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A Survey-Weighted Model-Assisted Framework for Indirect Under-Five Mortality Estimation from Summary Birth Histories

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Abstract Classical indirect estimation relies on Summary Birth Histories (SBH) where civil registration or full birth histories are unavailable, but Brass-type procedures pool maternal-age-group data before applying model-life-table transformations. In non-homogeneous populations, this ordering can provide a mix of fertility composition and mortality risk. This paper develops An Improved Indirect Estimation Approach (AIIEA), a survey-weighted, model-assisted approach that consists of a quasibinomial conditional mortality surface (ψ), followed by birth-weighted design aggregation (A) and ratio-preserving benchmark calibration (C). The framework formalizes non-identifiability of exact period under-five mortality from SBH, mixture distortion, non-commutativity, compositional bounds, and calibration preservation. In the Nigeria DHS 2023-24 internal application, calibrated AIIEA estimates had a zonal mean absolute discrepancy of 2.94 deaths per 1,000 relative to direct full birth history estimates, compared with 11.64 for Brass-Trussell Manual X, a 74.7% reduction interpreted as within-sample subnational reconciliation rather than out-of-sample prediction. In the 24 comparable country applications within a 27-country DHS archive, national MAE was 9.71 per 1,000 for raw AIIEA and 19.57 for the classical comparator. AIIEA extends, rather than rejects, the Brass tradition by making conditioning, survey design and calibration explicit.

Keywords Under-five mortality, Summary birth history, Indirect estimation, Brass-Trussell, Model-assisted survey estimation, Quasibinomial regression, Calibration, Nigeria DHS, AIIEA

1. Introduction

Under-five mortality, denoted ${}_5q_0$, is the probability that a live-born child dies before exact age five. It is a key demographic and public health indicator as it serves as an indicator of child survival, maternal health, childhood disease, nutrition, household welfare, health system performance and social inequality. Where the civil registration and vital statistics systems are adequate and complete, ${}_5q_0$ can be estimated directly from registered births and deaths data. In many low- and middle-income settings, however, the estimation of mortality remains largely based on the household surveys and censuses. [1], [2]

There are two survey data structures that are prevalent in the estimation of child mortality. Full birth histories record child-level dates of birth, survival status and age or date at death. They permit direct life-table estimation, but are lengthy, and may be associated with recall displacement, age-at-death heaping, omission and high sampling variance when broken down into small domains. Summary birth histories collect only the number of children ever born and the number surviving or deceased for each woman. In censuses and compact surveys, they are shorter and more feasible, but sacrifice the timing information that allows for the identification of period mortality, without the use of auxiliary assumptions. The classical solution to the SBH problem is the Brass tradition and its several modifications (by Sullivan, Trussell, Manual X, Feeney, Zlotnik-Hill, QFIVE, MORTPAK and other related demographic tool). These techniques convert maternal-age-group proportions dead into probabilities of dying before selected childhood ages using multipliers derived from model fertility and mortality schedules. The limitation addressed here is that these procedures typically pool heterogeneous women prior to transformation. [3], [4], [5], [6], [7], [8], [9], [10], [11]

However, in such a context as Nigeria, where fertility, educational attainment, wealth, residence and regional mortality risks differ sharply, pooling before conditioning can confound mortality risk with population composition. [12], [13], [14]

This paper develops the methodological foundations of AIIEA. The three-point idea is to condition, aggregate,

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calibrate. The paper is as a contribution to mathematical demography and applied survey statistics. It formalizes the reason why exact period ${}_5q_0$ cannot be identified with SBH, defines the estimand that is compatible with SBH, derives the operator framework for AIIEA, and demonstrates its empirical behavior using Nigeria DHS 2023-24.

