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Permutation Approach in Obtaining Control Limits and Interpretation of out of Control Signals in Multivariate Control Charts

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Abstract The use of multivariate control charts in manufacturing and service industries is often avoided because of the complexity in its development and interpretation of out of control signals. Most multivariate control charts require a specific distributional assumption to establish their control limits, but the bootstrap and permutation methods does not rely on such distributional assumption. Control limit obtain from the bootstrap method is approximate in nature while that of permutation is exact. This study introduces the permutation method in obtaining exact control limits as well as interpreting out of control signals in multivariate Hotelling's T^2 control charts. A performance study of the methods using empirical data sets shows that results from the proposed Permutation method when compared with other existing methods perform better in identifying out of control signal rather than stopping the entire processes.

Keywords Exact control limits, Hotelling's T^2, out-of-control signals, Permutation p-values

1. Introduction

An important part of statistical process control chart is the setting of control limits, which plays a major role in determining whether a given process is in control or not. Since control limits assists in establishing whether the observed data are statistically significant or not, any statistical method that will guarantee its proper computation should be developed and employed so that the desired false alarm rate (similar to probability of making Type 1 error) will be minimized or exactly α [1]. When a control chart signals, process quality managers must initiate a search for the cause of the process disturbance. The standard practice is to plot univariate control charts on the individual quality characteristics.

However, the use of separate charts does not allow information about the correlation of variables to be utilized, hence the multivariate control chart [2]. One common method of constructing multivariate control charts is based on Hotelling's T^2 statistics [3-6]. Traditionally, control chart is based on the assumption that monitoring statistics follow some form of distributional assumption. The modern time practice is that this assumption is usually violated and a control limit obtained through this process may be inaccurate thereby increasing the rate of false alarms [7-9].

To address the problem of distributional assumption, univariate bootstrap control charts were introduced to obtain control limits [10-16]. Generally, univariate control charts involves the computations of one variable, and any decision based on these charts when two or more variables are involved can increase false alarm rate, hence the nonparametric multivariate control charts, such as the bootstrap method may provides better alternative [17-21]. Most of the nonparametric multivariate Hotelling's T^2 control limit obtained in the literature is from the bootstrap method. However, the bootstrap method is approximate in nature since sampling is carried out with replacement. To reduce the problem of violating multivariate distributional assumption as well as avoiding the problem of approximation, this study proposed the permutation method so as to obtain exact control limit. The proposed permutation method is exact in nature because sampling is done without replacement.

2. Proposed Permutation Method in Obtaining Hotelling's T^2 Control Limit

To reduce abnormal behaviours when multivariate distributional assumption is violated as well as overcoming the problem of approximation [22, 1], the permutation procedure is proposed to obtain exact control limit. Suppose there are $\boldsymbol{p}$ quality characteristics with $\boldsymbol{n}$ set of

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observations (x_{ij}); $(i = 1,2,,,,,n; j = 1,2,\dots,p)$ as can be summarized in the variance covariance matrix below:

$$\begin{matrix} \mathbf{x_1} & \mathbf{x_2} & \cdots & \mathbf{x_p} \end{matrix}$$
$$\begin{pmatrix} x_{11} & x_{12} & \cdots & x_{1p} \\ x_{21} & x_{22} & \cdots & x_{2p} \\ & & \cdots & \\ x_{n1} & x_{n2} & \cdots & x_{np} \end{pmatrix}$$

The permutation algorithm for setting exact Hotelling's T^2 control limit is given by the following Steps:

STEP 1. Compute Hotelling's T^2 statistic with n observations from a given dataset as:

$$T_i^2 = (x_j - \bar{x})' S^{-1}(x_j - \bar{x}),\ i = 1,2,\dots,n; j = 1,2,\dots,d$$

STEP 2. Generate permutation sample from the initial T_i^2 statistics in Step 1 without replacement as:

$$T_1^{2(i)}, T_2^{2(i)}, \dots, T_n^{2(i)},\ (i^* = 1,2,\dots,B)$$

STEP 3. Compute the control limits by taking B average of $100(1-\alpha)^{th}$ percentile values as:

$$CL_{prop\ .perm\ .} = \frac{\sum_{i=1}^{B}(100(1-\alpha)^{th})}{B} \qquad (1)$$

STEP 4. Use the established control limit to monitor a new observation. That is, if the monitoring statistic of a new observation exceeds $CL_{prop\ .perm\ .}$, declare that specific observation as out of control.

