Rama Distribution and Its Application

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Abstract A one parameter lifetime distribution named "Rama distribution" for modeling lifetime data has been proposed and investigated. Its statistical and mathematical properties including its shape, moments, skewness, kurtosis, hazard rate function, mean residual life function, stochastic ordering, mean deviations, Bonferroni and Lorenz curves, stress-strength reliability have been discussed. The condition under which Rama distribution is over-dispersed, equi-dispersed, and under-dispersed have been studied and compared with other one parameter lifetime distributions. Maximum likelihood estimation and method of moments have been discussed for estimating its parameter. Finally the goodness of fit of the proposed distribution has been discussed with a real lifetime data and the fit has been compared with other one parameter lifetime distributions.

Keywords Lifetime distribution, Moments, Hazard rate function, Mean residual life function, Mean deviations, Bonferroni and Lorenz Curves, Stress-strength reliability, Estimation of parameter, Goodness of fit

1. Introduction

The modeling and analyzing lifetime data are essential in many applied sciences including biomedicine, engineering, insurance and finance, amongst others. The two one parameter lifetimes distribution used in statistics for modeling lifetime data are exponential and Lindley (1958) distributions. Recently, Shanker *et al* (2015) have comparative study on both exponential and Lindley distributions for modeling lifetime data and observed that there are many situations where these two distributions are not suitable to fit lifetime data due to their shape and other properties. In a search for continuous distribution which gives better fit than exponential and Lindley distributions, Shanker (2015 a, 2015 b, 2016 a, 2016 b, 2016 c) has introduced one parameter lifetime distributions namely Akash, Shanker, Aradhana, Amarendra and Sujatha distributions. Each of these lifetime distributions has advantages and disadvantages over one another due to its shape, hazard rate function and mean residual life function. Still there are many situations where these distributions are not suitable for modeling lifetime data from theoretical or applied point of view. Therefore, an attempt has been made in this paper to obtain a new distribution which is flexible than these lifetime distributions for modeling lifetime data in reliability and in terms of its hazard rate shapes. The new one parameter lifetime distribution is based on a two- component mixture of an exponential distribution having scale

parameter θ and a gamma distribution having shape parameter 4 and scale parameter θ with their mixing proportion $\dfrac{\theta^3}{\theta^3 + 6}$.

The probability density function (p.d.f.) of a new one parameter lifetime distribution can be introduced as

$$f(x;\theta) = \frac{\theta^4}{\theta^3 + 6}\left(1 + x^3\right)e^{-\theta x} \quad ; x > 0, \theta > 0 \quad (1.1)$$

We would call this distribution, "Rama distribution". This distribution can be easily expressed as a mixture of exponential(θ) and gamma $(4, \theta)$ with mixing proportion $\dfrac{\theta^3}{\theta^3 + 6}$. We have

$$f(x, \theta) = p\, g_1(x) + (1 - p)\, g_2(x)$$

where

$$p = \frac{\theta^3}{\theta^3 + 6}, \; g_1(x, \theta) = \theta e^{-\theta x}, \text{ and } g_2(x, \theta) = \frac{\theta^4 x^3 e^{-\theta x}}{6}.$$

The corresponding cumulative distribution function (c.d.f.) of (1.1) can be obtained as

$$F(x, \theta) = 1 - \left[1 + \frac{\theta^3 x^3 + 3\theta^2 x^2 + 6\theta x}{\theta^3 + 6}\right]e^{-\theta x};$$