2. Classical Indirect Estimation and the Estimation Problem

Direct full birth history estimation uses interval survival probabilities to obtain under-five mortality. If q_x denotes the probability of dying in standard age interval *x* below exact age five, then [2]

$$ {}_5q_0 = 1 - \frac{l_5}{l_0} = 1 - \prod_{x=0}^{4}(1 - q_x) \tag{1}$$

This is an estimator not tied to a model life table, but small-domain direct estimates might be noisy. For a summary birth history, let B_j denote the number of children woman *j* has ever born, and, let S_j denote the number of children woman *j* has survived and $D_j = B_j - S_j$ the number of children woman *j* has lost. The reported proportion dead (RPD) for the women was [4], [11]

$$Y_j = \frac{D_j}{B_j}, \text{ for } B_j > 0 \tag{2}$$

The classical pooled proportion dead is

$$P_i = \frac{D_i}{B_i} = \frac{\sum_{j \in g_i} D_j}{\sum_{j \in g_i} B_j} \tag{3}$$

The original Brass method converts to a childhood mortality probability denoted as: [5]

$$q(x_i) = k_i P_i \tag{4}$$

k_i is a multiplier dependent on the assumptions in the fertility and mortality schedules. The multipliers are given in Trussell and Manual *X* in terms of parity ratios, typically of the form [7], [8]

$$k_i = a_i + b_i \times \left(\frac{AP_1}{AP_2}\right) + c_i \times \left(\frac{AP_2}{AP_3}\right) \tag{5}$$

The reference time equation is similar in form and specifies the approximate time period to which the indirect estimate applies. This matters because an SBH-derived estimate does not necessarily correspond to the survey year; it usually refers to an earlier retrospective exposure window. [6], [9], [10], [11]

3. Estimand Hierarchy and Age-Time Interpretation

AIIEA begins by separating three objects that are often blurred in applied mortality estimation: the ideal period target ${}_5q_0, d(t)$, the functional target θ_d that is compatible with the SBH, and the calibrated operational target $\theta_{d,cal}$. The time periods ${}_5q_0, d(t)$ requires deaths and exposures for each child age and time period. SBH only gives information on cumulative births and survivorship outcomes reported by women at the time of the survey. The information gap is not just a computational gap but a structural gap.

AIIEA will therefore interpret the raw estimate as an SBH compatible mortality functional. A convenient representation uses an age-time exposure kernel:

$$\theta_{gd} = \int_0^5 \int_0^{\mathcal{T}_{max}} F_d(a,t) K_g(a,t) dt\, da \tag{6}$$

Here $F_d(a,t)$ is the mortality surface by child age a and retrospective time *t* in domain *d*, while $K_g(a,t)$ is the exposure-density kernel induced by fertility timing, child age, survival reporting and maternal age group g. Younger maternal age groups generate more recent and concentrated kernels, while older age groups generate more diffuse retrospective kernels. This motivates the primary empirical age window of women aged 20-34.

4. AIIEA as an Operator Framework

AIIEA is represented as a three-stage composite operator:

$$T_{AIIEA} = C \cdot A \cdot \psi \tag{7}$$

The conditional surface ψ is applied first to respondent-level SBH microdata. The aggregation operator A is applied second to produce raw national and domain-level mortality functionals. The calibration operator *C* is applied third to align the national level with an accepted benchmark. This order reverses the classical Brass sequence, which first pools and then transforms.

4.1. Stage 1: Survey-Weighted Quasibinomial Conditional Surface

For woman *j* with $B_j > 0$, AIIEA models the conditional expected number of reported child deaths as

$$E(D_j | B_j, X_j) = B_j p_j, logit(p_j) = \tilde{X}_j^T \beta \tag{8}$$

The quasibinomial variance is

$$Var(D_j | B_j, X_j) = \phi B_j p_j (1 - p_j), \phi > 0 \tag{9}$$

The estimator is obtained by solving the survey-weighted pseudo-score equation [15], [16]

$$U(\beta) = \sum_h \sum_c \sum_{j \in s_{hc}} w_{hcj} \tilde{X}_{hcj} \left[D_{hcj} - B_{hcj} expit(\tilde{X}_{hcj}^T \beta)\right] = 0 \tag{10}$$

The covariate vector contains maternal age group, geographic domain, place of residence, education and wealth (subject to the variables available in the DHS individual recode file). The model is not to be considered as a true biological mortality law. A working mean surface used to remove observed compositional heterogeneity before aggregation.

