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Estimators of Population Median Using New Parametric Relationship

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Abstract In this paper, we first establish the new parametric relationship for population median (M_d) and then propose conventional consistent, ratio-type and product-type estimators of M_d under different situations. The expression for their optimum biases and minimum mean square errors (MSE's) are obtained, up to terms of order n^{-1} and compared with each other. Empirically, we have checked the authenticity of the relationship for M_d and also the gain in efficiencies of the proposed estimators with each other and existing ones are illustrated.

Keywords Auxiliary variable, Bias, Empirical relationship for mode, Estimation, Mean squared error, Median, Mode, Product estimator, Ratio estimator, Skewness

1. Introduction

In survey sampling, statisticians are often interested in dealing with variables that have highly skewed distributions such as consumptions, expenditure, income, etc. In such situations median is regarded as a more appropriate measure of location than mean. It has been well recognized that use of auxiliary variable information results in efficient estimators of population parameters. Initially, estimation of median without auxiliary variable was analyzed, after that some authors including Kuk and Mak (1989), Mak and Kuk (1993), Garcia and Cebrian (2001), Singh et al. (2006), Al and Cingi (2010) used the auxiliary information in median estimation. Singh and Solanki (2013), proposed the four estimators and a generalized class of estimators of population median (M_d) using known information of population median of the auxiliary variable. Empirically, they studied the 22 estimators.

In this paper, we first establish the new parametric relationship for population median (M_d) and propose three estimators of M_d under different situations. This technique of estimation of population median is different from the existing one given by Kuk and Mak (1989), which is very simple and efficient. Up to terms of order n^{-1}, the optimum biases and minimum MSE's of the proposed estimators are obtained and compared with each other. Empirically, we have done two studies, first to check the authenticity of the relationship for M_d and to have the rough idea about the efficiencies of proposed estimators with each other. Secondly, to compare the efficiencies of the proposed estimators with existing ones as the techniques of estimation and parameters involved in their minimum MSE's are different.

2. Notations and Results

Suppose a simple random sample of size n is drawn from a finite population of size N without replacement and observations on both study variables y and auxiliary variable x are taken. Let the values of variable y and x be denoted by Y_i and X_i respectively on the i^{th} unit of the population $i = 1,2,\dots,N$ and the corresponding small letters y_i and x_i denote the sample values corresponding to the i^{th} unit in the sample.

Taking,

$$\bar{Y} = \frac{1}{N}\sum_{i=1}^{N} Y_i, \qquad \bar{X} = \frac{1}{N}\sum_{i=1}^{N} X_i,$$

$$S_y^2 = \frac{1}{N-1}\sum_{i=1}^{N}(Y_i - \bar{Y})^2, \qquad S_x^2 = \frac{1}{N-1}\sum_{i=1}^{N}(X_i - \bar{X})^2,$$

$$\mu_{rs} = \frac{1}{N}\sum_{i=1}^{N}(Y_i - \bar{Y})^r\,(X_i - \bar{X})^s, \qquad \lambda_{rs} = \frac{\mu_{rs}}{\mu_{20}^{r/2}\mu_{02}^{s/2}},$$

$$m_{30} = \frac{n}{(n-1)(n-2)}\sum_{i=1}^{n}(y_i - \bar{y})^3, \qquad C_y = \frac{S_y}{\bar{Y}},$$

$$C_x = \frac{S_x}{\bar{X}}, \qquad C_{yx} = \frac{S_{yx}}{\bar{Y}\bar{X}} = \rho C_y C_x,$$

$$S_{yx} = \frac{1}{N-1}\sum_{i=1}^{N}(Y_i - \bar{Y})(X_i - \bar{X})$$

Obviously

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$\lambda_{11} = \rho_{xy} = \rho$(Correlation between x and y);
$\lambda_{30} = \beta_{1y}$(Coefficient of skewness of y);
$\lambda_{40} = \beta_{2y}$(Coefficient of kurtosis of y).

Defining,

$$\delta_0 = \frac{\bar{y}}{\bar{Y}} - 1, \qquad \delta = \frac{s_y^2}{S_y^2} - 1,$$
$$\epsilon = \frac{\bar{x}}{\bar{X}} - 1, \qquad \eta_1 = \frac{m_{30}}{\mu_{30}} - 1.$$

For the sake of simplicity, assume that N is large enough as compares to n so that finite population correction (fpc) terms are ignored throughout.

