International Journal of Statistics and Applications 2018, 8(6): 323-331
DOI: 10.5923/j.statistics.20180806.06

Joint Analysis of Several Experiments Conducted via Orthogonal Sudoku Design of Odd Order

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Abstract Researchers conduct the same experiments at different environments or periods (seasons) with similar treatments purposely to study the interaction between the treatments and environments or between the treatments and periods (seasons). The interest is to carry out joint analysis of these multi-environment experiments, instead of analyzing each experiment separately. However, Joint analysis of these experiments when conducted using orthogonal (Graeco) Sudoku square design is still missing in the literature. This paper proposed a method of obtaining joint analysis of many experiments carried out using orthogonal Sudoku square design of odd order in which experimental treatments are similar for each experiment. A numerical example was presented to explain the proposed method.

Keywords Joint analysis, Sudoku square design, Graeco Sudoku design, Multi-environment experiments

1. Introduction

Many researchers conduct field experiments by randomized block design, Latin square design, balanced incomplete block design or Youden design over different environments such as locations or periods (seasons). Analysis of data from these experiments is usually carried out through joint analysis of all the experiments, instead of individual experiment. It was reported in [1] that one of the advantages of multi-environment analysis is that it increases the accuracy of evaluation (hence the accuracy of selection). There are two main reasons for multiple-environments trials as discussed in ([2], [3]). The reasons are neither mutually exclusive nor all inclusive. The first reason is to estimate the effects and comparative effects of treatments for specific environments or to estimate the effects of comparative effects of treatments over broad population of environments [2]. The second reason was to estimate the consistency of treatment effects for specific environments over broad populations of environments as in [3].

In [6] the method of joint intra-block analysis of a group of experiments in complete randomized block, where some common treatments applied to all the experiments, series of examples were given, separate tables of analysis and table of the combined analysis were also given. They further gave the condition that must be met before combined analysis combined analysis can be carried out, as such, if the residual variance estimates from separate analyses are not too different a joint analysis can be carried out for the whole set of the experiments. Subsequently [11] following the same approach and obtained the joint analysis of balanced incomplete block designs with some common treatments applied to all the experiments of the multiple-environment, he also obtained the methods of adjusted treatment of sum of square and illustration was given.

However, [5] discussed the combined analysis of Youden squares and Latin square designs with some common treatments as an extension of the work by [11]. [5] also said that the method can be used to analysis data from combined data set of Youden squares when some treatments are common while that of [6] can be used for combined analysis of a Latin square when some treatments are also common to all the experiments. The work of [9] discussed the appropriate analyses of combined experiments based on the constraint that the effects of random interactions with the Anova table and spilt-plot experiment combined over location or year. In [12], method of combined analysis when treatments are used to represents levels of quantitative factor but differ among experiments is presented. Multiple regression analysis was used when a continuous variable represents treatment levels, classification variables represents experiments and product of the continuous and classification variables represent differences among experiments. This assumption reflects a tradition in the analysis of fixed effects that, in most cases, is not appropriate for a random effect ([7], [10]). Recently, [4] presented a paper on combined analysis of Sudoku square designs with common treatments. The work in [4], also discussed combined analysis of data from multi- environment

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Published online at http://journal.sapub.org/statistics

experiments using Sudoku square designs.

Sudoku square design consists of treatments (Latin letters) which are arranged in a square array in such a way that each row, column and sub-square contains each of the treatments exactly once see [8]. In addition, [8] observed that Sudoku square design go beyond Latin square design with additional source of variation called box effect. An extension of the work of [8] is presented in [14], with additional terms in the models and in the source of variation namely; Row-block effect and column-block effect. For instance, [14] considered Sudoku of order $k = m^2$ and gave detail analysis with illustrative examples and the application of Sudoku square to agricultural experiment. In addition, [13] extended the work of [14] to the construction of orthogonal Sudoku square design with additional source of variation i.e treatments (Greek and Latin) effects. In Line with [13], two Sudoku squares of the same order are said to be orthogonal Sudoku squares, if we superimpose the two Sudoku squares then one may get a pair of treatments (Latin and Greek letters) in each row, column and sub-square only once. The work of [14] was used by [4] to carry out combined analysis of several experiments conducted in different environments using of Sudoku square design. However the Sudoku square design presented by [4] does not contains pair of treatments labeled with Greek and Latin letters occurring only once in each row, column and sub-square of the design. That is, the analysis in [4] is not on the orthogonal Sudoku square design.

This paper proposed joint analysis of multi-environment experiments when implemented by mean of orthogonal Sudoku square design of odd order for which experimental treatments common in each environment or experiment. However, only orthogonal Sudoku square of order nine is considered for the study.

2. Method of Analysis

Assume that we have two environments of which experiments were conducted using orthogonal Sudoku square design (see Fig.1) each with k treatments with each treatment occurs only once in each (row, column or sub-block) such that $k = m^2$ where $k = 9$ and m (number of row-block or column-block).

In this study, four models of orthogonal Sudoku square designs discussed by [13] were modified and combined analysis from each were discussed under "g" environments. For all the models considered in this study, it is assumed that all effects are fixed.