4.2. Stage 2: Birth-Weighted Design Aggregation

After estimating β, individual fitted risks are obtained as $\hat{p}_j = expit(\tilde{X}_j^T \hat{\beta})$. For domain *d*, the raw AIIEA functional is the birth-weighted Hájek design ratio [17], [18], [19]

$$\hat{\theta}_d^{raw} = \frac{\sum_{j\in s_d} w_j B_j \psi(X_j)}{\sum_{j\in s_d} w_j B_j} = \frac{\sum_{j\in s_d} w_j B_j \hat{p}_j}{\sum_{j\in s_d} w_j B_j} \tag{11}$$

Under five mortality is a child-level probability, therefore birth weighting is necessary as the SBH observations are reported by women. A woman with six births has 6 times more child exposure than a woman with one birth. A simple woman-level average would be used to approximate the average woman-level reported-death propensity, rather than the child-level SBH mortality functional.

4.3. Stage 3: Benchmark Calibration

Let q_N^{ref} be a defensible national benchmark, such as a direct FBH estimate from the same survey or an official national series. The calibration scalar is [19]

$$\hat{\gamma} = \frac{q_N^{ref}}{\hat{\theta}_{N,raw}} \tag{12}$$

The calibrated domain estimate is

$$\hat{\theta}_{d,cal} = \hat{\gamma} \cdot \hat{\theta}_{d,raw} \tag{13}$$

The total of the national responses is calibrated to match this total, but all raw AIIEA domain ratios are maintained. Thus, calibration is not used to create subnational rankings; it rescales an already estimated subnational structure.

5. Formal Properties

5.1. Non-Identifiability of Exact Period Mortality from SBH

Without auxiliary assumptions, exact period ${}_5q_0, d(t)$ is not uniquely determined by the child dates of birth and the dates of child–human exposure, if these are unobserved. The same values for B_j and D_j can be produced by different combinations of fertility and mortality history. Hence AIIEA does not claim to calculate the exact period mortality from SBH. It is an estimation of an SBH-compatible function followed by calibration to a benchmark.

5.2. Mixture-Distortion Decomposition

Suppose domains d and e contain R strata with birth shares λ_{rd} and λ_{re} and stratum-specific mortality summaries π_{rd} and π_{re}. The pooled difference decomposes as

$$\pi_{d,pool} - \pi_{e,pool} = \sum_r \lambda_{rd}(\pi_{rd} - \pi_{re}) + \sum_r (\lambda_{rd} - \lambda_{re})\pi_{re} \tag{14}$$

The first term is a within-stratum mortality contrast. The second term is a compositional birth-share contrast. Classical pooled estimation cannot distinguish these two mechanisms after aggregation. AIIEA addresses the problem by modeling the conditional surface before aggregating.

5.3. Non-Commutativity of Pooling and Transformation

Let $K_B(P; z)$ be an indirect transformation depending on a pooled proportion P and demographic parameters z, such as parity ratios or model-life-table coefficients. If K_B is nonlinear in P, or if z changes with the composition used to form the pooled statistic, then pooling and transformation generally do not commute:

$$K_B\left(\sum_r \lambda_r P_r; z_{pool}\right) \neq \sum_r \lambda_r K_B(P_r; z_r) \tag{15}$$

This statement is deliberately conditional. Equality can hold in special cases, such as a fixed linear transformation with common parameters. The substantive point is not that all pooling is mathematically invalid; rather, in heterogeneous populations with composition-dependent transformation parameters, pooling before transformation can alter the estimand.

5.4. Compositional Distortion Bound

The compositional term in Equation (14) satisfies the bound

$$\left|\sum_r (\lambda_{rd} - \lambda_{re})\pi_{re}\right| \leq \|\lambda_d - \lambda_e\|_1 \|\pi_e\|_\infty \tag{16}$$

The answer is following from the triangle inequality. The probabilities in π_e mean that $\|\pi_e\|_\infty$ will be at most times less than 1, but the same is not true for the distance between the probability vectors $|\|\lambda_d - \lambda_e\|_1|$ which can be as large as 2. The bound connects demographic intuition of compositional heterogeneity with a formal distance between the vectors of birth-shares of the subgroups.

