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Uncertainty and Sensitivity Analysis of Thin-layer Drying Models Based on Seed Maize Drying Data

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Abstract Thin-layer drying models (TDMs) have been widely utilized as predictive tools to guide the design, scale-up and optimization of food drying processes. However, these models are empirically calibrated using experimental data. As such, the variability in the experimental data directly affects the uncertainty margins in the model predictions and subsequently impact on the model reliability. In this work, model prediction uncertainty for five selected TDMs namely; Page, Henderson & Pabis, Logarithmic, Verma *et al* and Diffusion, was evaluated and profiled, using variance based methods. Also, the sensitivity of model outputs to model input parameters was assessed using Sobol' indices. The nominal values of the parameters were estimated by fitting the models to experimental data. An array of pseudo-random numbers was generated around the nominal value of the input parameter considered and within a defined domain range. Then for each model, the outputs corresponding to the array of values were computed. The moisture ratio displayed high sensitivity to both the drying rate coefficient (K) and the drying time (t_f) during the falling rate drying period. The Page model and Logarithmic model returned wider uncertainty margins, which is attributable to the model structure. The uncertainty importance of parameter K was relatively higher than parameter t_f in all models albeit Verma *et al*. The findings are of significance for advancing the understanding of uncertainty intervals and limitations of thin-layer drying models in prediction of drying kinetics and for scale-up of drying processes.

Keywords Uncertainty, Sensitivity analysis, Thin-layer models, Moisture

1. Introduction

Food drying is an intricate operation that requires accurate process design and control in order to achieve optimum product quality in a cost efficient manner. As a result, it has attracted continued research. In fact, in value addition processes, the food quality is strongly characterized by the drying conditions. It is generally accepted that the time and energy expended to dehydrate material in a dryer to the desired moisture level is an indication of the process efficiency and varies intimately with the drying kinetics [1-3]. Hence, it is important that the kinetics of drying processes are adequately understood and characterized with the highest degree of certainty as a keystone to achieving optimal and efficient drying.

Often, simulation models are deployed to generate critical information of the underlying drying kinetics and provide insights needed for design, scale-up and optimization of the complex drying processes such as the case of intermittent drying. The ultimate goal is to maximize the energy efficiency in drying and minimize the drying time, while preserving product quality. As there are many types of food dryers and the physical properties of the food products vary over a wide range, no single modelling approach can be regarded as '*fit-for-all'*. As such dryer designers consider model predictive behavior together with the sensitivity profile and uncertainty pattern in ascertaining the model adequacy. This serves to minimize on error propagation from model scale (laboratory scale) to full scale drying systems. Indeed uncertainty has always been inherent in laboratory experimental data due to noise, incompleteness and inconsistency.

Thin-layer drying models (TDMs) find popular application in design and optimization of drying processes due to their simplicity and ease of use as opposed to classical complex distributed models [4-6]. But notwithstanding the merits of TDMs, the empiricism and idealization in the modelling process impacts the parameter sensitivity and prediction accuracy of these models. Further, the calibration studies of thin-layer drying models have mostly been accomplished using laboratory data and the results extrapolated to industrial scale through scale-up methodologies. The inherent uncertainties in the measured variables and their levels of sensitivity would inevitably influence the model adequacy and cause performance mismatch in the full scale drying system. In this regard, it is essential to take these uncertainties into account at the onset

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of drying process design or optimization. Thus, for successful industrial scale-applications, it is necessary to ensure that TDMs projections are reliable and accurate with well-defined uncertainty intervals. Indeed, majority of researchers concur that knowledge of uncertainty margins and sensitivity profiles of models is necessary to ascertain the model validity [7-9].

Generally, sensitivity and uncertainty analyses are concomitantly implemented in model development process. The sensitivity analysis is performed to assess the variability in model output corresponding to uncertainties in the model structure and input parameters. [10,11] This helps to visualize the real model behavior when used in practice. The relative impact of measurement errors in model parameters and input data on model predictions can be established. This knowledge can assist in efforts to enhance the precision of model projections while also contributing to better understanding of the relationships between model assumptions, parameters, data and model predictions. Note that reliability of model predictions is the primary basis for decision making regarding model utilization. Therefore, sensitivity analysis serves as an essential guiding tool for determining model behavior when it is used for prediction or for decision support.

