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An Adaptive Incremental Algorithm for Distributed Filtering of Hypercomplex Processes

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Abstract An adaptive network is a collection of agents (nodes) that collaborate with each other through in-network local processing rules in order to perform distributed information processing tasks. In this paper we propose an adaptive incremental network (algorithm) for distributed adaptive filtering of three- and four-dimensional processes. The proposed algorithm exploits spatiotemporal diversity. Moreover, the proposed algorithm employs both the covariance and pseudocovariance terms within its update and can therefore cater for noncircularly symmetric quaternion data. We show that the distributed widely linear (equivalent to augmented term) model outperforms the distributed standard linear model and non-cooperative case. Simulations in the prediction setting on benchmark 4D and 3D signals show the results.

Keywords Multidimensional adaptive filters, Widely linear model, Quadrivariate processes, Incremental network

1. Introduction

Complex-valued random signals appear in great number of applications such as communications, radar, sonar, geophysics oceanography, optics, and electromagnetic [1]. The complex domain $\mathbb{C}$ is adequate to process two dimensional signals. A common assumption in complex signal processing is that they are proper or circular. A proper complex random signal is uncorrelated with its complex conjugate, and a circular complex random variable has a probability distribution that is rotationally invariant in the complex plane [2]. Although assumptions are convenient because and simplify many computations, however, there are also many situations where proper and circular signals are very poor models of the underlying physics [3]. To exploit the improper or noncircular nature of complex signals, we need to utilize the complete statistical characterization of complex-valued random signals. For example, to completely extract the second-order information available within the general complex signals we need to consider both the covariance and pseudo-covariance matrices.

To access the information contained in this correlation, we can employ widely linear model that take the full second order statistics into account. Widely linear model suitable for processing of general complex signals, both circular and noncircular, is introduced by Picinbono [4]. The augmented representation, i.e., $\mathrm{x}^a = [\mathrm{x}^{\mathrm{T}}, \mathrm{x}^{\mathrm{H}}]^{\mathrm{T}}$, is required to fully model the second order statistical information within the complex domain, that is, $\mathrm{C}_{xx} = \mathrm{E}[\mathrm{x}\mathrm{x}^{\mathrm{H}}]$ and $\mathrm{P}_{xx} = \mathrm{E}[\mathrm{x}\mathrm{x}^{\mathrm{T}}]$ respectively the covariance and pseudo-covariance matrices of complex-valued vector x.

Complex-valued adaptive filtering algorithms have been used in a wide range of application. Different complex-valued adaptive filtering algorithms based on the augmented complex statistics have been introduced in the literature [3]. These algorithms are usually termed widely linear or augmented, such as the widely linear LMS (WL-LMS) augmented CLMS (ACLMS) [5,6,7], augmented affine projection (AAPA) [8], widely linear Recursive Least Squares (WL-RLS) algorithms [9], regularized normalized augmented complex LMS (RN-ACLMS) algorithm [10].

To deal with three and four-dimensional processes, Quaternion domain $\mathbb{H}$ introduces a natural framework for a unified treatment of them. The quaternion algorithms have been developed in the signal processing community, consist of Kalman filtering [11], MUSIC spectrum estimation [12], singular value decomposition for vector sensing [13], and the least-mean-square estimation [14]. The quaternion domain can be significantly used in many multivariate problems based on vector sensors (motion body sensors, seismic, wind modeling). It is therefore natural to extend the so-called augmented complex statistics and widely linear modeling in the complex domain to the quaternion domain. Such widely linear models, include AQLMS [14], widely linear QLMS (WL-QLMS) [15] and WLIQLMS [16].

Distributed information processing tasks appear in many practical applications [17]. There are already several useful strategies for estimation over distributed networks. An example of a distributed method is the consensus strategy [18] in which each node performs a local estimation and

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fuses its estimate with those of its neighbors so that all nodes converge to the same estimate as the number of iterations increases.