5.5. Calibration Preservation

The calibration operator preserves national alignment, ratios and rank ordering. Because National alignment follows because $\hat{\gamma} \cdot \hat{\theta}_{N,raw} = q_N^{ref}$. Ratio preservation follows because

$$\frac{\hat{\theta}_{d_2,cal}}{\hat{\theta}_{d_1,cal}} = \frac{\hat{\theta}_{d_2,raw}}{\hat{\theta}_{d_1,raw}} \tag{17}$$

If $\hat{\gamma}$ is positive, rank preservation follows immediately. Also under common multiplicative discrepancy, calibration scale-invariance means that if raw estimates are on a common scale which is a multiple of the domain functional, then on the calibrated scale domain estimates get the benchmark-scaled estimates. Calibration does not address domain-specific bias, it does only address a common national scale discrepancy.

6. Estimation of Variance and Asymptotic Justifications

Under complex sampling, the survey-weighted quasibinomial estimator is considered to be a pseudo-likelihood or quasi-score estimator. Under standard regularity conditions for survey M-estimation, [18], [19]

$$\sqrt{n(\hat{\beta} - \beta_0)} \longrightarrow N(0, V_\beta) \tag{18}$$

The primary sampling unit (PSU), strata and sampling weights are taken into consideration in the estimation of the covariance matrix in a cluster-robust sandwich form. The delta-method approximation for the raw domain estimator $\left(\hat{\theta}_{d,raw} = h_d(\hat{\beta})\right)$ is

$$Var[\hat{\theta}_{d,raw}] \approx [\nabla h_d(\hat{\beta})]^T V_{\hat{\beta}} [\nabla h_d(\hat{\beta})] \quad (19)$$

For calibrated estimates, the relevant function is

$$g(\hat{\theta}_{d,raw}, \hat{\theta}_{N,raw}) = q_N^{ref} \cdot \left(\frac{\hat{\theta}_{d,raw}}{\hat{\theta}_{N,raw}}\right) \quad (20)$$

Hence, the variance of the calibrated estimator is a function of the domain numerator as well as the national denominator along with the covariance between the two. This is significant because calibration is a ratio operation and not just multiplication by a fixed value when sampling variability is taken into account.

When the benchmark q_N^{ref} is itself estimated rather than treated as fixed, its uncertainty should also enter the final variance. A first-order extension may be written as $Var(\hat{\theta}_{d,cal})$ approximately equal to the gradient contribution from $\hat{\theta}_{d,raw}$ *and* $\hat{\theta}_{N,raw}$ plus the contribution from $Var(\hat{q}_N^{ref})$, including covariance terms where the benchmark and AIIEA raw estimates are estimated from the same survey. In the present internal Nigeria comparison, the benchmark is used primarily to define the calibration scale; the point-estimate validation is therefore interpreted at the zonal level rather than as evidence of zero national error.

7. Empirical Application: Nigeria DHS 2023-24

The official Nigeria DHS 2023-24 Individual Recode (IR) microdata is used for the empirical illustration. The primary analytical sample consists of 13,739 women age 20-34 who had at least one child ever born and had no missing model or survey design variables. The v201 variable is used to measure the number of children born, $v206 + v207$ to report on deaths of children, $\frac{v005}{1,000,000}$ to measure the sampling weight, v001 to measure the primary sampling unit, and $\frac{v022}{v023}$ as defined in the release to report on the stratification variables supplied. The design-weighted sample comprises of 43,575.1 children ever born and 4,646.8 reported child deaths; the SBH proportion-dead statistic (unmodelled by design) has DEFF = 74.60. Maternal age group, geopolitical zone, urban-rural residence, maternal education and household wealth are included as conditional factors. The internal national calibration anchor for the within-survey direct FBH estimate is a FBH estimate corresponding to the dated birth histories of 100.6 deaths per 1,000 (Table 1). The classical indirect comparator is the Brass-Trussell Manual X. All reported estimates must be generated from the archived survey design object and analysis script and derived output tables. [20], [21]

