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On Two Random Variables and Archimedean Copulas

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Abstract The study focused on the likelihood of a pair of random variables having either an Archimedean copula or an Elliptical copula. The study involved simulating several pairs of random variables and the bicopselect () function in R was used to select an appropriate bivariate copula family for simulated pairs of random variables. The corresponding parameter estimates were obtained by maximum likelihood estimation. The method compared AICs of the various bivariate copulas under consideration. In all, about forty (40) bivariate copulas were considered ranging from one parameter models to two parameter models, three parameter models and in some cases rotations of some of these models. Fifty (50) different pairs of random variables were simulated for sample sizes 30, 300, 1000, 10000, 100000 and 1000000. For sample size thirty (30), 47 pairs had their copulas being Archimedean, for sample size 300, 47 had their copulas being Archimedean, for sample sizes 1000, 10000, 100000 and 1000000, 49, 44, 47 and 46 pairs respectively had their copulas being Archimedean. The results showed that between the Archimedean and Elliptical copulas, the Archimedean copulas were the most likely to fit the simulated pairs of random variables.

Keywords Archimedean copulas, Elliptical copulas, Simulation, Random variables

1. Introduction

A copula is a function which joins or couples a multivariate distribution function to its one-dimensional marginal distribution functions. Over the years, copulas have played an important role in several areas of statistics. According to Fisher (1997), specifically his notes in the Encyclopedia of Statistical Sciences, “Copulas are of interest to statisticians for two main reasons; First, as a way of studying scale-free measures of dependence; and secondly, as a starting point for constructing families of bivariate distributions.

One attractive property of copulas is their invariance under strictly increasing transformations of the margins. Copulas have been thoroughly reviewed in Nelsen (2006). Copula was first used in financial applications by Embrechts et. al. (2002). Since then the application on copula theory in finance and economics has grown tremendously. Moreover, practical applications of this modeling approach are found in fields such as finance (Nikoloulopoulos et. al. (2012); Fang and Madsen (2013)), hydrology (Genest et. al. (2007)), public health and medical (Winkelmann (2012)) and actuarial science (Frees and Valdez (1998); Otani and Imai (2013)).

An important class of copulas are the Archimedean copulas, they are discussed in [Genest and Mackay (1986), Joe (1997), McNeil and Neslehova (2009)]. Archimedean copulas are popular since they are easy to handle, have simple, closed-form expressions, and can be used to derive portfolio distributions (Crook and Moreira (2011)).

Trede and Savu (2013) suggested a new straightforward method to check whether a copula is an Archimedean copula without specifying its parametric family. Their approach was applied to (bivariate) joint distributions of stock asset returns and they discovered that in general, stock returns may have Archimedean copulas.

Archimedean copulas over the years have been successfully applied in various sectors (Louie (2014), Corbella and Stretch (2013) and Yee et. al (2014)).

This study is to serve as a support to the works that seek to tie two random variables with the Archimedean copulas.

2. Copulas

A copula is a multivariate cumulative distribution function (CDF) whose univariate marginal distributions are all Uniform (0, 1). Suppose that $Y = (Y_1, \ldots, Y_d)$ has a multivariate CDF F_Y with continuous marginal univariate CDFs $F_{Y_1}, \ldots, F_{Y_d}$. If Y has a continuous CDF F, then $F(Y)$ has a uniform (0, 1) distribution. $F(Y)$ is often called the probability transformation of Y. This fact is easy to see if F is strictly increasing, since then F^{-1} exists, so that

$$P\{F(Y) \leq y\} = P\{Y \leq F-1(y)\} = F\{F-1(y)\} = y \quad (1)$$

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Then, by the equation above each of $F_{Y_1}(Y_1), \dots, F_{Y_d}(Y_d)$ is distributed uniform (0, 1). Thus, the CDF of $\{F_{Y_1}(Y_1), \dots, F_{Y_d}(Y_d)\}$ is a copula. This CDF is called the copula of Y and denoted by C_Y. C_Y contains all information about dependencies among the components of Y but has no information about the marginal CDFs of Y.