Table 1. Summary of notations

Notation	Description
$\Re\{.\}$	real part
$\Im\{.\}$	imaginary part
$\Im_{i,j,k}\{.\}$	*i,j,k*-component of vector imaginary part of quaternion variable
$\times$	cross-product
$q^{i,j,k}$	i,j,k-involution of quaternion variable
$(.)^*$	conjugate operator
$(.)^T$	transpose operator
$(.)^H$	conjugate transpose operator
E[.]	Expectation operator

The main problem of mentioned methods is that the framework does not allow the network to undertake a continuous learning and optimization which motivated the development of adaptive networks. An adaptive network is a collection of agents (nodes) that collaborate with each other through in-network local processing rules in order to estimate and track parameters of interest [19]. Two major classes of adaptive networks are incremental strategy [20-22] and diffusion strategy [23-25]. In the incremental mode, nodes passing updates to each other in a Hamiltonian cycle in the network, while in diffusion mode, each node can communicate with a subset of neighboring nodes.

Adaptive networks based on the widely linear models have been considered in some recent works such as [26-30]. In this paper, we propose an incremental based adaptive network for distributed adaptive filtering of three- and four-dimensional processes, such as those observed in atmospheric modeling (wind, vector fields). The proposed algorithm has the advantage that exploits spatiotemporal diversity to improve the filtering performance. Moreover, it accounts for complex nonlinear dynamics and coupling between the dimensions, which are the main problems of mentioned multidimensional processes. The proposed algorithm employs both the covariance and pseudocovariance terms within its update and can therefore cater for noncircularly symmetric quaternion data. We evaluate the performance of the proposed algorithm with some simulation results.

The organization of the paper is as follows: in Section 2 we briefly review the elements of quaternion algebra. In section 3 we derive the proposed algorithm. In section 4 simulations will be presented and finally we conclude our work in section 5.

For convenience, a summary of notations is given in Table 1.

2. Quaternion Domain

A quaternion variable $q \in \mathbb{H}$ consists of a real part $\Re\{.\}$ (denoted by subscript a) and a vector part $\Im\{.\}$, also known as a pure quaternion, consisting of three imaginary components, and can be expressed as

$$\begin{aligned} q &= \Re\{q\} + \Im\{q\} \\ &= \Re\{q\} + i\Im_i\{q\} + \mathrm{j}\,\Im_j\{q\} + \mathrm{k}\,\Im_k\{q\} \\ &= q_a + iq_b + jq_c + kq_d \in \mathbb{H} \end{aligned} \tag{1}$$

the relationships between imaginary numbers, i, j, k (also known as orthogonal unit vectors) are described as follows

$$\begin{aligned} ij = k,\ jk = i, ki = j \\ ijk = i^2 = j^2 = k^2 = -1 \end{aligned} \tag{2}$$

multiplication for every quaternion $q_1, q_2 \in \mathbb{H}$, is defined as

$$q_1 q_2 = \Re\{q_1 q_2\} + \Im\{q_1 q_2\}$$

where

$$\begin{aligned} \Re\{q_1 q_2\} &= q_{1,a}q_{2,a} + q_{1,b}q_{2,b} + q_{1,c}q_{2,c} + q_{1,d}q_{2,d} \\ \Im\{q_1 q_2\} &= q_{1,a}\Im\{q_2\} + q_{2,a}\Im\{q_1\} + \Im\{q_1\} \times \Im\{q_2\} \end{aligned} \tag{3}$$

due to $q_1 q_2 = q_2 q_1 - 2\Im\{q_2\} \times \Im\{q_1\} \neq q_2 q_1$, the noncommutativity of the quaternion multiplication can be observed. The quaternion conjugate is also defined as