$$x > 0, \theta > 0 \quad (1.2)$$

The graph of the p.d.f. and the c.d.f. of Rama distribution for varying values of θ are shown in **figures 1 and 2**.
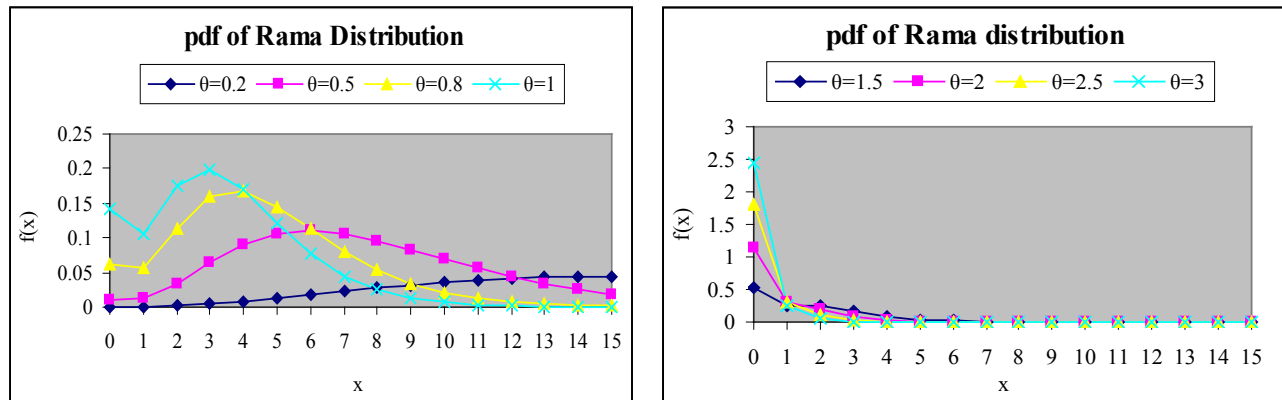
Figure 1. Graph of the pdf of Rama distribution for varying values of the parameter θ
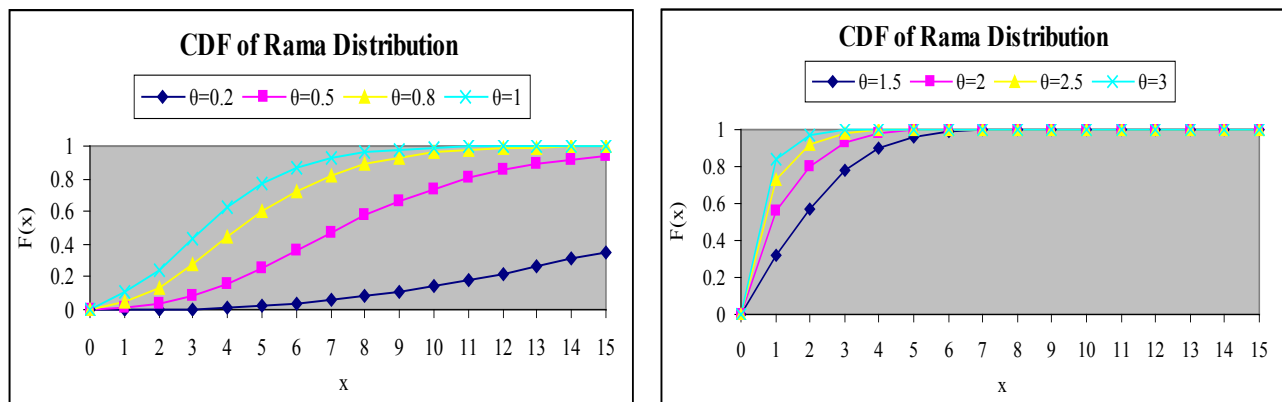

Figure 2. Graph of the cdf of Rama distribution for varying values of the parameter θ

2. Moments and Moments Based Measures

The moment generating function of Rama distribution (1.1) can be obtained as

$$M_X(t) = \frac{\theta^4}{\theta^3+6}\int_0^{\infty} e^{-(\theta-t)x}\left(1+x^3\right)dx$$

$$= \frac{\theta^4}{\theta^3+6}\left[\frac{1}{\theta-t}+\frac{6}{(\theta-t)^4}\right]$$

$$= \frac{\theta^4}{\theta^3+6}\left[\frac{1}{\theta}\sum_{k=0}^{\infty}\left(\frac{t}{\theta}\right)^k + \frac{6}{\theta^4}\sum_{k=0}^{\infty}\binom{k+3}{k}\left(\frac{t}{\theta}\right)^k\right]$$

$$= \sum_{k=0}^{\infty}\frac{\theta^3+(k+1)(k+2)(k+3)}{\theta^3+6}\left(\frac{t}{\theta}\right)^k$$

Thus the r th moment about origin of Rama distribution (1.1) can be obtained as

$$\mu_r{}' = \frac{r!\left[\theta^3+(r+1)(r+2)(r+3)\right]}{\theta^r\left(\theta^3+6\right)} \quad ;r=1,2,3,...$$