The national calibration scalar is

$$\hat{\gamma} = \frac{100.6}{106.64} = 0.94313 \quad (21)$$

This scalar is applied when multiplying the raw domain estimates. Equality between calibrated national AIIEA and the national FBH anchor is imposed by construction and is not treated as validation. The Nigeria comparison, on the other hand, is based on within-sample subnational reconciliation, i.e., the agreement between the domain structure estimated after conditioning but before calibration and the Brass-Trussell domain structure. Because geopolitical zone is included in Stage 1 and the same six zones are used for evaluation, the resulting discrepancies are descriptive internal-fit measures rather than out-of-sample prediction errors.

Table 1. Nigeria DHS 2023-24 analytical sample summary

Women	Weighted CEB	Weighted deaths	Observed proportion dead	Observed deaths per 1,000
13,739	43,575.1	4,646.8	0.107	106.6

Table 2. National comparison of direct FBH, Brass-Trussell and AIIEA

Estimator	U5MR per 1,000	Interpretation
Direct FBH	100.6	Within-survey benchmark
Brass-Trussell Manual *X*	114.6	Classical indirect estimate using preferred maternal age group
AIIEA raw	106.6	Uncalibrated SBH-compatible estimate
AIIEA calibrated	100.6	Nationally aligned by construction in internal validation

Table 3. Three-method geopolitical-zone comparison, Nigeria DHS 2023-24

Zone	Direct FBH	Brass-Trussell	AIIEA calibrated	|BT-FBH|	|AIIEA-FBH|
North West	129.0	152.5	126.5	23.5	2.5
North East	122.0	141.9	124.3	19.9	2.3
North Central	69.6	79.1	67.2	9.4	2.5
South East	66.1	71.4	60.6	5.3	5.4
South South	52.6	59.1	52.5	6.5	0.0
South West	50.1	44.9	45.2	5.2	4.9

Table 4. Validation metrics for Nigeria DHS 2023-24 geopolitical zones

Metric	Brass-Trussell	AIIEA calibrated	Interpretation
MAE	11.64	2.94	AIIEA reduces absolute zonal error by 74.7%
RMSE	13.75	3.45	AIIEA reduces squared-error penalty
Spearman rank correlation	1.000	1.000	Both preserve broad zone ordering in this application

Table 5. Maternal age-window sensitivity

Age window	Women	Raw national U5MR	Calibration scalar	Zonal MAE
20-29	8,646	105.0	0.95803	4.87
20-34	13,739	106.6	0.94313	2.94
15-34	14,543	107.2	0.93829	3.20
20-39	18,497	106.5	0.94405	3.50

Table 6. Covariate omission sensitivity

Specification	Raw national U5MR	Calibration scalar	Zonal MAE	Change in MAE
Full model	106.6	0.94313	2.94	0.00
Omit region	106.6	0.94313	17.80	+14.86
Omit education	106.6	0.94313	2.94	0.00
Omit wealth	106.6	0.94313	2.94	0.00
Omit residence	106.6	0.94313	2.94	0.00

Table 7. Multi-country national error summary from completed AIIEA applications

Country archive / comparable AIIEA outputs	MAE Brass-Trussell	MAE AIIEA raw	Median AE Brass-Trussell	Median AE AIIEA raw
27 archived / 24 comparable	19.57	9.71	15.80	6.94

8. Sensitivity, Manual Computability and Portability

The AIIEA was evaluated using age window sensitivity and covariate block omission. The preferred 20-34 window produced the lowest within-sample zonal MAE among the tested alternatives. This pattern is consistent with the timing-kernel argument that very young maternal ages provide limited exposure while older ages introduce more temporally diffuse exposure; it is not, by itself, an external test of predictive performance.