$$q^* = \Re\{q\} - \Im\{q\} = q_a - iq_b - jq_c - kq_d \tag{4}$$

2.1. Quaternion Involutions and Augmented Basis Vector

In the complex domain $\mathbb{C}$, elements of a complex number $x = x_a + ix_b$ in $\mathbb{R}^2$ can be described using `augmented' basis vector $\mathrm{x}^a = [\mathrm{x}, \mathrm{x}^*]^T$ as $x_a = (1/2)(x + x^*)$ and $x_b = (1/2i)(x - x^*)$. In the quaternion domain $\mathbb{H}$, we seek an `augmented' basis vector to relate its components to those of a quaternion valued variable in $\mathbb{R}^4$. For this reason, we define three perpendicular quaternion involutions as

$$\begin{aligned} q^i &= -iqi = q_a + iq_b - jq_c - kq_d \\ q^j &= -jqj = q_a - iq_b + jq_c - kq_d \\ q^k &= -kqk = q_a - iq_b - jq_c + kq_d \end{aligned} \tag{5}$$

whose conjugates q^{i*}, q^{j*} and q^{k*} are described as

$$\begin{aligned} q^{i*} &= q_a - iq_b + jq_c + kq_d \\ q^{j*} &= q_a + iq_b - jq_c + kq_d \\ q^{k*} &= q_a + iq_b + jq_c - kq_d \end{aligned} \quad (6)$$

Now, the elements of a quaternion valued variable q can be computed as

$$\begin{aligned} q_a &= \frac{1}{2}(q+q^{*}), q_b = \frac{1}{2i}(q-q^{i*}) \\ q_c &= \frac{1}{2j}(q-q^{j*}), q_d = \frac{1}{2k}(q-q^{k*}) \end{aligned} \quad (7)$$

Notice that the quaternion conjugate q^{*} is also an involution, and is given by

$$q^{*} = \frac{1}{2}(q^{i} + q^{j} + q^{k} - q) \quad (8)$$

We can use a combination of $[\mathrm{q}, \mathrm{q}^{*}, \mathrm{q}^{i}, \mathrm{q}^{j}, \mathrm{q}^{k}]$ to define an augmented quaternion vector. One augmented basis vector is $[\mathrm{q}, \mathrm{q}^{*}]$ and will be used in this work.

2.2. Second Order Circularity in $\mathbb{H}$ and $\mathbb{Q}$ -Properness

For a complex-valued variable $x = x_a + ix_b$ to be second order circular (or $\mathbb{C}$ -proper), its probability distribution should be rotation-invariant in complex plane, and therefore have two conditions

$$\begin{aligned} \sigma_{x_a}^2 &= \sigma_{x_b}^2 \\ \mathrm{E}[x_a x_b] &= 0 \end{aligned} \quad (9)$$

that is, the real and imaginary parts should be uncorrelated and have equal power, which leads to a vanishing pseudo-covariance matrix $\mathrm{P}_{xx} = \mathrm{E}[\mathrm{xx}^T]$ [1]. Second order circularity is established for a quaternion variable q, when its probability distribution be rotation-invariant with respect to the six pairs of axes: $\{1,\mathrm{i}\},\{1,\mathrm{j}\},\{1,\mathrm{k}\},\{\mathrm{i},\mathrm{j}\},\{\mathrm{k},\mathrm{j}\}$ and $\{\mathrm{k},\mathrm{i}\}$, where '1' denotes the real axis and i, j, k denote the corresponding imaginary axes. In other words, the probability distribution of a $\mathbb{Q}$ -proper variable is rotation-invariant with respect to all these six pairs of axes.

1- For a $\mathbb{C}$ -proper signal, the pseudo-covariance P_{xx} vanishes, that is, $\mathrm{E}[\mathrm{xx}^T] = \mathrm{E}[\mathrm{x}_r \mathrm{x}_r^T] - \mathrm{E}[\mathrm{x}_i \mathrm{x}_i^T] + i(\mathrm{E}[\mathrm{x}_r \mathrm{x}_i^T] - \mathrm{E}[\mathrm{x}_i \mathrm{x}_r^T]) = 0$, this is because for a $\mathbb{C}$ -proper signal, the real part x_r and the imaginary part x_i have the same power and are uncorrelated.