The first four moments about origin of Rama distribution (1.1) are as follows

$$\mu_1' = \frac{\theta^3 + 24}{\theta(\theta^3 + 6)}, \qquad \mu_2' = \frac{2(\theta^3 + 60)}{\theta^2(\theta^3 + 6)}, \qquad \mu_3' = \frac{6(\theta^3 + 120)}{\theta^3(\theta^3 + 6)}, \qquad \mu_4' = \frac{24(\theta^3 + 210)}{\theta^4(\theta^3 + 6)}$$

Using relationship between moments about mean and moments about origin, central moments of Rama distribution (1.1) are obtained as

$$\mu_2 = \frac{\theta^6 + 84\theta^3 + 144}{\theta^2(\theta^3 + 6)^2}$$

$$\mu_3 = \frac{2(\theta^9 + 198\theta^6 + 324\theta^3 + 864)}{\theta^3(\theta^3 + 6)^3}$$

$$\mu_4 = \frac{9(\theta^{12} + 312\theta^9 + 2304\theta^6 + 10368\theta^3 + 10368)}{\theta^4(\theta^3 + 6)^4}$$

The coefficient of variation $(C.V)$, coefficient of skewness $(\sqrt{\beta_1})$, coefficient of kurtosis (β_2) and index of dispersion (γ) of Rama distribution (1.1) are thus obtained as

$$C.V = \frac{\sigma}{\mu_1'} = \frac{\sqrt{\theta^6 + 84\theta^3 + 144}}{\theta^3 + 24}$$

$$\sqrt{\beta_1} = \frac{\mu_3}{\mu_2^{3/2}} = \frac{2(\theta^9 + 198\theta^6 + 324\theta^3 + 864)}{(\theta^6 + 84\theta^3 + 144)^{3/2}}$$

$$\beta_2 = \frac{\mu_4}{\mu_2^2} = \frac{9(\theta^{12} + 312\theta^9 + 2304\theta^6 + 10368\theta^3 + 10368)}{(\theta^6 + 84\theta^3 + 144)^2}$$

$$\gamma = \frac{\sigma^2}{\mu_1'} = \frac{\theta^6 + 84\theta^3 + 144}{\theta(\theta^3 + 6)(\theta^3 + 24)}$$

The condition under which Rama distribution is over-dispersed, equi-dispersed, and under-dispersed has been discussed along with condition under which Akash, Shanker, Aradhana, Amarendra Sujatha, Lindley and exponential distributions are over-dispersed, equi-dispersed, and under-dispersed and are presented in table 1.

Table 1. Over-dispersion, equi-dispersion and under-dispersion of Rama, Akash, Shanker, Aradhana, Amarendra, Sujatha, Lindley and exponential distributions for varying values of their parameter θ

Distribution	Over-dispersion $(\mu < \sigma^2)$	Equi-dispersion $(\mu = \sigma^2)$	Under-dispersion $(\mu > \sigma^2)$
Rama	$\theta < 1.950164618$	$\theta = 1.950164618$	$\theta > 1.950164618$
Akash	$\theta < 1.515400063$	$\theta = 1.515400063$	$\theta > 1.515400063$
Shanker	$\theta < 1.171535555$	$\theta = 1.171535555$	$\theta > 1.171535555$
Aradhana	$\theta < 1.283826505$	$\theta = 1.283826505$	$\theta > 1.283826505$
Amarendra	$\theta < 1.525763580$	$\theta = 1.525763580$	$\theta > 1.525763580$
Sujatha	$\theta < 1.364271174$	$\theta = 1.364271174$	$\theta > 1.364271174$
Lindley	$\theta < 1.170086487$	$\theta = 1.170086487$	$\theta > 1.170086487$
Exponential	$\theta < 1$	$\theta = 1$	$\theta > 1$

3. Hazard Rate Function and Mean Residual Life Function

Let $f(x)$ and $F(x)$ be the p.d.f. and c.d.f of a continuous random variable. The hazard rate function $h(x)$ (also known as the failure rate function) and the mean residual life function $m(x)$ of X are respectively defined as

$$h(x) = \lim_{\Delta x \to 0} \frac{P(X < x + \Delta x | X > x)}{\Delta x} = \frac{f(x)}{1 - F(x)} \tag{3.1}$$

and

$$m(x) = E[X - x | X > x] = \frac{1}{1 - F(x)} \int_x^\infty [1 - F(t)] \, dt \tag{3.2}$$