For example, if there are $T_i^2, i = 1,2,3,4$; there are $4! = 24$ ways of arrangements as follows:

1.	T_1^2	T_2^2	T_3^2	T_4^2
2.	T_1^2	T_2^2	T_4^2	T_3^2
3.	T_1^2	T_3^2	T_2^2	T_4^2
4.	T_1^2	T_3^2	T_4^2	T_2^2
5.	T_1^2	T_4^2	T_2^2	T_3^2
6.	T_1^2	T_4^2	T_3^2	T_2^2
7.	T_2^2	T_1^2	T_3^2	T_4^2
8.	T_2^2	T_1^2	T_4^2	T_3^2
9.	T_2^2	T_3^2	T_1^2	T_4^2
10.	T_2^2	T_3^2	T_4^2	T_1^2
11.	T_2^2	T_4^2	T_1^2	T_3^2
12.	T_2^2	T_4^2	T_3^2	T_1^2
13.	T_3^2	T_1^2	T_2^2	T_4^2
14.	T_3^2	T_1^2	T_4^2	T_2^2
15.	T_3^2	T_2^2	T_1^2	T_4^2
16.	T_3^2	T_2^2	T_4^2	T_1^2
17.	T_3^2	T_4^2	T_1^2	T_2^2
18.	T_3^2	T_4^2	T_2^2	T_1^2
19.	T_4^2	T_1^2	T_2^2	T_3^2
20.	T_4^2	T_1^2	T_3^2	T_2^2
21.	T_4^2	T_2^2	T_1^2	T_3^2
22.	T_4^2	T_2^2	T_3^2	T_1^2
23.	T_4^2	T_3^2	T_1^2	T_2^2
24.	T_4^2	T_3^2	T_2^2	T_1^2

From the arrangement above, there are $T_i^2! * T_i^2 = 96$ **possible terms** and $T_i^2 * \left(2^{T_i^2-1}\right) = 32$ **distinct terms** and the procedure is obtained as follows:

1. Obtain $\left(\frac{T_i^2!}{T_i^2}\right)$ ways each for the 1st column, i.e. $\frac{4!}{4} = 6$ for $T_i^2, i = 1,2,3,4$
2. Obtain $\left(\frac{T_i^2!}{T_i^2(T_i^2-1)}\right)$ way for each of the ways in 1st column to have 2nd column, i.e. $\left(\frac{4!}{4(4-1)}\right) = 2$ for each of the 6 ways in 1st column
3. Obtain $\left(\frac{T_i^2!}{T_i^2(T_i^2-1)(T_i^2-2)}\right)$ way for each of the 2ways in 2nd column to have 3rd column, i.e. $\left(\frac{4!}{4(4-1)(4-2)}\right) = 1$ way for each of the way in 2nd column.
4. Obtain $\left(\frac{T_i^2!}{T_i^2(T_i^2-1)(T_i^2-2)(T_i^2-3)}\right)$ way for each of the way in 3rd column to have 4th column, i.e. $\left(\frac{4!}{4(4-1)(4-2)(4-3)}\right) = 1$ way for each of the way in 3rd column. Or complete the 4th (last) column by filling the value of T_i^2 that is yet to appear in each of the row.
5. Obtain 100(1-α) percentile for each row.
6. Obtain the permutation control limit by taking the average of the 100(1-α) percentile.

2.1. Proposed Permutation P-values Method in Identifying out of Control Signal

The problem of identifying quality characteristics that is responsible for an out of control signals has been an issue in multivariate control charts [23-30]. The introduced of Multivariate statistical process control charts based on the approximate sequential χ^2 test to detect a change in the non-centrality parameter was by [31]. The skipping strategy to reduce the effect of the autocorrelation on the T^2 chart's performance was proposed by [32]. A very useful approach to the diagnosis of an out of control signal is to decompose the Hotelling's T^2 statistic into components that reflect the contribution of each of the d-dimensional vector of quality characteristics [23]. The following Steps Modified [23] in identifying out of control signal in multivariate control chart.