Despite for a $\mathbb{Q}$ -proper signal, the scalar/real part x_a and the vector/imaginary part $x_{b,c,d}$ have equal power and are uncorrelated, but the pseudo-covariance P_{xx} does not vanish, that is,

$$\begin{aligned} \mathrm{E}[\mathrm{xx}^T] &= \mathrm{E}[\mathrm{x}_a \mathrm{x}_a^T] - \mathrm{E}[\mathrm{x}_b \mathrm{x}_b^T] - \mathrm{E}[\mathrm{x}_c \mathrm{x}_c^T] - \mathrm{E}[\mathrm{x}_d \mathrm{x}_d^T] \\ &+ i2\,\mathrm{E}[\mathrm{x}_a \mathrm{x}_b^T] + j2\,\mathrm{E}[\mathrm{x}_a \mathrm{x}_c^T] + k2\,\mathrm{E}[\mathrm{x}_a \mathrm{x}_d^T] \neq 0 \end{aligned} \quad (10)$$

3. I-AQLMS Algorithm

3.1. The Augmented QLMS (AQLMS)

For the Quaternion LMS, where the output $y(\mathrm{n}) = h^T(\mathrm{n})\mathrm{x}(\mathrm{n})$, with $\mathrm{x}(\mathrm{n})$ and $h(\mathrm{n})$ denoting respectively the filter input and the adaptive weight vectors, the update of the adaptive weight vector of QLMS can be obtained as [14]

$$h(\mathrm{n}+1) = h(\mathrm{n}) + \mu(2e(\mathrm{n})\mathrm{x}^{*}(\mathrm{n}) - \mathrm{x}^{*}(\mathrm{n})e^{*}(\mathrm{n})) \quad (11)$$

where the error $e(\mathrm{n}) = d(\mathrm{n}) - h^T(\mathrm{n})\mathrm{x}(\mathrm{n})$, with $d(\mathrm{n})$ denoting the desired signal. Following the augmented CLMS, augmented CRTRL, and augmented statistics for wind profile [14, 15], we can employ a quaternion-valued widely linear model to cater for both circular and noncircular quaternion processes, given by

$$y(\mathrm{n}) = h^T(\mathrm{n})\mathrm{x}(\mathrm{n}) + g^T(\mathrm{n})\mathrm{x}^{*}(\mathrm{n}) \quad (12)$$

This model contains the information in both covariance C_{xx} and pseudocovariance P_{xx}. Similarly to update for vector h, that for vector g in (12) can be obtained as follows

$$g(\mathrm{n}+1) = g(\mathrm{n}) + \mu(2e(\mathrm{n})\mathrm{x}(\mathrm{n}) - \mathrm{x}(\mathrm{n})e^{*}(\mathrm{n})) \quad (13)$$

the non-commutativity of the quaternion products has been taken into account during the derivation of the updates. Finally, (11) and (13) can be expressed into a compact `augmented' form as

$$w(\mathrm{n}) = [h^T(\mathrm{n})g^T(\mathrm{n})]^T \quad (14)$$

and the weight update of the Augmented QLMS (AQLMS), can be obtained as

$$w(\mathrm{n}+1) = w(\mathrm{n}) + \mu(2e(\mathrm{n})\mathrm{x}^{a*}(\mathrm{n}) - \mathrm{x}^{a*}(\mathrm{n})e^{*}(\mathrm{n})) \quad (15)$$

where the augmented input vector $\mathrm{x}^a(\mathrm{n}) = [\mathrm{x}^T(\mathrm{n})\mathrm{x}^H(\mathrm{n})]^T$ and error $e(\mathrm{n}) = d(\mathrm{n}) - w^T(\mathrm{n})\mathrm{x}^a(\mathrm{n})$.

