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Transient Analysis of a Limited Capacity Markovian Queueing System Subjected to Varying Catastrophic Intensity and Restoration

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Abstract In this paper, we have obtained the time- dependent solution of a varying catastrophic intensity-cum- restorative Markovian queueing model with finite capacity. The transient solution has been obtained recursively. The simulation of the model has also been performed and various measures of performance have been computed. The steady-state solution is also derived. Further, some particular cases of the queuing model have been derived and discussed.

Keywords Varying Catastrophic Intensity, Restoration Time, Markovian Queueing System, Steady State Solution

1. Introduction

During the last three decades the attention of the queueing models has been focused on certain extension that includes the effect of catastrophes, in particulars, birth and death models. A large number of research papers have been published on population processes under the influence of catastrophes, for instance, Swift[12], Kyriakidis[3], Brockwell[9, 10, and 11] among other have discussed birth and death models with catastrophes. Queuing models continue to be one of the most important area of computer networks and have played a vital role in performance evaluation of computer systems. In computer system, if a job is infected, this job may transmit virus which may be transferred to the other processors. Hence computer network with virus may be modeled by queuing networks with catastrophes[2]. In the above mentioned research work the authors has assumed that the random occurrence of catastrophe destroys all the customers in a queuing system. But it is not always the case. So, necessary amendment is incorporated by Jain and Bura[6, 7] in the form of varying catastrophic intensity to destroy a finite number of customers at a time. The number of customers in a queuing system is instantly reset to zero or not depends upon the intensity of catastrophe. Queuing models with varying catastrophic intensity are applied in various field of biological sciences and agriculture. The concept of restoration time is considered by Jain and Kumar[8]. If the catastrophic intensity destroys all the customers in a queuing system then the system will require some sort of time to function in a normal way, which is taken as restoration time. Hence, an assumption of restoration is added in addition to the assumption of varying catastrophic intensity. In restoration case, we assume that during the restoration time no arrival is allowed to occur.

2. Queuing Model

The queuing model investigated in this paper is based on the following assumptions:-

(i) The customers arrive in the system one by one in accordance with a Poisson process at a single service station with rate $\lambda > 0$.

(ii) The customers are served one by one by a single server and the service times are independently, identically, and exponentially distributed with rate $\mu > 0$.

(iii) When the system is not empty, catastrophes occur according to a Poisson process with rate ξ and intensity C_r, $\left(r=1,2,3,....,N\right), \sum_{r=1}^{N} C_r = 1$. It depends upon the intensity of the catastrophe whether it destroys all the customers or not. If it destroys all the customers, then the system will require some sort of time to re-function in a normal way, which is taken as restoration time.

(iv) The restoration time is independently, identically and exponentially distributed With parameter β. During the restoration time no arrival is allowed to occur.

(v) The queue discipline is first- come first- served.

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(vi) The capacity of the system is limited to N. i.e., if at any instant there are N customers in the system, then the customers arriving in the duration for which the system remains in state N are not permitted to join the queue and considered lost for the system with probability one.

(vii) Initially, there are zero customers in the system.

Define

$P_{00}(t)$=the probability that there are zero customer in the system at time t without the occurrence of catastrophe.

$Q_{00}(t)$=the probability that there are zero customer in the system at time t with the occurrence of catastrophe destroying all the customers.

$P_n(t)$=the probability that there are n customers in the system at time t.

3. Transient Solution

The differential-difference equations governing the system are:

$$P_{00}^{/}(t) = -\lambda P_{00}(t) + \mu P_1(t) + \beta Q_{00}(t) \tag{1}$$

$$Q_{00}^{/}(t) = -\beta Q_{00}(t) + \xi\left(\sum_{n=1}^{N}\sum_{r=n}^{N} c_r P_n(t)\right) \tag{2}$$

$$P_1^{/}(t) = -(\lambda+\mu+\xi)P_1(t) + \lambda P_{00}(t) + \mu P_2(t) + \xi\sum_{r=1}^{N-1} c_r\, P_{(1+r)}(t) \tag{3}$$

$$P_n^{/}(t) = -(\lambda+\mu+\xi)P_n(t) + \lambda P_{(n-1)}(t) + \mu P_{(n+1)}(t) + \xi\sum_{r=1}^{N-n} c_r\, P_{(n+r)}(t), \quad n = 2,3,\ldots\ldots,N-1 \tag{4}$$

$$P_N^{/}(t) = -(\mu+\xi)P_N(t) + \lambda P_{(N-1)}(t) \tag{5}$$

Taking, Laplace Transform of equations (1), to (5) w.r.t.'t', we have

$$sP_{00}^{*}(s) = 1 - \lambda P_{00}^{*}(s) + \mu P_1^{*}(s) + \beta Q_{00}^{*}(s) \tag{6}$$

$$sQ_{00}^{*}(s) = -\beta Q_{00}^{*}(s) + \xi\left(\sum_{n=1}^{N}\sum_{r=n}^{N} c_r P_n^{*}(s)\right) \tag{7}$$

$$sP_1^{*}(s) = -(\lambda+\mu+\xi)P_1^{*}(s) + \lambda P_{00}^{*}(s) + \mu P_2^{*}(s) + \xi\sum_{r=1}^{N-n} c_r\, P_{(1+r)}^{*}(s) \tag{8}$$

$$sP_n^{*}(s) = -(\lambda+\mu+\xi)P_n^{*}(s) + \lambda P_{(n-1)}^{*}(s) + \mu P_{(n+1)}^{*}(s) + \xi\sum_{r=1}^{N-n} c_r\, P_{(n+r)}^{*}(s) \tag{9}$$