Literature on uncertainty and sensitivity of TDMs is limited. Available studies [10,12] are largely addressed to static drying conditions with no considerations for the dynamic drying conditions that are characteristic of industrial drying processes. Under static drying conditions the moisture diffusivity is unchanged during drying, the drying product is homogeneous, the physical and thermodynamic properties are time independent and no temperature gradients occur within the product [13]. Strictly speaking, a complete drying profile comprises the first stage of drying (the equilibrium air temperature is greater than the temperature of the product), a constant-rate period, and a falling-rate period. In grain and crop drying, the dominant stage is the falling rate period, in which the drying conditions are dynamic [13]. It is worth mentioning that moisture diffusivity and activation energy are the principal kinetic parameters in thin-layer modelling [4,14,15]. In practice, these parameters vary dynamically along the drying profile and viciously influence the characteristic drying time and energy consumption in the falling rate period of drying. It is imperative that the sensitivity behavior and uncertainty ranges of thin layer drying models under dynamic kinetic conditions is established as a prerequisite for process design and scale-up application.

The objective of this study is to evaluate the sensitivity of TDMs and quantify the uncertainty levels of these models in prediction of the moisture ratio under dynamic kinetic conditions, relevant to industrial drying processes. The primary model parameters considered in this work are the drying coefficient and the characteristic drying time. The latter corresponds to the falling rate period of drying. As a contribution of this work, the sensitivity analysis will provide improved understanding of the limitations and uncertainty intervals of thin-layer drying models in prediction of drying kinetics. This would help to keep the propagation of model prediction errors to within the acceptable level of accuracy for reliable model based simulation and scale-up of drying processes. The net outcome shall be improved product quality and energy efficient drying processes.

2. Theoretical Background and Context

2.1. Theory of Thin-layer Drying

With reference to grain drying, a thin-layer can be considered as a single kernel freely suspended in the drying air or one layer of grain kernels [16,17]. Also it could be considered as a poly-layer of many grain thicknesses in which the temperature and the relative humidity of the drying air is in the same thermodynamic state at any time of drying. Premised on this definition, it can be deduced that the thickness of a thin layer may change with the velocity, temperature and relative humidity of the drying air and that the thermodynamic state of the drying air approaches the equilibrium state in heat and mass transfer with the grain dried in this layer [18]. Generally, industrial dryers are designed based on the theory of thin layer drying.

2.2. Thin-layer Modelling and Dynamic Drying Kinetics

TDMs are semi-theoretical in form, derived from the Fick's second law of diffusion and Newton's law of cooling by integrating theoretical and practical approaches [19,20]. Due to the usage of experimental data, they are easier and require fewer assumptions, but have limited validity tied to the process conditions applied. Equation 1 represents the general description of thin-layer drying [21]. The moisture content (x) of the drying product varies in proportion to the equilibrium moisture (x_e). The equilibrium moisture is attained when the moisture flux between the wet product and the drying air is zero for a given thermodynamic state of the air (i.e. vapor pressures at the product surface and air are equal).

$$\frac{dx}{dt} = -k\left(x - x_e\right) \quad (1)$$

The parameter k in equation 1 is the drying coefficient (per unit time) which is influenced by material moisture content, temperature, and size, as well as air humidity and velocity. Equation 2 represents the moisture ratio (x_R) in the thin layers of the drying product under static drying conditions [17].

$$x_R = \frac{x(t) - x_e}{x_o - x_e} = \frac{6}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^2}\exp(-n^2 k\, t) \approx Z\left(k,t\right) \quad (2)$$

Where x is the moisture content at any time (kg water/kg dry matter), x_o is the initial moisture content (kg water/kg dry matter), x_R is the moisture ratio, n is the number of terms (1, 2, 3, …) and t is the drying time (seconds). The term *Z(k,t)* is an approximation correlation for the variation of moisture ratio as a function of time and drying coefficient. The drying coefficient (k) is defined in terms of an effective diffusion coefficient, D and the geometric parameter, L of the drying product as $k = \pi^2 D/L$. The inherent relation between moisture diffusivity and activation energy is expressed in form of equation 3.

$$D = D_o \exp\left(-E_a / RT\right) \tag{3}$$

Where E_a is the activation energy (kJ/mol), R is the universal gas constant (kJ/mol.K), T is the absolute air temperature and D_o a factor in the Arrhenius equation (m^2/s). Therefore, k is defined by both the diffusivity and activation energy as follows:

$$k = \frac{\pi^2}{r} D_o \exp\left(-\frac{E_a}{RT}\right) \tag{4}$$

Diffusivity in drying varies with the moisture concentration and hence, the drying curve can be assumed to correspond to the total diffusivity of the product. The differentiable falling rate drying periods observed with most products point to the fact that drying kinetics change with the progressively reducing product moisture concentration [22–24]. With this consideration, equation 2 can be represented as,

$$x(t) = x_e + Z(k,t)\left[x_o - x_e\right] \tag{5}$$

Differentiating equation 5 with respect to time and then diving through by equation 1 yields the drying rate expression representative of the dynamic drying kinetics:

$$\frac{dx(t)}{dt} = \frac{X_R}{Z(k,t)} \frac{dZ(k,t)}{dt}\left(x_o - x_e\right) = -k^*(t_c)\left[x(t) - x_e\right] \tag{6}$$