2.1. Archimedean Copulas

An Archimedean copula with a strict generator has the form;

$$C(u_1, \dots, u_d) = \varphi^{-1}\{\varphi(u_1) + \cdots + \varphi(u_d)\} \quad (2)$$

where the generator function φ satisfies the following conditions:

1. φ is a continuous, strictly decreasing, and convex function mapping [0,1] onto [0,∞].
2. $\varphi(0) = \infty$ and
3. $\varphi(1) = 0$

2.2. Elliptical Copulas

Elliptical copulas are the copulas of elliptically contoured distributions. The multivariate and the Student-t are the most commonly used elliptical distributions. The Normal copula is an elliptical copula given by:

$$C_\rho(u, v) = \int_{-\infty}^{\phi^{-1}(u)} \int_{-\infty}^{\phi^{-1}(v)} \frac{1}{2\pi(1-\rho^2)^{\frac{1}{2}}} \exp\left\{-\frac{x^2 - 2\rho xy + y^2}{2(1-\rho^2)}\right\} dxdy \quad (3)$$

The Student-t copula is an elliptical copula defined by:

$$C_{\rho,v}(u, v) = \int_{-1\infty}^{t_v^{-1}(u)} \int_{-\infty}^{t_v^{-1}(v)} \frac{1}{2\pi(1-\rho^2)^{\frac{1}{2}}} \left\{1 + \frac{x^2 - 2\rho xy + y^2}{v(1-\rho^2)}\right\}^{-\frac{(v+2)}{2}} dxdy \quad (4)$$

3. Simulations and Results

To assess the argument that the Copula of two random variables is more often than not Archimedean, several simulations are performed. Considering pairs of random variables of size, n each, the Vuong and Clarke tests for selecting a bivariate copula is used to assign copulas to each of the 50 pairs of random variables.

The choice of a bivariate copula is between the elliptical (Gaussian and Student-t) and the Archimedean (Clayton, Gumbel, Frank, Joe, BB1, BB6, BB7 and BB8) copulas to cover a large range of dependence patterns. For Archimedean copula families, rotated versions were included to cover negative dependence as well. The Tawn copula being an asymmetric extension of the Gumbel copula with three parameters was added to the Archimedean. For simplicity, two versions of the Tawn copula with two parameters each were employed. Each type has one of the asymmetry parameters fixed to 1, so that the corresponding copula density is either left- or right-skewed (in relation to the main diagonal). For each of the possible pairs, the tests decide which family best fits the given data.

3.1. Algorithm

Step 1:

Simulate two random variables x1 and x2 of equal length n, with uniform margins.

Step 2:

Using the Vuong and Clarke tests for selecting a bivariate copula, select a copula.

Step 3:

Repeat Step 1 for different values of n for 49 other pairs of simulated x1 and x2 and apply step 2 in each case.

Example using R;
set.seed(1)

Step 1: Nsim=10000 #number of random numbers

```
x1=runif(Nsim)
x2=runif(Nsim) #vectors
```

Step 2:

```
selectedCopula <- BiCopSelect(x1,x2,familyset=NA)
selectedCopula
```

Tables 1, 2, 3, 4, 5 and 6 give the results of simulated pairs of random variables, selected copulas and in some cases their parameter values.

4. Results

The results of the whole work is summarized in Tables 1, 2, 3, 4, 5, 6 and 7.

Table 1. Pairs of Random Variables and Their Copulas (n=30)

Pairs of Random Variables	Bi-Copula Number (Family)	Copula Type
1	134	Rotated Tawn Type 1 (270 degrees)
2	5	Frank
3	26	Rotated Joe (90 degrees)
4	1	Gaussian
5	124	Rotated Tawn Type 1 (90 degrees)
6	124	Rotated Tawn Type 1 (90 degrees)
7	114	Rotated Tawn Type 1 (180 degrees)