A_α	B_β	C_γ	D_λ	E_θ	F_μ	G_π	H_ϕ	I_δ
D_π	E_ϕ	F_δ	G_α	H_β	I_γ	A_λ	B_θ	C_μ
G_λ	H_θ	I_μ	A_π	B_ϕ	C_δ	D_α	E_β	F_γ
E_δ	F_π	D_ϕ	H_γ	I_α	G_β	B_μ	C_λ	A_θ
H_μ	I_λ	G_θ	B_δ	C_π	A_ϕ	E_γ	F_α	D_β
B_γ	C_α	A_β	E_μ	F_λ	D_θ	H_δ	I_π	G_ϕ
I_θ	G_μ	H_λ	C_ϕ	A_δ	B_π	F_β	D_γ	E_α
C_β	A_γ	B_α	F_θ	D_μ	E_λ	I_ϕ	G_δ	H_π
F_ϕ	D_δ	E_π	I_β	G_γ	H_α	C_θ	A_μ	B_λ

Figure 1. Orthogonal Sudoku square of order 9

Model 1: The modification of Type I of [13] is as follows:

The model assume row, column and treatment effects as in complete Latin square designs. In addition the row-block, column-block and square effects are in Sudoku square design:

$$Y_{ij\,(lnpqw\,)x} = \mu + \theta_x + \alpha_{ix} + \beta_{jx} + \tau_w + \tau_n + \gamma_{lx} + C_{px} + S_{qx} + (\tau\theta\,)_{wx} + (\tau\theta\,)_{nx} + \varepsilon_{ij\,(lnpqw\,)x}$$

Where $i, j = 1,2,3;\ w, l, n, p, q = 1, 2, \ldots, m^2;\ m = 3;\ k = m^2;\ and\ x = 1, 2, \ldots, g$

Where $Y_{ij\,(lpqw\,)x}$ = Observation on experimental unit.

$\mu =$ Overall mean,

$\alpha_{ix} = i^{th}$ Row block effect in xth environment experiment,

$\beta_{jx} = j^{th}$ Column block effect in xth environment experiment,

$\tau_w =$ Latin letter w^{th} Treatment effect,

$\tau_n =$ Greek letter n^{th} Treatment effect,

$\gamma_{lx} = l^{th}$ Row effect in xth environment experiment,

$c_{px} = p^{th}$ Column effect in xth environment experiment,

$s_{qx} = q^{th}$ Sub-Square effect in xth environment experiment,

$\theta_x = x^{th}$ Environment-experiment,

$(\tau\theta)_{wx} =$ Interaction between w^{th} Latin Treatment and xth environment experiment,

$(\tau\theta)_{nx} =$ Interaction between n^{th} Greek Treatment and xth environment experiment,

$\varepsilon_{ij\,(lnpqw\,)x} =$ Error component with mean zero and variance σ^2

$$G = \sum_{l=1}^{m^2}\sum_{p=1}^{m^2} Y_{lp} \ and \ N = gk^2, \quad TSS = \sum_{l=1}^{m^2}\sum_{p=1}^{m^2}\sum_{x=1}^{g} Y^2{}_{lpx} - \frac{G^2}{gk^2}, \quad SST_w = \sum_{w=1}^{m^2}\frac{\tau_w^2}{k} - \frac{G^2}{gk^2}$$

$$SST_n = \sum_{n=1}^{m^2}\frac{\tau_n^2}{k} - \frac{G^2}{gk^2}, \quad SSRB = \sum_{i=1}^{m}\sum_{x=1}^{g}\frac{RB_{ix}^2}{gm^3} - \frac{G^2}{gk^2}, \quad SSCB = \sum_{j=1}^{m}\sum_{x=1}^{g}\frac{CB_{jx}^2}{gm^3} - \frac{G^2}{gk^2}$$

$$SSR = \sum_{l=1}^{m^2}\sum_{x=1}^{g}\frac{R_{lx}^2}{gk} - \frac{G^2}{gk^2}, \quad SSC = \sum_{p=1}^{m^2}\sum_{x=1}^{g}\frac{C_{px}^2}{gk} - \frac{G^2}{gk^2}, \quad SSS = \sum_{q=1}^{m^2}\sum_{x=1}^{g}\frac{S_{qx}^2}{gk} - \frac{G^2}{gk^2}$$

$$SSEE = \sum_{x=1}^{g}\frac{\theta_x^2}{k^2} - \frac{G^2}{gk^2}, \quad SS_{EXL} = \sum_{w=1}^{m^2}\sum_{x=1}^{g}\frac{\tau\theta_{wx}^2}{gk} - \sum_{w=1}^{m^2}\frac{\tau_w^2}{k} - \sum_{x=1}^{g}\frac{\theta_x^2}{k^2} + \frac{G^2}{gk^2}, \quad SS_{EXG} = \sum_{n=1}^{m^2}\sum_{x=1}^{g}\frac{\tau\theta_{nx}^2}{gk} - \sum_{n=1}^{m^2}\frac{\tau_n^2}{k} - \sum_{x=1}^{g}\frac{\theta_x^2}{k^2} + \frac{G^2}{gk^2}$$

$$ESS = \sum_{l=1}^{m^2}\sum_{p=1}^{m^2} Y^2{}_{lp} + \sum_{x=1}^{g}\frac{\theta_x^2}{k^2} - \sum_{i=1}^{m}\sum_{x=1}^{g}\frac{RB_{ix}^2}{gm^3} - \sum_{j=1}^{m}\sum_{x=1}^{g}\frac{CB_{jx}^2}{gm^3} - \sum_{l=1}^{m^2}\sum_{x=1}^{g}\frac{R_{lx}^2}{gk} - \sum_{p=1}^{m^2}\sum_{x=1}^{g}\frac{C_{px}^2}{gk} - \sum_{q=1}^{m^2}\sum_{x=1}^{g}\frac{S_q^2 x}{gk} - \sum_{w=1}^{m^2}\sum_{x=1}^{g}\frac{\tau\theta_{wx}^2}{gk} - \sum_{n=1}^{m^2}\sum_{x=1}^{g}\frac{\tau\theta_{nx}^2}{gk} + \frac{5G^2}{gk^2}$$