3.2. Derivation of I-AQLMS Algorithm

Consider a network with N distributed nodes over an area, as shown in Figure 3. Each node k has access to local measurement $d(\text{n})$ and a regressor vector $\text{x}_k(\text{n}) = [x_k(\text{n}-1), ..., x_k(\text{n}-M)]^T$, at time instant n. the flow of data in a sequential manner from one node to the adjacent node by an incremental protocol can be described as follows: assume that $w_k(\text{n})$ denotes the local estimate of optimum weight vector w^o at node k at time n, and node k has access to local estimate $w_{k-1}(\text{n})$ at previous node $k-1$ in the defined cycle. by starting with the initial condition $w_0(\text{n}) = \phi(\text{n}-1)$ in node 1, and going across the network, then at the end of cycle, the local estimate at node N, $w_N(\text{n})$, will be considered as $w_0(\text{n}+1)$ for next iteration. This algorithm for AQLMS is summarized in Table 2, where $\text{x}_k^a(\text{n}) = [\text{x}_k^T(\text{n})\,\text{x}_k^H(\text{n})]^T$ and $w_k(\text{n}) = [h_k^T(\text{n})\,g_k^T(\text{n})]^T$.

Table 2. Incremental AQLMS algorithm

I-AQLMS
Repeat for n=1,2,...,m(number of iterations):
1. $w_0(\text{n}) \leftarrow \phi(\text{n}-1)$
2. for k = 1, 2, ... ,N,
$w_k(\text{n}) \leftarrow w_{k-1}(\text{n}) + \mu_k(2e_k(\text{n})\,\text{x}_k^{a^*}(\text{n}) - \text{x}_k^{a^*}(\text{n})e_k^*(\text{n}))$
3. $\phi(\text{n}) \leftarrow w_N(\text{n})$