STEP 1.

For a d-dimensional vector of quality characteristics, the first row is expressed as:

$$T^2 = T_{j.i;\ \forall j=1, i=j-1}^2, T_{j.i;\ \forall j=2, i=j-1, j-2}^2, \dots, T_{j.i;\ \forall j=d, i=j-1, j-2, j-3, \dots, j-d}^2$$
$$= T_1^2, T_{2.1}^2, T_{3.1,2}^2, T_{4.1,2,3}^2, \dots, T_{d.1,2,3,\dots,d-1}^2$$

STEP 2.

Obtain the Critical Values from f-distribution for each of T_j^2 and $T_{j.i}^2$ terms in [23] such that:

$$T_j^2 \sim \frac{c(n+1)(n-1)}{n(n-c)} f_{(c, n-c, \alpha)}, c = 1$$

and

$$T_{j.i}^{2} \sim \frac{c(n+1)(n-1)}{n(n-c)} f_{(c,n-c,\alpha)}, c = 2, 3, \ldots, j-1$$

are used to check whether the jth variable is conforming to the relationship with other variables or not.

Table 1. FDPSOP Data and $\boldsymbol{T^2}$ Statistic

Sample	X_1	X_2	X_3	X_4	T^2
1	3000	94	30	5.3	2.5324
2	2850	90	28	5.6	0.9443
3	2300	92	24	5.4	1.0588
4	2500	80	25	5.2	2.499
5	2750	45	27	7.5	**24.9818**
6	2400	82	26	5.8	0.581
7	1550	80	20	5.1	**10.5469**
8	2950	100	30	4.2	8.2502
9	2850	93	29	6.1	3.271
10	2300	85	25	5.9	0.7641
11	2250	95	24	5.5	1.3214
12	2900	80	26	5.2	3.4108
13	2550	87	27	5.7	0.1649
14	2100	98	28	5.4	2.0423
15	2000	86	29	5	4.3045
16	1050	70	20	6.2	**15.6408**
17	3000	82	30	6	3.5346
18	2850	80	30	5.2	3.7166
19	2000	95	31	5	5.9975
20	2050	86	26	5.8	1.0593
21	2150	91	25	5.7	0.803
22	2060	83	28	5.4	2.1454
23	2700	90	24	5.6	1.723
24	2800	94	25	5.3	1.7413
25	2950	85	27	5.4	1.7658
26	2250	86	29	5.4	1.549
27	2005	97	32	5.9	8.2599
28	2010	100	24	5.6	3.0063
29	3010	98	23	5	6.2249
30	2500	84	28	4.8	3.5793
31	2450	88	24	5.3	1.3496
32	2680	96	26	4.9	2.0374
33	2750	100	22	6	7.3517
34	2900	87	29	6.3	3.9443
35	2850	89	30	5.1	2.4777
36	2000	96	26	5.3	1.7141
37	3000	99	27	6.1	5.2445
38	2150	100	28	6	5.9041
39	2300	101	22	5.8	5.1479
40	2400	102	25	5.7	2.717
41	2600	80	28	5.2	2.2793
42	2015	94	29	5.9	3.5712
43	2225	90	30	6	3.3235
44	2450	98	27	5.4	0.7971
45	2900	81	26	5.5	2.2492

STEP 3.

Repeat Steps 1 and 2 for other rows based on the number of quality characteristics (d!) and obtain the distinct terms ($d*2^{d-1}$) for both the unconditional (T_j^2) and conditional ($T_{j.i}^2$) terms.

STEP 4.

Obtain the permutation p-values for each of T_j^2 and $T_{j.i}^2$ terms such that:

$$P_{value\ (Perm.)} = \frac{1}{B}\{(T_{Perm.}^{2*}) \geq (T_j^2)\};$$

$$P_{value\ (Perm.)} = \frac{1}{B}\{(T_{Perm.}^{2*}) \geq (T_{j.i}^2)\}$$

STEP 5.

Use the various $P_{-values}$ in Step 4 to assess whether there is a significant difference or not. If $(P_{-values\ (\ Perm)} > \alpha)$ value, it means that T_j^2 or $T_{j.i}^2$ is (are) not responsible for the out of control signal(s). But when $(P_{-values\ (\ Perm} \leq \alpha)$ value, it means that T_j^2 or $T_{j.i}^2$ is (are) responsible for the out of control signal(s).