Table 1. ANOVA Table for Combined Graeco Sudoku square designs for model 1

Source	Sum of squares	Degrees of Freedom	Mean square
Environments	$SSEE$	$g-1$	$MSEE = {}^{SSEE}/_{df}$
Treatments (Latin)	SSt_w	$k-1$	$MST_w = {}^{SSt_w}/_{df}$
Treatments (Greek)	SSt_n	$k-1$	$MST_n = {}^{SSt_n}/_{df}$
Row Blocks	$SSbr$	$g(m-1)$	$MSBR = {}^{SSbr}/_{df}$
Column Blocks	$SSbc$	$g(m-1)$	$MSBC = {}^{SSbc}/_{df}$
Row	SSr	$g(k-1)$	$MSR = {}^{SSr}/_{df}$
Column	SSc	$g(k-1)$	$MSC = {}^{SSc}/_{df}$
Sub-Squares (Boxes)	SSs	$g(k-1)$	$MSS = {}^{SSs}/_{df}$
Env x treatments (Latin)	SS_{EXL}	$(g-1)(k-1)$	$MS_{EXL} = {}^{SS_{EXL}}/_{df}$
Env x treatments (Greek)	SS_{EXG}	$(g-1)(k-1)$	$MS_{EXG} = {}^{SS_{EXG}}/_{df}$
Error	Sse	$g[(k-2)(k-3)-2m]$	$MSE = {}^{Sse}/_{df}$
Total	SST	gk^2-1	

Model 2: The modified linear model Type II

The model assumed that row effects are nested in the row-block effects and the column effects are nested in the column-block effects.

$$Y_{ij\,(lnpqw\,)x} = \mu + \alpha_{ix} + \beta_{jx} + \tau_w + \tau_n + \gamma(\alpha)_{l(i)x} + c(\beta)_{p(j)x} + s_{qx} + \theta_x + (\tau\theta)_{wx} + (\tau\theta)_{nx} + e_{ij\,(lnpqw\,)x}$$

Where $i, j, l, p = 1, 2, 3$; $n, q, w = 1, 2, \ldots, m^2$ and $x = 1, 2, \ldots, g$.

Where $Y_{ij\,(lnpqw\,)x}$ = Observation on experimental unit.

$\mu =$ Overall mean

$\alpha_{ix} = i^{th}$ Row block effect in xth environment experiment,

$\beta_{jx} = j^{th}$ Column block effect in xth environment experiment,

$\tau_w =$ Latin letter w^{th} Treatment effect,

$\tau_n =$ Greek letter n^{th} Treatment effect,

$\gamma(\alpha)_{l(i)x} = l^{th}$ Row effect nested in i^{th} block (row) in xth environment experiment,

$C(\beta)_{p(j)x} = p^{th}$ Column effect nested in j^{th} block (column) in xth environment experiment,

$S_{qx} = q^{th}$ Sub-Square effect in xth environment experiment,

$\theta_x = xth$ Environment-experiment,

$(\tau\theta\,)_{wx} =$ Interaction between w^{th} Latin Treatment and xth environment experiment,

$(\tau\theta)_{nx} =$ Interaction between n^{th} Greek Treatment and xth environment experiment,
$\varepsilon_{ij\,(lnpqw)x} =$ Error component with mean zero and variance σ^2

$$TSS = \sum_{i=1}^{m^2}\sum_{j=1}^{m^2}\sum_{x=1}^{g} Y^2{}_{ijx} - \frac{G^2}{gk^2}, \quad SST_w = \sum_{w=1}^{m^2}\frac{\tau_w^2}{k} - \frac{G^2}{gk^2}, \quad SST_n = \sum_{n=1}^{m^2}\frac{\tau_n^2}{k} - \frac{G^2}{gk^2}$$

$$SSRB = \sum_{i=1}^{m}\sum_{x=1}^{g}\frac{RB_{ix}^2}{gm^3} - \frac{G^2}{gk^2}, \quad SSCB = \sum_{j=1}^{m}\sum_{x=1}^{g}\frac{CB_{jx}^2}{gm^3} - \frac{G^2}{gk^2}, \quad SSR = \sum_{l=1}^{m}\sum_{l=1}^{m}\sum_{x=1}^{g}\frac{R_{l(i)x}^2}{gk} - \sum_{i=1}^{m}\sum_{x=1}^{g}\frac{RB_{ix}^2}{gm^3}$$

$$SSC = \sum_{j=1}^{m}\sum_{p=1}^{m}\sum_{x=1}^{g}\frac{C_{p(j)x}^2}{gk} - \sum_{i=1}^{m}\sum_{x=1}^{g}\frac{CB_{jx}^2}{gm^3}, \quad SSS = \sum_{q=1}^{m^2}\sum_{x=1}^{g}\frac{S_{qx}^2}{gk} - \frac{G^2}{gk^2}, \quad SSEE = \sum_{x=1}^{g}\frac{\theta_x^2}{k^2} - \frac{G^2}{gk^2}$$