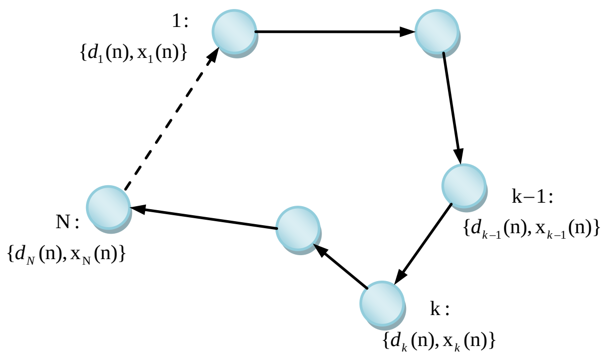
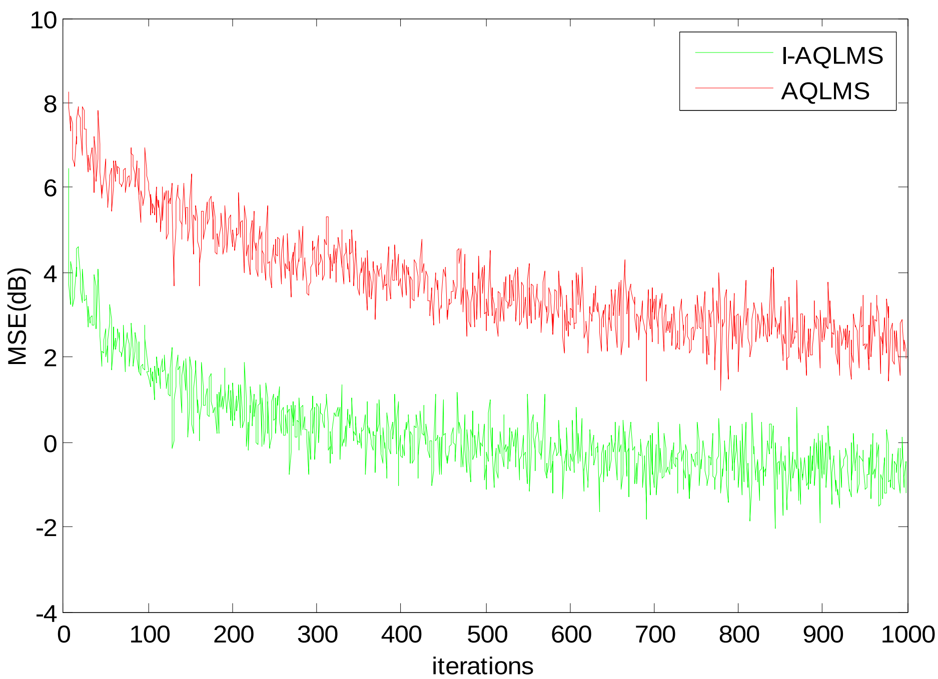
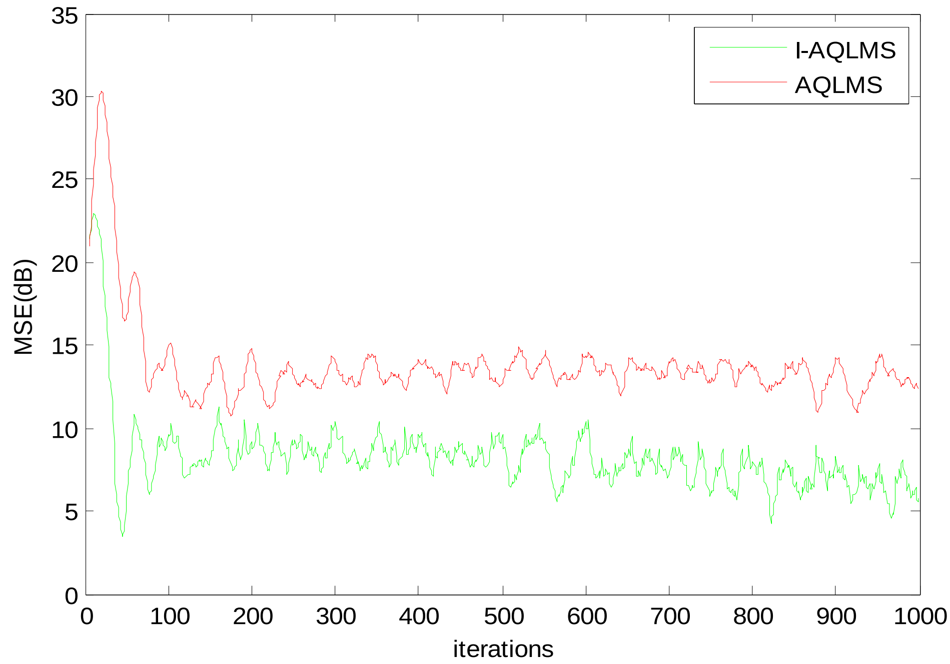


Figure 1. Incremental mode of cooperation structure



(a)



(b)

Figure 2. Comparison of MSEs of non-cooperative AQLMS and Incremental AQLMS algorithms for prediction of (a) circular 4D AR(4) process with $\mu = 10^{-4}$, and (b) noncircular 3D Lorenz signal with $\mu = 10^{-6}$

