed in the literature to estimate the CFO of OFDM systems [3-8]. In [4] a CFO estimation scheme has been developed which uses a training symbol with two identical halves. The given algorithm in [5] utilizes a training symbol with more than two identical halves which increases the estimation range twice that of the scheme in [4]. The algorithm in [6] relies on the maximum-likelihood (ML) criterion and same training symbol as in [5]. A periodogram-based CFO estimation scheme has been proposed in [6], whose estimation range is as large as the bandwidth of the OFDM signal while maintaining the same level of the estimation performance as those of [4, 5].

Most of available works assume CFO as a stationary parameter and do not consider the variation of the offset caused by the Doppler shift. Thus these types of algorithms fail when frequency offset changes from time to time. In [8] an adaptive LMS-based filter has been proposed to estimate CFO, which can track the variation of the offset caused by the Doppler shift. The proposed algorithm in [8] works well under the assumption of the Gaussian distributed noise. However, in many wireless channels, it has been observed that the noise often follows non-Gaussian distribution [9]. Thus, the conventional estimators could suffer from performance degradation in the non-Gaussian noise environments.

To address this issue, in this paper our aim is to develop a robust and adaptive algorithm to estimate the CFO which is robust in presence of impulsive noise. To this end we need to go beyond mean squared error to exploit higher order moments of the error. Information theoretic quantities have also been proposed as cost functions in adaptive filters (algorithms). For example a family of Minimum Error Entropy (MEE) based adaptive filters has been proposed [10], [11], wherein the weights are adapted such that the entropy or the information content of the error signal is minimized. Although MEE based algorithm shows robustness in presence of non-Gaussian and impulsive noise, however, it

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has high computational complexity. The use of Correntropy as a cost function in order to train the filter weights has been proposed in [12]. The given algorithm in [12] which uses maximum correntropy as the cost function has lower computational complexity than MEE, while its robustness is similar to the MEE based algorithm. Thus, in this paper we develop a MCC based adaptive algorithm for CFO estimation. To derive the proposed algorithm, we firstly formulate the CFO estimation according to the MCC, and then we utilize the gradient-ascent method with suitable instantaneous approximations to solve it. We compare the performance of the proposed algorithm with the LMS-based algorithm in [8] in different conditions including the Gaussian noise, impulsive noise, and time varying CFO, where the results show the superior performance of the proposed algorithm.

Notation: Through the paper we use lower case bold letters to denote vectors, capital bold letters for matrices. We also use $(\cdot)^T$ to denote transposition, and $(\cdot)^H$ to denote Hermitian transpose. Moreover $\mathrm{Im}\{\}$ denotes the imaginary part of its argument and E stands for the statistical expectation.

The rest of this paper is organized as follows: In section 2, we present some background on CFO estimation problem and also introduce the MCC. The proposed algorithm is presented in section 3. Simulation results are presented in section 4. Section 5 concludes the paper.

2. Background

2.1. CFO Estimation Problem

Let us define $\mathbf{s}(n) = [s_0(n), s_1(n), ..., s_{N-1}(n)]^T$ as the nth OFDM data block to be transmitted, where *N* is the number of subcarriers in the OFDM system. The data are used to modulate orthogonal subcarriers and this modulation can be implemented by inverse discrete Fourier transform (IDFT). Using the matrix representation, the *n*th block of the modulated signal is [14]

$$\mathbf{d}(n) = [d_0(n), d_1(n), ..., d_{N-1}(n)]^T = \mathbf{W}\mathbf{s}(n) \quad (1)$$

where $\mathbf{W}$ is the $N \times N$ IDFT matrix which is given by

$$\mathbf{W} = \frac{1}{\sqrt{N}} \begin{bmatrix} 1 & 1 & \cdots & 1 \\ 1 & e^{j\omega} & \cdots & e^{j(N-1)\omega} \\ \vdots & \vdots & & \vdots \\ 1 & e^{j(N-1)\omega} & \cdots & e^{j(N-1)(N-1)\omega} \end{bmatrix} \quad (2)$$