3. Application to Numerical Example

By way of illustration, the set of data were obtained from the production processing of Owel Industries Nig. Ltd., a Family Delight Pure Soya Oil Production (FDPSOP) Company in Ekpoma, Edo State, Nigeria. Four quality characteristics (X_1, X_2, X_3 and X_4) representing phosphoric acid (milliliters), water (liters), caustic soda solution (kg) and industrial salt (kg) respectively at the neutralizer stage, under which forty five samples were recorded as shown in Columns (X_1, X_2, X_3 and X_4) of Table 1. The choice of data used in this study is the presence of sub standard product of cooking oil displayed in the local market in Nigeria. Another motivation is the challenges faced by Quality Control Managers to identify the variable that is responsible for out of control signals or stop the entire production process. Stopping the process will result to waste of material resources and continuing with the process without identifying the variable will lead to sub standard product. The urge to solve these problems gave rise to this research work.

3.1. Test of Correlation Coefficient

To apply any multivariate control chart methodology, there is need to know whether there exists inter-correlation among the four quality characteristics. From the given data, the statistics of mean vector $(\bar{x})$, variance covariance matrix (s) and correlation matrix (r) are given as follows:

$$\bar{x} = \begin{bmatrix} 2451.2222 \\ 89.0889 \\ 26.6444 \\ 5.5489 \end{bmatrix}$$

$$s = \begin{bmatrix} 186363.813 & 172.389 & 370.672 & -16.141 \\ 172.389 & 101.674 & 1.260 & -2.027 \\ 370.672 & 1.260 & 7.962 & -0.112 \\ -16.141 & -2.027 & -0.112 & 0.269 \end{bmatrix}$$

and

$$r == \begin{bmatrix} 1.0000 & 0.040 & 0.304^{*} & -0.072 \\ 0.040 & 1.000 & 0.044 & -0.388^{**} \\ 0.304^{*} & 0.044 & 1.0000 & -0.076 \\ -0.072 & -0.388^{**} & -0.076 & 1.0000 \end{bmatrix}$$

* Significant at 0.05 ** Significant at 0.01

The correlation matrix shows that there exists inter – correlation among the four quality characteristics, hence the need for multivariate control chart. The values of Hotelling's T^2 statistic is computed for each sample and summarized in the last column of Table 1. The proposed permutation procedures presented in Session 2.0 was translated to Permutation Generic Code, permutation samples were replicated 13,948 times from the Hotelling's T^2 statistic, and control limit is determined to be 10.0895 by taken the B average of 100(1-α) percentile value computed for each sample. Table 2 shows the results of control limits obtained from two existing methods (F-distribution and Phaladigalon's [19]) with the proposed permutation methods at α = 0.05 level of significant and the control chart to monitor the observations is shown in Figure 1.

Table 2. Control Limits for the Different Methods at α Level of 0.05

Alpha level (α)	F-Distribution	Existing Bootstrap Method	Proposed Permutation Method
0.05	11.4089	11.3336	10.0895