Pairs of Random Variables	Bi-Copula Number (Family)	Copula Type
8	214	Rotated Tawn Type 2 (180 degrees)
9	224	Rotated Tawn Type 2 (90 degrees)
10	114	Rotated Tawn Type 1 (180 degrees)
11	5	Frank
12	5	Frank
13	3	Clayton
14	224	Rotated Tawn Type 2 (90 degrees)
15	26	Rotated Joe (90 degrees)
16	6	Joe
17	23	Rotated Clayton (90 degrees)
18	16	Rotated Joe (180 degrees)
19	5	Frank
20	134	Rotated Tawn Type 1 (270 degrees)
21	114	Rotated Tawn Type 1 (180 degrees)
22	104	Tawn Type 1
23	6	Joe
24	36	Rotated Joe (270 degrees)
25	16	Rotated Joe (180 degrees)
26	36	Rotated Joe (270 degrees)
27	224	Rotated Tawn Type 2 (90 degrees)
28	134	Rotated Tawn Type 1 (270 degrees)
29	5	Frank
30	214	Rotated Tawn Type 2 (180 degrees)
31	104	Tawn Type 1
32	36	Rotated Joe (270 degrees)
33	214	Rotated Tawn Type 2 (180 degrees)
34	26	Rotated Joe (90 degrees)
35	204	Tawn Type 2
36	204	Tawn Type 2
37	204	Tawn Type 2
38	36	Rotated Joe (270 degrees)
39	134	Rotated Tawn Type 1 (270 degrees)
40	16	Rotated Joe (180 degrees)
41	204	Tawn Type 2
42	5	Frank
43	224	Rotated Tawn Type 2 (90 degrees)
44	1	Gaussian
45	5	Frank
46	224	Rotated Tawn Type 2 (90 degrees)
47	16	Rotated Joe (180 degrees)
48	204	Tawn Type 2
49	204	Tawn Type 2
50	1	Gaussian

NB: Archimedean Copulas are in "RED".

Table 1 constitutes simulation 1 for 50 different simulated pairs of random variables subjected to the process in 4.1. Out of the 50 pairs of random variables simulated following the process in 4.1, 47 of the pairs had their copula being Archimedean Copulas. Only 3 (simulated pair 4, 44 and 50) had their copula being the Elliptical copulas (Gaussian).

Table 2. Pairs of Random Variables and Their Copulas (n = 300)

Pairs of Random Variables	Bi-Copula Number (Family)	Copula Type	Parameter 1	Parameter 2
1	204	Tawn Type 2	14.48416	0.008408582
2	214	Rotated Tawn Type 2 (180 degrees)	6.343869	0.01747199
3	204	Tawn Type 2	20	0.01181782
4	104	Tawn Type 1	1.379204	0.1255907
5	5	Frank	-0.2013011	0
6	114	Rotated Tawn Type 1 (180 degrees)	1.70345	0.04993857
7	5	Frank	-0.8130363	0
8	13	Clayton	0.02826758	0
9	2	Student t	0.05698729	7.253719
10	5	Frank	0.2621446	0
11	5	Frank	0.544137	0
12	204	Tawn Type 2	16.98002	0.001705523
13	1	Gaussian	-0.04184862	0
14	204	Tawn Type 2	1.514891	0.07073632
15	204	Tawn Type 2	1.475668	0.08042687
16	23	Rotated Clayton (90 degrees)	-0.0695994	0
17	224	Rotated Tawn Type 2 (90 degrees)	-1.270983	0.04145445
18	134	Rotated Tawn Type 1 (270 degrees)	-3.764414	0.003714084
19	104	Tawn Type 1	6.491484	0.03209095
20	3	Clayton	0.04233865	0
21	3	Clayton	0.06988842	0
22	234	Rotated Tawn Type 2 (270 degrees)	-5.845296	0.003570964
23	234	Rotated Tawn Type 2 (270 degrees)	-1.742632	0.06856019
24	5	Frank	-0.2059542	0
25	1	Gaussian	-0.0207488	0
26	5	Frank	0.5949366	0
27	5	Frank	-0.4023436	0
28	26	Rotated Joe (90 degrees)	-1.040715	0
29	3	Clayton	0.07450558	0
30	124	Rotated Tawn Type 1 (90 degrees)	-1.548161	0.02793191
31	134	Rotated Tawn Type 1 (270 degrees)	-5.917137	0.01321439
32	6	Joe	1.074425	0
33	204	Tawn Type 2	7.83952	0.002106726
34	134	Rotated Tawn Type 1 (270 degrees)	-2.288161	0.02840188
35	104	Tawn Type 1	20	0.002495725
36	33	Rotated Clayton (270 degrees)	-0.1225876	0
37	5	Frank	-0.5201417	0
38	204	Tawn Type 2	12.14455	0.00132292
39	214	Rotated Tawn Type 2 (180 degrees)	2.478545	0.007805447
40	124	Rotated Tawn Type 1 (90 degrees)	-19.47738	0.001608896
41	26	Rotated Joe (90 degrees)	-1.021392	0
42	114	Rotated Tawn Type 1 (180 degrees)	4.973694	0.002261282
43	104	Tawn Type 1	20	0.006014855
44	104	Tawn Type 1	1.471994	0.05110982
45	5	Frank	0.9312155	0
46	34	Rotated Gumbel (270 degrees)	-1.029469	0
47	6	Joe	1.075419	0
48	26	Rotated Joe (90 degrees)	-1.030669	0
49	214	Tawn Type 2	9.1177	0.00751996
50	224	Rotated Tawn Type 2 (90 degrees)	-2.594418	0.02043922