Where $k^*(t_c)$ is the dynamic drying coefficient and t_c is the characteristic time that is correlated to the thin layer drying time [13,25]. In the present work, t_c is substituted for t_f, which is the drying duration corresponding to the falling rate period on the drying curve (Figure 1). Equation 6 can then be rewritten as follows:

$$\frac{dx(t)}{dt} = -k^*(t_f)\left[x(t) - x_e\right] \tag{7}$$

The falling drying rate period comprises two phases. The first falling drying rate occurs when wetted regions on the surface of the drying product continuously decrease until the surface is dried (region III). The second falling rate period (region IV) sets in when the surface is completely dry. The plane of evaporation recedes from the surface, hence the vapour moves through the solid into air stream [17].

This study involved selected thin-layer models presented in Table 1, namely; Page model, Henderson & Pabis model, Logarithmic model, Verma *et al* model and approximation of diffusion model. These models have been successfully used to describe the drying characteristics of crops and grains [1,15,19,24,26,27].

Table 1. Selected thin-layer models adjusted for dynamic drying kinetics

Model Name	Model Equation
Page	$X_R = \exp\left(-k^*(t_f)t_f^n\right)$
Henderson & Pabis	$X_R = \lambda\, e\mathrm{xp}\left(-k^*(t_f)t_f\right)$
Logarithmic	$X_R = \lambda \exp\left(-k^*(t_f)t_f\right) + \gamma$
Verma et al	$X_R = \lambda \exp\left(-k^*(t_f)t_f\right) + (1-\lambda)\exp\left(-\varphi^*(t_f)t_f\right)$
Diffusion	$X_R = \lambda \exp\left(-k^*(t_f)t_f\right) + (1-\lambda)\exp\left(-\beta k^*(t_f)t_f\right)$

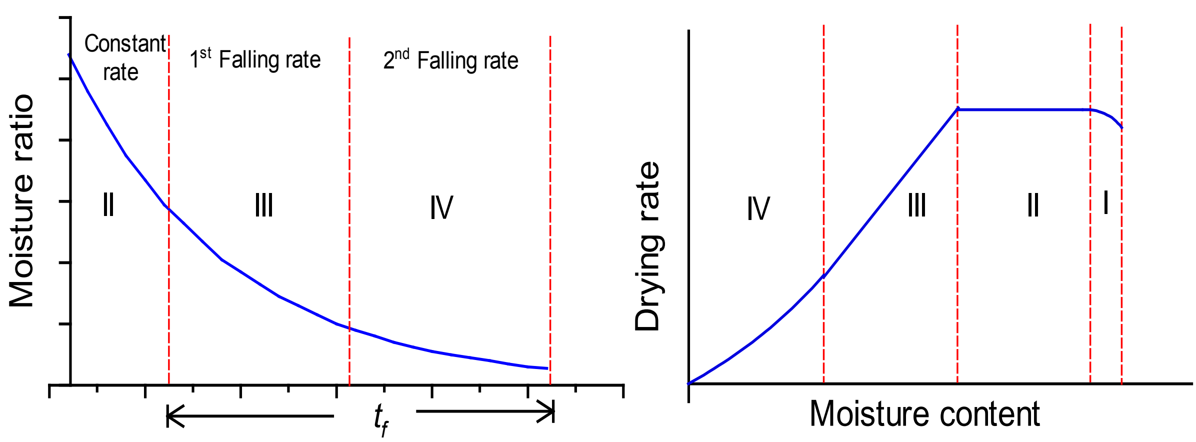

Figure 1. Representation of a complete drying curve showing the constant rate and falling rate periods

3. Materials and Methods

3.1. Experimental Data

The experimental equipment used for data collection is described in detail in the recent work by Makokha et al [28]. The equipment was manufactured by PETKUS Technologies, Germany. It is designed to operate in double pass reversing system for drying of seed maize with ability to handle input grain moisture of up to 30%. The plant has capacity of drying 600 tons of maize on cob over duration of 48 to 72 hours in 12 drying bins. The maize grain was dried at low temperature of 35 to 42 degrees. The moisture content of the grain was measured using Dolmar-MM400 Agromatic digital moisture meter with resolution of 0.1% and error tolerance of 0.05% of moisture content conforming to International Organization of Legal Metrology (OIML) margin of tolerances for Class 1 meters. The maize grain was dried from the moisture level of 19.1 to 13%. The data obtained of moisture ratio was fitted to five selected thin-layer models to estimate the drying coefficients and shape coefficients. The fit results are presented in Figure 2.