$$sP_N^{*}(s) = -(\mu+\xi)P_N^{*}(s) + \lambda P_{(N-1)}^{*}(s) \tag{10}$$

Where

$$P_n^{*}(s) = \int_0^{\infty} e^{-st}P_n(t)dt \text{ and } P_{00}(0) = 1$$

Solving this set of equations recursively, we have

$$P_n^{*}(s) = \rho^{-N}\gamma^{n}P_N^{*}(s) \tag{11}$$

$$P_1^{*}(s) = \frac{\lambda(s+\beta)}{H} + \frac{\rho^{-N}}{H}\left[\begin{array}{l}\mu(s+\lambda)(s+\beta)\gamma_2 + \lambda\beta\xi\displaystyle\sum_{n=2}^{N}\sum_{r=n}^{N} C_r\gamma_n \\ +\xi(s+\lambda)(s+\beta)\displaystyle\sum_{r=1}^{N-1} C_r\,\gamma_{(1+r)}\end{array}\right]P_N^{*}(s) \tag{12}$$

$$Q_{00}^{*}(s) = \frac{\lambda\xi(s+\beta)}{H(s+\beta)} + \frac{\rho^{-N}}{(s+\beta)H}\left[\begin{array}{l}\mu\xi(s+\lambda)(s+\beta)\gamma_2 + (\lambda\beta\xi+H)\xi\displaystyle\sum_{n=2}^{N}\sum_{r=n}^{N} C_r\,\gamma_n \\ +\xi^2(s+\lambda)(s+\beta)\displaystyle\sum_{r=1}^{N-1} C_r\,\gamma_{(1+r)}\end{array}\right]P_N^{*}(s) \tag{13}$$

and

$$P_{00}^*(s) = \frac{(\lambda E+H)}{(s+\lambda)H} + \frac{\rho^{-N}}{(s+\lambda)(s+\beta)H}\left[\begin{array}{l}\mu E(s+\lambda)(s+\beta)\gamma_2 + (\lambda E+H)\xi\beta\sum_{n=2}^{N}\sum_{r=n}^{N} C_r\gamma_n \\ +\xi E(s+\lambda)(s+\beta)\sum_{r=1}^{N-1} C_r\,\gamma_{(1+r)}\end{array}\right] P_N^*(s) \tag{14}$$

Where $\gamma_n = \rho^n + \sum_{i=1}^{\left[\frac{-3+\sqrt{9+8(N-n)}}{2}\right]} \sum_{l_0=i}^{\left[\frac{2(N-n)-(i(i-1))}{4}\right]} \prod_{j=1}^{i-1}\left(\sum_{l_j=(i-j)}^{l_{j-1}-1}\right)\prod_{m=0}^{i-1}\left(\sum_{k_{(m+1)}=k_m+1}^{[A_m]}\right)\eta\ \rho^{L_i+n} D_i$

$$\rho = \left(\frac{\lambda}{s+\mu+\xi}\right), \quad \eta = \prod_{j=1}^{i}\left(\frac{s+\xi\left(1-\sum_{r_j=1}^{k_j} C_{r_j}\right)}{s+\mu+\xi}\right)^{l_{(i-j)}-l_{(i-(j-1))}}, \quad [k] \rightarrow \text{An integral function.}$$

$$\prod_{j}^{i} = 1 \text{ and } \sum_{j}^{i} = 0 \text{ for } i<j,\ k_0 = 0,$$

$$A_m = \frac{N-n-(i-m)l_0 + \sum_{a=1}^{i-m-1} l_a - k_m l_{(i-m)} - \sum_{b=1}^{m-1}\left(k_{(m-b)} - k_{(m-(b-1))}\right) l_{(i+b-m)}}{l_0 - l_{(i-m)}}$$

$$L_i = \sum_{j=1}^{i}\left(l_{(i-j)} - l_{(i-(j-1))}\right)k_j,\quad l_j = \begin{cases}0 & \text{if } j=i \\ l_j & \text{if } 1\le j<i\end{cases},\quad D_i = \prod_{j=1}^{i}\binom{N-n-L_i-l_{(i-(j-1))}}{l_{(i-j)}-l_{(i-(j-1))}}$$

$$H = (s+\lambda)(s+\beta)(s+\lambda+\mu+\xi) - \lambda\mu(s+\beta) - \lambda\beta\xi \quad \text{and} \quad E = \mu(s+\beta)+\beta\xi$$

Using normalization condition, we have

$$P_N^*(s) = \frac{H_1}{s\rho^{-N}\left[\mu(s+\lambda)(s+\beta)E_1\gamma_2 + \sum_{n=2}^{N}\left(\xi E_2\sum_{r=n}^{N} C_r + (s+\lambda)(s+\beta)H\right)\gamma_n + \xi(s+\lambda)(s+\beta)E_3\sum_{r=1}^{N-1} C_r\gamma_{(1+r)}\right]} \tag{15}$$

Where

$$H_1 = (s+\lambda)(s+\beta)H - s\left((s+\beta)H + \lambda(s+\beta)\left(E+(s+\lambda)(\xi+(s+\beta))\right)\right),$$
$$E_1 = \left(E+(s+\lambda)\xi+(s+\lambda)(s+\beta)\right),\ E_2 = \lambda\beta E + H\beta + \lambda\beta\xi(s+\lambda) + H(s+\lambda) + \lambda\beta(s+\lambda)(s+\beta),$$
$$E_3 = E + \xi(s+\lambda) + (s+\lambda)(s+\beta)$$