$$SS_{EXL} = \sum_{w=1}^{m^2}\sum_{x=1}^{g}\frac{\tau\theta_{wx}^2}{gk} - \sum_{w=1}^{m^2}\frac{\tau_w^2}{k} - \sum_{x=1}^{g}\frac{\theta_x^2}{k^2} + \frac{G^2}{gk^2}, \quad SS_{EXG} = \sum_{n=1}^{m^2}\sum_{x=1}^{g}\frac{\tau\theta_{nx}^2}{gk} - \sum_{n=1}^{m^2}\frac{\tau_n^2}{k} - \sum_{x=1}^{g}\frac{\theta_x^2}{k^2} + \frac{G^2}{gk^2}$$

$$ESS = \sum_{i=1}^{m^2}\sum_{j=1}^{m^2}\sum_{x=1}^{g} Y^2{}_{ijx} + \sum_{x=1}^{g}\frac{\theta_x^2}{k^2} - \sum_{l=1}^{m}\sum_{l}^{m}\sum_{x=1}^{g}\frac{R_{l(i)x}^2}{gk} - \sum_{j=1}^{m}\sum_{p}^{m}\sum_{x=1}^{g}\frac{C_{p(j)x}^2}{gk} - \sum_{q=1}^{m^2}\sum_{x=1}^{g}\frac{S_q^2 x}{gk} - \sum_{w=1}^{m^2}\sum_{x=1}^{g}\frac{\tau\theta_{wx}^2}{gk} - \sum_{n=1}^{m^2}\sum_{x=1}^{g}\frac{\tau\theta_{nx}^2}{gk} + \frac{3G^2}{gk^2}$$

Table 2. ANOVA Table for Combined Graeco Sudoku square designs for model 2

Source	Sum of squares	Degrees of Freedom	Mean squares
Environments	$SSEE$	$g-1$	$MSEE = {SSEE}/{df}$
Treatments (Latin)	SSt_w	$k-1$	$MST_w = {SSt_w}/{df}$
Treatments (Greek)	SSt_n	$k-1$	$MST_n = {SSt_n}/{df}$
Rows within row-Block	$SSrrb$	$gm(m-1)$	$MSRRB = {SSrrb}/{df}$
Column within column-Blocks	$SSccb$	$gm(m-1)$	$MSCCB = SSccb$
Sub-Squares	$SSss$	$g(k-1)$	$MSSS = {SSss}/{df}$
Row-block	$SSrb$	$g(m-1)$	$MSRB = {SSrb}/{df}$
Columns-block	$SScb$	$g(m-1)$	$MSCB = {SScb}/{df}$
Env x treatments (Latin)	SS_{EXl}	$(g-1)(k-1)$	$MS_{EXl} = {SS_{EXl}}/{df}$
Env x treatments (Greek)	SS_{EXG}	$(g-1)(k-1)$	$MS_{EXG} = {SS_{EXG}}/{df}$
Error	SSe	$g[(k^2-3k+4)-2m^2]$	$MSE = {SSe}/{df}$
TOTAL	SST	gk^2-1	

Model 3: The modified linear model Type III

In this model it is assumed that the horizontal square effects are nested in the row block effects and the vertical square effects are nested in the column block effects.

$$Y_{ij\,(w,l,n,p,q,r)x} = \mu + \alpha_{ix} + \beta_{jx} + \tau_w + \tau_n + \gamma_{lx} + c_{px} + s(\alpha)_{q(i)x} + \pi(\beta)_{r(j)x} + \theta_x + (\tau\theta)_{wx} + (\tau\theta)_{nx} + \varepsilon_{i,j\,(w,l,n,p,q,r)x}$$

Where $i,j,q,r = 1,2,3;\quad l,n,p,w = 1,2,\dots m^2,\ x = 1,2,\dots,g$

Where $Y_{ij\,(lnpqrw)x}$ = Observation on experimental unit.

$\mu =$ Overall mean,
$\alpha_{ix} = i^{th}$ Row block effect in xth environment experiment,
$\beta_{jx} = j^{th}$ Column block effect in xth environment experiment,
$\tau_w =$ Latin letter w^{th} Treatment effect,
$\tau_n =$ Greek letter n^{th} Treatment effect,
$\gamma_{lx} = l^{th}$ Row effect in xth environment experiment,
$C_{px} = p^{th}$ Row effect in xth environment experiment,
$S(\alpha)_{q(i)x} = q^{th}$ Row effect nested in i^{th} block (row) in xth environment experiment,
$\pi(\beta)_{r(j)x} = r^{th}$ Column effect nested in j^{th} block (column) in xth environment experiment,
$\theta_x = xth$ Environment-experiment,
$(\tau\theta)_{wx} =$ Interaction between w^{th} Latin Treatment and xth environment experiment,