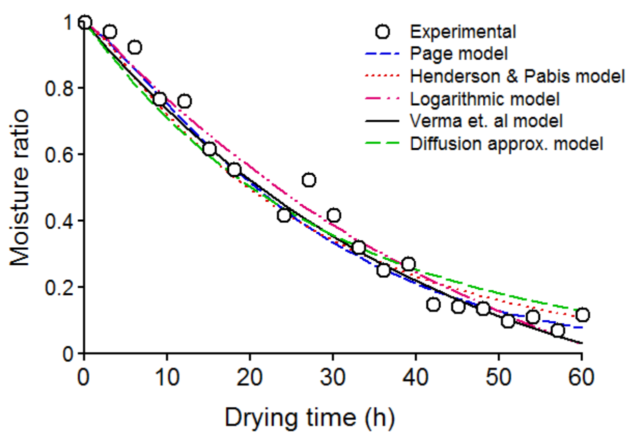

Figure 2. The measured and model predictions of moisture ratio for maize grain at low temperature drying (35 – 42°C)

The values of the parameters obtained by fitting the models to the experimental data are presented in Table 2. They were used as nominal values in the sensitivity tests. The total drying time was 60 h comprising a constant rate period of 20 h and a falling rate period of 40 h.

Table 2. Model parameters used in the simulation tests estimated from the industrial drying data for maize grain

Model	Parameter	Value	R^2	t_f
Page	k^*	0.017 h^{-1}	95.8%	40h
	n	1.224		
Henderson & Pabis	λ	1.045	94.7%	40h
	k^*	0.037 h^{-1}		
Logarithmic	λ	1.398	90.3%	40h
	k^*	0.021 h^{-1}		
	γ	-0.233		
Verma et. al	λ	1.121	96.5%	40h
	k^*	0.022 h^{-1}		
	φ	0.003 h^{-1}		
Diffusion	λ	0.163	92.1%	40h
	k^*	0.027 h^{-1}		
	β	1.341		

3.2. Sensitivity Analysis

Various methods of sensitivity analysis exist that are employed in various modelling situations. The simplified form of the Smirnov test statistics [29,30] based on relative mean deviation (RMD) and the Sobol's method [31] were adopted in the current analysis to assess the sensitivity of the models. Sensitivity analysis was performed on five selected thin-layer models presented in Table 1 to assess the relative influence of the two input uncertainty parameters considered (i.e. -drying coefficient and the drying time). Since the region of interest for this study is the falling rate drying period, the drying time considered here corresponds to the falling rate period on the drying curve. The Monte Carlo technique was used to generate a set of random model outputs for each pseudo-random input parameter through repeated simulations. The values were uniformly distributed in the defined domain of ±50% of the nominal values as presented in Table 3. This provides an equal likelihood of occurrence for each combination of model parameter values. The reported ranges of drying coefficient (*K*) for different products at drying temperatures of 10 – 80°C, air velocity of 0 - 10m/s and relative humidity of 10 – 70% are as follows [4,32,33]; Cereals (0.005 – 0.3), Fruits (0.01 – 0.08), Vegetables (0.003 – 0.12) and Others (0.001 – 0.44).

Table 3. Estimated nominal values of model parameters and their corresponding interval ranges used in generation of random pseudo input values

Model	Drying coefficient K (h^{-1})			Drying time t_f (h)		
	Min	*Nominal*	*Max*	*Min*	*Nominal*	*Max*
Page	0.009	0.017	0.026	20	40	60
Henderson & Pabis	0.019	0.037	0.056	20	40	60
Logarithmic	0.011	0.021	0.032	20	40	60
Verma et. al	0.012	0.024	0.036	20	40	60
Diffusion	0.014	0.027	0.041	20	40	60

The output variability is evaluated when the input factors vary in their whole uncertainty domains. To eliminate the effect of different input parameter scales, the sensitivity index was calculated relative to the mean deviation in the predicted values of moisture ratio. The relative mean deviation (RMD) was evaluated for each parameter perturbation as a characteristic of the response. The objective function values for N observations (data points) were obtained as,

$$RMD(y)|_{x_i} = \left. \frac{1}{N}\left(\sum_{k=1}^{N} |y_k - \bar{y}| \Big/ |\bar{y}|\right)\right|_{x_{\sim i}} \quad (8)$$