For the given SRSWOR, we have the following expectations,

$$E(\delta_0) = E(\delta) = E(\epsilon) = E(\eta_1) = 0,$$
$$E(\delta_0^2) = \frac{1}{n}C_y^2, \qquad E(\epsilon^2) = \frac{1}{n}C_x^2,$$
$$E(\delta_0\epsilon) = \frac{1}{n}C_{yx}, \qquad E(\epsilon\delta) = \frac{1}{n}\lambda_{21}C_x,$$
$$E(\delta_0\delta) = \frac{1}{n}\lambda_{30}C_y = \frac{1}{n}\beta_{1y}C_y,$$

and up to terms of order n^{-1}

$$E(\delta^2) = \frac{1}{n}(\lambda_{40} - 1) = \frac{1}{n}\left(\beta_{2y} - 1\right),$$
$$E(\eta_1^2) = \frac{1}{n}\frac{(\lambda_{60} - 6\lambda_{40} - \lambda_{30}^2 + 9)}{\lambda_{30}^2}$$
$$= \frac{1}{n}\frac{\left(\lambda_{60} - 6\beta_{2y} - \beta_{1y}^2 + 9\right)}{\beta_{1y}^2},$$
$$E(\delta_0\eta_1) = \frac{1}{n}\frac{(\lambda_{40} - 3)}{\lambda_{30}}C_y = \frac{1}{n}\frac{\left(\beta_{2y} - 3\right)}{\beta_{1y}}C_y,$$
$$E(\delta\eta_1) = \frac{1}{n}\frac{(\lambda_{50} - 4\lambda_{30})}{\lambda_{30}} = \frac{1}{n}\frac{\left(\lambda_{50} - 4\beta_{1y}\right)}{\beta_{1y}},$$
$$E(\epsilon\eta_1) = \frac{1}{n}\frac{(\lambda_{31} - 3\rho)}{\lambda_{30}}C_x = \frac{1}{n}\frac{(\lambda_{31} - 3\rho)}{\beta_{1y}}C_x.$$

3. Proposed Estimators and Their Biases and MSE's

Sharma et al. (2016) established the parametric relation for population mode as

$$M_o = \bar{Y} - k\frac{\mu_{30}}{S_y^2},$$

where k is unknown constant to be determined.

Sharma et al. (2016) also proposed three estimators of M_o under the three different situations as

$$\widehat{M}_{o1} = \bar{y} - k_1\frac{m_{30}}{s_y^2},$$
$$\widehat{M}_{o2} = \bar{y}\frac{\bar{X}}{\bar{x}} - k_2\frac{m_{30}}{s_y^2}\frac{\bar{X}}{\bar{x}},$$

and

$$\widehat{M}_{o3} = \bar{y}\frac{\bar{x}}{\bar{X}} - k_3\frac{m_{30}}{s_y^2}\frac{\bar{x}}{\bar{X}}.$$

They determined the optimum value of k_1, k_2 and k_3 by minimizing the MSE's of respective estimators as

$$k_{1opt} = \frac{\beta_y}{\lambda_{60} - 6\beta_y + \beta_{0y}},$$
$$k_{2opt} = \frac{\beta_y C_y + \beta_{1y}\left(C_x^2 - C_{yx}\right) + B}{C_y\left(\lambda_{60} - 6\beta_y + \beta_{0y} + \beta_{1y}\left(\beta_{1y}C_x^2 + 2B\right)\right)},$$
$$k_{3opt} = \frac{\beta_y C_y + \beta_{1y}\left(C_x^2 + C_{yx}\right) - B}{C_y\left(\lambda_{60} - 6\beta_y + \beta_{0y} + \beta_{1y}\left(\beta_{1y}C_x^2 - 2B\right)\right)}.$$

where,

$$\beta_y = \beta_{2y} - \beta_{1y}^2 - 3,$$
$$\beta_{0y} = \beta_{1y}^2\beta_{2y} - 2\beta_{1y}\lambda_{50} - 9,$$
$$B = \beta_{1y}\lambda_{21}C_x - \lambda_{31}C_x + 3\rho C_x.$$

Note that k_{iopt} , $i = 1,2,3$ are further functions of unknown population parameters. Srivastava and Jhajj (1983) have shown that if we replace the parameters involved in the optimum value of the unknown constant by their consistent estimators then up to terms of order n^{-1}, the MSE remains the same. Therefore, the presence of unknown parameters in the optimum values of the constants will not create any problem for practical use of the proposed estimators.