$(\tau\theta)_{nx} =$ Interaction between n^{th} Greek Treatment and xth environment experiment,
$\varepsilon_{ij\,(lnpqrw)x} =$ Error component with mean zero and variance σ^2

$$TSS = \sum_{i=1}^{m}\sum_{j=1}^{m}\sum_{x=1}^{g} Y_{ijx}^2 - \frac{G^2}{gk^2}, \quad SST_w = \sum_{w=1}^{m^2}\frac{T_{w..}^2}{k} - \frac{G^2}{gk^2}, \quad SST_n = \sum_{n=1}^{m^2}\frac{T_{n..}^2}{k} - \frac{G^2}{gk^2}$$

$$SSRB = \sum_{i=1}^{m}\sum_{x=1}^{g}\frac{RB^2 i\,.x}{gm^3} - \frac{G^2}{gk^2}, \quad SSCB = \sum_{j=1}^{m}\sum_{x=1}^{g}\frac{CB^2\,.j\,x}{gm^3} - \frac{G^2}{gk^2}, \quad SSR = \sum_{l=1}^{m^2}\sum_{x=1}^{g}\frac{R_{lx}^2}{gk} - \frac{G^2}{gk^2}$$

$$SSC = \sum_{p=1}^{m^2}\sum_{x=1}^{g}\frac{C_{px}^2}{gk} - \frac{G^2}{gk^2}, \quad SSHS = \sum_{i=1}^{m}\sum_{q=1}^{m^2}\sum_{x=1}^{g}\frac{S_{q(i)x}^2}{gk} - \sum_{i=1}^{m}\sum_{x=1}^{g}\frac{RB^2 i\,.x}{gm^3}, \quad SSVS = \sum_{j=1}^{m}\sum_{r=1}^{m^2}\sum_{x=1}^{g}\frac{\pi_{r(j)x}^2}{gk} - \sum_{j=1}^{m}\sum_{x=1}^{g}\frac{CB^2\,.j\,x}{gm^3}$$

$$SSEE = \sum_{x=1}^{g}\frac{\theta^2{}_x}{k^2} - \frac{G^2}{gk^2}, \quad SS_{EXL} = \sum_{w=1}^{m^2}\sum_{x=1}^{g}\frac{T\theta_{(w)x}}{gk} - \sum_{w=1}^{m^2}\frac{T_w^2}{k} - \sum_{x=1}^{g}\frac{\theta^2{}_x}{k^2} + \frac{G^2}{gk^2}, \quad SS_{EXG}$$
$$= \sum_{n=1}^{m^2}\sum_{x=1}^{g}\frac{T\theta_{(n)x}}{gk} - \sum_{n=1}^{m^2}\frac{T_n^2}{k} - \sum_{x=1}^{g}\frac{\theta^2{}_x}{k^2} + \frac{G^2}{gk^2}$$

$$SSE = \sum_{i=1}^{m}\sum_{j=1}^{m}\sum_{x=1}^{g} Y_{ij.x}^2 + \sum_{x=1}^{g}\frac{\theta^2{}_x}{k^2} - \sum_{l=1}^{m^2}\sum_{x=1}^{g}\frac{R_{lx}^2}{gk} - \sum_{p=1}^{m^2}\sum_{x=1}^{g}\frac{C_{px}^2}{gk} - \sum_{i=1}^{m}\sum_{q=1}^{m^2}\sum_{x=1}^{g}\frac{S_{q(i)x}^2}{gk} - \sum_{j=1}^{m}\sum_{r=1}^{m^2}\sum_{x=1}^{g}\frac{\pi_{r(j)x}^2}{gk} - \sum_{w=1}^{m^2}\sum_{x=1}^{g}\frac{T\theta_{(w)x}}{gk}$$
$$- \sum_{n=1}^{m^2}\sum_{x=1}^{g}\frac{T\theta_{(n)x}}{gk} + \frac{4G^2}{gk^2}$$

Table 3. ANOVA Table for Combined Graeco Sudoku square designs for model 3

Source	Sum of squares	Degrees of Freedom	Mean squares
Environments	$SSEE$	$g-1$	$MSEE = {SSEE}/{df}$
Treatments (Latin)	SSt_w	$k-1$	$MST_w = {SSt_w}/{df}$
Treatments (Greek)	SSt_n	$k-1$	$MST_n = {SSt_n}/{df}$
Rows	SSr	$g(k-1)$	$MSR = {SSr}/{df}$
Columns	SSc	$g(k-1)$	$MSC = {SSc}/{df}$
Row-block	$SSrb$	$g(m-1)$	$MSRB = {SSrb}/{df}$
Columns-block	$SScb$	$g(m-1)$	$MSCB = {SScb}/{df}$
Horizontal sq. within row-block	$SShrb$	$gm(m-1)$	$MSHRB = {SShrb}/{df}$
Vertical sq. within column-block	$SSvcb$	$gm(m-1)$	$MSVRB = {SSvrb}/{df}$
Env. x treatments (Latin)	SS_{EXL}	$(g-1)(k-1)$	$MS_{EXL} = {SS_{EXL}}/{df}$
Env. x treatments (Greek)	SS_{EXG}	$(g-1)(k-1)$	$MS_{EXG} = {SS_{EXG}}/{df}$
Error	SSe	$g[(k^2-4k+5)-2m^2]$	$MSEE = {SSe}/{df}$
TOTAL	SST	gk^2-1	

Model 4: The modified linear model type IV

In this model it is assumed that the row effects and the horizontal square effects are nested in the row block effects and the column effects and the vertical square effects are nested in the column block effects.