Where y is the output (response) at each kth level of the model input parameters, x. In this study, y is the moisture ratio while x is a set of model input parameters. The magnitude of the deviation measures the strength of impact of model parameters on model response or output. The interquartile range (i.e. 75th and 25th percentiles) of the RMD values depicts the degree of variability in the model predictions and so is considered a good measure of the sensitivity of the model response to changes in parameter levels. The interquartile range (IQR) is thus computed as $(3(N+1)/4)^{th} - (N+1/4)^{th}$ value of RMD where N is the total number of data points/observations.

The other approach employed in this study uses Sobol' indices, which is a variance based technique for sensitivity analysis of a model output. Given a model of the form $y = f(x_i)\ \forall i$, the total-order Sobol' index (S_i^T) can be estimated using equation 9 [31]. The index S_i^T represents the contribution of each input parameter x_i on the response y considering interaction of x_i with other model input parameters.

$$S_i^T = \frac{E\left[Var(\mathrm{y} \mid x_{\sim i})\right]}{Var(\mathrm{y})} \quad (9)$$

Where, E denotes expected value, $x_{\sim i}$ is a vector containing all input parameters except x_i while the expression $Var(y|x_{\sim i})$ computes the variance of y taken over all possible x_i values. For non-purely additive models, the sum of the Sobol' indices equals to unity, $\sum_{i=1}^{m} S_i^T = 1$.

3.3. Analysis of Uncertainty Propagation

Quantification of uncertainty and uncertainty propagation, particularly the characterization of model output uncertainty due to input uncertainty is an important part of any modelling process. The quantification of uncertainty in thin layer model predictions is both a notable challenge and an important goal. Majority of the methods presented in literature for quantifying uncertainty are statistical due to stochastic nature of uncertainty and among them the polynomial chaos approximation and variance based techniques are commonly used. Uncertainty in model input values can result from the statistical error in analysis of data or experimental errors in measurements. Consequently, input may be treated as a random variable with a probability distribution. There is a large choice of probability distributions available for mapping the propagation of uncertainty of inputs to model outputs. The uniform distribution puts equal weight on each value in the uncertainty range. In most cases, however, the extreme values of the uncertainty ranges are less likely than the middle values. The well-known Gaussian and Normal distribution are often convenient since they require only the specification of a mean value and a standard deviation. Uncertainty quantification based on covariance analysis has been applied in our analysis [12]. This method is sensitive to the variability in input parameters and propagates this effect through the model to the response. Here, the model response is the moisture ratio.

The uncertainty (u_y) in the response y at the confidence interval of (1-α) for ν degrees of freedom is obtained as a product of the standard deviation of the input parameters and the t-test value as shown by equations 10 [12].

$$\sigma_y = \sqrt{\left(\frac{\partial y}{\partial x_i}\sigma_{ii}\right)^2 + \left(\frac{\partial y}{\partial x_j}\sigma_{jj}\right)^2 + 2\frac{\partial y}{\partial x_i}\cdot\frac{\partial y}{\partial x_j}\sigma_{ij}^2} \quad (10)$$
$$u_y = \pm t_{(1-\alpha)/2,\nu} \times \sigma_y$$

Where y denotes the moisture ratio and x_i and x_j are the input parameters. The terms σ_{ii}^2, σ_{jj}^2 and σ_{ij}^2 represent the covariance, $Cov(x_i, x_i), Cov(x_j, x_j)$ and $Cov(x_i, x_j)$ respectively.

The uncertainty importance (I_u) is used here as an index that characterizes the degree of reliability of model output. The uncertainty importance (I_u) of the input parameter x_i is defined as the expected reduction in the variance of the output y attributable to ascertaining the value of x_i [7]. It is mathematically expressed as:

$$I_u = \sqrt{Var(y) - E\left[Var(y|x_i)\right]} \quad (11)$$

Relatively lower values of I_u mean less sensitivity of the model response to variability pattern of input parameters, which implies reliable model predictions.

4. Results and Discussion

The results and discussion of sensitivity and uncertainty evaluation for five selected thin-layer drying models are presented in this section. In order to compare the sensitivity of individual model parameters and their level of contribution to the model uncertainty, the moisture ratios were computed for all randomly-generated values of model parameters in the defined range.