Let $\hat{k}_{iopt}$, $i = 1,2,3$ be the values when parameters are replaced by their consistent estimators in k_{iopts} . So then the estimators reduces to

$$\widehat{M}_{o1}' = \bar{y} - \hat{k}_{1opt}\frac{m_{30}}{s_y^2},$$
$$\widehat{M}_{o2}' = \bar{y}\frac{\bar{X}}{\bar{x}} - \hat{k}_{2opt}\frac{m_{30}}{s_y^2}\frac{\bar{X}}{\bar{x}},$$
$$\widehat{M}_{o3}' = \bar{y}\frac{\bar{x}}{\bar{X}} - \hat{k}_{3opt}\frac{m_{30}}{s_y^2}\frac{\bar{x}}{\bar{X}},$$

and

$$MSE\left(\widehat{M}_{oi}\right) = MSE\left(\widehat{M}'_{oi}\right), i = 1,2,3.$$

For moderately skewed distribution, Doodson (1917) suggested the empirical relationship as

$$Mode = 3Median - 2Mean,$$

which is also known as Karl Pearson empirical relation between mean, median and mode.

That implied

$$Median = \frac{Mode + 2Mean}{3}$$

or

$$M_d = \frac{M_o + 2\bar{Y}}{3}$$

Using above, obviously, we have

$$M_d = \bar{Y} - \frac{k_{iopt}}{3}\frac{\mu_{30}}{S_y^2}, i = 1,2,3. \tag{3.1}$$

where for univariate population, $i = 1$ and for the bivariate population (y, x), $i = 2$ if ρ is highly positive and $i = 3$ if ρ is highly negative, which is a new parametric relationship for population median.

Using above parametric relationship for population median, we here propose three different estimators of M_d as

$$\widehat{M}_{d_1}' = \bar{y} - \frac{\hat{k}_{1opt}}{3}\frac{m_{30}}{s_y^2}, \tag{3.2}$$

$$\widehat{M}_{d_2}' = \bar{y}\frac{\bar{X}}{\bar{x}} - \frac{\hat{k}_{2opt}}{3}\frac{m_{30}}{s_y^2}\frac{\bar{X}}{\bar{x}}, \tag{3.3}$$

and

$$\widehat{M}_{d_3}' = \bar{y}\frac{\bar{x}}{\bar{X}} - \frac{\hat{k}_{3opt}}{3}\frac{m_{30}}{s_y^2}\frac{\bar{x}}{\bar{X}}. \tag{3.4}$$

Up to terms of order n^{-1}, the biases and MSE's of $\widehat{M}_{d_1}$, $\widehat{M}_{d_2}$ and $\widehat{M}_{d_3}$ are given as

$$B_{k_{1opt}}\left(\widehat{M}_{d_1}\right) = \frac{1}{3n}\bar{Y}C_y\frac{\beta_y\{\lambda_{50} - \beta_{1y}(\beta_{2y} + 3)\}}{\{\lambda_{60} - 6\beta_y + \beta_{0y}\}},$$

$$B_{k_{2opt}}\left(\widehat{M}_{d_2}\right) = \frac{1}{n}\bar{Y}\left[\{C_x^2 - C_{yx}\} - \frac{1}{3}\frac{\{\beta_{1y}(C_x^2 + \beta_{2y} + 3) + B - \lambda_{50}\}\{\beta_y C_y + \beta_{1y}(C_x^2 - C_{yx}) + B\}}{\{\lambda_{60} - 6\beta_y + \beta_{0y} + \beta_{1y}(\beta_{1y}C_x^2 + 2B)\}}\right],$$

$$B_{k_{3opt}}\left(\widehat{M}_{d_3}\right) = \frac{1}{n}\bar{Y}\left[C_{yx} - \frac{1}{3}\frac{\{\beta_{1y}(\beta_{2y} + 3) - B - \lambda_{50}\}\{\beta_y C_y + \beta_{1y}(C_x^2 + C_{yx}) - B\}}{\{\lambda_{60} - 6\beta_y + \beta_{0y} + \beta_{1y}(\beta_{1y}C_x^2 - 2B)\}}\right],$$

