International Journal of Statistics and Applications 2019, 9(1): 8-20
DOI: 10.5923/j.statistics.20190901.02

Weighted Quasi Lindley Distribution with Properties and Applications

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Abstract In this paper a three-parameter weighted quasi Lindley distribution which includes two-parameter quasi Lindley distribution, weighted Lindley distribution and gamma distribution and one parameter Lindley and exponential distributions as particular cases has been proposed and its statistical properties including moments and moments based measures, hazard rate function, and mean residual life function have been discussed. Maximum likelihood estimation has been discussed for estimating its parameters. Applications of the distribution have explained with two real failure time data and the goodness of fit shows quite satisfactory fit over other lifetime distributions.

Keywords Weighted distribution, Quasi Lindley distribution, Moments, Hazard rate function, Mean residual life function, Maximum Likelihood estimation, Lifetime data, Goodness of fit

1. Introduction

The study of weighted distributions are useful in distribution theory because it provides a new understanding of the existing standard probability distributions and it provides methods for extending existing standard probability distributions for modeling lifetime data due to the introduction of additional parameter in the model which creates flexibility in their nature. Weighted distributions occur in modeling clustered sampling, heterogeneity, and extraneous variation in the dataset.

The concept of weighted distributions were firstly introduced by Fisher (1934) to model ascertainment biases which were later formulized by Rao (1965) in a unifying theory for problems where the observations fall in non-experimental, non-replicated and non-random manner. When observations are recorded by an investigator in the nature according to certain stochastic model, the distribution of the recorded observations will not have the original distribution unless every observation is given an equal chance of being recorded. For instance, let the original observation x_0 comes from a distribution having probability density function $f_0(x,\theta_1)$, where θ_1 may be a parameter vector and the observation x is recorded according to a probability re-weighted by weight function $w(x,\theta_2)>0$, θ_2 being a new parameter vector, then x comes from a distribution having pdf

$$f(x;\theta_1,\theta_2)=A\,w(x;\theta_2)\,f_0(x;\theta_1), \tag{1.1}$$

where A is a normalizing constant. Note that such types of distribution are known as weighted distributions. The weighted distributions with weight function $w(x,\theta_2)=x$ are called length biased distributions or simple size-biased distributions. Patil and Rao (1977, 1978) have examined some general probability models leading to weighted probability distributions, discussed their applications and showed the occurrence of $w(x;\theta_2)=x$ in a natural way in problems relating to sampling.

Shanker and Mishra (2013) introduced a two-parameter quasi Lindley distribution (QLD) having parameters θ and α and defined by its probability density function (pdf) and cumulative distribution function (cdf)

$$f_1(x;\theta,\alpha)=\frac{\theta}{\alpha+1}(\alpha+\theta x)e^{-\theta x}\ ;x>0,\theta>0,\alpha>-1 \tag{1.2}$$

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Published online at http://journal.sapub.org/statistics

$$F_1(x;\theta,\alpha)=1-\left[1+\frac{\theta x}{\alpha+1}\right]e^{-\theta x}\quad ; x>0,\theta>0,\alpha>-1 \tag{1.3}$$

It can be easily verified that at $\alpha=\theta$, QLD reduces to Lindley distribution, introduced by Lindley (1958). Also exponential distribution is a limiting case of QLD for $\alpha\to\infty$. Shanker and Mishra (2016) also proposed a quasi Poisson-Lindley distribution and discussed its statistical properties, estimation of parameters and applications. Shanker *et al* (2016) discussed various important statistical properties and applications of QLD.

Ghitany *et al* (2011) introduced a two-parameter weighted Lindley distribution (WLD) with parameters θ and α defined by its pdf and cdf

$$f_2(x;\theta,\alpha)=\frac{\theta^{\alpha+1}}{\theta+\alpha}\frac{x^{\alpha-1}}{\Gamma(\alpha+1)}(1+x)e^{-\theta x}\ ; x>0,\theta>0,\alpha>0 \tag{1.4}$$

$$F_2(x;\theta,\alpha)=1-\frac{(\theta+\alpha)\Gamma(\alpha,\theta x)+(\theta x)^{\alpha}e^{-\theta x}}{(\theta+\alpha)\Gamma(\alpha)}\quad ; x>0,\theta>0,\alpha>0\,, \tag{1.5}$$

where

$$\Gamma(\alpha,z)=\int_z^{\infty}e^{-y}y^{\alpha-1}\,dy\quad ; y\ge 0,\alpha>0 \tag{1.6}$$

is the upper incomplete gamma function. It can be easily verified that the Lindley distribution introduced by Lindley (1958) having pdf and cdf given by

$$f_3(x;\theta)=\frac{\theta^2}{\theta+1}(1+x)e^{-\theta x}\quad ; x>0,\theta>0 \tag{1.7}$$

$$F_3(x;\theta)=1-\left[1+\frac{\theta x}{\theta+1}\right]e^{-\theta x}\quad ; x>0,\theta>0 \tag{1.8}$$

is a particular case of (1.3) for $\alpha=1$. Ghitany *et al* (2008) have detailed study regarding its statistical and mathematical properties, estimation of parameter and application. Shanker *et al* (2015) have comparative study on applications of exponential and Lindley distribution for modeling lifetime data. Shanker and Shukla (2018) has obtained a Poisson mixture of weighted Lindley distribution and discussed its statistical properties, estimation of parameters and applications for discrete data.

In the present paper, a three - parameter weighted quasi Lindley distribution which includes Lindley (1958) distribution, weighted Lindley distribution and quasi Lindley distribution as particular cases, has been proposed and discussed. Its moments about origin and central moments, coefficient of variation, skewness, kurtosis and index of dispersion have been derived. The hazard rate function and the mean residual life function of the distribution have been derived and their shapes have been discussed for varying values of the parameters. The estimation of its parameters has been discussed using maximum likelihood method. Finally, the goodness of fit and the applications of the distribution have been explained through two failure time data and the fit has been compared with one parameter Lindley distribution and the two-parameter weighted Lindley distribution and quasi Lindley distribution.