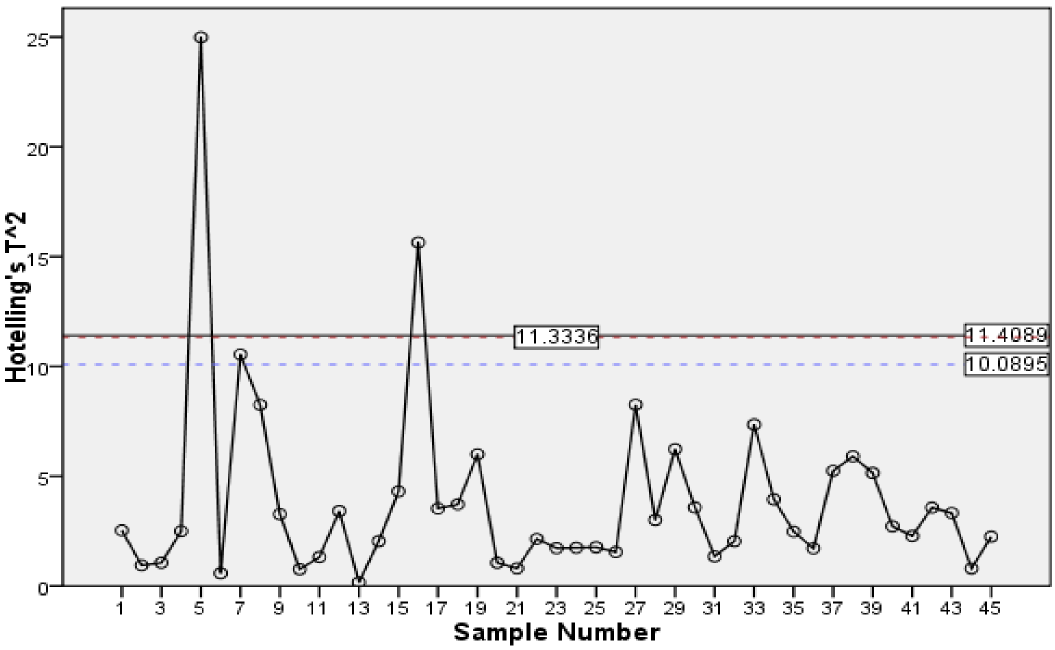

Figure 1. Multivariate Hotelling's T^2 Control Chart for the given Data

Table 3. Unconditional $\boldsymbol{T^2}$ Terms with p-values (Number of $\boldsymbol{T^2_{Sortd} \geq T^2_j}$ in Parenthesis)

		Existing Methods		Proposed Methods	
$\boldsymbol{T_j^2}$ Component	Computed $\boldsymbol{T_j^2}$ Value	Manson Critical values	Phaladigalon's Bootstrap P-Value	Permutation P-Value	$\boldsymbol{T_j^2}$ Reduced (Sig./Not)
T_1^2	0.4790	4.1519	1.0000 **(3000)**	1.0000 **(13948)**	N/S
T_2^2	19.1183*	„	0.0783 **(235)**	0.000 **(0)*****	5.8635 (N/S)
T_3^2	0.0159	„	1.0000 **(3000)**	1.0000 **(13948)**	N/S
T_4^2	14.1516*	„	0.3087 **(926)**	0.0000*** **(0)**	10.830 (Sig)

***Out of Control Signals ***Significant at 0.01 N/S (Not Significant) (Sig) Significant ($\boldsymbol{T_j^2 > CL}$)**

Table 4. 1st Conditional T^2 Terms with p-values (Number of $T^2_{Sortd} \geq T^2_{j.i}$ in Parenthesis)

		Existing Methods		Proposed Methods	
$T^2_{j.i}$ Component	Computed $T^2_{j.i}$ Value	Manson Critical values	Phaladigalon's Bootstrap P-Value	Permutation P-Value	$T^2_{j.i}$ Reduced (Sig./Not)
$T^2_{1.2}$	0.7498	6.7247	1.0000 **(3000)**	1.0000 **(13948)**	N/S
$T^2_{1.3}$	0.4710	,,	1.0000 **(3000)**	1.0000 **(13948)**	N/S
$T^2_{1.4}$	0.9327	,,	1.0000 **(3000)**	1.0000 **(13948)**	N/S
$T^2_{2.1}$	19.3818*	,,	0.0783 **(235)**	0.0000*** **(0)**	5.6 (N/S)
$T^2_{2.3}$	19.2047*	,,	0.0783 **(235)**	0.0000*** **(0)**	5.7771 (N/S)
$T^2_{2.4}$	9.9951*	,,	0.565 **(1695)**	1.0000 **(13948)**	14.986 (Sig)
$T^2_{3.1}$	0.0079	,,	1.0000 **(3000)**	1.0000 **(13948)**	N/S
$T^2_{3.2}$	0.1023	,,	1.0000 **(3000)**	1.0000 **(13948)**	N/S
$T^2_{3.4}$	0.1723	,,	1.0000 **(3000)**	1.0000 **(13948)**	N/S
$T^2_{4.1}$	14.6054*	,,	0.259 **(777)**	0.0000*** **(0)**	10.376 (Sig)
$T^2_{4.2}$	5.0285	,,	0.999 **(2997)**	1.0000 **(13948)**	N/S
$T^2_{4.3}$	14.3080*	,,	0.259 **(777)**	0.0000*** **(0)**	10.673 (Sig)