NB: Archimedean Copulas are in “RED”.

Table 3. Pairs of Random Variables and Their Copulas (n = 1000)

Pairs of Random Variables	Bi-Copula Number (Family)	Copula Type	Parameter 1	Parameter 2
1	224	Rotated Tawn Type 2 (90 degrees)	-10.07035	0.005185978
2	224	Rotated Tawn Type 2 (90 degrees)	-2.717438	0.01378937
3	234	Rotated Tawn Type 2 (270 degrees)	-4.631492	0.002813678
4	114	Rotated Tawn Type 1 (180 degrees)	7.839527	0.004410584
5	134	Rotated Tawn Type 1 (270 degrees)	-2.334564	0.006195052
6	23	Rotated Clayton (90 degrees)	-0.006750176	0
7	36	Rotated Joe (70 degrees)	-1.02562	0
8	5	Frank	-0.1620883	0
9	234	Rotated Tawn Type 2 (270 degrees)	-1.325352	0.04519359
10	23	Rotated Clayton (90 degrees)	-0.04660569	0
11	13	Rotated Clayton (180 degrees)	0.02568463	0
12	6	Joe	1.029129	0
13	5	Frank	-0.4024086	0
14	204	Tawn Type 2	8.510368	0.001689399
15	34	Rotated Gumbel (270 degrees)	-1.025861	0
16	33	Rotated Clayton (270 degrees)	-0.05531397	0
17	5	Frank	-0.1864237	0
18	5	Frank	-0.1337674	0
19	234	Rotated Tawn Type 2 (270 degrees)	-12.74722	0.002591045
20	14	Rotated Gumbel (180 degrees)	1.013389	0
21	224	Rotated Tawn Type 2 (90 degrees)	-17.56683	0.004400793
22	5	Frank	-0.06560941	0
23	204	Tawn Type 2	13.86403	0.003532782
24	36	Rotated Joe (270 degrees)	-1.03173	0
25	33	Rotated Clayton (270 degrees)	-0.0532508	0
26	234	Rotated Tawn Type 2 (270 degrees)	-16.96935	0.00410771
27	5	Frank	-0.3608203	0
28	124	Rotated Tawn Type 1 (90 degrees)	-2.063189	0.009526584
29	114	Rotated Tawn Type 1 (180 degrees)	1.394521	0.05358823
30	13	Rotated Clayton (180 degrees)	0.0520688	0
31	5	Frank	-0.2281523	0
32	5	Frank	0.5071022	0
33	14	Rotated Gumbel (180 degrees)	1.017152	0
34	204	Tawn Type 2	11.35934	0.002482861
35	34	Rotated Gumbel (270 degrees)	-1.022759	0
36	114	Rotated Tawn Type 1 (180 degrees)	6.4634	0.002659584
37	3	Clayton	0.06359287	0
38	16	Rotated Joe (180 degrees)	1.053157	0
39	36	Rotated Joe (270 degrees)	-1.028122	0
40	34	Rotated Gumbel (270 degrees)	-1.035152	0
41	204	Tawn Type 2	4.413874	0.001551874
42	1	Gaussian	-0.003686919	0
43	33	Rotated Clayton (270 degrees)	-0.05730067	0
44	3	Clayton	0.05894495	0
45	224	Rotated Tawn Type 2 (90 degrees)	-8.398277	0.006663609
46	114	Rotated Tawn Type 1 (180 degrees)	1.409001	0.03178957
47	16	Rotated Joe (180 degrees)	1.046323	0
48	5	Frank	-0.1209836	0
49	5	Tawn Type 2	-0.1656696	0
50	36	Rotated Joe (270 degrees)	-1.011151	0

NB: Archimedean Copulas are in “RED”.