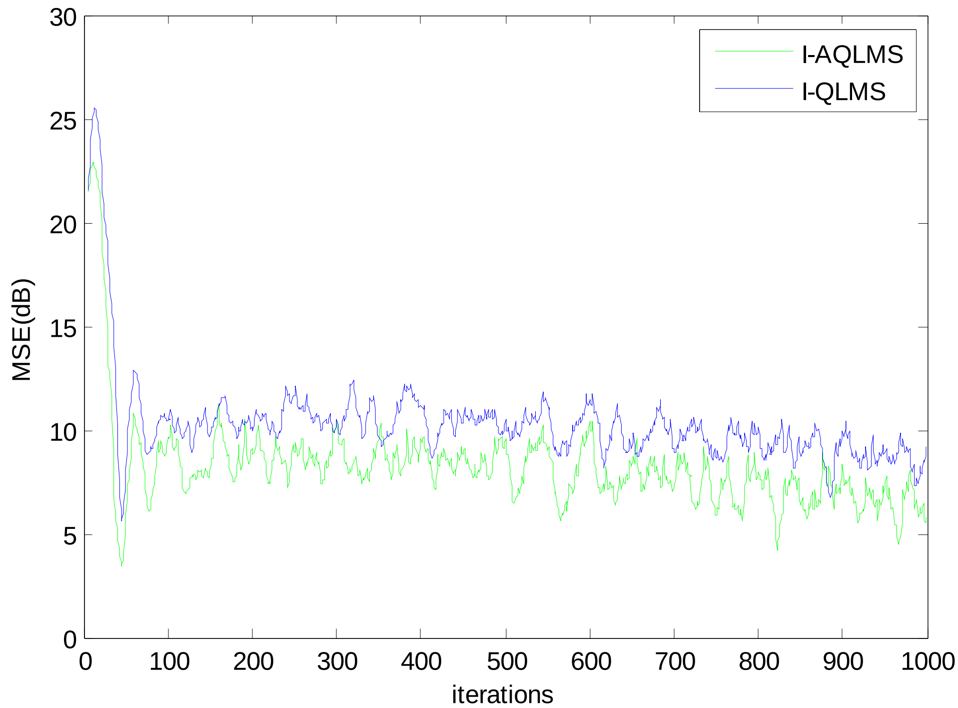


Figure 3. Comparison of MSEs of I-AQLMS and I-QLMS algorithms for prediction of noncircular 3D Lorenz signal with $\mu = 10^{-6}$

4. Simulation

The one step ahead prediction performance of proposed algorithms I-AQLMS is compared with its standard linear counterpart (I-QLMS) and non-cooperative case. The network consist of N = 10 nodes, and filter order is set to M = 5. For evaluating the algorithms performance, we employ two signals, one is a three-dimensional nonlinear system known as the Lorenz attractor used originally to model atmospheric turbulence, but also to model lasers, dynamos, and the motion of waterwheel [5]. Mathematically, the Lorenz system can be expressed as a system of coupled differential equations:

$$\frac{\partial x}{\partial t} = \alpha(y - x)$$
$$\frac{\partial y}{\partial t} = x(\rho - z) - y \quad (16)$$
$$\frac{\partial z}{\partial t} = xy - \beta z$$

where $\alpha, \beta, \rho > 0$, and the other signal is generated by a linear AR(4) model as follows

$$y(0) = 0$$
$$y(n) = 1.79y(n-1) - 1.85y(n-2) + 1.27y(n-3) - 0.41y(n-4) + v(n), n \geq 1 \quad (17)$$

where $v(n)$ denotes the quaternion-valued circular white Gaussian noise. Figure 2 illustrates the evaluation of mean square error (MSE) of proposed algorithm in analogy to non-cooperative case for one step ahead prediction of the mentioned signals. Observe that as expected the proposed cooperative algorithm outperforms the non-cooperative case. Figure 3 shows the evaluation of MSE for one step ahead prediction of the noncircular Lorenz signal for comparing the proposed algorithm with its standard linear counterpart. It is worth noting that due to non-circular nature of the Lorenz signal, the widely linear distributed algorithm has better performance than standard linear distributed one.

5. Conclusions

We have proposed an Incremental Augmented QLMS (I-AQLMS) algorithm for processing of both circular and noncircular quaternion-valued signals collaboratively. The advantage of the widely linear incremental algorithm over standard linear incremental and noncooperative case in terms of convergence speed and steady state performance has been shown in simulations on synthetic signals.

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