Using (15) in (11) to (14) we have

$$P_n^*(s) = \frac{H_1\gamma_n}{s\left[\mu(s+\lambda)(s+\beta)E_1\gamma_2 + \sum_{n=2}^{N}\left(\xi E_2\sum_{r=n}^{N} C_r + (s+\lambda)(s+\beta)H\right)\gamma_n + \xi(s+\lambda)(s+\beta)E_3\sum_{r=1}^{N-1} C_r\gamma_{(1+r)}\right]} \tag{16}$$

$$P_1^*(s) = \frac{\lambda(s+\beta)}{H} + \frac{H_1\left[\mu(s+\lambda)(s+\beta)\gamma_2 + \lambda\beta\xi\sum_{n=2}^{N}\sum_{r=n}^{N} C_r\gamma_n + \xi(s+\lambda)(s+\beta)\sum_{r=1}^{N-1} C_r\gamma_{(1+r)}\right]}{sH\left[\mu(s+\lambda)(s+\beta)E_1\gamma_2 + \sum_{n=2}^{N}\left(\xi E_2\sum_{r=n}^{N} C_r + (s+\lambda)(s+\beta)H\right)\gamma_n + \xi(s+\lambda)(s+\beta)E_3\sum_{r=1}^{N-1} C_r\gamma_{(1+r)}\right]} \tag{17}$$

$$Q_{00}^{*}(s) = \frac{\lambda\xi}{H} + \frac{H_1\left[\mu\xi(s+\lambda)(s+\beta)\gamma_2 + (\lambda\beta\xi+H)\xi\sum_{n=2}^{N}\sum_{r=n}^{N}C_r\gamma_n + \xi^2(s+\lambda)(s+\beta)\sum_{r=1}^{N-1}C_r\gamma_{(1+r)}\right]}{s(s+\beta)H\left[\mu(s+\lambda)(s+\beta)E_1\gamma_2 + \sum_{n=2}^{N}\left(\xi E_2\sum_{r=n}^{N}C_r + (s+\lambda)(s+\beta)H\right)\gamma_n + \xi(s+\lambda)(s+\beta)E_3\sum_{r=1}^{N-1}C_r\gamma_{(1+r)}\right]} \tag{18}$$

$$P_{00}^{*}(s) = \frac{(\lambda E+H)}{(s+\lambda)H} + \frac{H_1\left[\mu E(s+\lambda)(s+\beta)\gamma_2 + (\lambda E+H)\xi\beta\sum_{n=2}^{N}\sum_{r=n}^{N}C_r\gamma_n + \xi E(s+\lambda)(s+\beta)\sum_{r=1}^{N-1}C_r\gamma_{(1+r)}\right]}{s(s+\lambda)(s+\beta)H\left[\mu(s+\lambda)(s+\beta)E_1\gamma_2 + \sum_{n=2}^{N}\left(\xi E_2\sum_{r=n}^{N}C_r + (s+\lambda)(s+\beta)H\right)\gamma_n + \xi(s+\lambda)(s+\beta)E_3\sum_{r=1}^{N-1}C_r\gamma_{(1+r)}\right]} \tag{19}$$

After taking Laplace inverse of (16) to (19), we can find all the probabilities.

4. Simulation Results

We obtain numerically the various measures of performance i.e. average delay in queue, average no. in queue; sever utilization time and average restoration time of this model by using simulation technique. The simulation analysis of the queueing model under investigation is carried out by using a computer program written in C language. The simulation results have been shown in tables 1-6. In tables 1-3, the simulation results are obtained by assuming that the catastrophic intensity follows the uniform distribution while in tables 4-6, the simulation results are obtained by assuming that the catastrophic intensity follows the modified binomial distribution.

Table 1.

Mean inter arrival time	Average Delay in queue	Average No. in queue	Server Utilization	Average Restoration Time
1	21.572	1.5482	0.3338	0.9546
2	16.3314	1.3798	0.3304	5.1508
3	11.1366	2.1634	0.6684	2.0092
4	9.9548	0.6096	0.1507	5.9084
5	10.2908	1.3336	0.3877	7.1034
6	6.1038	0.3666	0.1500	4.4040
7	7.6528	0.8144	0.4402	6.1506
8	7.2618	0.6722	0.4576	0.0136
9	9.1406	0.2672	0.1300	20.6696
10	2.6568	0.0286	0.0882	4.4738

In Table- 1we obtained the various measures of an M/M/1/N queueing model with uniformly distributed catastrophic intensity subjected to restoration for the effect of change in means inter arrival time $\left(1/\lambda\right)$ for fixed mean service time = 5 minutes, mean inter catastrophe time= 100 minutes, simulation length= 480 and N=5.

Table 2.

Mean Service Time	Average Delay in queue	Average No. in queue	Server Utilization	Average Restoration Time
1	0.195	0.020	0.122	5.906
2	1.383	0.222	0.251	6.499
3	4.969	0.130	0.158	4.625
4	3.361	0.272	0.271	6.962
5	10.2908	1.3336	0.3877	7.1034
6	7.2978	0.5878	0.2800	2.0933
7	12.5606	1.5232	0.5812	1.1806
8	19.3102	1.8182	0.5174	3.9818
9	17.3234	1.1662	0.3490	5.3244
10	16.1036	0.6386	0.2054	3.0650

In Table- 2 we obtained the various measures of an M/M/1/N queueing model with uniformly distributed catastrophic intensity subjected to restoration for the effect of change in means inter service time $\left(1/\mu\right)$ for fixed mean inter arrival time = 5 minutes, mean inter catastrophe time= 100 minutes, simulation length= 480 and N=5.