where in (2) $\omega = 2\pi / N$. Then, a cyclic prefix (CP) is inserted where it is assumed that the length of CP is longer than the maximum delay spread of the channel to avoid intersymbol interference (ISI). Finally, the resultant baseband signal is up-converted to the radio frequency (RF) before transmission. At the receiver side, the signal is firstly down-converted and demodulated using discrete Fourier transform (DFT) to recover the desired signal. Without CFO, the received signal can be expressed by

$$\mathbf{r}(n) = \mathbf{W}\mathbf{C}\mathbf{s}(n) + \mathbf{q}(n) \quad (3)$$

where $\mathbf{q}(n)$ denotes the additive white Gaussian noise (AWGN) and

$$\mathbf{C} = \mathrm{diag}\{C_0(n), C_1(n), ..., C_{N-1}(n)\} \quad (4)$$

represents the channel characteristics in the frequency domain. In this paper, we assume AWGN channel which means we have $\mathbf{C} = \mathrm{diag}\{1, 1, ..., 1\}$. In the presence of CFO the received signal, after the removal of the CP, is given as

$$\mathbf{r}(n) = \mathbf{\Phi}\mathbf{W}\mathbf{C}\mathbf{s}(n) + \mathbf{q}(n) \quad (5)$$

where

$$\mathbf{\Phi} = \mathrm{diag}\left\{1, e^{\frac{j2\pi v}{N}}, ..., e^{\frac{j2\pi(N-1)v}{N}}\right\} \quad (6)$$

represents the CFO matrix and is the frequency offset normalized to subcarrier spacing. To guarantee the system performance, the CFO must be estimated and compensated before the DFT demodulation. In the presence of the CFO, the received signal, after DFT demodulation, is given by

$$\mathbf{x}(n) = \mathbf{W}^H\mathbf{C}\mathbf{r}(n) = \mathbf{W}^H\mathbf{\Phi}\mathbf{W}\mathbf{C}\mathbf{s}(n) + \mathbf{q}'(n) \quad (7)$$

where $\mathbf{W}^H$ represents the DFT demodulation matrix.

2.2. Maximum Correntropy Criterion

Given two random variable X and Y, the correntropy is defined as follows

$$\begin{aligned} V(X,Y) &= \mathrm{E}\big[\kappa(X,Y)\big] \\ &= \int \kappa(x,y)dF_{X,Y}(x,y) \end{aligned} \quad (8)$$

where $\kappa(\cdot\,,\cdot)$ is a shift-invariant Mercer kernel, and $F_{X,Y}(x,y)$ denotes the joint distribution function of (X,Y). The Gaussian kernel is the most widely used kernel in correntropy

$$\kappa(x,y) = \frac{1}{\sqrt{2\pi\sigma^2}}\exp\left(-\frac{\xi^2}{2\sigma^2}\right) \quad (9)$$

Where $\xi = x - y$, and $\sigma > 0$ is the kernel width. The maximum correntropy cost function is given by

$$J_{MCC} = \mathrm{E}\left[\exp\left(-\frac{\xi^2(i)}{2\sigma^2}\right)\right] \quad (10)$$

In the sequel, we will use this cost function to derive the proposed algorithm.

3. Proposed CFO Estimation Algorithm

In this section, we present our adaptive CFO estimation algorithm. To this end, let us denote ith sample of recovered signal on subcarrier k as x_i (we ignore the subcarrier index for the sake of simplicity). We also use the training symbols in d_i where every $s_i(n)$ in (2) is a quadrature amplitude modulation (QAM) or phase-shift-keying (PSK) symbol. Then, by comparing the estimation with the training symbols d_i, we define the error signal as

$$\varepsilon_i = d_i - e^{j2\pi w_i i/N} x_i \tag{11}$$

where w_i is a real-valued. We now define the cost function in terms of the error signal as

$$J(i) = E\left[\exp\left(-\frac{|\varepsilon_i|^2}{2\sigma^2}\right)\right] \tag{12}$$

we have

$$|\varepsilon_i|^2 = |d_i|^2 - e^{\frac{-j2\pi w_i i}{N}} x_i d_i^* - e^{\frac{-j2\pi w_i i}{N}} x_i^* d_i + |x_i|^2 \tag{13}$$