The 20-34 restriction is therefore a primary analytical window rather than a claim that women aged 15-19 or 35-49 are irrelevant. In full operational use, analysts may report supplementary estimates for wider maternal-age windows or develop age-group-specific calibration strategies. The present paper reports the restriction transparently because the aim is to evaluate the most temporally interpretable SBH functional rather than to force all reproductive ages into a single period-like estimate.

The omission results show that the Nigeria zonal agreement is driven primarily by explicit geographic conditioning: removing region raises MAE from 2.94 to 17.80, whereas removing education, wealth or residence does not change the zone-level MAE at the reported precision. This does not imply that the socioeconomic covariates are demographically irrelevant. Rather, their marginal contribution is not identified by an evaluation criterion defined on the same six zone categories used in the fitted model. Education, wealth and residence remain theoretically appropriate because they may improve individual-level risk estimation, finer-domain applications, subgroup analyses, and applications in countries where socioeconomic composition is less spatially aligned. The result therefore supports the mixture-distortion argument while also limiting the 74.7% figure to internal subnational reconciliation.

The method can be manually audited after model prediction. In Stage 2, the fitted probabilities are multiplied by birth counts and survey weights and then a ratio is taken. In Stage 3, the national calibration scalar is applied by ordinary multiplication. R software is required mainly for Stage 1: survey-weighted quasibinomial estimation, complex design objects and sandwich linearisation. To facilitate adoption, the analysis code, derived output tables and a simplified manual aggregation/calibration template can be supplied as supplementary materials, subject to DHS data-use restrictions.

The multi-country evidence is treated as a portability assessment and not as external predictive validation. Direct FBH and Brass-Trussell results are available for 27 DHS countries; raw AIIEA outputs are available for 24 comparable country applications. For these comparable applications, the national MAE is 19.57 per 1,000 for Brass-Trussell and 9.71 for raw AIIEA, while the median absolute errors are 15.80 and 6.94, respectively. The country-level values are provided in Appendix Table A1 and transparently mark Burundi, Cote d'Ivoire and Namibia as pending AIIEA extension cases. These three cases are excluded from the AIIEA aggregate error calculation rather than imputed. Their pending status reflects the need for additional file harmonisation and quality-control checks before the same conditional-surface workflow is applied, not a substantive failure of the estimator. The aggregate statistics therefore demonstrate computational portability and agreement among completed settings, but not unbiased agreement throughout all archived countries.

9. Discussion

The results are consistent with the theoretical explanation that the order of operation is important. In classical approaches, the pooling is often done prior to demographic transformations, while AIIEA conditions first, aggregates second, and calibrates third. Conditional on the presence of a pooled birth-share structure, conditioning prior to aggregation ensures that the observed spatial structure is maintained, but should it be present in a heterogeneous population. The Nigeria results are an example of this mechanism and the 24 country applications completed are a wider portability check.

The Nigeria application shows close within-sample alignment with direct FBH at the zonal level while retaining the data economy of SBH. This agreement should not be interpreted as out-of-sample prediction because zone enters the fitted conditional surface. AIIEA also cannot recover exact child-level timing and remains dependent on reporting quality, model specification and calibration benchmarks. Its more specific contribution is a survey-weighted, model-assisted and auditable use of SBH microdata that retains observed heterogeneity until the aggregation stage.

This method is not intended to replace the direct FBH estimation, the Bayesian small area estimation or the modelling performed by IGME official. When there is a history of a child or sample sizes are adequate, direct FBH is useful. The smoothing of spatial and temporal variation is still possible with the Bayesian approach, as well as with the small-area approach. AIIEA is filling a different void: in situations where only summary histories are available, or subnational SBH estimates are required as an operational monitoring tool. [22], [23], [24], [25]

9.1. Relationship to the Brass Tradition

This proposed estimator is not meant to be a criticism of Brass, Sullivan, Trussell and Manual *X*. These techniques addressed a long-standing problem, that of estimating childhood mortality from a limited amount of fertility and survivorship data when event histories are not available. Their ongoing contribution comes from their demographic parsimony and transparency. When the analyst has both respondent-level SBH microdata and sampling weights, but not all child-level timing, AIIEA asks if the classical order of operations is still sufficient.