$$Y_{ij\,(lnpqrw)x} = \mu + \alpha_{ix} + \beta_{jx} + \tau_w + \tau_n + \gamma(\alpha)_{l(i)x} + c(\beta)_{p(jx)} + s(\alpha)_{q(i)x} + \pi(\beta)_{r(j)x} + \theta_x + (\tau\theta)_{wx} + (\tau\theta)_{nx} + \varepsilon_{i,j\,(lnpqrw)x}$$

Where $i,j,l,p,q,r = 1,2,3;\quad n,w = 1\,...\,m^2$ and $x = 1,\dots,g$
Where $Y_{ij\,(lnpqrw)x}$ = Observation on experimental unit.
$\mu =$ General mean effect
$\alpha_{ix} = i^{th}$ Row block effect in xth environment experiment,
$\beta_{jx} = j^{th}$ Column block effect in xth environment experiment,

$\tau_w =$ Latin letter w^{th} Treatment effect,
$\tau_n =$ Greek letter n^{th} Treatment effect,
$\gamma(\alpha)_{l(i)x} = l^{th}$ Row effect nested in i^{th} block (row) in xth environment experiment,
$C(\beta)_{p(j)x} = p^{th}$ Column effect nested in j^{th} block (column) in xth environment experiment,
$S(\alpha)_{q(i)x} = q^{th}$ Horizontal Square effect nested in i^{th} block (row) in xth environment experiment,
$\pi(\beta)_{r(j)x} = r^{th}$ Vertical Square effect nested in j^{th} block (column) in xth environment experiment,
$\theta_x = xth$ Environment-experiment,
$(\tau\theta)_{wx} =$ Interaction between w^{th} Latin Treatment and xth environment experiment,
$(\tau\theta)_{nx} =$ Interaction between n^{th} Greek Treatment and xth environment experiment,
$\varepsilon_{ij(wlnpqr)x} =$ Error component with mean zero and variance σ^2

$$TSS = \sum_{i=1}^{m}\sum_{j=1}^{m}\sum_{x=1}^{g} Y_{ijx}^2 - \frac{G^2}{gk^2}, \qquad SST_w = \sum_{w=1}^{m^2}\frac{T_{w..}^2}{k} - \frac{G^2}{gk^2}, \qquad SST_n = \sum_{n=1}^{m^2}\frac{T_{n..}^2}{k} - \frac{G^2}{gk^2}$$

$$SSRB = \sum_{i=1}^{m}\sum_{x=1}^{g}\frac{RB^2 i.x}{gm^3} - \frac{G^2}{gk^2}, \qquad SSCB = \sum_{j=1}^{m}\sum_{x=1}^{g}\frac{CB^2 .j\, x}{gm^3} - \frac{G^2}{gk^2}, \qquad SSR = \sum_{l=1}^{m^2}\sum_{x=1}^{g}\frac{R_{lx}^2}{gk} - \sum_{i=1}^{m}\sum_{x=1}^{g}\frac{RB^2 i.x}{gm^3}$$

$$SSC = \sum_{p=1}^{m^2}\sum_{x=1}^{g}\frac{C_{px}^2}{gk} - \sum_{j=1}^{m}\sum_{x=1}^{g}\frac{CB^2 .j\, x}{gm^3}, \qquad SSHS = \sum_{i=1}^{m}\sum_{q=1}^{m^2}\sum_{x=1}^{g}\frac{S_{q(i)x}^2}{gk} - \sum_{i=1}^{m}\sum_{x=1}^{g}\frac{RB^2 i.x}{gm^3}, \qquad SSVS = \sum_{j=1}^{m}\sum_{r=1}^{m^2}\sum_{x=1}^{g}\frac{\pi_{r(j)x}^2}{gk} - \sum_{j=1}^{m}\sum_{x=1}^{g}\frac{CB^2 .j\, x}{gm^3}$$

$$SSEE = \sum_{x=1}^{g}\frac{\theta^2{}_x}{k^2} - \frac{G^2}{gk^2}, \qquad SS_{EXL} = \sum_{w=1}^{m^2}\sum_{x=1}^{g}\frac{T\theta_{(w)x}}{gk} - \sum_{w=1}^{m^2}\frac{T_w^2}{k} - \sum_{x=1}^{g}\frac{\theta^2{}_x}{k^2} + \frac{G^2}{gk^2}, \qquad SS_{EXG} = \sum_{n=1}^{m^2}\sum_{x=1}^{g}\frac{T\theta_{(n)x}}{gk} - \sum_{n=1}^{m^2}\frac{T_n^2}{k} - \sum_{x=1}^{g}\frac{\theta^2{}_x}{k^2} + \frac{G^2}{gk^2}$$

$$SSE = \sum_{i=1}^{m}\sum_{j=1}^{m}\sum_{x=1}^{g} Y_{ij.x}^2 + \sum_{i=1}^{m}\sum_{x=1}^{g}\frac{RB^2 i.x}{gm^3} + \sum_{x=1}^{g}\frac{\theta^2{}_x}{k^2} + \sum_{j=1}^{m}\sum_{x=1}^{g}\frac{CB^2 .j\, x}{gm^3} - \sum_{l=1}^{m^2}\sum_{x=1}^{g}\frac{R_{lx}^2}{gk} - \sum_{p=1}^{m^2}\sum_{x=1}^{g}\frac{C_{px}^2}{gk} - \sum_{i=1}^{m}\sum_{q=1}^{m^2}\sum_{x=1}^{g}\frac{S_{q(i)x}^2}{gk} - \sum_{j=1}^{m}\sum_{r=1}^{m^2}\sum_{x=1}^{g}\frac{\pi_{r(j)x}^2}{gk} - \sum_{w=1}^{m^2}\sum_{x=1}^{g}\frac{T\theta_{(w)x}}{gk} - \sum_{n=1}^{m^2}\sum_{x=1}^{g}\frac{T\theta_{(n)x}}{gk} + \frac{2G^2}{gk^2}$$