To find the optimum weight that maximizes the cost function in (12), we can use iterative gradient ascent approach as

$$w_{i+1} = w_i + \mu \nabla_{w_i} J(i) \tag{14}$$

where μ is the step-size parameter and ∇_{w_i} denotes the gradient with respect to w. By computing the gradient $J(i)$ with respect w_i to we obtain

$$\nabla_{w_i} J(i) = \left(\frac{4\pi i}{2\sigma^2 N}\operatorname{Im}\{e^{-j2\pi w_i i/N} x_i^* d_i\}\right) \times E\left[\exp(-\frac{|\varepsilon_i|^2}{2\sigma^2})\right] \tag{15}$$

Substituting $\nabla_{w_i} J(i)$ from (15) in (14) gives

$$w_{i+1} = w_i + \mu\left(\frac{4\pi i}{2\sigma^2 N}\operatorname{Im}\{e^{-j2\pi w_i i/N} x_i^* d_i\}\right) \times E\left[\exp(-\frac{|e_i|^2}{2\sigma^2})\right] \tag{16}$$

Then, the proposed algorithm can be obtained by replacing the statistical moment in (16) by instantaneous approximation as

$$w_{i+1} = w_i + \mu\left(\frac{4\pi i}{2\sigma^2 N}\operatorname{Im}\{e^{-j2\pi w_i i/N} x_i^* d_i\}\right) \times \left(\exp(-\frac{|\varepsilon_i|^2}{2\sigma^2})\right) \tag{17}$$

The estimate of CFO is given in terms of w_i as $\widehat{v} = -w_i$, where the number of iterations equals the number of pilot symbols.

4. Simulation Results

In this section, we evaluate the performance of the proposed algorithm and compare it with the given LMS-based algorithm in [8]. We consider the number of subcarriers in the OFDM system as $N = 1024$. We consider the mean-square-error (MSE) as the performance metric which is defined as

$$\mathrm{MSE} = \frac{1}{P}\sum_{t=1}^{P} |v - \hat{v}|^2 \tag{18}$$

where P is the number of Monte Carlo runs, which in our simulations was $P = 1000$. The CFO is assumed as $v = 0.2$ in the following set of simulations. In the first set of simulation we consider Gaussian noise condition where $E_b / N_0 = 20\,\mathrm{dB}$. Fig. 1 shows the learning curves of the proposed algorithm and the LMS-based algorithm for $\mu = 0.005$, $\sigma = 2$. We can see that the proposed algorithm exhibits a better performance than the algorithm LMS algorithm since it has smaller estimation variance. Fig. 2 presents the MSE versus Eb/N0 for various step-size parameters. It is observed that for both algorithms, the MSE decreases as E_b / N_0 increases or when the step size parameter decreases.

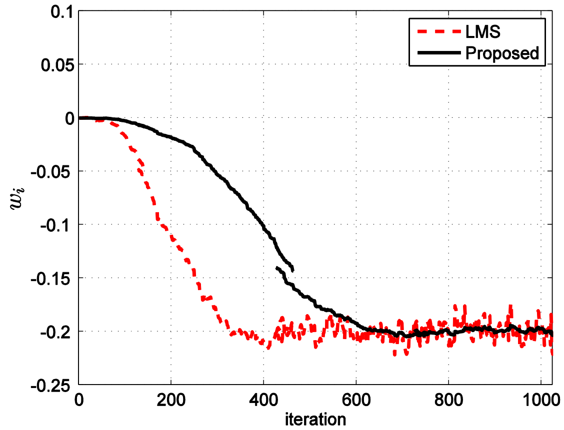


Figure 1. The learning curves of the proposed algorithm and the LMS-based algorithm for E_b/N_0=20 dB