The biases of above estimators are of order n^{-1} so their contribution to the MSE's will be of the order n^{-2}. Up to terms of order n^{-1}, the MSE's are

$$MSE_{min}\left(\widehat{M}_{d_1}\right) = \frac{1}{n}\bar{Y}^2C_y^2\left[1 - \frac{5}{9}\frac{\beta_y^2}{\{\lambda_{60} - 6\beta_y + \beta_{0y}\}}\right],$$

$$MSE_{min}\left(\widehat{M}_{d_2}\right) = \frac{1}{n}\bar{Y}^2\left[\left(C_y^2 + C_x^2 - 2C_{yx}\right) - \frac{5}{9}\frac{\{\beta_y C_y + \beta_{1y}(C_x^2 - C_{yx}) + B\}^2}{\{\lambda_{60} - 6\beta_y + \beta_{0y} + \beta_{1y}(\beta_{1y}C_x^2 + 2B)\}}\right],$$

$$MSE_{min}\left(\widehat{M}_{d_3}\right) = \frac{1}{n}\bar{Y}^2\left[\left(C_y^2 + C_x^2 + 2C_{yx}\right) - \frac{5}{9}\frac{\{\beta_y C_y + \beta_{1y}(C_x^2 + C_{yx}) - B\}^2}{\{\lambda_{60} - 6\beta_y + \beta_{0y} + \beta_{1y}(\beta_{1y}C_x^2 - 2B)\}}\right].$$

Among the three estimators, the most efficient is

i. $\widehat{M}_{d_2}$ iff

$$\rho\frac{C_y}{C_x} > \frac{1}{2} + \frac{5}{18C_x^2}\left[\frac{\beta_y^2C_y^2}{(\lambda_{60} - 6\beta_y + \beta_{0y})} - \frac{\left(\beta_y C_y + \beta_{1y}(C_x^2 - C_{yx}) + B\right)^2}{\left(\lambda_{60} - 6\beta_y + \beta_{0y} + \beta_{1y}(\beta_{1y}C_x^2 + 2B)\right)}\right]$$

ii. $\widehat{M}_{d_3}$ iff

$$\rho\frac{C_y}{C_x} < -\frac{1}{2} - \frac{5}{18C_x^2}\left[\frac{\beta_y^2C_y^2}{(\lambda_{60} - 6\beta_y + \beta_{0y})} - \frac{\left(\beta_y C_y + \beta_{1y}(C_x^2 + C_{yx}) - B\right)^2}{\left(\lambda_{60} - 6\beta_y + \beta_{0y} + \beta_{1y}(\beta_{1y}C_x^2 - 2B)\right)}\right]$$

iii. $\widehat{M}_{d_1}$ iff

$$-\frac{1}{2} - \frac{5}{18C_x^2}\left[\frac{\beta_y^2C_y^2}{(\lambda_{60} - 6\beta_y + \beta_{0y})} - \frac{\left(\beta_y C_y + \beta_{1y}(C_x^2 + C_{yx}) - B\right)^2}{\left(\lambda_{60} - 6\beta_y + \beta_{0y} + \beta_{1y}(\beta_{1y}C_x^2 - 2B)\right)}\right] < \rho\frac{C_y}{C_x}$$

$$< \frac{1}{2} + \frac{5}{18C_x^2}\left[\frac{\beta_y^2C_y^2}{(\lambda_{60} - 6\beta_y + \beta_{0y})} - \frac{\left(\beta_y C_y + \beta_{1y}(C_x^2 - C_{yx}) + B\right)^2}{\left(\lambda_{60} - 6\beta_y + \beta_{0y} + \beta_{1y}(\beta_{1y}C_x^2 + 2B)\right)}\right].$$

4. Empirical and Simulation Studies

4.1. Empirical Studies

To illustrate the result numerically, we have made computations for 10 populations taken from literature, by using Microsoft Excel 2010.

Empirical study-I

The source of the populations, the nature of the variables, the values of $\bar{Y}$, k_{1opt}, μ_{20}, β_{1y} and ρ are listed in Table 1.

In Table 2, we have given the true value of population Median (M_d) and values of the population median obtained by using the new parametric functional relationship of M_d and the efficiencies of proposed estimators are given in Table 3.