Table 3.

Mean inter catastrophic time	Average Delay in queue	Average No. in queue	Server Utilization	Average Restoration Time
101	12.5908	1.5316	0.5774	1.8162
102	13.1660	1.2016	0.4278	2.1920
103	13.4366	1.5608	0.5634	2.2536
104	13.1624	1.0170	0.3840	7.7728
105	18.5368	2.6252	0.8274	1.3162
106	17.8952	2.4418	0.7798	1.4400
107	17.8952	2.4464	0.7788	1.2068
108	17.3086	2.3282	0.7474	0.7280
109	17.2610	1.7912	0.5590	2.5438
110	15.9628	1.6606	0.5354	0.6216

In Table- 3 we obtained the various measures of an M/M/1/N queueing model with uniformly distributed catastrophic intensity subjected to restoration for the effect of change in means inter catastrophe time $\left(\frac{1}{\xi}\right)$ for mean inter arrival time = 2 minutes, mean service time= 5 minutes, simulation length= 480 and N=5.

Table 4.

Mean inter arrival time	Average Delay in queue	Average No. in queue	Server Utilization	Average Restoration Time
1	21.8921	1.1972	0.2597	1.0467
2	16.3507	1.4027	0.3417	5.8731
3	11.3900	2.4816	0.7949	1.4095
4	10.3448	0.5520	0.1478	6.7884
5	10.5879	1.3843	0.3972	6.8965
6	6.5949	0.3525	0.1396	4.8243
7	7.3515	0.8106	0.4535	5.2916
8	7.3861	0.8616	0.5697	1.1682
9	9.1290	0.2267	0.1064	21.6721
10	2.6418	0.0285	0.0868	4.4032

In Table- 4 we obtained the various measures of an M/M/1/N queueing model with modified binomially distributed catastrophic intensity subjected to restoration for the effect of change in means inter arrival time $\left(\frac{1}{\lambda}\right)$ for fixed mean service time = 5 minutes, mean inter catastrophe time= 100 minutes, simulation length= 480 and N=5.

Table 5.

Mean Service time	Average Delay in queue	Average No. in queue	Server Utilization	Average Restoration Time
1	0.1948	0.0200	0.1218	5.9058
2	1.3827	0.2222	0.2514	6.4989
3	4.9687	0.1300	0.1579	4.6248
4	3.3609	0.2723	0.2713	6.9623
5	10.5879	1.3843	0.3972	6.8965
6	7.6033	0.6422	0.2839	3.1263
7	13.2269	1.9131	0.7351	0.2178
8	20.6556	2.2770	0.6448	3.1682
9	18.6970	1.4763	0.4277	4.3962
10	16.0795	0.7553	0.2385	3.6333

In Table- 5 we obtained the various measures of an M/M/1/N queueing model with modified binomially distributed catastrophic intensity subjected to restoration for the effect of change in means inter service time $\left(\frac{1}{\mu}\right)$ for fixed mean inter arrival time = 5 minutes, mean inter catastrophe time= 100 minutes, simulation length= 480 and N=5.

In Table-6 we obtained the various measures of an M/M/1/N queueing model with modified binomially distributed catastrophic intensity subjected to restoration for the effect of change in means inter catastrophe time $\left(\frac{1}{\xi}\right)$ for mean inter arrival time = 2 minutes, mean service time= 5 minutes, simulation length= 480 and N=5.

Table 6.

Mean inter catastrophic time	Average Delay in queue	Average No. in queue	Server Utilization	Average Restoration Time
101	13.6542	2.0175	0.7404	0.4412
102	14.3749	1.4505	0.4979	1.0442
103	14.0400	2.0341	0.7261	0.6586
104	13.8649	1.5969	0.5825	5.2152
105	19.1817	2.8505	0.9082	0.2126
106	19.0782	2.8291	0.9005	0.2326
107	19.0782	2.8381	0.9004	0.1949
108	18.9834	2.8271	0.8953	0.1176
109	16.6162	1.6146	0.5432	3.0329
110	15.2659	1.4961	0.5074	0.5550

5. Steady State Solution

Using the property $\lim_{s\to 0} s P_n^*(s) = P_n$, We have from (16) to (19)

$$P_n = \frac{\beta\gamma_n}{\left\{\beta \sum_{n=0}^{N} \gamma_n + \xi \sum_{n=1}^{N} \sum_{r=n}^{N} C_r \gamma_n\right\}} \quad (20)$$

$n = 1, 2, \ldots, N$

$$Q_{00} = \frac{\xi \sum_{n=1}^{N} \sum_{r=n}^{N} C_r \gamma_n}{\left\{\beta \sum_{n=0}^{N} \gamma_n + \xi \sum_{n=1}^{N} \sum_{r=n}^{N} C_r \gamma_n\right\}} \quad (21)$$

$$P_{00} = \frac{\beta\gamma_0}{\left\{\beta \sum_{n=0}^{N} \gamma_n + \xi \sum_{n=1}^{N} \sum_{r=n}^{N} C_r \gamma_n\right\}} \quad (22)$$