Presented in Figures 3 (a-j) are surface and contour plots of model response data, demonstrating the trends in predicted moisture ratio with changes in the values of input parameters in a defined domain (Table 2), for each of the five thin-layer models (Table 1). Notice that the scatter plots have been superimposed onto contour plots to display the distribution of random values of the model input parameters

(K and t_f) as generated from Monte Carlo simulation.

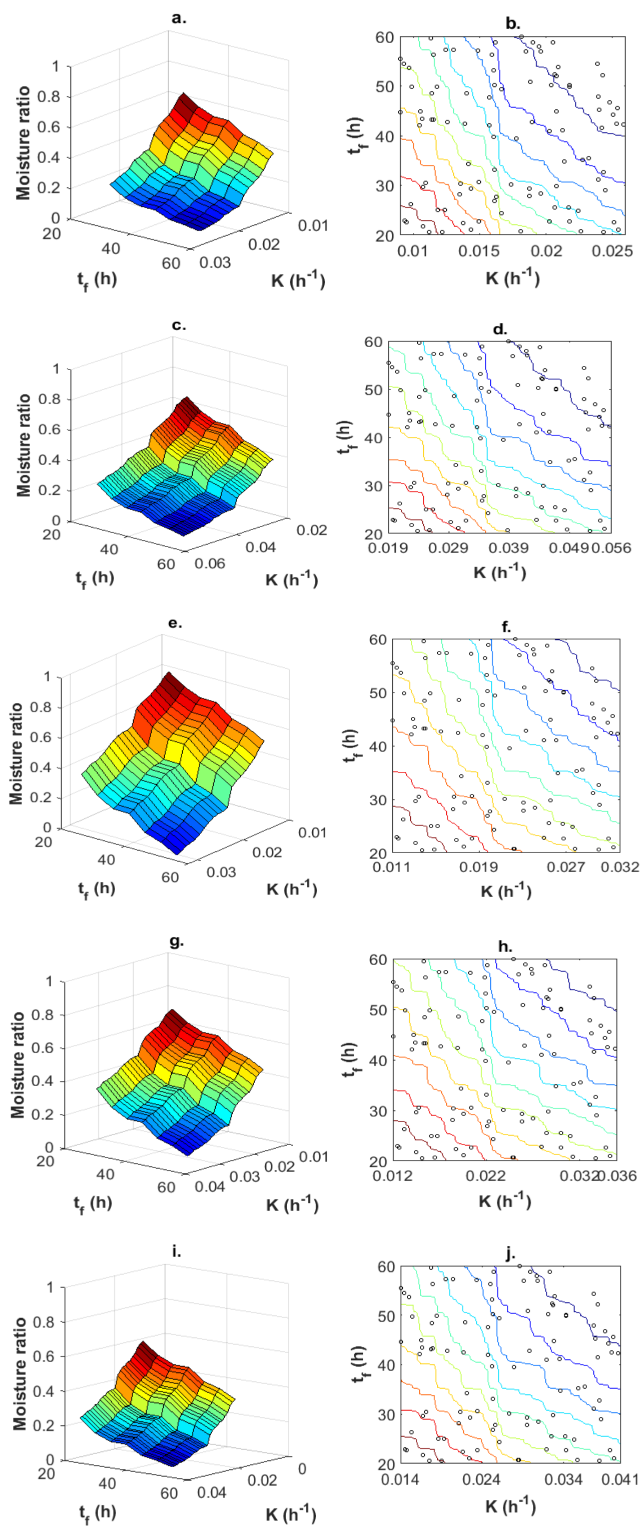

Figure 3. Profile of predicted moisture ratio at varying levels of input parameters, K and t_f for five models tested: (a, b) Page model; (c, d) Henderson & Pabis model; (e, f) Logarithmic model; (g, h) Verma et al model; (i, j) Diffusion model

A close inspection of the data leads to two important observations. Firstly, the moisture ratio decreases with increase in both drying time (t_f) and drying coefficient (K) in all cases, which conforms to our expectations and literature findings. Also, the moisture ratio changes rapidly with change in parameter K, which indicates that the estimation of the moisture content relies heavily on accurate values of the drying coefficient. The second important observation is displayed in the data between 40 and 60 h of drying time. Here, the surface plots depicting model response profile reveal a rapid drop followed by a steady decline in the drying rate in all the five models presented. It is a speculated indication that drying has entered the second falling rate drying period. This pattern is characteristic of the dynamic moisture activity in the falling rate zone. Indeed, one should bear in mind that drying in the falling rate period is controlled by combined external–internal resistances or by either external or internal resistance to heat and mass transfer as reiterated by Araya-Farias and Ratti [34].