Table 1. Description of populations

Sr. No.	Source	y	x	$\bar{Y}$	k_{1opt}	μ_{20}	β_{1y}	ρ
1	Murthy (1967) p-398	No. of absentees	No. of workers	9.6512	0.0670	42.1341	1.5575	0.6608
2	Chakravarty et al. (1967) p-183	Length (cm) measured by 1st person	Length (cm) measured by 2nd person	4.9737	-0.2875	0.1346	-0.0546	0.9317
3	Chakravarty et al. (1967) p-207	Weight (kg) of male	Height (cm) of male	29.2625	-0.0841	6.5836	0.3670	0.7709
4	Chakravarty et al. (1967) p-207	Weight (kg) of female	Height (cm) of female	28.5313	-0.4080	1.8109	0.1099	0.2306
5	Chakravarty et al. (1967) p-185 (1-35)	Weight (lb) of Kayastha males	Stature (cm) of Kayastha males	82.2000	-0.4975	191.7029	0.0439	0.8578
6	Chakravarty et al. (1967) p-185 (1-76)	Weight (lb) of Kayastha males	Stature (cm) of Kayastha males	89.4211	0.0503	278.4806	0.6068	0.4361
7	Chochran (2007) p-325	Total number of persons	Average persons per room	101.1000	-0.4229	214.6900	0.3248	0.6515
8	Maddala and Lahiri (1992) p-316	Consumption per capital of Lamb	Deflated prices of Lamb	4.5188	-0.1696	0.2103	-0.6578	-0.7517
9	Gujarati (2009) p-27, (1-50)	Egg production in 1991(million)	Price per dozen(cent) in 1990	78.2880	0.0034	445.3787	0.9959	-0.3096
10	http://content.hccfl.edu	highway fuel efficiency of vehicles (in miles)	weight of vehicles (in 1000 lbs.)	30.6154	-0.5950	15.6213	0.0549	-0.8978

Table 2. Values of population median (M_d) and their values obtained from new parametric relationship

Pop. No.	M_d	M_{d_1}
1	8.0000	9.4254
2	4.9500	4.9718
3	29.2000	29.2889
4	28.5000	28.5514
5	82.0000	82.3007
6	88.0000	89.2512
7	100.0000	101.7709
8	4.6000	4.5017
9	75.3500	78.2642
10	30.0000	30.6584

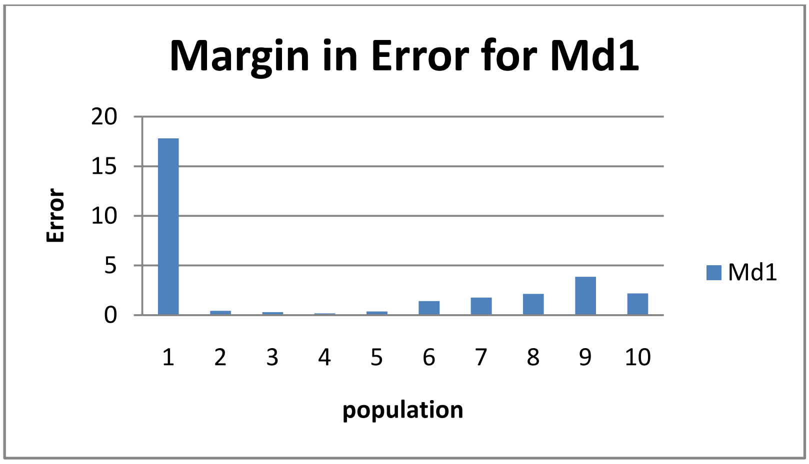
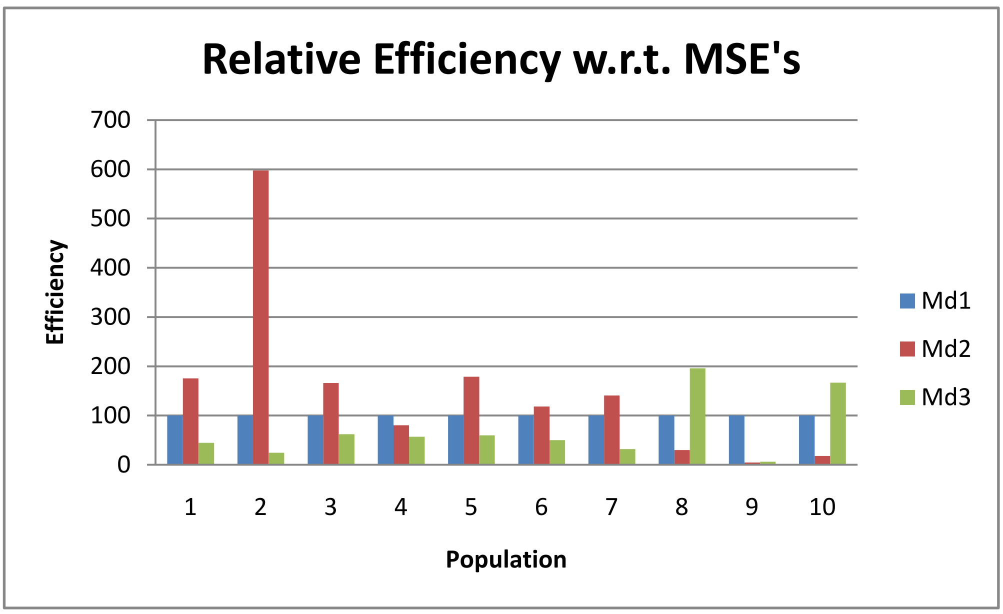