6. Measures of Effectiveness

The steady state probability distribution for the system size allows us to calculate what are commonly called measures of effectiveness. Two, of immediate interest are the expected number of customers in the system and the expected number of customers in the queue. To derive the foregoing measures, let L_s represents the expected number in the system and L_q represents the expected number in the queue. Thus we have

$$L_s = \frac{\sum_{n=1}^{N} n\beta\gamma_n}{\left\{\beta\sum_{n=0}^{N}\gamma_n + \xi\sum_{n=1}^{N}\sum_{r=n}^{N} C_r\gamma_n\right\}}$$

and

$$L_q = \frac{\sum_{n=1}^{N} (n-1)\beta\gamma_n}{\left\{\beta\sum_{n=0}^{N}\gamma_n + \xi\sum_{n=1}^{N}\sum_{r=n}^{N} C_r\gamma_n\right\}}$$

7. Important Particular Cases of the Model

(I) In case the catastrophic intensity follows the uniform distribution then we get:

$$P_n = \frac{\beta\gamma 1_n}{\left\{\beta\sum_{n=0}^{N}\gamma 1_n + \xi\sum_{n=1}^{N}\sum_{r=n}^{N} C_r\gamma 1_n\right\}}$$

$$Q_{00} = \frac{\xi\sum_{n=1}^{N}\sum_{r=n}^{N} C_r\gamma 1_n}{\left\{\beta\sum_{n=0}^{N}\gamma 1_n + \xi\sum_{n=1}^{N}\sum_{r=n}^{N} C_r\gamma 1_n\right\}}$$

$$P_{00} = \frac{\beta\gamma 1_0}{\left\{\beta\sum_{n=0}^{N}\gamma 1_n + \xi\sum_{n=1}^{N}\sum_{r=n}^{N} C_r\gamma 1_n\right\}}$$

$$L_s = \frac{\sum_{n=1}^{N} n\beta\gamma 1_n}{\left\{\beta\sum_{n=0}^{N}\gamma 1_n + \xi\sum_{n=1}^{N}\sum_{r=n}^{N} C_r\gamma 1_n\right\}}$$

$$L_q = \frac{\sum_{n=1}^{N} (n-1)\beta\gamma 1_n}{\left\{\beta\sum_{n=0}^{N}\gamma 1_n + \xi\sum_{n=1}^{N}\sum_{r=n}^{N} C_r\gamma 1_n\right\}}$$

Where

$$\gamma 1_n = \rho^n + \sum_{i=1}^{\left[\frac{-3+\sqrt{9+8(N-n)}}{2}\right]} \sum_{l_0=i}^{\left[\frac{2(N-n)-(i(i-1))}{4}\right]} \prod_{j=1}^{i-1}\left(\sum_{l_j=(i-j)}^{l_{j-1}-1}\right) \prod_{m=0}^{i-1}\left(\sum_{k_{(m+1)}=k_m+1}^{[A_m]}\right) \rho^{L_i+n} D_i \prod_{j=1}^{i}\left(\frac{\xi\left(\frac{N-k_j}{N}\right)}{\mu+\xi}\right)^{l_{(i-j)}-l_{(i-(j-1))}}$$

(II) In case the catastrophic intensity follows the modified binomially distribution then we get:

$$P_n = \frac{\beta\gamma 2_n}{\left\{\beta\sum_{n=0}^{N}\gamma 2_n + \xi\sum_{n=1}^{N}\sum_{r=n}^{N} C_r\gamma 2_n\right\}}$$

$$Q_{00} = \frac{\xi\sum_{n=1}^{N}\sum_{r=n}^{N} C_r\gamma 2_n}{\left\{\beta\sum_{n=0}^{N}\gamma 2_n + \xi\sum_{n=1}^{N}\sum_{r=n}^{N} C_r\gamma 2_n\right\}}$$

$$P_{00} = \frac{\beta\gamma 2_n}{\left\{\beta\sum_{n=0}^{N}\gamma 2_n + \xi\sum_{n=1}^{N}\sum_{r=n}^{N} C_r\gamma 2_n\right\}}$$

$$L_s = \frac{\sum_{n=1}^{N} n\beta\gamma 2_n}{\left\{\beta\sum_{n=0}^{N}\gamma 2_n + \xi\sum_{n=1}^{N}\sum_{r=n}^{N} C_r\,\gamma 2_n\right\}}$$

$$L_q = \frac{\sum_{n=1}^{N} (n-1)\beta\gamma 2_n}{\left\{\beta\sum_{n=0}^{N}\gamma 2_n + \xi\sum_{n=1}^{N}\sum_{r=n}^{N} C_r\,\gamma 2_n\right\}}$$

Where

$$\gamma 2_n = \rho^n + \sum_{i=1}^{\left[\frac{-3+\sqrt{9+8(N-n)}}{2}\right]}\ \sum_{l_0=i}^{\left[\frac{2(N-n)-(i(i-1))}{4}\right]}\ \prod_{j=1}^{i-1}\left(\sum_{l_j=(i-j)}^{l_{j-1}-1}\right)\prod_{m=0}^{i-1}\left(\sum_{k_{(m+1)}=k_m+1}^{[A_m]}\right)\rho^{L_i+n} D_i \prod_{j=1}^{i}\left(\frac{\xi\left(1-\sum_{r_j=1}^{k_j}\frac{{}^N C_{r_j} a^{r_j} b^{N-r_j}}{1-b^N}\right)}{\mu+\xi}\right)^{l_{(i-j)}-l_{(i-(j-1))}}$$

8. Graphical Presentation

So far, we have obtained explicit expression for P_{00}, Q_{00}, P_n n=1, 2… N. The steady state probabilities of the system size, L_s, the mean number of customers in the system, L_q, the mean number of customers in the queue for both the cases in which the catastrophic intensity follows (I) the uniform distribution (II) the modified binomial distribution. Now here, we present some graphical presentation to highlight the effects of the catastrophe parameter ξ, the arrival rate λ and the service rate μ on P_{00}, Q_{00} and Ls for N = 8.