***Out of Control Signals ***Significant at 0.01 N/S (Not Significant) (Sig) Significant ($T^2_{j.i} > CL$)**

3.2. Identification of Out of Control Signal by the Critical and P-values Methods

Table 2 and Figure 1 shows that samples 5, 7 and 16 are out of control, but we do not know which or set of quality characteristic(s) that is(are) responsible for the signals, hence the need to identify those quality characteristics by using the proposed p-values methods. To explain this procedure, focus shall be on Samples 5. Table 3 shows all the unconditional and conditional T^2 values computed for Sample 5 and compared with the various critical and p-values from both existing and proposed methods.

3.3. Discussion and Interpretation of Out of Control Results

Results in Table 2 shows that control limits obtained from both F-distribution and Phaladiganon's methods where able to detect out of control signal for Samples 5 and 16, but consider Sample 7 to be under control. This has demonstrated the ability of both methods to detect large shift in the process mean vector. However, the proposed permutation method was able to detect out of control signals in additional one Sample (7), this has demonstrated the ability of the proposed method to detect shift better in any process. Table 3 shows all the unconditional and conditional T^2 values for Sample 5 that were computed and compared with their respective critical and p-values. From Table 3, T_2^2, T_4^2 of the four unconditional T^2 terms associated with Sample 5 are significant, which means X_2 (water in liters) and X_4 (industrial salt in kg) are responsible for the out of control signals individually. However, results from Phaladiganon's p-values did not support this finding. This has further demonstrated the ability of the proposed method to performed better in setting control limits and identifying out of control signals over the existing method. To reduce the problem of out of control signal facing variables X_2 and X_4, remove T_2^2, T_4^2 separately from $T^2 = 24.9818$ of Sample 5 and compare with the control limits whether they are significant or not. i.e.

$$T^2 - T_2^2 = 24.9818 - 19.1183 = 5.8635 < 11.4689, 10.0895, 11.3336$$

Hence, we conclude that variable X_2 is not significant. However, result obtain when T_4^2 is removed from T^2 shows that X_4 (industrial salt in kg) is significant when compared with Control Limits from the proposed methods in Table 2, hence we move to the next step, i.e.

$$T^2 - T_4^2 = 24.9818 - 14.1516 = 10.8302 < 11.4689, 11.3336; > 10.089$$

Table 5. 2nd Conditional T^2 Terms with p-values (Number of $T^2_{j.i} \geq T^2$ in Parenthesis)

		Existing Methods		Proposed Methods	
$T^2_{j.i}$ Component	Computed $T^2_{j.i}$ Value	Manson Critical values	Phaladigalon's Bootstrap P-Value	Permutation P-Value	$T^2_{j.i}$ Reduced (Sig./Not)
$T^2_{1.23}$	0.6511	9.0824	1.0000 **(3000)**	1.0000 **(13948)**	N/S
$T^2_{1.24}$	0.8018	,,	1.0000 **(3000)**	1.0000 **(13948)**	N/S
$T^2_{1.34}$	0.7776	,,	1.0000 **(3000)**	1.0000 **(13948)**	N/S
$T^2_{2.13}$	19.3849*	,,	0.0783 **(235)**	0.0000*** **(0)**	5.5969 (N/S)
$T^2_{2.14}$	10.0744*	,,	0.565 **(1695)**	1.0000 **(13948)**	14.9074 **(Sig)**
$T^2_{2.34}$	10.0395*	,,	0.565 **(1695)**	1.0000 **(13948)**	14.9423 **(Sig)**
$T^2_{3.12}$	0.0036	,,	1.0000 **(3000)**	1.0000 **(13948)**	N/S
$T^2_{3.14}$	0.0172	,,	1.0000 **(3000)**	1.0000 **(13948)**	N/S
$T^2_{3.24}$	0.2177	,,	1.0000 **(3000)**	1.0000 **(13948)**	N/S
$T^2_{4.12}$	5.2907	,,	0.9977 **(2993)**	1.0000 **(13948)**	N/S
$T^2_{4.13}$	14.6157*	,,	0.259 **(777)**	0.0000*** **(0)**	10.366 **(Sig)**
$T^2_{4.23}$	5.1428	,,	0.9987 **(2966)**	1.0000 **(13948)**	N/S