Table 4. Pairs of Random Variables and Their Copulas (n = 10000)

Pairs of Random Variables	Bi-Copula Number (Family)	Copula Type	Parameter 1	Parameter 2
1	114	Rotated Tawn Type 1 (180 degrees)	1.155907	0.008112878
2	124	Rotated Tawn Type 1 (90 degrees)	-1.478397	0.0019798
3	1	Gaussian	0.008873408	0
4	1	Gaussian	-0.00268579	0
5	13	Rotated Clayton (180 degrees)	0.008725757	0
6	36	Rotated Joe (270 degrees)	-1.013391	0
7	1	Gaussian	-0.01122607	0
8	204	Tawn Type 2	1.316691	0.001950984
9	33	Rotated Clayton (270 degrees)	-0.01331629	0
10	224	Rotated Tawn Type 2 (90 degrees)	-1.114805	0.007540145
11	3	Clayton	0.02731143	0
12	204	Tawn Type 2	1.280833	0.001327157
13	26	Rotated Joe (90 degrees)	-1.007268	0
14	6	Joe	1.010018	0
15	1	Gaussian	-0.003200067	0
16	23	Rotated Clayton (90 degrees)	-0.02488021	0
17	214	Rotated Tawn Type 2 (180 degrees)	1.485505	0.002267112
18	23	Rotated Clayton (90 degrees)	-0.006219675	0
19	224	Rotated Tawn Type 2 (90 degrees)	-1.414551	0.001361773
20	14	Rotated Gumbel (180 degrees)	1.013389	0
21	3	Clayton	0.004166482	0
22	5	Frank	-0.06560941	0
23	214	Rotated Tawn Type 2 (180 degrees)	1.304129	0.004259981
24	16	Rotated Joe (180 degrees)	1.007396	0
25	5	Frank	0.09887132	0
26	23	Rotated Clayton (90 degrees)	-0.009136288	0
27	114	Rotated Tawn Type 1 (180 degrees)	1.456298	0.00122227
28	234	Rotated Tawn Type 2 (270 degrees)	-1.112805	0.01208872
29	36	Rotated Joe (270 degrees)	-1.012406	0
30	13	Rotated Clayton (180 degrees)	0.01051188	0
31	5	Frank	-0.02330692	0
32	14	Rotated Gumbel (180 degrees)	1.005536	0
33	5	Frank	-1.008751	0
34	26	Rotated Joe (90 degrees)	11.35934	0.002482861
35	3	Clayton	0.01928258	0
36	6	Joe	1.00516	0
37	33	Rotated Clayton (270 degrees)	-0.01585007	0
38	5	Frank	0.099009	0
39	1	Gaussian	-0.00780174	0
40	214	Rotated Tawn Type 2 (180 degrees)	13.95408	0.0001
41	224	Rotated Tawn Type 2 (90 degrees)	-1.591775	0.002713813
42	23	Rotated Clayton (90 degrees)	-0.002984755	0
43	24	Rotated Gumbel (90 degrees)	-1.008299	0
44	3	Clayton	0.05894495	0
45	214	Rotated Tawn Type 2 (180 degrees)	7.60397	0.0001
46	204	Tawn Type 2	1.069604	0.02780733
47	33	Rotated Clayton (270 degrees)	-0.01872496	0
48	114	Rotated Tawn Type 1 (180 degrees)	1.282865	0.002601514
49	1	Gaussian	0.003311384	0
50	104	Tawn Type 1	1.073471	0.02430817

NB: Archimedean Copulas are in “RED”.