In fig. 1, we plot the behavior of the probability P_{00}, the probability of zero customers in the system without the occurrence of catastrophe, as a function of λ. P_{00} decreases as the arrival rate λ increases.

In fig. 2, we plot the behavior of the probability P_{00}, the probability of zero customers in the system without the occurrence of catastrophe, as a function of μ. P_{00} increases with the increase in service rate μ

In fig. 3, we plot the behavior of the probability P_{00}, the probability of zero customers in the system without the occurrence of catastrophe, as a function of ξ. P_{00} decreases as the catastrophe rate ξ increases.

In fig. 4, we illustrate the effect of the arrival rate λ, on Q_{00}, the probability of zero customers in the system with the occurrence of catastrophe. It has been observed that Q_{00} is an increasing function of λ

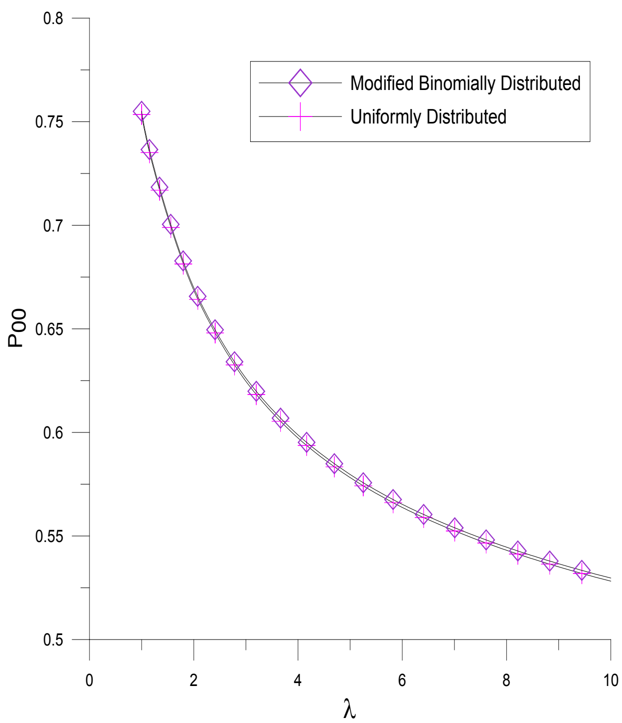

Figure 1. P_{00} as a function of λ for $\mu = 5, \xi = 100$