2. Weighted Quasi Lindley Distribution

The pdf of a three-parameter weighted quasi Lindley distribution (WQLD) can be obtained as

$$f_4(x;\theta,\alpha,\beta)=A\,x^{\beta-1}f_0(x;\theta,\alpha)\ ;\ x>0,\ \theta>0,\alpha>-1,\beta>0\,, \tag{2.1}$$

where A is a normalizing constant and $f_0(x;\theta,\alpha)$ is the pdf of quasi Lindley distribution introduced by Shanker and Mishra (2013) given in (1.2). Thus, the pdf of WQLD can be expressed as

$$f_4(x;\theta,\alpha,\beta)=\frac{\theta^{\beta}}{(\alpha+\beta)}\frac{x^{\beta-1}}{\Gamma(\beta)}(\alpha+\theta x)e^{-\theta x}\quad ;\ x>0,\theta>0,\alpha>-1,\beta>0 \tag{2.2}$$

The pdf in (2.1) can be easily expressed as a convex combination of gamma (θ,β) and gamma $(\theta,\beta+1)$ distributions. We have

$$f_3(x;\theta,\alpha,\beta)=p\,g_1(x;\theta,\beta)+(1-p)\,g_2(x;\theta,\beta+1)\,, \tag{2.3}$$

where $p=\dfrac{\alpha}{\alpha+\beta}$, $g_1(x;\theta,\beta)=\dfrac{\theta^{\beta}}{\Gamma(\beta)}e^{-\theta x}x^{\beta-1}$, $g_2(x;\theta,\beta+1)=\dfrac{\theta^{\beta+1}}{\Gamma(\beta+1)}e^{-\theta x}x^{\beta+1-1}$.

It can be easily verified that one parameter Lindley and exponential distributions, two-parameter weighted Lindley distribution, quasi Lindley distribution and gamma distribution are particular cases of WQLD at $(\alpha=\theta,\beta=1)$, $(\alpha\to\infty,\beta=1)$ $(\alpha=\theta)$, $(\beta=1)$ and $(\alpha\to\infty)$, respectively.

The survival (reliability) function of WQLD can be obtained as

$$S(x;\theta,\alpha,\beta)=P(X>x)=\int_x^{\infty} f_4(t;\theta,\alpha,\beta)\,dt$$

$$=\frac{\theta^{\beta}}{(\alpha+\beta)\Gamma(\beta)}\int_x^{\infty} t^{\beta-1}(\alpha+\theta t)e^{-\theta t}dt$$

$$=\frac{\theta^{\beta}}{(\alpha+\beta)\Gamma(\beta)}\left[\alpha\int_x^{\infty} e^{-\theta t}t^{\beta-1}dt+\theta\int_x^{\infty} e^{-\theta t}t^{\beta}dt\right]$$

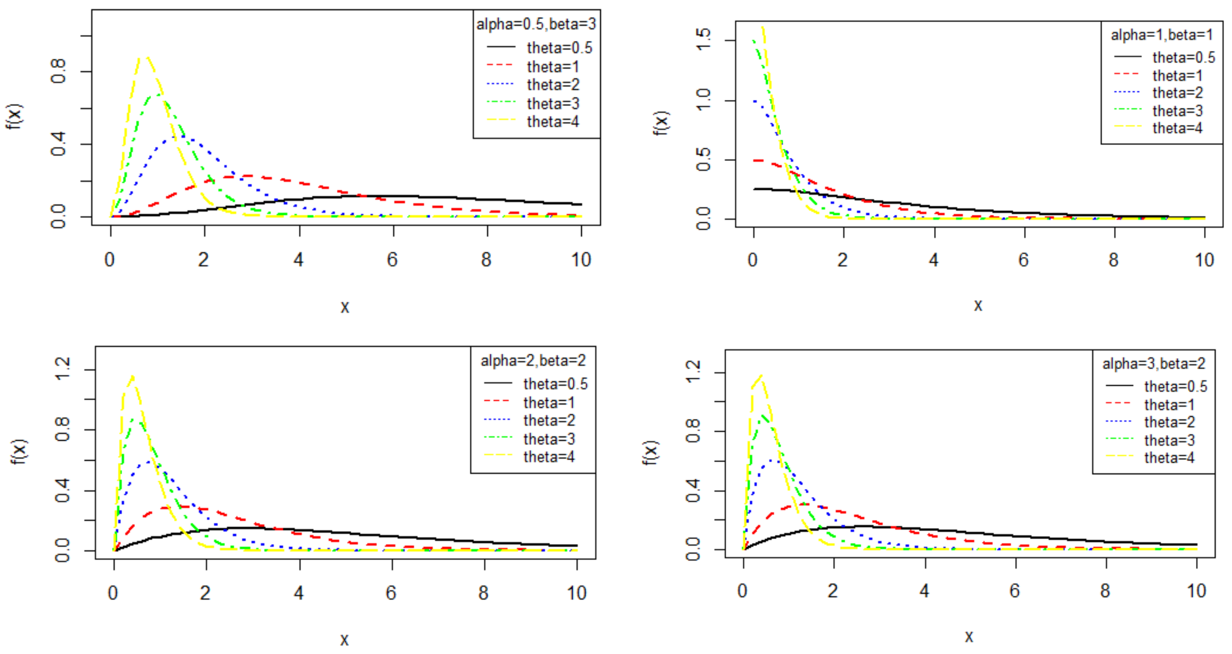
$$=\frac{\theta^{\beta}}{(\alpha+\beta)\Gamma(\beta)}\left[\frac{\alpha\Gamma(\beta,\theta x)}{\theta^{\beta}}+\frac{\theta\left\{e^{-\theta x}(\theta x)^{\beta}+\beta\Gamma(\beta,\theta x)\right\}}{\theta^{\beta+1}}\right]$$

$$=\frac{(\alpha+\beta)\Gamma(\beta,\theta x)+(\theta x)^{\beta}e^{-\theta x}}{(\alpha+\beta)\Gamma(\beta)}.$$

where $\Gamma(\alpha,z)$ is the upper incomplete gamma function defined in (1.3). The corresponding cdf of WQLD can thus be given by

$$F(x;\theta,\alpha,\beta)=1-S(x;\theta,\alpha,\beta)=1-\frac{(\alpha+\beta)\Gamma(\beta,\theta x)+(\theta x)^{\beta}e^{-\theta x}}{(\alpha+\beta)\Gamma(\beta)}.$$

The behaviour of the pdf of WQLD for varying values of parameters has been shown in figure 1. Further, the behaviour of the cdf of WQLD has also been shown in figure 2.