The distribution of relative mean deviation (RMD) depicting the variance in the predicted moisture ratio (model response) for each of the five models is presented with a box plot. Figure 4a shows the contribution of parameter K while Figure 4b shows the contribution of parameter t_f, to the variance of the moisture ratio. Values for Quartile 1 (Q1), Median, and Quartile 3 (Q3) are delineated. The line within the box marks the median while the whiskers above and below the box indicate the furthest points that are within 1.5 times the interquartile range from the end of the box.

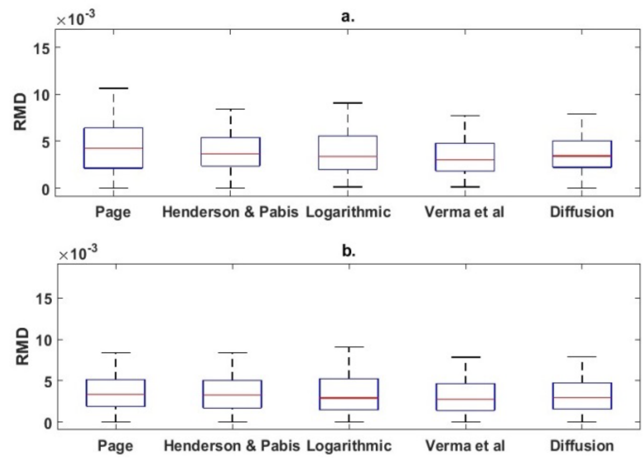

Figure 4. Box plots of the relative mean deviation (RMD) in the model response data for the five thin-layer models for two scenarios (a) effect of parameter K and (b) effect of parameter t_f

By examining the plots, it is clear that the difference between Q1 and Q3 varies between the different models. Also, the effect of the model parameters K and t_f. It is clearly evident that parameter K has a relatively stronger influence on uncertainty level of the Page model while parameter t_f appears to exert more influence on the Logarithmic model. On the other hand, Henderson & Pabis model, Verma *et al* model and Diffusion model all exhibit nearly the same level of uncertainty in both cases, implying equal influence by the two parameters, K and t_f. The possible explanation for higher level of variability exhibited by Page model relates to the structure of the model (see Table 1). The page model contains a power exponent (n), which in effect (for n>1) would allow errors to grow by a factor of n. The Logarithmic model has an extra additive parameter γ that also contributes to the total variance of the model response. On the premise of these observations, it can be deduced that the both Page and Logarithmic model could be less adequate in the scale-up applications where drying conditions are dynamic.

In order to quantify the impact of individual model input parameters on the variance of the moisture ratio, a sensitivity analysis was carried out using Sobol's total sensitivity index (S^T). Figure 5 shows the results of the sensitivity analysis for the five models investigated. The advantage of using total sensitivity index lies in the ability to estimate the contribution of individual parameters including interaction effects. From the results presented, the respective contribution by the parameters $K\ and\ t_f$ to uncertainty in the predicted moisture ratio is evaluated as, 67 and 34% for Page model; 55 and 46% for Henderson & Pabis; 50 and 53% for Logarithmic, 50.5 and 51% for Verma *et al*, and 54 and 45% for Diffusion model.

It is apparent that the moisture ratio is more sensitive to parameter K than t_f for Page, Henderson & Pabis and Diffusion model. Further, the results show a marginally higher sensitivity of moisture ratio to the parameter t_f than K for Logarithmic model, but display equal sensitivity of both parameters for Verma et al model. Notice that for Verma *et al* model, the drying constant has been partitioned into two i.e. K^* and φ^* (see Table 1), corresponding to constant rate and falling rate drying periods respectively. From a quantitative point of view, it can be deduced that the moisture ratio is sensitive to both parameters, $K\ and\ t_f$.

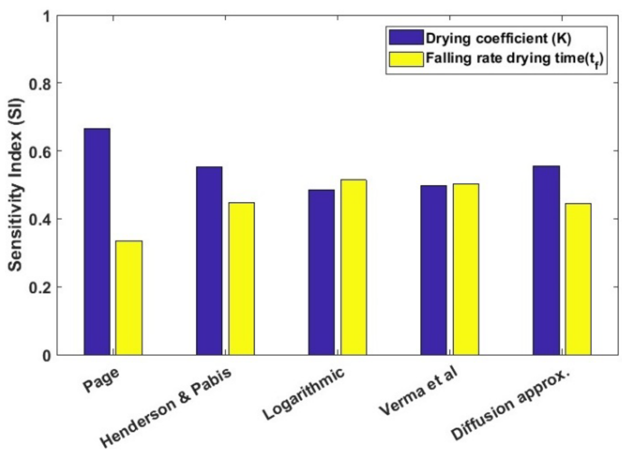

Figure 5. Bar plots showing the relative sensitivity of the model parameters, $K\ and\ t_f$ based on Sobol's index

Uncertainty assessment is important for enhancing the model results interpretation and usability. [8] Table 4 presents results of uncertainty observed in the predicted moisture ratio data characterized at 95% confidence interval and 9999 degrees of freedom (10000 data points -1). The maximum and minimum values are based on the student t-test values.