Table 5. Pairs of Random Variables and Their Copulas (n = 100000)

Pairs of Random Variables	Bi-Copula Number (Family)	Copula Type	Parameter 1	Parameter 2
1	1	Gaussian	-0.006875427	0
2	114	Rotated Tawn Type 1 (180 degrees)	1.078467	0.00164116
3	124	Rotated Tawn Type 1 (90 degrees)	-1.130366	0.0003145505
4	5	Frank	-0.02287776	0
5	5	Frank	0.03985872	0
6	23	Rotated Clayton (90 degrees)	-0.004914904	0
7	13	Rotated Clayton (180 degrees)	0.002127076	0
8	23	Rotated Clayton (90 degrees)	-0.01035591	0
9	224	Rotated Tawn Type 2 (90 degrees)	-1.012644	0.1094078
10	23	Rotated Clayton (90 degrees)	-0.00150629	0
11	5	Frank	0.01562637	0
12	23	Rotated Clayton (90 degrees)	-0.004579322	0
13	234	Rotated Tawn Type 2 (270 degrees)	-1.386011	0.0001
14	5	Frank	-0.01935665	0
15	23	Rotated Clayton (90 degrees)	-0.006430406	0
16	134	Rotated Tawn Type 1 (270 degrees)	-1.026183	0.0226184
17	224	Rotated Tawn Type 2 (90 degrees)	-1.11159	0.001197463
18	6	Joe	1.002038	0
19	4	Gumbel	1.00342	0
20	33	Rotated Clayton (270 degrees)	-0.004264028	0
21	23	Rotated Clayton (90 degrees)	-0.002700568	0
22	1	Gaussian	0.001044793	0
23	124	Rotated Tawn Type 1 (90 degrees)	-1.984708	0.0002354378
24	13	Rotated Clayton (180 degrees)	0.003314794	0
25	33	Rotated Clayton (270 degrees)	-0.002008442	0
26	13	Rotated Clayton (180 degrees)	0.002977579	0
27	3	Clayton	0.003536868	0
28	13	Rotated Clayton (180 degrees)	0.004923761	0
29	26	Rotated Joe (90 degrees)	-1.001959	0
30	5	Frank	0.04275088	0
31	33	Rotated Clayton (270 degrees)	-0.0031626	0
32	33	Rotated Clayton (270 degrees)	-0.005001467	0
33	1	Gaussian	-0.00517704	0
34	13	Rotated Clayton (180 degrees)	0.001681644	0
35	104	Tawn Type 1	5.794707	1e-04
36	134	Rotated Tawn Type 1 (270 degrees)	-2.442294	1e-04
37	6	Joe	1.00166	0
38	5	Frank	-0.02092695	0
39	204	Tawn Type 2	2.409549	1e-04
40	16	Rotated Joe (180 degrees)	1.003354	0
41	224	Rotated Tawn Type 2 (90 degrees)	-2.560946	1e-04
42	23	Rotated Clayton (90 degrees)	-0.003804658	0
43	3	Clayton	0.003493326	0
44	114	Clayton	1.974587	1e-04
45	5	Frank	-0.02166793	0
46	5	Tawn Type 2	0.02274893	0
47	16	Rotated Joe (180 degrees)	1.001687	0
48	24	Rotated Gumbel (90 degrees)	-1.001685	0
49	214	Rotated Tawn Type 2 180 degrees)	1.023572	0.0169742
50	224	Rotated Tawn Type 2 (90 degrees)	-1.539881	1e-04

NB: Archimedean Copulas are in “RED”.

Table 6. Pairs of Random Variables and Their Copulas (n = 1000000)