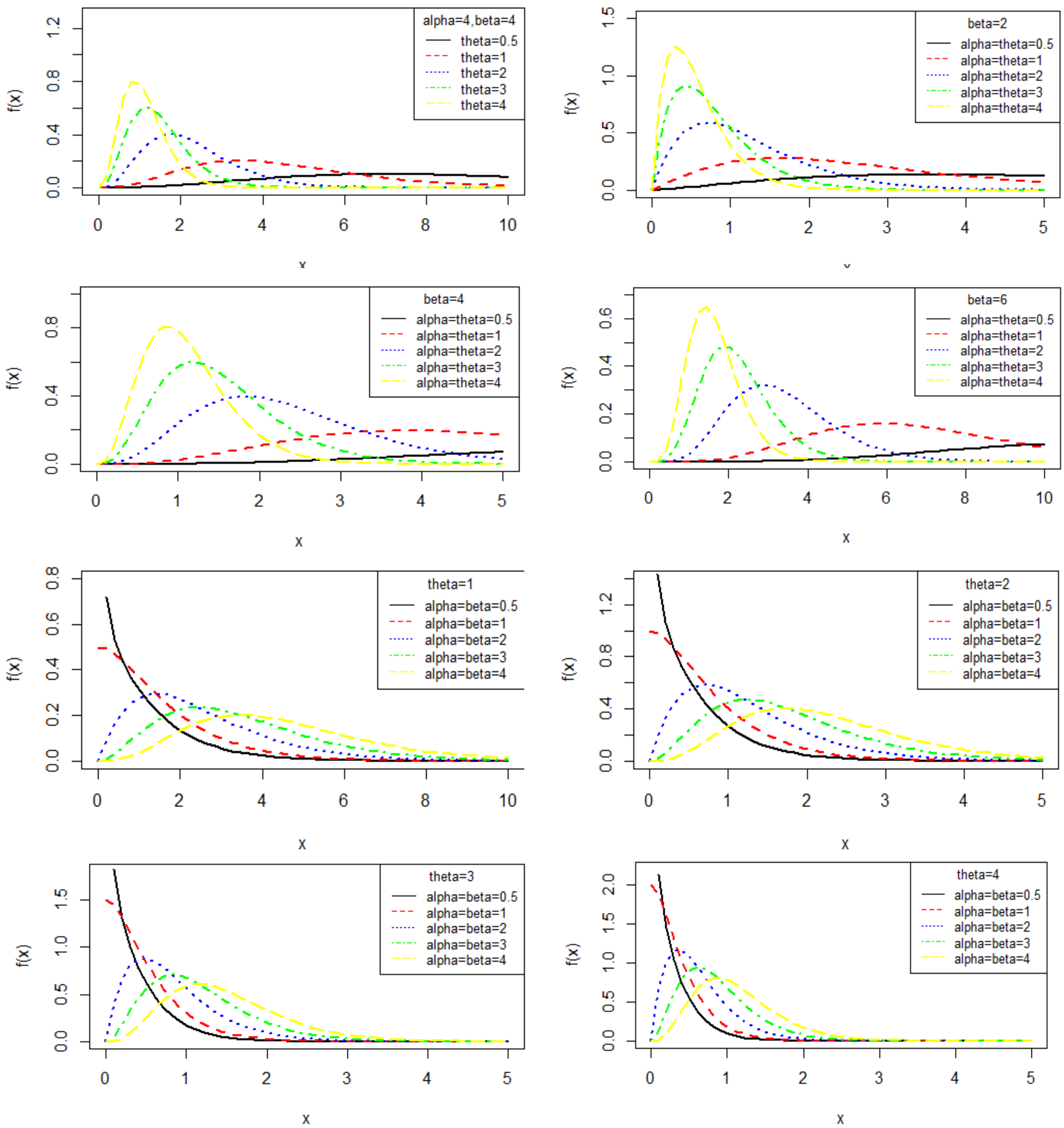
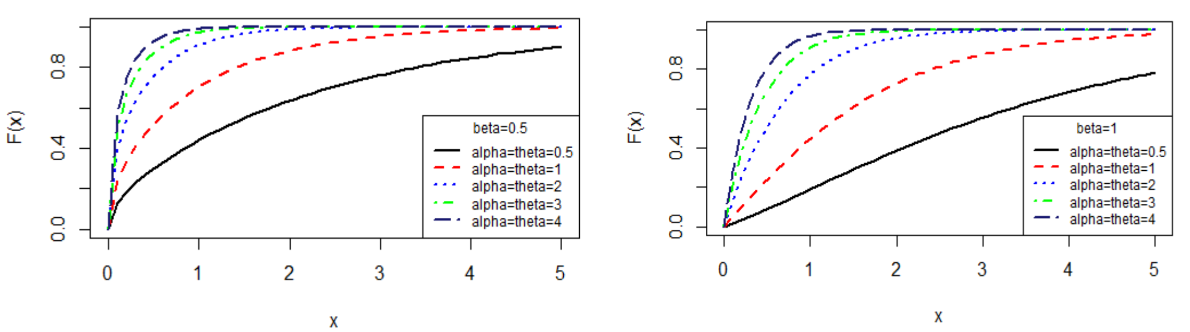