Table 4. ANOVA Table for Combined Graeco Sudoku square designs for model 4

Source	Sum of squares	Degrees of Freedom	Mean squares
Environments	$SSEE$	$g-1$	$MSEE = {SSEE}/{df}$
Treatment (Latin)	SSt_w	$k-1$	$MST_w = {SSt_w}/{df}$
Treatment (Greek)	SSt_n	$k-1$	$MST_n = {SSt_n}/{df}$
Rows within row-block	$SSrrb$	$gm(m-1)$	$MSRRB = {SSrrb}/{df}$
Column within column-block	$SSccb$	$gm(m-1)$	$MSCCB = {SSccb}/{df}$
Horizontal sq. within row-block	$SShrb$	$gm(m-1)$	$MShrb = {SShrb}/{df}$
Ver. sq. within column-block	$SSvrb$	$gm(m-1)$	$MSvrb = {SSvrb}/{df}$
Row-block	$SSrb$	$g(m-1)$	$MSRB = {SSrb}/{df}$
Column-block	$SScb$	$g(m-1)$	$MSCB = {SScb}/{df}$
Env. x treatments (Latin)	SS_{EXL}	$(g-1)(k-1)$	$MSS_{EXL} = {SS_{EXL}}/{df}$
Env. x treatments (Greek)	SS_{EXG}	$(g-1)(k-1)$	$MSS_{EXG} = {SS_{EXG}}/{df}$
Error	SSe	$g[(k^2-2k+3)-2m(2m-1)]$	$MSE = {SSe}/{df}$
TOTAL	SST	gk^2-1	

Illustration: Tables 5 and 6 give hypothetical datasets for two environments (E_1 *and* E_2) obtained using orthogonal Sudoku square designs of order 9. Data for E_1 are presented in [13] for analysis of single orthogonal Sudoku design. Combined ANOVA tables obtained using R software for models 1 to 4 are presented in tables 7 to 10, respectively.

Table 5. Hypothetical data of a Graeco Sudoku square design of order 9 (Environment I)

		Column block 1			*Column block* 2			*Column block* 3		
		$C1$	$C2$	$C3$	$C1$	$C2$	$C3$	$C1$	$C2$	$C3$
Row block-1	**R1**	A_α	B_β	C_γ	D_λ	E_θ	F_μ	G_π	H_ϕ	I_δ
		15	11	16	21	22	21	14	19	15
	R2	D_π	E_ϕ	F_δ	G_α	H_β	I_γ	A_λ	B_θ	C_μ
		18	23	20	18	20	23	17	22	15
	R3	G_λ	H_θ	I_μ	A_π	B_ϕ	C_δ	D_α	E_β	F_γ
		15	10	22	15	18	19	24	15	1
Row block-1	**R1**	E_δ	F_π	D_ϕ	H_γ	I_α	G_β	B_μ	C_λ	A_θ
		17	23	18	15	16	21	15	12	16
	R2	H_μ	I_λ	G_θ	B_δ	C_π	A_ϕ	E_γ	F_α	D_β
		20	25	12	26	21	17	18	25	19
	R3	B_γ	C_α	A_β	E_μ	F_λ	D_θ	H_δ	I_π	G_ϕ
		21	17	18	22	16	23	22	19	25
Row block-1	**R1**	I_θ	G_μ	H_λ	C_ϕ	A_δ	B_π	F_β	D_γ	E_α
		12	25	20	19	22	16	19	13	18
	R2	C_β	A_γ	B_α	F_θ	D_μ	E_λ	I_ϕ	G_δ	H_π
		21	22	13	22	21	22	20	19	14
	R3	F_ϕ	D_δ	E_π	I_β	G_γ	H_α	C_θ	A_μ	B_λ
		14	17	11	16	18	23	15	23	16

Table 6. Hypothetical data of a Graeco Sudoku square design of order 9 (Environment II)

		Column block 1			*Column block* 2			*Column block* 3		
		$C1$	$C2$	$C3$	$C1$	$C2$	$C3$	$C1$	$C2$	$C3$
Row block-1	**R1**	A_α	B_β	C_γ	D_λ	E_θ	F_μ	G_π	H_ϕ	I_δ
		14	16	12	15	17	20	12	20	12
	R2	D_π	E_ϕ	F_δ	G_α	H_β	I_γ	A_λ	B_θ	C_μ
		10	21	10	21	22	16	22	21	15
	R3	G_λ	H_θ	I_μ	A_π	B_ϕ	C_δ	D_α	E_β	F_γ
		15	18	19	16	12	17	17	12	11
Row block-1	**R1**	E_δ	F_π	D_ϕ	H_γ	I_α	G_β	B_μ	C_λ	A_θ
		17	15	14	12	25	20	18	20	13
	R2	H_μ	I_λ	G_θ	B_δ	C_π	A_ϕ	E_γ	F_α	D_β
		20	17	17	13	20	14	21	19	15
	R3	B_γ	C_α	A_β	E_μ	F_λ	D_θ	H_δ	I_π	G_ϕ
		19	20	18	19	16	21	17	20	21
Row block-1	**R1**	I_θ	G_μ	H_λ	C_ϕ	A_δ	B_π	F_β	D_γ	E_α
		11	16	22	19	16	19	17	18	14
	R2	C_β	A_γ	B_α	F_θ	D_μ	E_λ	I_ϕ	G_δ	H_π
		16	21	13	11	23	16	11	16	22
	R3	F_ϕ	D_δ	E_π	I_β	G_γ	H_α	C_θ	A_μ	B_λ
		12	14	14	15	17	22	16	20	15