Henderson & Pabis model displays the lowest average value of uncertainty. This has to be attributable to the model structure having fewer parameters and no additive empirical terms. Conversely, Logarithmic model has an empirical additive parameter that has to be estimated from experimental data. This would increase the uncertainty in the model predictions as evidenced in the results in Table 4. Page model is formulated with a power exponent. As a consequence of this, the uncertainty propagates more rapidly.

Table 4. Average uncertainty in the model response data (based on eqn 10)

Model	Uncertainty (u_y)		
	Max	*Ave*	*Min*
Page	0.0844	0.0816	0.0742
Henderson & Pabis	0.0780	0.0748	0.0673
Logarithmic	0.0996	0.0895	0.0751
Verma et al	0.0858	0.0783	0.0661
Diffusion	0.0793	0.0764	0.0689

Last, we report in Figure 6 the evaluation results of uncertainty importance for the parameters $K\ and\ t_f$. The respective values of uncertainty importance of the parameters $K\ and\ t_f$ to model predictions of moisture ratio are evaluated as, 0.14 and 0.10 for Page model; 0.12 and 0.11 for Henderson and Pabis; 0.17 and 0.15 for Logarithmic, 0.13 and 0.13 for Verma et al and 0.12 and 0.11 for Diffusion model. It is apparent that the uncertainty importance of parameter K is higher relative to t_f in all cases except for Verma et al. Again, this is reminiscent of the pattern observed for sensitivity indices in Figure 4. The level of uncertainty importance for K relative to t_f is the same for both Henderson & Pabis model and Diffusion model. The highest values of uncertainty importance of K and t_f are associated with the Logarithmic model.

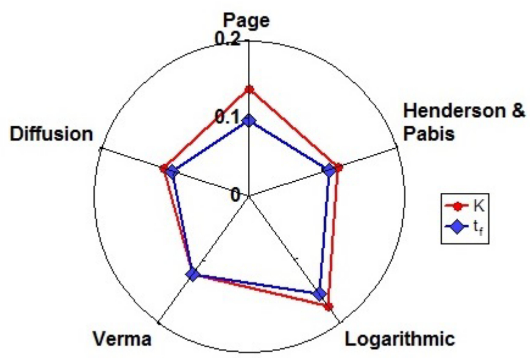

Figure 6. Radar plot showing the relative uncertainty importance (I_u) of the model parameters $K\ and\ t_f$ for the five thin-layer models tested

Trends observed in Figures 5 and 6 seem to indicate that the sensitivity as well as uncertainty importance of parameters K and t_f would vary across the different thin-layer models. Therefore, the predictive applications of thin-layer models in dynamic drying processes should be performed judiciously guided by data of the sensitivity indices of model parameters and the magnitude of uncertainty that could be propagated from model parameters to the model output.

5. Conclusions

The objective of this work was to analyze sensitivities of thin-layer models and comparatively evaluate the extent to which the model parameters influence the level of uncertainty in the predicted values of moisture ratio. Two fundamental parameters were evaluated for five selected thin-layer models. The investigation of parameter sensitivity and uncertainty propagation was accomplished through variance based statistical methods. In all the cases tested, the results revealed that moisture ratio is highly sensitive to both the drying rate coefficient (parameter K) and the falling rate drying time (parameter t_f), and that parameter K has a higher uncertainty importance than parameter t_f, which is a salient facet of our results. The highest level of uncertainty was displayed by Page and Logarithmic models, which is attributable to the model structure. Further, it was observed that the moisture ratio has equal sensitivity to both parameters $K\ and\ t_f$ in the predictions by Verma *et al* model. Also, the uncertainty importance of $K\ and\ t_f$ was noted to be indifferent for Verma et al Model.

The findings of this study have provided indicative insight on the reliability of thin-layer drying models under dynamic drying kinetics at low temperature drying of 35 to 42°C. This information is fundamental to accurate design and scale-up, effective monitoring and better control of dynamic drying processes.

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