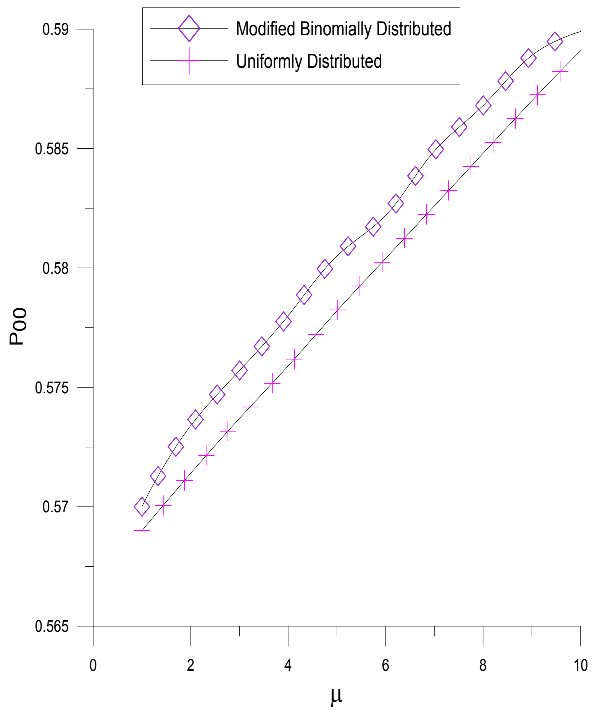

Figure 2. P_{00} as a function of μ for $\lambda = 5,\ \xi = 100$

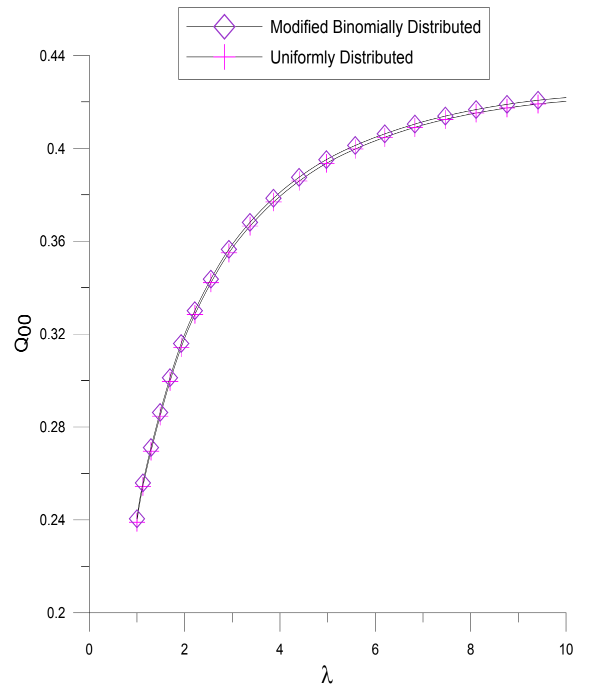

Figure 4. Q00 as a function of λ for $\mu = 5, \beta = 2, \xi = 100$

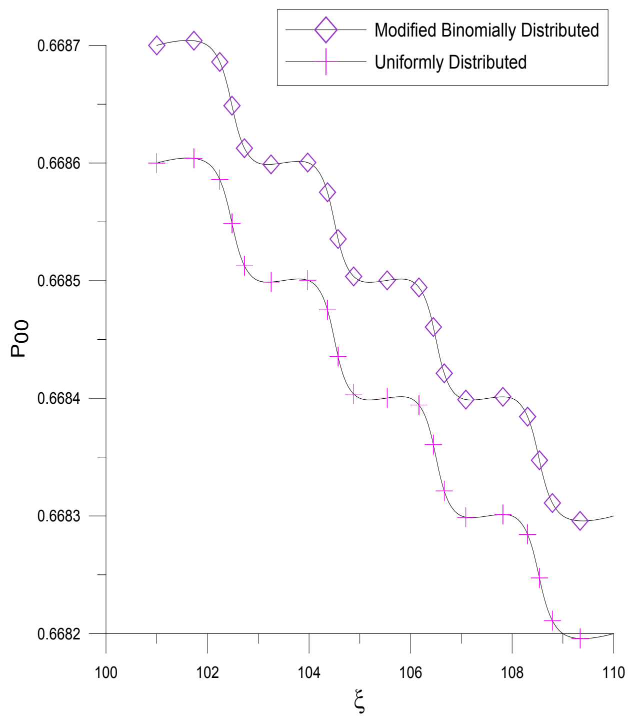

Figure 3. P_{00} as a function of ξ for $\mu = 5, \lambda = 2$

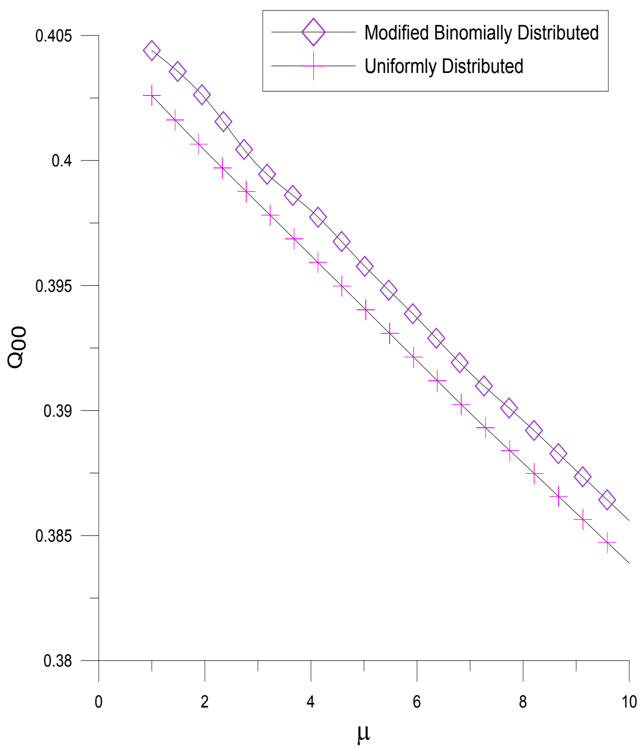

Figure 5. Q_{00} as a function of μ for $\lambda = 5, \beta = 2, \xi = 100$

In fig. 5 we illustrate the effect of service rate μ on Q_{00}, the probability of zero customers in the system with the occurrence of catastrophe. It has been observed that Q_{00} is a decreasing function of μ.

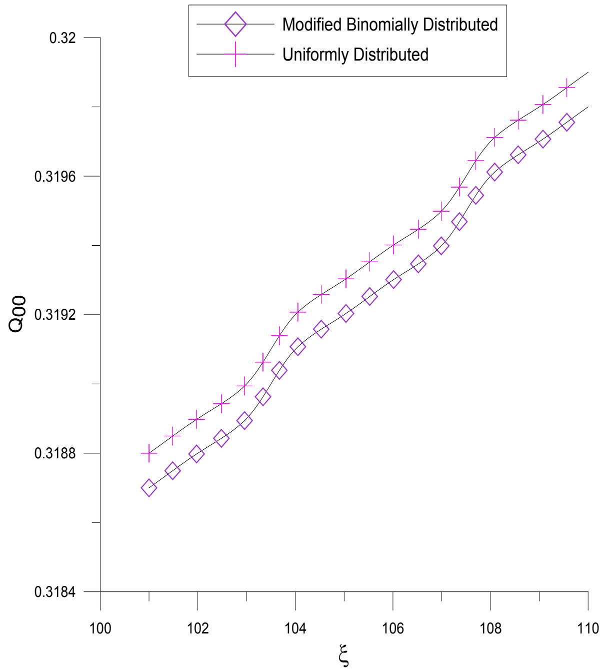

Figure 6. Q_{00} as a function of ξ for $\mu = 5, \lambda = 2, \beta = 2$

In fig. 6 we illustrate the effect of the catastrophe rate ξ on Q_{00}, the probability of zero customers in the system with the occurrence of catastrophe. It has been observed that Q_{00} is an increasing function of ξ.