Figure 1. Behaviour of the pdf of WQLD for varying values of parameters



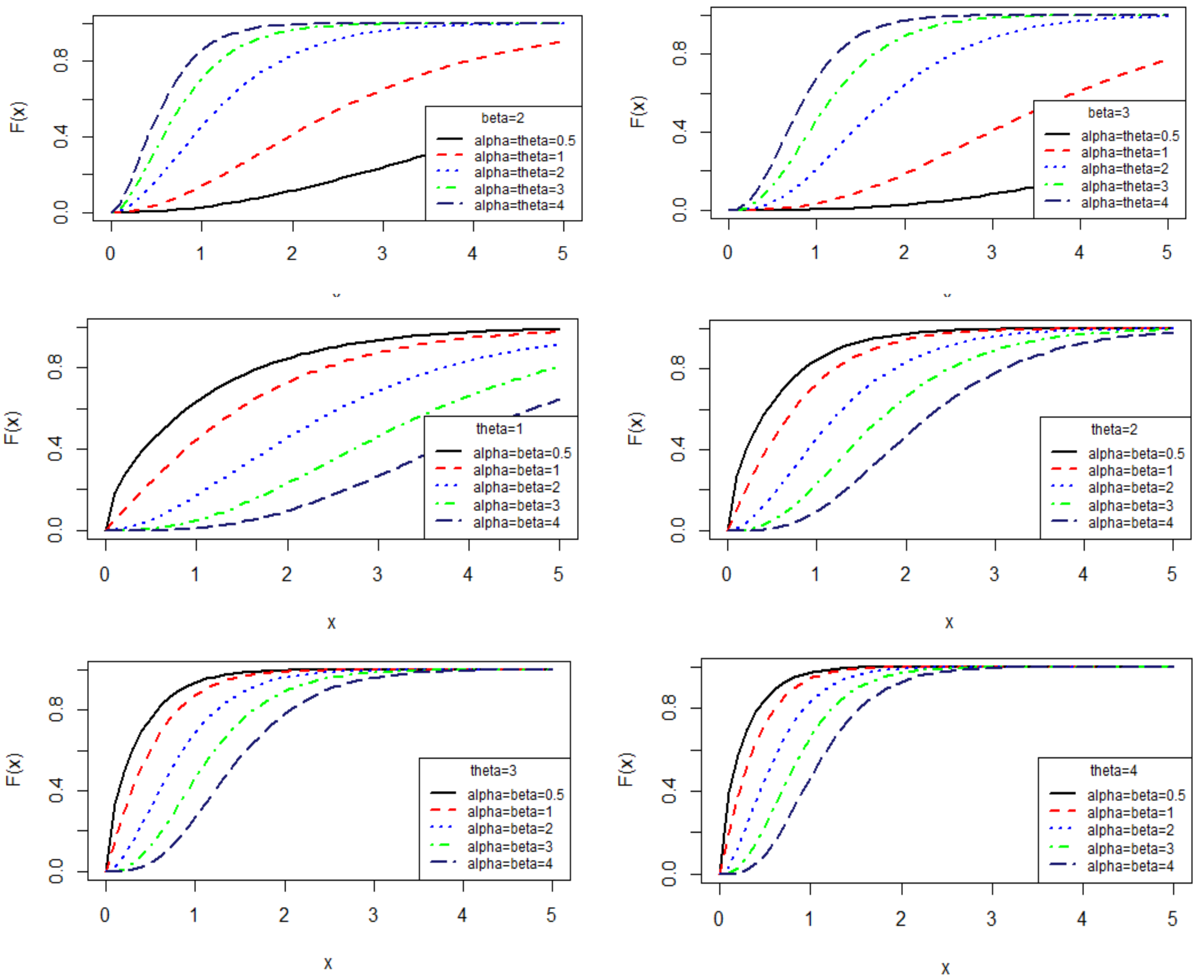


Figure 2. Behaviour pf the cdf of WQLD for varying values of parameters

3. Statistical Characteristics

The r th moment about origin μ_r' of the WQLD) (2.2) can be obtained as

$$\mu_r' = E\left(X^r\right) = \frac{\Gamma(\beta+r)}{\Gamma(\beta)}\,\frac{\alpha+\beta+r}{\theta^r(\alpha+\beta)};\; r=1,2,3,... \tag{3.1}$$

Substituting $r=1,2,3,$ and 4 in (3.1), the first four moments about origin of the WQLD are obtained as

$$\mu_1' = \frac{\beta(\alpha+\beta+1)}{\theta(\alpha+\beta)}$$

$$\mu_2' = \frac{\beta(\beta+1)(\alpha+\beta+2)}{\theta^2(\alpha+\beta)}$$

$$\mu_3' = \frac{\beta(\beta+1)(\beta+2)(\alpha+\beta+3)}{\theta^3(\alpha+\beta)}$$

$$\mu_4' = \frac{\beta(\beta+1)(\beta+2)(\beta+3)(\alpha+\beta+4)}{\theta^4(\alpha+\beta)}$$

Again using relationship $\mu_r = E\left(Y-\mu_1'\right)^r = \sum_{k=0}^{r}\binom{r}{k}\mu_k'\left(-\mu_1'\right)^{r-k}$ between central moments and moments about origin, the central moments of WQLD are obtained as

$$\mu_2 = \frac{\beta\left\{\beta^2+(2\alpha+1)\beta+\alpha(\alpha+2)\right\}}{\theta^2(\alpha+\beta)^2}$$

$$\mu_3 = \frac{2\beta\left\{\beta^3+(3\alpha+1)\beta^2+3(\alpha+1)\alpha\beta+\alpha^2(\alpha+3)\right\}}{\theta^3(\alpha+\beta)^3}$$

$$\mu_4 = \frac{3\beta\left\{\begin{array}{l}\beta^5+4(\alpha+1)\beta^4+\left(6\alpha^2+16\alpha+3\right)\beta^3+2\left(2\alpha^2+11\alpha+6\right)\alpha\beta^2\\+\left(\alpha^2+12\alpha+16\right)\alpha^2\beta+2\alpha^3(\alpha+4)\end{array}\right\}}{\theta^4(\alpha+\beta)^4}$$

The expressions for coefficient of variation (C.V.) coefficient of skewness $\left(\sqrt{\beta_1}\right)$, coefficient of kurtosis (β_2), and index of dispersion (γ) of the WQLD (2.1) are thus obtained as

$$C.V. = \frac{\sigma}{\mu_1'} = \frac{\sqrt{\beta^2+(2\alpha+1)\beta+\alpha(\alpha+2)}}{\sqrt{\beta}(\alpha+\beta+1)}$$

$$\sqrt{\beta_1} = \frac{\mu_3}{\mu_2^{3/2}} = \frac{2\left\{\beta^3+(3\alpha+1)\beta^2+3(\alpha+1)\alpha\beta+\alpha^2(\alpha+3)\right\}}{\sqrt{\beta}\left[\left\{\beta^2+(2\alpha+1)\beta+\alpha(\alpha+2)\right\}\right]^{3/2}}$$

$$\beta_2 = \frac{\mu_4}{\mu_2^2} = \frac{3\left\{\begin{array}{l}\beta^5+4(\alpha+1)\beta^4+\left(6\alpha^2+16\alpha+3\right)\beta^3+2\left(2\alpha^2+11\alpha+6\right)\alpha\beta^2\\+\left(\alpha^2+12\alpha+16\right)\alpha^2\beta+2\alpha^3(\alpha+4)\end{array}\right\}}{\beta\left[\left\{\beta^2+(2\alpha+1)\beta+\alpha(\alpha+2)\right\}\right]^2}$$

$$\gamma = \frac{\sigma^2}{\mu_1'} = \frac{\beta^2+(2\alpha+1)\beta+\alpha(\alpha+2)}{\theta(\alpha+\beta)(\alpha+\beta+1)}$$

The nature of the coefficient of variation, coefficient of skewness, coefficient of kurtosis and index of dispersion of WQLD for varying values of the parameters have been shown in figure 3.