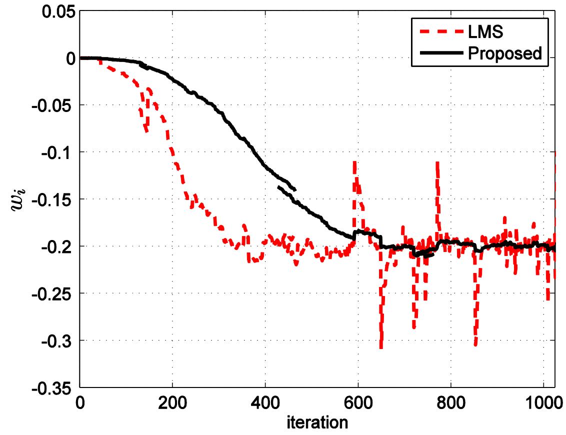


Figure 3. The MSE performance of both algorithms in terms of the Eb/N0 for different step size parameters

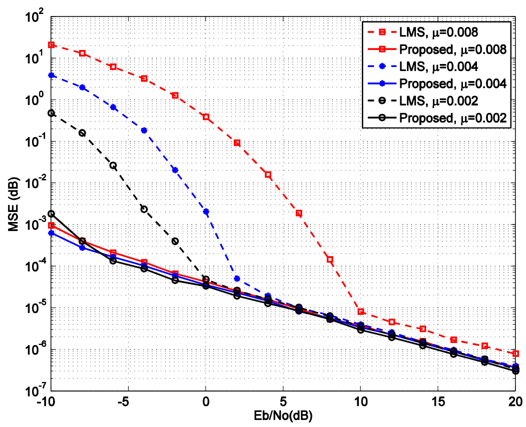


Figure 2. The MSE performance of both algorithms in terms of the E_b/N_0 for different step size parameters

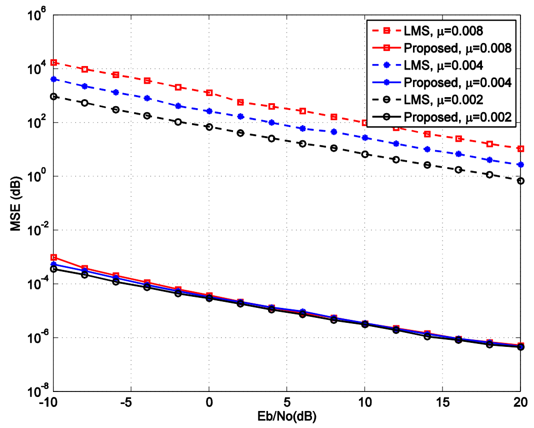


Figure 4. The MSE versus Eb/N0 for various step size parameters under impulsive noise condition

For small $E_b\ /\ N_0$ values, the proposed algorithm performs better than the LMS- based algorithm for all step-size values. As $E_b\ /\ N_0$ increases, the performance of both algorithms becomes similar, specially for small step-size values.

In the next simulation setup, we assume again an OFDM system described above but with impulsive noise. To model the impulsive noise we add a zero-mean complex-valued doubly white Gaussian noise at 0.2dB SNR with probability Pr = 0.03, where Pr denotes the probability that the impulsive noise occurs. The learning curves of the proposed algorithm and the LMS-based algorithm is shown in Fig. 3. The MSE versus Eb/N0 for various step-size parameters indifferent ranges of Eb/N0 are shown in Fig. 4 and Fig. 5. We can see that in the presence of impulsive noise, the performance of LMS-based algorithm decreases, while the proposed algorithm works well in this condition. In Fig. 6, we compare the estimation range of both the proposed LMS-based algorithm and the proposed algorithm under the Gaussian noise and impulsive noise conditions. It is clear that the proposed algorithm has better performance for the whole range of CFO than the LMS-based algorithm.