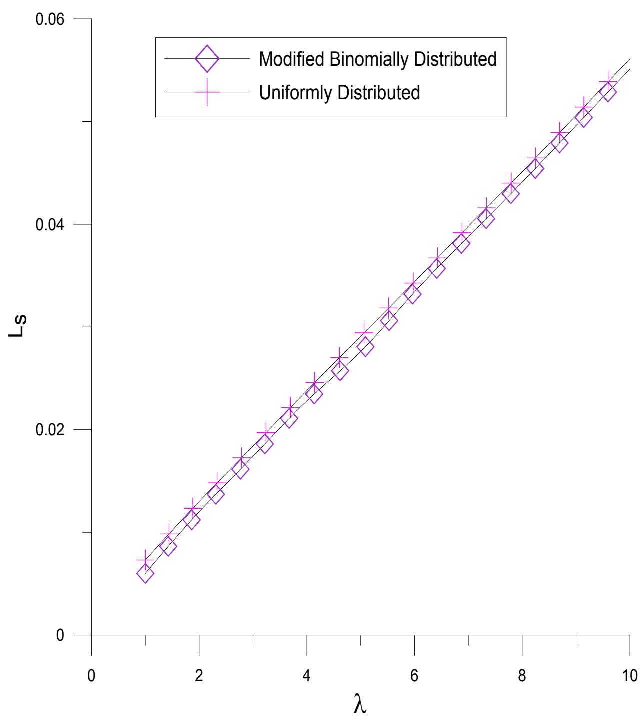

Figure 7. L_s as a function of λ for $\mu = 5, \xi = 100$

In fig. 7 we illustrate the effect of the arrival rate λ on L_s. It has been noticed that L_s is an increasing function of λ.

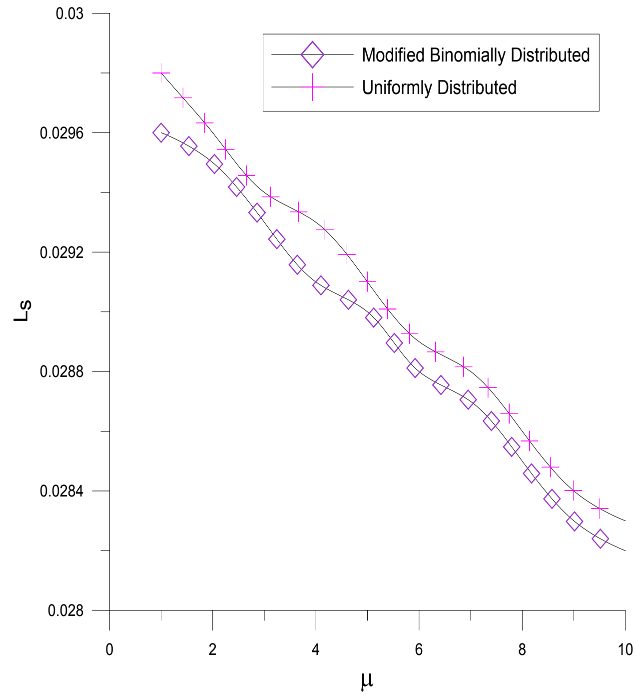

Figure 8. L_s as a function of μ of for $\lambda = 5, \xi = 100$

In fig. 8 we illustrate the effect of the service rate μ on L_s. It has been noticed that L_s is a decreasing function of μ.

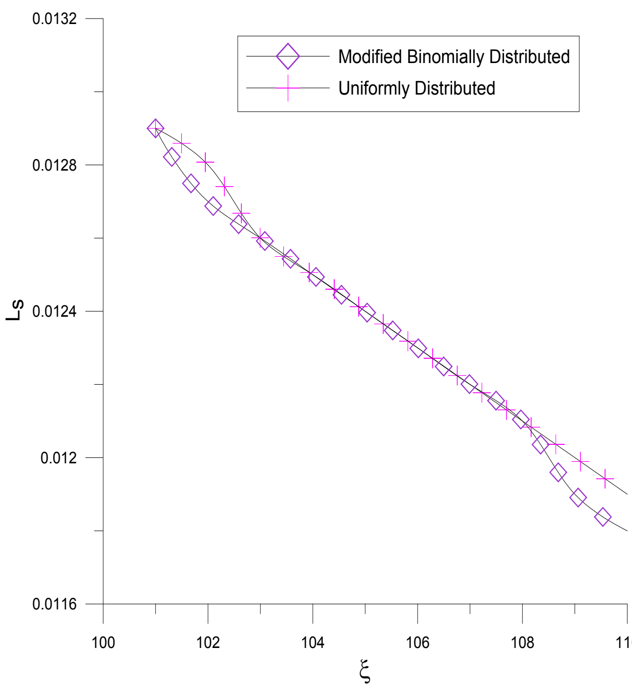

Figure 9. L_s as a function ξ for $\mu = 5, \lambda = 2$

In figs. 9 we illustrate the effect of the catastrophe rate ξ on L_s. It has been noticed that L_s is a decreasing function of ξ.

9. Conclusions

In the present paper, we have studied a varying catastrophic intensity-cum- restorative Markovian queueing model with finite capacity. The concept of varying catastrophic intensity has numerous applications in a wide variety of areas particularly in agriculture and biosciences etc. In agriculture, if a crop is infected with some disease then for the treatment of such type of disease we use some chemicals. The destruction of the number of bacteria present in the crops depends upon the intensity of the chemicals used, that is, the application of the chemical may destroy all or a part there of. The use of the chemicals is like the occurrence of catastrophe. Therefore the infected crops with use of chemical are modeled by the birth and death queue with varying catastrophic intensity. With the introduction of restoration time the practical utility of the model is considerably increased.

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