The classical estimator begins with a maternal-age group i, forms the pooled proportion dead P_i, and then applies a multiplier k_i. In a homogeneous population this is sensible. In a heterogeneous population, however, the pooled P_i is already a mixture of fertility composition and mortality experience. AIIEA therefore keeps the Brass insight that SBH can be transformed into childhood mortality information, but changes the timing of the transformation: it conditions on heterogeneity before aggregating.

9.2. Interpretation of the SBH Functional

One of the criticisms of indirect estimation is that the resulting estimate might be a period mortality rate when no actual period exposure exists in the SBH data. AIIEA's solution is to break down the ideal demographic target from the identifiable SBH functional. The raw AIIEA estimator is an SBH-compatible mortality functional summarizing the reported risk of child-death over the retrospective exposure kernel of the process of fertility timing and maternal age composition. Calibration then maps this raw functional onto a benchmark-aligned scale.

9.3. Quasibinomial Working-Mean Specification

The quasibinomial model is used since the response is number of child deaths out of children ever born for each woman. If the outcome of child deaths was a simple binomial conditional on the woman's set of covariates, child-death outcomes in a woman's birth history would be independent Bernoulli trials with variance $B_j p_j\left(1-p_j\right)$. That assumption is too narrow for demographic survey data. Children born to the same woman share maternal characteristics, household circumstances, community risk, period conditions and reporting mechanisms. The quasibinomial variance is an overdispersion scale φ, which does not need to be the binomial variance but allows the mean model to be logistic. [15], [16]

9.4. Survey Design as Part of the Estimator

AIIEA treats complex survey design as part of the estimator rather than as an optional post-estimation correction. DHS surveys use stratification, clustering and unequal selection probabilities. Failure to account for these features can lead to biased point estimates and uncertainty estimates, especially for subnational mortality outcomes where cluster composition is strongly related to geography and socioeconomic conditions. The survey weight w_j enters both the quasi-score equations and the aggregation operator. The primary sampling unit and stratum enter the sandwich covariance estimation. [18], [21]

9.5. Relationship to Bayesian Small-Area Models

AIIEA should be distinguished from Bayesian small-area mortality models. The goal of the Bayesian approaches is to borrow strength across the areas, to quantify the uncertainty hierarchically and to stabilize the sparse estimates of the direct ones. AIIEA addresses a different problem: how to extract more defensible indirect estimates from SBH microdata before aggregation. It does not require spatial priors, adjacency matrices or time-series smoothing, although it can complement Bayesian models by producing improved SBH-based inputs or diagnostics. [22]-[25]

10. Limitations

First, AIIEA is constrained by the reporting error inherent in SBH, such as the lack of reporting for children who have died and problems with recalling children's ages. Second, the quasibinomial specification is a working mean model rather than a full generative mortality process. Third, a national benchmark must be defensible in order for it to be used in calibration. Fourth, the Nigeria zonal comparison is internal: geopolitical zone enters Stage 1 and the same six zones define the validation criterion. The 74.7% reduction therefore measures within-sample reconciliation, not out-of-sample predictive accuracy. Fifth, direct FBH comparators are themselves subject to recall and sampling error. Sixth, the multi-country archive contains 27 countries but only 24 completed raw AIIEA outputs; the remaining three are transparently marked in Appendix Table A1. Finally, observed covariates cannot remove unobserved heterogeneity, and uncertainty in the benchmark should be propagated when the benchmark is estimated rather than fixed. [3], [4]

11. Conclusions

In this paper, AIIEA is proposed as a survey-weighted, model-assisted framework for indirect under-five mortality estimation from summary birth histories. The method extends the Brass tradition further by conditioning on observed heterogeneity prior to birth-weighted aggregation and applying transparent, ratio-preserving calibration only after the raw domain structure is estimated. The Nigeria DHS 2023-24 application exhibits substantially closer within-sample zonal reconciliation with direct FBH than Brass-Trussell Manual *X*, while the sensitivity analysis shows that this gain is driven mainly by explicit geographic conditioning. Also, across the 24 completed country applications in the 27-country archive, and when comparing the raw AIIEA to the classical comparator, the national absolute discrepancy is lower. These results provide an empirical basis for the proposition that AIIEA has potential as an extension of indirect estimation, and are illuminated by a mathematically explicit treatment of the method, but suggest that there is room for further research on out-of-sample validation, completion of the remaining country runs, and propagation of uncertainty of the benchmark.