The corresponding hazard rate function, $h(x)$ and the mean residual life function, $m(x)$ of the Rama distribution are thus obtained as

$$h(x) = \frac{\theta^4 (1 + x^3)}{\theta^3 (x^3 + 1) + 3\theta^2 x^2 + 6(\theta x + 1)} \tag{3.3}$$

and

$$m(x) = \frac{1}{\left[\theta^3 (x^3 + 1) + 3\theta^2 x^2 + 6(\theta x + 1)\right] e^{-\theta x}} \int_x^\infty \left[\theta^3 (t^3 + 1) + 3\theta^2 t^2 + 6(\theta t + 1)\right] e^{-\theta t} dt$$

$$= \frac{\theta^3 (x^3 + 1) + 6\theta^2 x^2 + 18\theta x + 24}{\theta \left[\theta^3 (x^3 + 1) + 3\theta^2 x^2 + 6(\theta x + 1)\right]} \tag{3.4}$$
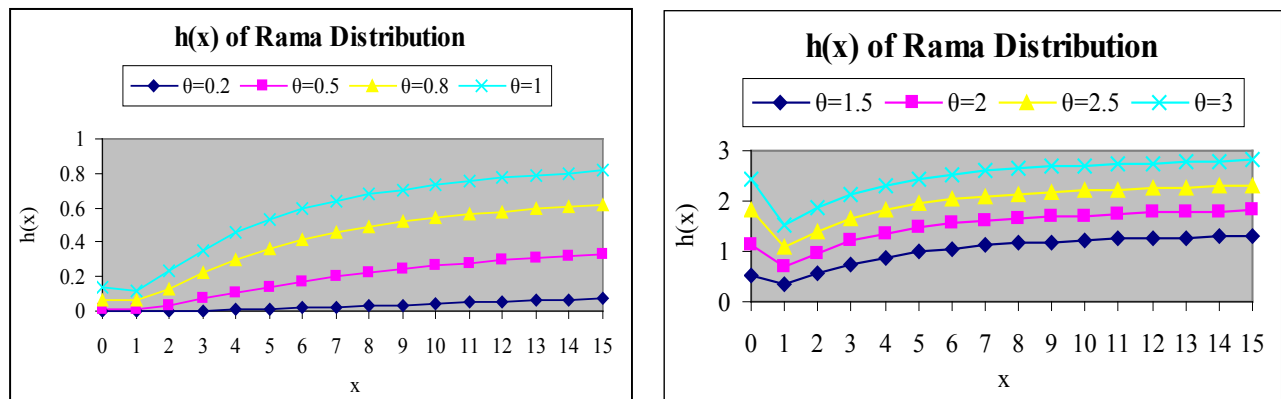

Figure 3. Graphs of hazard rate function of Rama distribution for different values of parameter θ
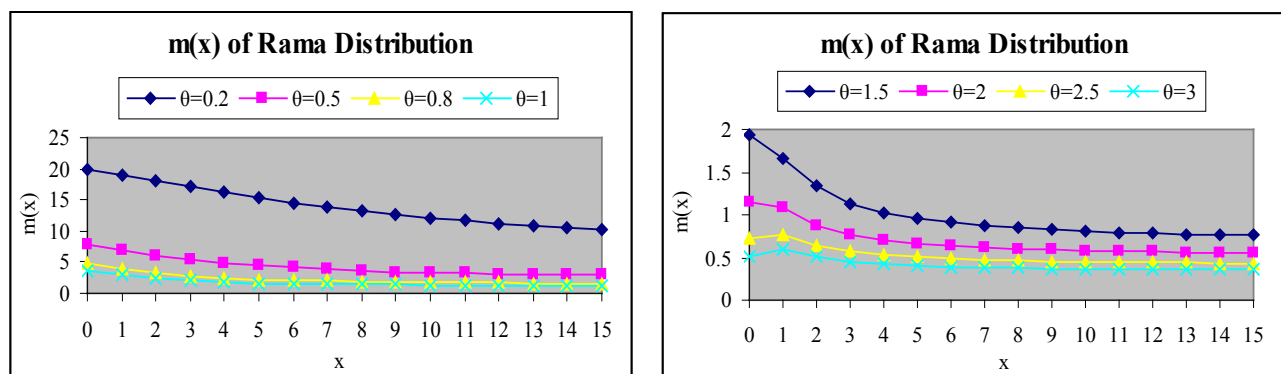