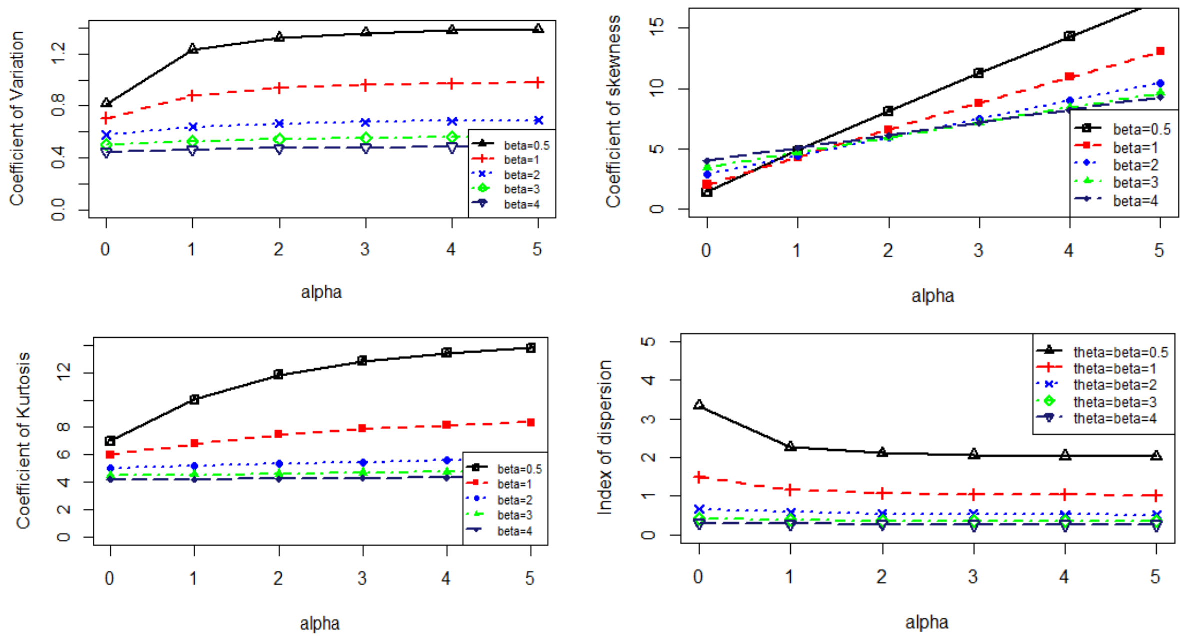


Figure 3. Nature of coefficient of variation, coefficient of skewness, coefficient of kurtosis and index of dispersion of WQLD for varying values of the parameters

4. Reliability Measures

4.1. Hazard Rate Function

The hazard (or failure) rate function, $h(x;\theta,\alpha,\beta)$ of WQLD is thus obtained as

$$h(x;\theta,\alpha,\beta)=\frac{f(x;\theta,\alpha,\beta)}{S(x;\theta,\alpha,\beta)}=\frac{\theta^{\beta}x^{\beta-1}(\alpha+\theta x)e^{-\theta x}}{(\alpha+\beta)\Gamma(\beta,\theta x)+(\theta x)^{\beta}e^{-\theta x}}\ ;x>0$$

The shapes of the hazard rate function, $h(x;\theta,\alpha,\beta)$ of the WQLD for varying values of the parameters are shown in the figure 4.