Table 7. ANOVA Table for Combined analysis of Graeco Sudoku square design of type I of order 9

Source	Sum of Squares	Degrees of Freedom	Mean Squares	F-Ratio	P-Value
Environments	110.8395	1	110.8395	8.7008	0.0009
Treatments (Latin)	105.9383	8	13.2423	1.0404	0.0084
Treatments (Greek)	129.6049	8	16.2006	1.2717	0.0019
Row block	69.4938	4	17.3735	1.3638	0.1053
Column block	96.1605	4	24.0401	1.8871	0.0518
Row	218.9383	16	13.6836	1.0742	0.0587
Column	249.0494	16	15.5656	1.2219	0.0950
Square	129.2716	16	08.0795	0.6342	0.4468
Env x Treatments (Latin)	100.1605	8	12.5201	0.9828	0.0146
Env. x Treatments (Greek)	133.6050	8	16.7006	1.3110	0.1042
Error	917.2098	72	12.7390	_	_
Total	2260.2716	161	_	_	_

Table 8. ANOVA Table for Combined analysis of Graeco Sudoku square design of type II of order 9

Source	Sum of Squares	Degrees of Freedom	Mean Squares	F-Ratio	P-Value
Environments	110.8395	1	110.8395	8.1886	0.0009
Treatments (Latin)	105.9383	8	13.2423	0.9783	0.0084
Treatments (Greek)	129.6049	8	16.2006	1.1969	0.0019
Row block	69.4938	4	17.3735	1.2835	0.1053
Column block	96.1605	4	24.0401	1.7760	0.0518
Row within RB	149.4445	12	12.4537	0.9201	0.0087
Column within CB	152.8889	12	12.7407	0.9413	0.0090
Square	129.2716	16	08.0795	0.5969	0.4468
Env x Treatments (Latin)	100.1605	8	12.5201	0.9250	0.0146
Env. x Treatments (Greek)	133.6050	8	16.7006	1.2338	0.1042
Error	1082.8641	80	13.5358	_	_
Total	2260.7284	161	_	_	_

Table 9. ANOVA Table for Combined analysis of Graeco Sudoku square design of type III of order 9

Source	Sum of Squares	Degrees of Freedom	Mean Squares	F-Ratio	P-Value
Environments	110.8395	1	110.8395	7.4390	0.0009
Treatments (Latin)	105.9383	8	13.2423	0.8888	0.0084
Treatments (Greek)	129.6049	8	16.2006	1.0873	0.0019
Row block	69.4938	4	17.3735	1.1660	0.1053
Column block	96.1605	4	24.0401	1.6135	0.0518
Row	218.9383	16	13.6836	0.9184	0.0587
Column	249.0494	16	15.5656	1.0447	0.0950
H_Square within RB	59.7778	12	04.9815	0.3343	0.2468
V_Square within CB	33.1111	12	02.7593	0.1852	0.1090
Env x Treatments (Latin)	100.1605	8	12.5201	0.8403	0.0146
Env. x Treatments (Greek)	133.6050	8	16.7006	1.1209	0.1042
Error	953.5880	64	14.8998	_	_
Total	2260.7284	161	_	_	_

Table 10. ANOVA Table for Combined analysis of Graeco Sudoku square design of type IV of order 9

Source	Sum of Squares	Degrees of Freedom	Mean Squares	F-Ratio	P-Value
Environments	110.8395	1	110.8395	7.1302	0.0009
Treatments (Latin)	105.9383	8	13.2423	0.8519	0.0084
Treatments (Greek)	129.6049	8	16.2006	1.0417	0.0019
Row block	69.4938	4	17.3735	1.1176	0.1053
Column block	96.1605	4	24.0401	1.5465	0.0518
Row within RB	149.4445	12	12.4537	0.8011	0.0587
Column within CB	152.8889	12	12.7407	0.8196	0.0950
H_Square. within RB	59.7778	12	04.9815	0.3205	0.2468
V_Square. within CB	33.1111	12	02.7593	0.1775	0.1090
Env x Treatments (Latin)	100.1605	8	12.5201	0.8054	0.0146
Env. x Treatments (Greek)	133.6050	8	16.7006	1.0743	0.1042
Error	1119.2468	72	15.5451	_	_
Total	2260.7284	161	_	_	_

3. Conclusions

In this paper, joint analysis of orthogonal Sudoku design conducted for g environments is proposed. The method allows experimenter to tests/investigates the interaction effect between environments and treatment (Latin and Greek treatment). In addition, the models of the orthogonal Sudoku designs were modified to contain the interaction between treatments (Latin and Greek) and environments effects.

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