Pairs of Random Variables	Bi-Copula Number (Family)	Copula Type	Parameter 1	Parameter 2
1	13	Rotated Clayton (180 degrees)	0.001874919	0
2	24	Rotated Gumbel (90 degrees)	-1.000554	0
3	36	Rotated Joe (270 degrees)	-1.00101	0
4	13	Rotated Clayton (180 degrees)	0.001114317	0
5	5	Frank	0.002921803	0
6	13	Rotated Clayton (180 degrees)	0.0003793721	0
7	4	Gumbel	1.000804	0
8	1	Gaussian	-0.001522547	0
9	1	Gaussian	-0.001605963	0
10	6	Joe	1.000456	0
11	34	Rotated Gumbel (270 degrees)	-1.000444	0
12	5	Frank	-0.007937841	0
13	36	Rotated Joe (270 degrees)	-1.000376	0
14	114	Rotated Tawn Type 1 (180 degrees)	1.020453	0.002053239
15	3	Clayton	0.0007436717	0
16	5	Frank	0.005235035	0
17	13	Rotated Clayton (180 degrees)	0.002060722	0
18	13	Rotated Clayton (180 degrees)	0.001075413	0
19	4	Gumbel	1.000282	0
20	234	Rotated Tawn Type 2 (270 degrees)	-1.001493	0.1002489
21	14	Rotated Gumbel (180 degrees)	1.000487	0
22	6	Joe	1.000381	0
23	5	Frank	0.009592589	0
24	33	Rotated Clayton (270 degrees)	-0.001753083	0
25	5	Frank	-0.01688023	0
26	1	Gaussian	-0.002061574	0
27	4	Gumbel	1.000285	0
28	6	Joe	1.00063	0
29	3	Clayton	0.001117876	0
30	104	Tawn Type 1	1.004084	0.0611272
31	1	Gaussian	0.0003216839	0
32	16	Rotated Joe (180 degrees)	1.000446	0
33	26	Rotated Joe (90 degrees)	-1.000897	0
34	24	Rotated Gumbel (90 degrees)	-1.000973	0
35	36	Rotated Joe (270 degrees)	-1.000724	0
36	234	Rotated Tawn Type 2 (270 degrees)	-1.037894	1e-04
37	5	Frank	-0.002373456	0
38	3	Clayton	0.0006730301	0
39	14	Rotated Gumbel (180 degrees)	1.00071	0
40	16	Rotated Joe (180 degrees)	1.000416	0
41	224	Rotated Tawn Type 2 (90 degrees)	-1.114766	1e-04
42	13	Rotated Clayton (180 degrees)	0.001378867	0
43	5	Frank	-0.003207516	0
44	23	Rotated Clayton (90 degrees)	-0.0008759278	0
45	33	Rotated Clayton (270 degrees)	-0.001228216	0
46	13	Rotated Clayton (180 degrees)	0.0008822748	0
47	5	Frank	0.008797257	0
48	3	Clayton	0.000924666	0
49	16	Rotated Joe (180 degrees)	1.000573	0
50	204	Tawn Type 2	1.039686	0.0009705741

NB: Archimedean Copulas are in "RED".

Table 7. Summary of simulation (for 50 runs)

Simulation	value of n	Number of Archimedean	Number of Elliptcal	Percentage of Archimedean	Percentage of Elliptical
1	30	47	3	94%	6%
2	300	47	3	94%	6%
3	1000	49	1	98%	2%
4	10000	44	6	88%	12%
5	100000	47	3	94%	6%
6	1000000	46	4	92%	8%

Table 2 shows results for simulation 2 made up of 50different pairs of simulated random variables. Out of the 50 pairs of random variables, 47 of them (in red) had their copula being Archimedean Copulas. Pairs 9, 13 and 25 however had their copula being the Elliptical copula (Gaussian and student t).

From Table 3, out of the 50 pairs of random variables simulated, 49 of them (in red) had their copula being Archimedean Copulas and only one (pair 42) had its copula being Elliptical (Gaussian).

From Table 4, out of the 50 pairs of random variables simulated, 44 of them (in red) had their copula being Archimedean Copulas and 6 of them (pairs 2, 3, 7, 15, 39 and 49) being Elliptical copulas (Gaussian).

From Table 5, out of the 50 pairs of random variables simulated, 47 of them (in red) had their copula being Archimedean Copulas whereas 3 of the pairs (pair 1, 2 and 33) had their copula being Elliptical (Gaussian).

From Table 6, out of the 50 pairs of random variables simulated, 46 of them (in red) had their copula being Archimedean Copulas and 4 of them (pairs 8, 9, 26 and 31) had their copula being Elliptical (Gaussian).

Table 7 above indicates that for a run of 50 pairs of random variables with different number of data points (n=30, 300, 1000, 10000 and 1000000 respectively), the Archimedean copulas were the most likely to fit those pairs.