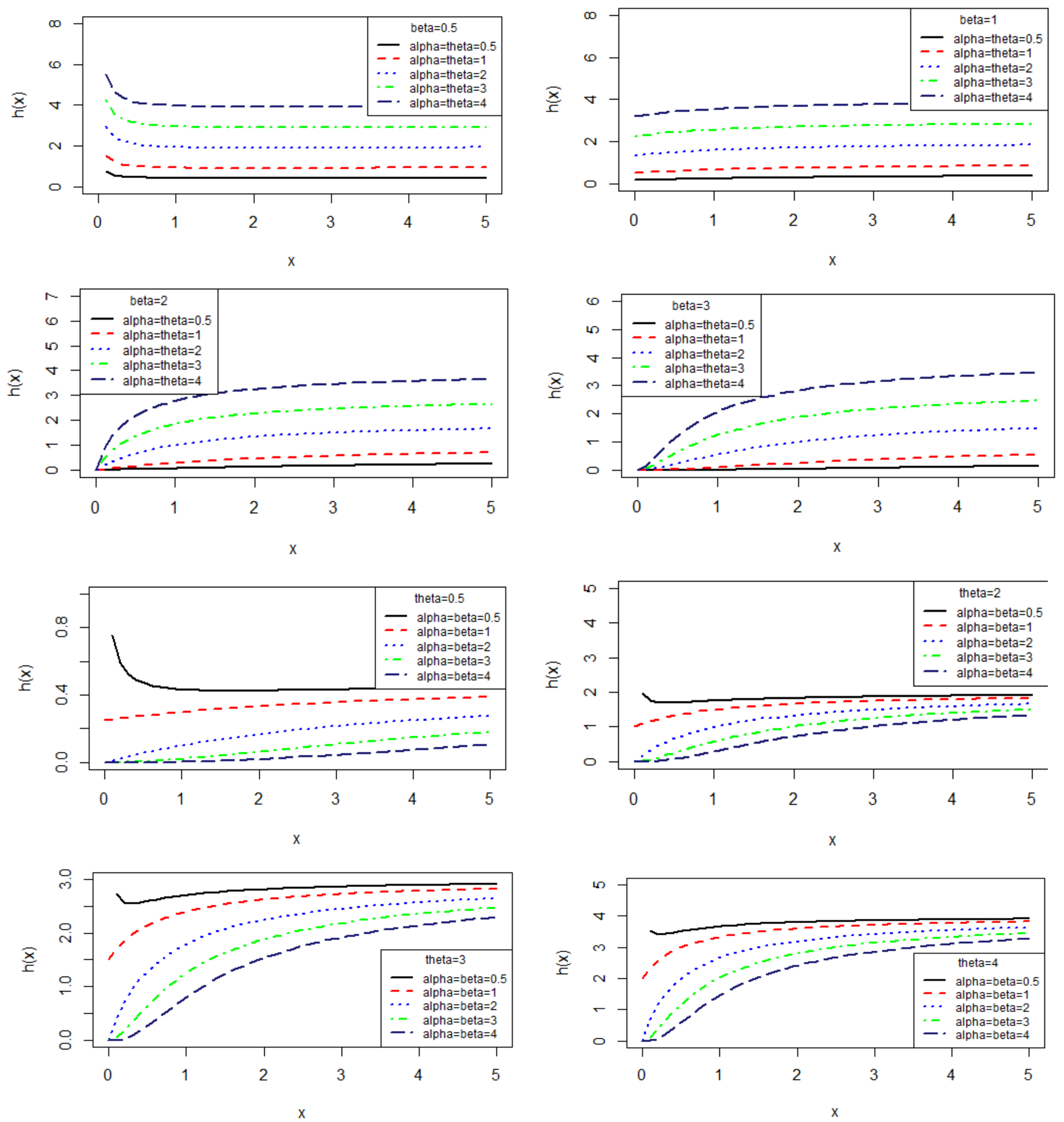


Figure 4. Nature of the hazard rate function, $h(x)$ of the WQLD for varying values of the parameters

4.2. Mean Residual Life Function

The mean residual life function $m(x)=E(X-x \mid X>x)$ of the WQLD can be obtained as

$$m(x;\theta,\alpha,\beta)=\frac{1}{S(x;\theta,\alpha,\beta)}\int_x^\infty t\, f_4(t;\theta,\alpha,\beta)\,dt-x$$

$$=\frac{\theta^\beta}{(\alpha+\beta)\Gamma(\beta,\theta x)+(\theta x)^\beta e^{-\theta x}}\int_x^\infty t^\beta(\alpha+\beta t)e^{-\theta t}dt-x$$

$$=\frac{\theta^\beta}{(\alpha+\beta)\Gamma(\beta,\theta x)+(\theta x)^\beta e^{-\theta x}}\left[\alpha\int_x^\infty e^{-\theta t}t^\beta\,dt+\theta\int_x^\infty e^{-\theta t}\,t^{\beta+1}\,dt\right]-x$$

$$=\frac{\theta^\beta}{(\alpha+\beta)\Gamma(\beta,\theta x)+(\theta x)^\beta e^{-\theta x}}\left[\begin{array}{l}\dfrac{\alpha\left\{e^{-\theta x}(\theta x)^\beta+\beta\Gamma(\beta,\theta x)\right\}}{\theta^{\beta+1}}\\+\dfrac{\theta\left\{e^{-\theta x}(\theta x)^\beta(\theta x+\beta+1)+\beta(\beta+1)\Gamma(\beta,\theta x)\right\}}{\theta^{\beta+2}}\end{array}\right]-x$$

$$=\frac{(\theta x)^\beta(\theta x+\alpha+\beta+1)e^{-\theta x}+\{\alpha\beta+\beta(\beta+1)\}\Gamma(\beta,\theta x)}{\theta\left\{(\alpha+\beta)\Gamma(\beta,\theta x)+(\theta x)^\beta e^{-\theta x}\right\}}-x.$$

$$=\frac{(\theta x)^\beta(\alpha+\beta+1)e^{-\theta x}+\{\alpha\beta+\beta(\beta+1)-\theta x(\alpha+\beta)\}\Gamma(\beta,\theta x)}{\theta\left\{(\alpha+\beta)\Gamma(\beta,\theta x)+(\theta x)^\beta e^{-\theta x}\right\}}$$

The shapes of the mean residual life function, $m(x)$ of the WQLD for varying values of the parameters are shown in figure 5.

5. Maximum Likelihood Estimation

Let $(x_1,x_2,x_3,\ldots,x_n)$ be a random sample of size n from WQLD. The natural log likelihood function of WQLD can be expressed as

$$\ln L=n\left[\beta\ln\theta-\ln(\alpha+\beta)-\ln\Gamma(\beta)-\theta\bar{x}\right]+(\beta-1)\sum_{i=1}^n\ln(x_i)+\sum_{i=1}^n\ln(\alpha+\theta x_i)$$

The maximum likelihood estimates (MLEs) $(\hat{\theta},\hat{\alpha},\hat{\beta})$ of parameters (θ,α,β) of WQLD are the solution of the following nonlinear equations

$$\frac{\partial\ln L}{\partial\theta}=\frac{n\beta}{\theta}-n\bar{x}+\sum_{i=1}^n\frac{x_i}{\alpha+\theta x_i}=0$$

$$\frac{\partial\ln L}{\partial\alpha}=-\frac{n}{\alpha+\beta}+\sum_{i=1}^n\frac{1}{\alpha+\theta x_i}=0$$

$$\frac{\partial\ln L}{\partial\beta}=n\ln\theta-\frac{n}{\alpha+\beta}-n\psi(\beta)+\sum_{i=1}^n\ln x_i=0,$$

where $\bar{x}$ is the sample mean and $\psi(\beta)=\frac{d}{d\beta}\ln\Gamma(\beta)$ is the digamma function.

These three natural log likelihood equations do not seem to be solved directly, because they cannot be expressed in closed forms. The (MLE's) $(\hat{\theta},\hat{\alpha},\hat{\beta})$ of (θ,α,β) can be computed directly by solving the natural log likelihood equation using

Newton-Raphson iteration using R-software till sufficiently close values of $(\hat{\theta}, \hat{\alpha}, \hat{\beta})$ are obtained.