Figure 4. Graphs of mean residual life function of Rama distribution for different values of parameter θ

It can be easily verified that $h(0) = \dfrac{\theta^4}{\theta^3 + 6} = f(0)$ and $m(0) = \dfrac{\theta^3 + 24}{\theta(\theta^3 + 6)} = \mu_1'$. The graphs of the hazard rate

function and mean residual life function of Rama distribution for varying values of the parameter are shown in figures 3 and 4.

4. Stochastic Orderings

Stochastic ordering of positive continuous random variables is an important tool for judging their comparative behavior. A random variable X is said to be smaller than a random variable Y in the

(i)   stochastic order $(X \leq_{st} Y)$ if $F_X(x) \geq F_Y(x)$ for all x

(ii) hazard rate order $(X \leq_{hr} Y)$ if $h_X(x) \geq h_Y(x)$ for all x

(iii) mean residual life order $(X \leq_{mrl} Y)$ if $m_X(x) \leq m_Y(x)$ for all x

(iv) likelihood ratio order $(X \leq_{lr} Y)$ if $\dfrac{f_X(x)}{f_Y(x)}$ decreases in x.

The following results due to Shaked and Shanthikumar (1994) are well known for establishing stochastic ordering of distributions

$$X \leq_{lr} Y \Rightarrow X \leq_{hr} Y \Rightarrow X \leq_{mrl} Y \tag{4.1}$$
$$\Downarrow$$
$$X \leq_{st} Y$$

The Rama distribution is ordered with respect to the strongest 'likelihood ratio' ordering as shown in the following theorem:

Theorem: Let $X \sim$ Rama distribution (θ_1) and $Y \sim$ Rama distribution (θ_2). If $\theta_1 > \theta_2$, then $X \leq_{lr} Y$ and hence $X \leq_{hr} Y$, $X \leq_{mrl} Y$ and $X \leq_{st} Y$.

Proof: We have

$$\frac{f_X(x)}{f_Y(x)} = \frac{\theta_1^4 \left(\theta_2^3 + 6 \right)}{\theta_2^4 \left(\theta_1^3 + 6 \right)} e^{-(\theta_1 - \theta_2)x} \quad ; x > 0$$

Now

$$\ln \frac{f_X(x)}{f_Y(x)} = \ln \left[\frac{\theta_1^4 \left(\theta_2^3 + 6 \right)}{\theta_2^4 \left(\theta_1^3 + 6 \right)} \right] - (\theta_1 - \theta_2)x$$

This gives

$$\frac{d}{dx} \ln \frac{f_X(x)}{f_Y(x)} = -(\theta_1 - \theta_2)$$

Thus for $\theta_1 > \theta_2$, $\dfrac{d}{dx} \ln \dfrac{f_X(x)}{f_Y(x)} < 0$. This means that $X \leq_{lr} Y$ and hence $X \leq_{hr} Y$, $X \leq_{mrl} Y$ and $X \leq_{st} Y$.

5. Mean Deviations

The amount of scatter in a population is measured to some extent by the totality of deviations usually from mean and median. These are known as the mean deviation about the mean and the mean deviation about the median and are defined as

$$\delta_1(X) = \int_0^\infty |x - \mu| f(x) dx \quad \text{and} \quad \delta_2(X) = \int_0^\infty |x - M| f(x) dx \quad, \quad \text{respectively,} \quad \text{where} \quad \mu = E(X) \quad \text{and}$$