***Out of Control Signals ***Significant at 0.01 N/S (Not Significant) (Sig) Significant ($T^2_{j.i} > CL$)**

Table 6. 3rd Conditional T^2 Terms with p-values (Number of $T^2_{j.i} \geq T^2$ in parenthesis)

		Existing Methods		Proposed Methods	
$T^2_{j.i}$ Component	Computed $T^2_{j.i}$ Value	Manson Critical values	Phaladigalon's Bootstrap P-Value	Permutation P-Value	$T^2_{j.i}$ Reduced (Sig./Not)
$T^2_{1.234}$	0.8247	**11.4088**	1.0000 **(3000)**	1.0000 **(13948)**	N/S
$T^2_{2.134}$	5.0433	,,	0.5650 **(1695)**	1.0000 **(13948)**	N/S
$T^2_{3.124}$	0.0293	,,	1.0000 **(3000)**	1.0000 **(13948)**	N/S
$T^2_{4.123}$	5.3164	,,	0.9977 **(2993)**	1.0000 **(13948)**	N/S

***Out of Control Signals N/S (Not Significant)**

In Table 4, the first conditional T^2 terms associated with Sample 5 shows that $T^2_{2.1}$, $T^2_{2.3}$, $T^2_{2.4}$, $T^2_{4.1}$, and $T^2_{4.3}$ of the twelve conditional T^2 terms have significant values, which means the relationship between X_2 (water) and X_1 (phosphoric acid); X_2 (water) and X_3 (caustic soda); X_2 (water) and X_4 (industrial salt); X_4 (industrial salt) and X_1 (phosphoric acid); X_4 (industrial salt) and X_3 (caustic soda) respectively are responsible for out of control signals. To reduced the problem of out of control signal facing these 1st conditional variables, remove $T^2_{2.1}$, $T^2_{2.3}$, $T^2_{2.4}$, $T^2_{4.1}$, and $T^2_{4.3}$ separately from $T^2 = 24.9818$ of Sample 5 and compare with the control limits whether they are significant or not. $T^2_{2.1}$, $T^2_{2.3}$ are not significant as shown in the last columns of Table 5, while $T^2_{2.4}$, $T^2_{4.1}$ and $T^2_{4.3}$ are significant, hence we move to the next step, i.e.

$$T^2 - T^2_{2.4} = 24.9818 - 9.9951 = 14.9867 > 11.4689, 11.3336, 10.0895$$

$$T^2 - T^2_{4.1} = 24.9818 - 14.6054 = 10.3764 < 11.4689, 11.3336 > 10.0895$$

$$T^2 - T^2_{4.3} = 24.9818 - 14.3080 = 10.6738$$
$$< 11.4689, 11.3336 > 10.0895$$

A similar interpretation of results from Table 5 shows that $T^2_{2.14}$, $T^2_{2.34}$ and $T^2_{4.13}$ of the second conditional terms are significant, hence the last step in Table 6 shown no significant difference.

4. Conclusions

This study specifically considered the permutation method as a means of determining exact Hotelling's T^2 control limits. Procedures that can carry out a systematic generation of permutation replications of two or more quality characteristics have been proposed in this work; it is straight forward but computer intensive. However, to identify the root cause of change when multivariate control charts signals, this work also considered the p-value method in identifying the variable(s) that is(are) responsible for the out of control signals. It is of importance to note that the univariate control chart practice is to stop the entire process as a result of out of control signal at variable X_2 and X_4, this will result to waste of material resources or low quality or sub standard products if the process continuous. With the proposed multivariate methods, one variable is being conditioned on the other(s) as shown in Table 3. The implication of these findings is the advantages of multivariate control charts; by combining variable X_2 or X_4 with any other variables until there is no out of control signals as observed in Table 6. This was achieved when variable X_2 was combined with X_1, X_3 and X_4 on one hand, and X_4 with X_1, X_2, and X_3 on the other hand. This finding will enhance production process and avoid waste of material resources and improve the quality of product.

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