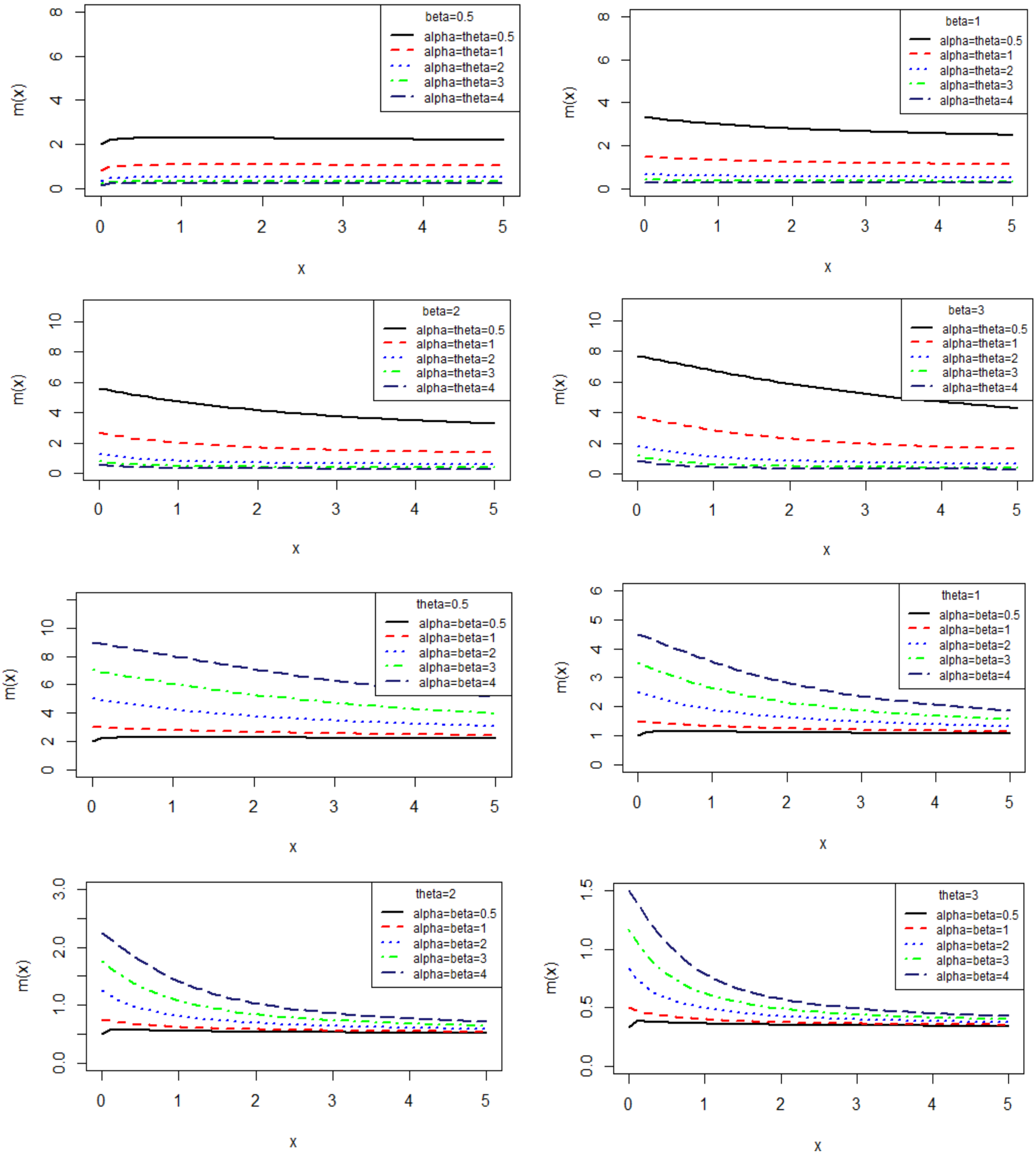


Figure 5. Nature of the mean residual life function, $m(x)$ of the WQLD for varying values of the parameters

6. Applications

The applications of the WQLD have been explained with two real lifetime datasets from engineering which are available in Lawless (2003, pp. 204 and 263). The two datasets are as follows

Data Set 1: The following data set represents the failure times (in minutes) for a sample of 15 electronic components in an accelerated life test, Lawless (2003, pp. 204)

1.4 5.1 6.3 10.8 12.1 18.5 19.7 22.2 23.0 30.6 37.3 46.3 53.9 59.8 66.2

Data Set 2: The following data set represents the number of cycles to failure for 25 100-cm specimens of yarn, tested at a particular strain level, Lawless (2003, pp. 263)

15 20 38 42 61 76 86 98 121 146 149 157 175 176
180 180 198 220 224 251 264 282 321 325 653

Table 1. The pdf and the cdf of the fitted distributions

Distributions		pdf and cdf
GLD	pdf	$f(x;\theta,\alpha,\beta)=\frac{\theta^{\alpha+1}}{(\beta+\theta)}\frac{x^{\alpha-1}}{\Gamma(\alpha+1)}(\alpha+\beta x)e^{-\theta x};x>0,\theta>0,\alpha>0,\beta>0$
	cdf	$F(x;\theta,\alpha,\beta)=1-\frac{\alpha(\beta+\theta)\Gamma(\alpha,\theta x)+\beta(\theta x)^{\alpha}e^{-\theta x}}{(\beta+\theta)\Gamma(\alpha+1)};x>0,\theta>0,\alpha>0,\beta>0$
GED	pdf	$f(x;\theta,\alpha)=\theta\alpha\left(1-e^{-\theta x}\right)^{\alpha-1}e^{-\theta x};x>0,\theta>0,\alpha>0$
	cdf	$F(x;\theta,\alpha)=\left(1-e^{-\theta x}\right)^{\alpha};x>0,\theta>0,\alpha>0$
Weibull	pdf	$f(x;\theta,\alpha)=\theta\alpha x^{\alpha-1}e^{-\theta x^{\alpha}};x>0,\theta>0,\alpha>0$
	cdf	$F(x;\theta,\alpha)=1-e^{-\theta x^{\alpha}};x>0,\theta>0,\alpha>0$
Gamma	pdf	$f(x;\theta,\alpha)=\frac{\theta^{\alpha}}{\Gamma(\alpha)}e^{-\theta x}x^{\alpha-1};\ x>0,\theta>0,\alpha>0$
	cdf	$F(x;\theta,\alpha)=1-\frac{\Gamma(\alpha,\theta x)}{\Gamma(\alpha)};x>0,\theta>0,\alpha>0$
Lognormal	pdf	$f(x;\theta,\alpha)=\frac{1}{\sqrt{2\pi}\alpha x}e^{-\frac{1}{2}\left(\frac{\log x-\theta}{\alpha}\right)^{2}};x>0,\theta>0,\alpha>0$
	cdf	$F(x;\theta,\alpha)=\phi\left(\frac{\log x-\theta}{\alpha}\right);x>0,\theta>0,\alpha>0$
Lindley	pdf	$f(x;\theta)=\frac{\theta^{2}}{\theta+1}(1+x)e^{-\theta x};x>0,\theta>0$
	cdf	$F(x;\theta)=1-\left[1+\frac{\theta x}{\theta+1}\right]e^{-\theta x};x>0,\theta>0$
Exponential	pdf	$f(x;\theta)=\theta e^{-\theta x};\ x>0,\theta>0$
	cdf	$F(x;\theta)=1-e^{-\theta x};x>0,\theta>0$