5. Conclusions

This study sought to check which bivariate copula (between the Archimedean and Elliptical copulas) was the most likely to fit two random variables. 50 pairs of random variables were simulated for sample sizes n = 30, n= 300, n= 1000, n = 10000, n = 100000 and n = 1000000. For all sample sizes under consideration, no regular pattern for the copula selection is observed. The question then is, “Is there a way to detect a relationship between the random variables and the copula selected?”. The study revealed that for all simulations under study, the most likely copula to best handle pairs of random variables were the Archimedean copulas. This study supports the argument that the copula of two random variables is Archimedean. It will however be interesting to delve deeper into what characteristics of the pairs of random variables that account for they being Archimedean or elliptical.

REFERENCES

[1] Belgorodski, N. (2010), Selecting pair-copula families for regular vines with application to the multivariate analysis of European stock market indices, Diploma Thesis, http://mediatum.ub.tum.de/?id=1079284.

[2] Clarke, K. A. (2007), A Simple Distribution-Free Test for Non nested Model Selection, Political Analysis, 15, 347-363.

[3] Vuong, Q. H. (1989), Ratio tests for model selection and non-nested hypotheses, Econometrica 57 (2), 307-333.

[4] R. B. Nelsen, An Introduction to Copulas, 2nd ed., Springer, New York, 2006.

[5] P. Embrechts, A. McNeil and D. Straumann, “Correlation and Dependence in Risk Management: Properties and Pitfalls,” Risk management: value at risk and beyond, pp. 176-223, 2002.

[6] K. Nikoloulopoulos, H. Joe and H. Li. ”Vine copulas with asymmetric tail dependence and applications to financial return data.” Computational Statistics & Data Analysis, vol. 56, no. 11, pp. 36593673, 2012.

[7] Y. Fang and L. Madsen, “Modified Gaussian Pseudo-Copula: Applications in Insurance and Finance,” Insurance Mathematics and Economics, vol. 53, pp. 292-301, 2013.

[8] C. Genest, A. C. Favre, J. B´eliveau and C. Jacques, “Meta elliptical Copulas and Their Use in Frequency Analysis of Multivariate Hydrological Data,” Water Resources Research, vol. 43, no. 9, 2007.

[9] E. W. Frees and E. A. Valdez, “Understanding Relationships Using Copulas,” North American Actuarial Journal, vol. 2, pp. 1-25, 1998.

[10] Y. Otani and J. Imai, “Pricing Portfolio Credit Derivatives with Stochastic Recovery and Systematic Factor.” IAENG International Journal of Applied Mathematics, vol. 43, no. 4, pp. 176-184, 2013.

[11] R. Winkelmann, “Copula bivariate probit models: with an application to medical expenditures.” Health Economics, vol. 21, no. 12, pp. 14441455, 2012.

[12] Mark Trede and Cornelia Savu, “Do stock returns have an Archimedean copula?”, Journal of Applied Statistics Vol. 40 , Iss. 8, 2013.

[13] Henry Louie (2014), “Evaluation of bivariate Archimedean and elliptical copulas to model wind power dependency

structures; Wind Energy, Pages 225-240: 10.1002/we.1571.

[14] S. Corbella, D. D. Stretch (2013), “Simulating a multivariate sea storm using Archimedean copulas”, Coastal Engineering, 76, 68 -78, https://doi.org/10.1016/j:coastaleng.2013.01.011.

[15] K. C. Yee, J. Suhaila, F. Yusof, F. H. Mean, “Bivariate copula in fitting rainfall data, AIP conference Proceedings 1605, 986 (2014); http://dx.doi.org/10.1063/1.4887724.

[16] Fisher, N. I. (1997). Copulas. In: Encyclopedia of Statistical Sciences, Update Vol.1, 159-163. John Wiley Sons, New York.

[17] C. Genest and J. MacKay, The joy of copulas: Bivariate distributions with uniform marginals, Amer. Statist. 40(4) (1986), pp. 280–283.

[18] H. Joe, Multivariate Models and Dependence Concepts, Chapman and Hall, London, 1997.

[19] A.J. McNeil and J. Neslehová, Multivariate Archimedean copulas, d-monotone functions and l-norm symmetric distributions Ann. Statist. 37 (2009), pp. 3059–3097.

[20] J. Crook and F. Moreira, Checking for asymmetric default dependence in a credit card portfolio: A copula approach, J. Empir. Finance 18(4) (2011), pp. 728–742.