$M = \text{Median}\ (X)$. The measures $\delta_1(X)$ and $\delta_2(X)$ can be calculated using the relationships

$$\delta_1(X) = \int_0^\mu (\mu - x) f(x) dx + \int_\mu^\infty (x - \mu) f(x) dx$$

$$= \mu F(\mu) - \int_0^\mu x f(x) dx - \mu[1 - F(\mu)] + \int_\mu^\infty x f(x) dx$$

$$= 2\mu F(\mu) - 2\mu + 2\int_\mu^\infty x f(x) dx$$

$$= 2\mu F(\mu) - 2\int_0^\mu x f(x) dx \tag{5.1}$$

and

$$\delta_2(X) = \int_0^M (M - x) f(x) dx + \int_M^\infty (x - M) f(x) dx$$

$$= M F(M) - \int_0^M x f(x) dx - M[1 - F(M)] + \int_M^\infty x f(x) dx$$

$$= -\mu + 2\int_M^\infty x f(x) dx$$

$$= \mu - 2\int_0^M x f(x) dx \tag{5.2}$$

Using p.d.f. (1.1) and its mean, we get

$$\int_0^\mu x f(x) dx = \mu - \frac{\left\{ \theta^4\left(\mu^4 + \mu\right) + \theta^3\left(4\mu^3 + 1\right) + 12\theta\mu(\theta\mu + 2) + 24 \right\} e^{-\theta\mu}}{\theta\left(\theta^3 + 6\right)} \tag{5.3}$$

$$\int_0^M x f(x) dx = \mu - \frac{\left\{ \theta^4\left(M^4 + M\right) + \theta^3\left(4M^3 + 1\right) + 12\theta M(\theta M + 2) + 24 \right\} e^{-\theta M}}{\theta\left(\theta^3 + 6\right)} \tag{5.4}$$

Using expressions from (5.1), (5.2), (5.3), and (5.4), the mean deviation about mean, $\delta_1(X)$ and the mean deviation about median, $\delta_2(X)$ of Rama distribution are obtained as

$$\delta_1(X) = \frac{2\left\{ \theta^3\mu^3 + 6\theta^2\mu^2 + 18\theta\mu + \left(\theta^3 + 24\right) \right\} e^{-\theta\mu}}{\theta\left(\theta^3 + 6\right)} \tag{5.5}$$

$$\delta_2(X) = \frac{2\left\{ \theta^4\left(M^4 + M\right) + \theta^3\left(4M^3 + 1\right) + 12\theta M(\theta M + 2) + 24 \right\} e^{-\theta M}}{\theta\left(\theta^3 + 6\right)} - \mu \tag{5.6}$$

6. Bonferroni and Lorenz Curves

The Bonferroni and Lorenz curves (Bonferroni, 1930) and Bonferroni and Gini indices have applications not only in economics to study income and poverty, but also in other fields like reliability, demography, insurance and medicine. The Bonferroni and Lorenz curves are defined as

$$B(p) = \frac{1}{p\mu}\int_0^q x f(x)dx = \frac{1}{p\mu}\left[\int_0^\infty x f(x)dx - \int_q^\infty x f(x)dx\right] = \frac{1}{p\mu}\left[\mu - \int_q^\infty x f(x)dx\right] \tag{6.1}$$

and

$$L(p) = \frac{1}{\mu}\int_0^q x f(x)dx = \frac{1}{\mu}\left[\int_0^\infty x f(x)dx - \int_q^\infty x f(x)dx\right] = \frac{1}{\mu}\left[\mu - \int_q^\infty x f(x)dx\right] \tag{6.2}$$

respectively or equivalently

$$B(p) = \frac{1}{p\mu}\int_0^p F^{-1}(x)dx \tag{6.3}$$

and

$$L(p) = \frac{1}{\mu}\int_0^p F^{-1}(x)dx \tag{6.4}$$

respectively, where $\mu = E(X)$ and $q = F^{-1}(p)$.

The Bonferroni and Gini indices are thus defined as

$$B = 1 - \int_0^1 B(p)dp \tag{6.5}$$

and

$$G = 1 - 2\int_0^1 L(p)dp \tag{6.6}$$

respectively.