Table 3. Up to terms of n^{-1} MSE of $\bar{y}$, $\hat{M}_{d_1}$, $\hat{M}_{d_2}$, $\hat{M}_{d_3}$, $\hat{\bar{Y}}_R$ and $\hat{\bar{Y}}_P$

Pop. No.	$\bar{y}$	$\hat{M}_{d_1}$	$\hat{M}_{d_2}$	$\hat{M}_{d_3}$	$\hat{\bar{Y}}_R$	$\hat{\bar{Y}}_P$
1	42.1341	40.3890	**22.9990**	90.9751	23.7459	-
2	0.1346	0.1201	**0.0201**	0.4947	0.0201	-
3	6.5836	6.5145	**3.9238**	10.5213	3.9590	-
4	1.8109	**1.5012**	1.8675	2.6333	-	-
5	191.7029	142.0275	**79.4387**	237.1650	105.5227	-
6	278.4806	270.2905	**228.2034**	539.9383	237.2253	-
7	214.6900	176.6954	**125.6235**	554.3948	135.1725	-
8	0.2103	0.2005	0.6691	**0.1023**	-	0.1023
9	445.3787	**445.3506**	10052.1851	7317.9041	-	-
10	15.6213	10.6760	59.4203	**6.4059**	-	6.7647



From Table 2, we observe that the values obtained from the new parametric functional relationship for median i.e. $M_{d_1} = \bar{Y} - \frac{k_{iopt}}{3}\frac{\mu_{30}}{S_y^2}$, are very close to the true value of the population median (M_d). The small amount of errors $(M_d - M_{di})$ is due to the fact that we are using the optimum value of k (by minimizing MSE up to order n^{-1}) not the exact one. So, empirically, this relation is verified and the percentage of relative margin of these errors presented in bar graph.

From Table 3, we observe that in the sampling theory, for the bivariate population, the known value of $\bar{X}$, increases the accuracy of the estimators of population median (M_d). We note that the MSE's of all $\widehat{M}_{d_1}$, $\widehat{M}_{d_2}$ and $\widehat{M}_{d_3}$ are less than $V(\bar{y})$, at the same time MSE's of $\widehat{M}_{d_2}$ and $\widehat{M}_{d_3}$ are less than $V(\hat{\bar{Y}}_R)$ and $V(\hat{\bar{Y}}_P)$ respectively when conditions (i), (ii) and (iii) are satisfied and the relative efficiencies of $\widehat{M}_{d_2}$ and $\widehat{M}_{d_3}$ are also shown through bar graph.

Empirical Study-II

Theoretically, the proposed estimators of M_d cannot be compared with the existing estimators because the techniques of estimators of M_d and the parameters involved in MSE's are different. So an empirical study has been done to check the performance of the proposed estimators over the existing ones, we consider the two populations, which are considered by Singh and Solanki (2013).

The minimum MSE's of proposed estimators and estimators considered by Singh and Solanki (2013) are given in Table 4.