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DISCLOSURE

Data Availability Statement

The DHS datasets used in the current study were accessed from the DHS Program repository (https://dhsprogram.com) by registering and approving for research purposes. The author can provide derived tables and code on reasonable request, with restriction on data use by DHS.

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Conflict of Interest

The author has no competing financial interests and no competing non-financial interests to declare.

Ethics Statement

Secondary, anonymized DHS microdata is used in this study. Ethical approval for DHS survey protocols is obtained by the national ethics authorities and by ICF Institutional Review Board procedures. The author did not collect any primary data.

Appendix A. Multi-Country Portability Archive

Appendix Table A1 reports the country-level direct FBH, Brass-Trussell and raw AIIEA comparisons. Direct FBH and Brass-Trussell results are available for all 27 archived countries. Raw AIIEA outputs are complete for 24 countries; dagger-marked rows are excluded from AIIEA summary metrics.

Appendix Table A1. National comparison across the 27-country DHS archive

Country	Survey	Direct FBH	BT $q(5)$	AIIEA Raw	|BT-FBH|	|AIIEA-FBH|
Angola	2023–24	48.90	54.51	51.17	5.61	2.27
Benin	2017–18	88.02	122.76	99.24	34.74	11.22
Burkina Faso	2021	44.41	74.36	58.44	29.95	14.03
Burundi †	2016–17	72.09	102.25	N/A	30.16	N/A
Cameroon	2018	74.67	93.56	83.87	18.89	9.20
Côte d'Ivoire †	2021	69.53	84.15	N/A	14.62	N/A
DR Congo	2023–24	86.30	85.58	80.32	0.72	5.98

Country	Survey	Direct FBH	BT $q(5)$	AIIEA Raw	|BT-FBH|	|AIIEA-FBH|
Ethiopia	2019	55.66	77.77	71.22	22.11	15.57
Gabon	2019–21	37.52	35.59	37.73	1.93	0.20
Ghana	2022	36.21	54.01	45.73	17.80	9.52
Guinea	2018	102.16	111.71	104.19	9.56	2.04
Kenya	2022	38.78	50.13	43.08	11.34	4.30
Liberia	2019–20	87.49	117.34	101.70	29.85	14.22
Malawi	2024	45.32	62.95	51.91	17.63	6.59
Mozambique	2022–23	56.04	66.27	62.00	10.23	5.96
Namibia †	2013	49.86	63.66	N/A	13.80	N/A
Niger	2012	111.19	186.80	156.15	75.61	44.96
Nigeria	2024	100.6	114.6	106.64	13.98	6.07
Rwanda	2019–20	42.75	52.11	47.07	9.36	4.32
Senegal	2023	37.99	47.60	42.13	9.61	4.14
Sierra Leone	2019	109.68	141.10	124.30	31.42	14.62
South Africa	2016	40.42	49.79	47.36	9.37	6.93
Tanzania	2022	40.93	58.82	47.88	17.88	6.95
Togo	2013–14	81.38	112.80	94.01	31.42	12.63
Uganda	2016	59.50	96.30	76.20	36.80	16.70
Zambia	2024	40.05	51.90	45.08	11.86	5.04
Zimbabwe	2015	63.76	75.86	73.31	12.10	9.55
MAE (24 AIIEA countries)	-	-	-	-	19.57	9.71
Median AE (24 AIIEA)	-	-	-	-	14.62	6.94

Note: Burundi, Cote d'Ivoire and Namibia have direct FBH and Brass-Trussell estimates but no completed raw AIIEA output in the archived comparison. Aggregate AIIEA metrics therefore use the 24 completed countries. Values should be distributed with the corresponding country-level output files and analysis code.

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