Using p.d.f. (1.1), we get

$$\int_q^\infty x f(x)dx = \frac{\left\{\theta^4\left(q^4 + q\right) + \theta^3\left(4q^3 + 1\right) + 12\theta q(\theta q + 2) + 24\right\}e^{-\theta q}}{\theta\left(\theta^3 + 6\right)} \tag{6.7}$$

Now using equation (6.7) in (6.1) and (6.2), we get

$$B(p) = \frac{1}{p}\left[1 - \frac{\left\{\theta^4\left(q^4 + q\right) + \theta^3\left(4q^3 + 1\right) + 12\theta q(\theta q + 2) + 24\right\}e^{-\theta q}}{\theta^3 + 24}\right] \tag{6.8}$$

and

$$L(p) = 1 - \frac{\left\{\theta^4\left(q^4 + q\right) + \theta^3\left(4q^3 + 1\right) + 12\theta q(\theta q + 2) + 24\right\}e^{-\theta q}}{\theta^3 + 24} \tag{6.9}$$

Now using equations (6.8) and (6.9) in (6.5) and (6.6), the Bonferroni and Gini indices are obtained as

$$B = 1 - \frac{\left\{\theta^4\left(q^4 + q\right) + \theta^3\left(4q^3 + 1\right) + 12\theta q\left(\theta q + 2\right) + 24\right\}e^{-\theta q}}{\theta^3 + 24} \tag{6.10}$$

$$G = -1 + \frac{2\left\{\theta^4\left(q^4 + q\right) + \theta^3\left(4q^3 + 1\right) + 12\theta q\left(\theta q + 2\right) + 24\right\}e^{-\theta q}}{\theta^3 + 24} \tag{6.11}$$

7. Stress-Strength Reliability

The stress- strength reliability describes the life of a component which has random strength X that is subjected to a random stress Y. When the stress applied to it exceeds the strength, the component fails instantly and the component will function satisfactorily till $X > Y$. Therefore, $R = P(Y < X)$ is a measure of component reliability and in statistical literature it is known as stress-strength parameter. It has wide applications in almost all areas of knowledge especially in engineering such as structures, deterioration of rocket motors, static fatigue of ceramic components, aging of concrete pressure vessels etc.

Let X and Y are independent strength and stress random variables having Rama distribution (1.1) with parameter θ_1 and θ_2, respectively. Then, the stress-strength reliability R of Rama distribution can be obtained as

$$R = P(Y < X) = \int_0^\infty P(Y < X \mid X = x) f_X(x) dx$$

$$= \int_0^\infty f(x;\theta_1) \, F(x;\theta_2) \, dx$$

$$= 1 - \frac{\theta_1^4 \begin{bmatrix} \theta_2^9 + 6\theta_1\theta_2^8 + 15\theta_1^2\theta_2^7 + 2\left(10\theta_1^3 + 9\right)\theta_2^6 + 3\left(5\theta_1^4 + 30\theta_1 + 411\right)\theta_2^5 \\ +6\left(\theta_1^4 + 31\theta_1 + 6\right)\theta_1\theta_2^4 + \left(\theta_1^6 + 204\theta_1^3 + 36\theta_1^2 + 1260\right)\theta_1\theta_2^3 \\ +2\left(63\theta_1^3 + 6\theta_1^2 + 378\right)\theta_1\theta_2^2 + 42\left(\theta_1^3 + 6\right)\theta_1^2\theta_2 + 6\left(\theta_1^3 + 6\right)\theta_1^3 \end{bmatrix}}{\left(\theta_1^3 + 6\right)\left(\theta_2^3 + 6\right)\left(\theta_1 + \theta_2\right)^7}.$$

8. Estimation of Parameter

8.1. Maximum Likelihood Estimation (MLE)

Let $\left(x_1, x_2, x_3, \dots, x_n\right)$ be a random sample from Rama distribution (1.1). The likelihood function, L of (1.1) is given by

$$L = \left(\frac{\theta^4}{\theta^3 + 6}\right)^n \prod_{i=1}^n \left(1 + x_i^3\right) e^{-n\theta\bar{x}}$$

The natural log likelihood function is thus obtained as

$$\ln L = n \ln\left(\frac{\theta^4}{\theta^3 + 6}\right) + \sum_{i=1}^n \ln\left(1 + x_i^3\right) - n\theta\bar{x}$$

Now $\quad \dfrac{d \ln L}{d\theta} = \dfrac{4n}{\theta} - \dfrac{3n\theta^2}{\theta^3 + 6} - n\bar{x}$

where $\bar{x}$ is the sample mean.