Table 2. ML estimates and summary of goodness of fit for dataset 1

Distributions	ML Estimates	Std Errors	$-2\log L$	K-S	p-value
WQLD	$\hat{\theta}=0.06421$	0.02139	128.16	0.095	0.9967
	$\hat{\alpha}=0.94171$	3.55165			
	$\hat{\beta}=0.81591$	0.81591			
GLD	$\hat{\theta}=0.06415$	0.02132	128.16	0.095	0.9967
	$\hat{\alpha}=1.20258$	0.81310			
	$\hat{\beta}=0.08329$	0.27068			

Distributions	ML Estimates	Std Errors	$-2\log L$	K-S	p-value
WLD	$\hat{\theta}=0.05900$	0.01988	128.40	0.098	0.9795
	$\hat{\alpha}=0.70275$	0.41991			
QLD	$\hat{\theta}=0.06226$	0.01720	128.21	0.095	0.9961
	$\hat{\alpha}=0.39833$	0.68811			
GED	$\hat{\theta}=0.04529$	0.01372	128.47	0.108	0.9868
	$\hat{\alpha}=1.44347$	0.51301			
Weibull	$\hat{\theta}=0.01190$	0.01124	128.04	0.098	0.9950
	$\hat{\alpha}=1.30586$	0.24925			
Gamma	$\hat{\theta}=0.05235$	0.02066	128.37	0.100	0.9570
	$\hat{\alpha}=1.44219$	0.47771			
Lognormal	$\hat{\theta}=2.93059$	0.26472	131.23	0.161	0.9135
	$\hat{\alpha}=1.02527$	0.18718			
Lindley	$\hat{\theta}=0.07022$	0.01283	128.81	0.1103	0.9830
Exponential	$\hat{\theta}=0.03631$	0.00936	129.47	0.156	0.8069

Table 3. ML estimates and summary of goodness of fit for dataset 2

Distributions	ML Estimates	Std Errors	$-2\log L$	K-S	p-value
WQLD	$\hat{\theta}=0.01187$	0.00302	304.73	0.128	0.8071
	$\hat{\alpha}=1.03661$	3.81055			
	$\hat{\beta}=1.52322$	0.87441			
GLD	$\hat{\theta}=0.01018$	0.00301	304.88	0.137	0.7370
	$\hat{\alpha}=0.81866$	0.48587			
	$\hat{\beta}=3.97404$	63.12878			
WLD	$\hat{\theta}=0.01036$	0.00294	304.90	0.137	0.7351
	$\hat{\alpha}=0.85499$	0.45329			
QLD	$\hat{\theta}=0.01082$	0.00196	304.91	0.132	0.7690
	$\hat{\alpha}=0.08102$	0.26179			
GED	$\hat{\theta}=0.00817$	0.00176	304.98	0.145	0.6641
	$\hat{\alpha}=1.88642$	0.54466			
Weibull	$\hat{\theta}=0.00256$	0.00068	306.57	0.697	0.0000
	$\hat{\alpha}=1.14807$	0.05897			
Gamma	$\hat{\theta}=0.01008$	0.00294	304.87	0.138	0.7210
	$\hat{\alpha}=1.79528$	0.45901			
Lognormal	$\hat{\theta}=4.87956$	0.17468	308.16	0.155	0.7128
	$\hat{\alpha}=0.87344$	0.12351			
Lindley	$\hat{\theta}=0.01118$	0.00157	305.01	0.129	0.7980
Exponential	$\hat{\theta}=0.00565$	0.00109	309.18	0.202	0.2571

The goodness of fit of WQLD has been compared with several distributions including generalized Lindley distribution (GLD) introduced by Zakerzadeh and Dolati (2009), generalized exponential distribution (GED) proposed by Gupta and Kundu (1999), weighted Lindley distribution (WLD) suggested by Ghitany *et al* (2011), Quasi Lindley distribution (QLD) proposed by Shanker and Mishra (2013), Weibull distribution introduced by Weibull (1951), gamma distribution, lognormal distribution, Lindley distribution, and exponential distribution. The pdf and the cdf of these distributions are given in table 1. The maximum likelihood estimates of parameters, standard errors of parameters, $-2\ln L$, K-S (Kolmogorov-Smirnov Statistics) and p-values of the fitted distributions for datasets 1 and 2 have been shown in tables 2 and 3 respectively. The best distribution is the distribution which corresponds to the lower value of $-2\ln L$ and K-S.

It is obvious from the goodness of fit of WQLD that it gives a better fit as compared to several one parameter, two-parameter and three-parameter lifetime distributions and hence it can be considered as an important three-parameter lifetime distribution in statistics literature.

7. Concluding Remarks

A three-parameter weighted quasi Lindley distribution (WQLD), which includes two-parameter quasi Lindley distribution, weighted Lindley distribution and gamma distribution and one parameter Lindley and exponential distributions as particular cases, has been proposed in the present paper. Some statistical properties of WQLD including moments and moments based measures, hazard rate function, and mean residual life function have been discussed. Maximum likelihood estimation has been discussed for estimating the parameters of the proposed distribution. Applications of the distribution have explained with two real failure time data.

ACKNOWLEDGEMENTS

Authors are grateful to the editor-in-chief and the anonymous reviewer for constructive comments which enhanced the quality and the presentation of the paper.

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