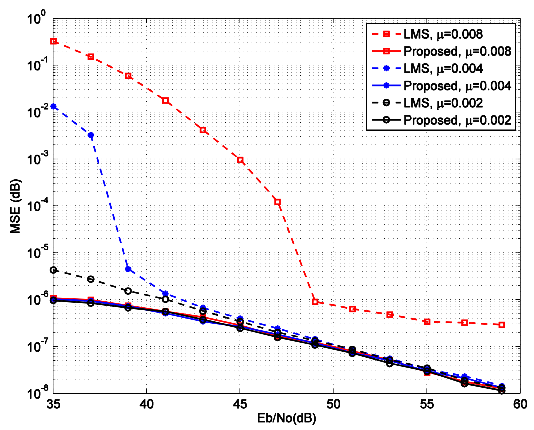
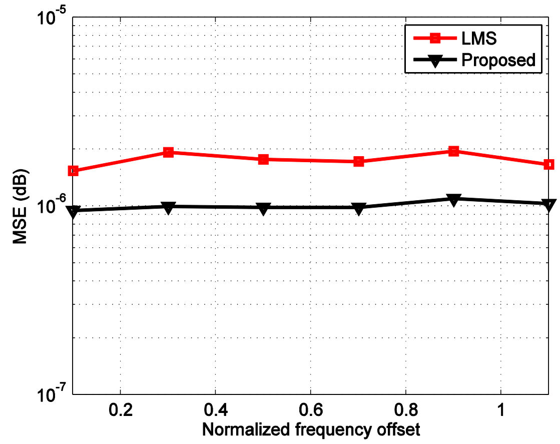


Figure 5. The MSE versus Eb/N0 for various step size parameters under impulsive noise condition

In Fig. 7, we compare the tracking performance of both algorithms in the presence of impulsive noise. In this case we assume that the normalized frequency offset changes in time according to the following model

$$v_i = v_{i-1} + z_i \tag{19}$$

where z_i is a zero-mean Gaussian noise with variance 0.005. As we can see the proposed algorithm can track the variation of the CFO and the estimation accuracy is not affected by the impulsive noise



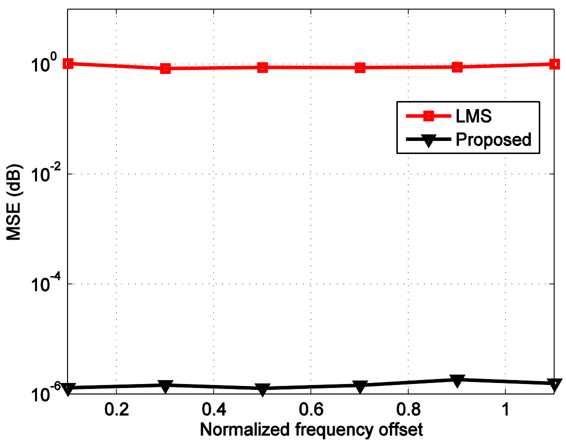


Figure 6. The estimation range of the proposed LMS-based algorithm and the proposed algorithm under the Gaussian noise (top) and impulsive noise (bottom) conditions

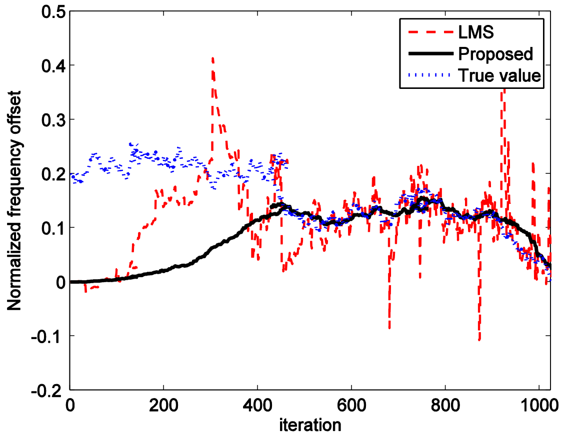


Figure 7. Tracking performance of both algorithms in the presence of impulsive noise

5. Conclusions

The carrier frequency offset degrades the performance of OFDM systems. In this paper, we have proposed a correntropy-based adaptive algorithm to estimate the CFO simultaneously in OFDM systems. The proposed algorithm achieves comparable performance and a wider estimation range in comparison with the LMS-based algorithm. The time-varying nature of Doppler shift causes the frequency offset to change from time to time, which affects the estimation accuracy of many existing estimators. The proposed algorithm can track the variation of the Doppler shift well and the estimation accuracy is not affected in the presence of impulsive noise.

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