The MLE $\hat{\theta}$ of θ is the solution of the equation $\dfrac{d\ln L}{d\theta} = 0$ and so it can be obtained by solving the following fourth degree polynomial equation

$$\overline{x}\theta^4 - \theta^3 + 6\overline{x}\theta - 24 = 0 \qquad (8.1.1)$$

8.2. Method of Moment Estimation (MOME)

Equating the population mean of the Rama distribution to the corresponding sample mean, MOME $\tilde{\theta}$ of θ is the same as given by equation (8.1.1).

9. Goodness of Fit

The goodness of fit of Rama distribution has been conducted with several lifetime data from biomedical science and engineering and in majority of data sets the goodness of fit of Rama distribution has been found to be quite satisfactory. In this section, the goodness of fit of one parameter Rama distribution has been discussed with a real lifetime data set and fit has been compared with one parameter Akash, Shanker, Aradhana, Amarendra, Sujatha, Lindley and exponential distributions. The following data set has been considered for the goodness of fit of Rama distribution.

The data set is the strength data of glass of the aircraft window reported by Fuller *et al* (1994)

18.83, 20.80, 21.657, 23.03, 23.23, 24.05, 24.321, 25.50, 25.52, 25.80, 26.69,
26.77, 26.78, 27.05, 27.67, 29.90, 31.11, 33.20, 33.73, 33.76, 33.89,
34.76, 35.75, 35.91, 36.98, 37.08, 37.09, 39.58, 44.045, 45.29, 45.381

In order to compare lifetime distributions, $-2\ln L$, AIC (Akaike Information Criterion) and K-S Statistics (Kolmogorov-Smirnov Statistics) for the above data set have been computed. The formulae for computing AIC and K-S Statistics are as follows:

$AIC = -2\ln L + 2k$, $K\text{-}S = \underset{x}{\mathrm{Sup}}\left| F_n(x) - F_0(x) \right|$, where k = the number of parameters, n = the sample size and

$F_n(x)$ is the empirical distribution function.

The best distribution is the distribution which corresponds to lower values of $-2\ln L$, AIC, and K-S statistics. The MLE $\left(\hat{\theta}\right)$ of θ, S.E $\left(\hat{\theta}\right)$, standard error of θ, $-2\ln L$, AIC and K-S Statistic of the fitted distributions are presented in the following table 2.

It can be easily seen from above table that the Rama distribution gives better fit than Akash, Shanker, Aradhana, Amarendra, Sujatha, Lindley and exponential distributions and hence can be considered an important lifetime distribution for modeling lifetime data.

Table 2. MLE's, S.E $\left(\hat{\theta}\right)$ - 2ln L, AIC and K-S Statistics of the fitted distributions of data set

Distributions	MLE $\left(\hat{\theta}\right)$	S.E $\left(\hat{\theta}\right)$	$-2\ln L$	AIC	K-S statistic
Rama	0.129781	0.011651	232.79	234.79	0.253
Akash	0.097065	0.010048	240.68	242.68	0.298
Shanker	0.647164	0.008200	252.35	254.35	0.358
Aradhana	0.094319	0.009780	242.22	244.22	0.306
Amarendra	0.128294	0.012413	233.41	235.41	0.257
Sujatha	0.095613	0.009904	241.50	243.50	0.303
Lindley	0.062992	0.008001	253.98	255.98	0.365
Exponential	0.032449	0.005822	274.52	276.53	0.458

10. Concluding Remarks

A one parameter lifetime distribution named, "Rama distribution" has been proposed. Its statistical properties including shape, moments, skewness, kurtosis, hazard rate function, mean residual life function, stochastic ordering, mean deviations, Bonferroni and Lorenz curves, stress-strength reliability have been discussed. The condition under which Rama distribution is over-dispersed, equi-dispersed, and under-dispersed are presented along other one parameter lifetime distributions. Maximum likelihood estimation and method of moments have been discussed for estimating its parameter. Finally, the goodness of fit test using K-S Statistics (Kolmogorov-Smirnov Statistics) for a real lifetime data set has been presented and the fit has been compared with other one parameter lifetime distributions.

ACKNOWLEDGEMENTS

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