Table 4. MSE and Relative Efficiencies of Population Median

	MSE		Relative	Efficiency
Estimators	**Pop.I**	**Pop.II**	**Pop.I**	**Pop.II**
$V(\widehat{M}_y)$	565443.57	565443.57	100.00	100.00
$MSE(\widehat{M}_r)$	988372.76	536149.50	57.21	105.46
$MSE_{min}(\widehat{M}_d)$				
$MSE_{min}(\widehat{M}_y^{(G)})$	552636.13	508766.02	102.32	111.14
$MSE_{min}(\widehat{M}_i)$				
$MSE_{min}(t_4)$	630993.68	478781.74	89.61	118.10
$MSE_{min}(t_5)$	499412.60	499412.60	113.22	113.22
$MSE_{min}(t_6)$	630979.49	478784.18	89.61	118.10
$MSE_{min}(t_7)$	630367.71	478806.00	89.70	118.09
$MSE_{min}(t_8)$	522345.11	488388.99	108.25	115.78
$MSE_{min}(t_9)$	630993.63	478781.75	89.61	118.10
$MSE_{min}(t_{10})$	489754.69	493940.28	115.45	114.48
$MSE_{min}(t_{11})$	630993.67	478781.74	89.61	118.10
$MSE_{min}\{\grave{M}_d^{(1)}\}$	489569.06	495484.97	115.50	114.12
$MSE_{min}\{\grave{M}_d^{(2)}\}$	489395.24	454675.78	115.54	124.36
$MSE_{min}\{\grave{M}_d^{(3)}\}$	3220.01	51355.17	17560.30	1101.05
$MSE_{min}\{\grave{M}_{d1}^{(4)}\}$	480458.29	454616.16	117.69	124.38
$MSE_{min}\{\grave{M}_{d2}^{(4)}\}$	489395.24	454675.78	115.54	124.36
$MSE_{min}\{\grave{M}_{d3}^{(4)}\}$	480459.82	454616.17	117.69	124.38
$MSE_{min}\{\grave{M}_{d4}^{(4)}\}$	480525.30	454616.32	117.67	124.38
$MSE_{min}\{\grave{M}_{d5}^{(4)}\}$	487375.11	454660.89	116.02	124.37
$MSE_{min}\{\grave{M}_{d6}^{(4)}\}$	480458.30	454616.16	117.69	124.38
$MSE_{min}\{\grave{M}_{d7}^{(4)}\}$	489260.97	454672.34	115.57	124.36
$MSE_{min}\{\grave{M}_{d8}^{(4)}\}$	480458.29	454616.16	117.69	124.38
$\boldsymbol{MSE_{min}(\grave{M}_{d_1})}$	2155601.93	2155601.93	26.23	26.23
$\boldsymbol{MSE_{min}(\grave{M}_{d_2})}$	**187364.86**	**241764.01**	**301.79**	**233.88**
$\boldsymbol{MSE_{min}(\grave{M}_{d_3})}$	6887379.49	7187700.83	8.21	7.87

From Table 4, we observe that, the efficiency of the proposed estimator $\widehat{M}_{d_2}$ (appropriate estimator because ρ is highly positive), which is very much high as compared to existing ones.

4.2. Simulation Study

A simulation study as been carried out using the software R that verifies the theoretical results of our work. For this purpose, 100,000 samples have drawn from different gamma distributions, which are skewed. In this case the actual value of k is used which is equal to 0.5. For these distributions, we check the behavior of the estimator for different sample sizes. The values of population mean, median and mode for different gamma distributions are shown in Table 5.

Using simulation study we compared the existing estimator of sample median (M_{ed}) with the proposed estimator $\widehat{M}_{d_1}$. The expected values and MSE's of these estimators are shown in Table 6.

Some results based on simulation study

1. Biases and MSE's for both the estimators decrease as the sample size in- creases.
2. The proposed estimator slightly more biased than the existing estimator.
3. Variance of proposed estimator always less than the variance of existing estimator, which shows that the proposed estimator is more stable than the existing estimator (sample median).

Table 5. Descriptive Statistics

Distribution	Gamma (1.5, 0.5)	Gamma (2, 0.5)	Gamma (2.5, 0.5)
Mean	3	4	5
Mode	1	2	3
Median	2.66	3.36	4.35

Table 6. The Expected and MSE values of the Estimators

	$E(M_{ed})$	$E(\widehat{M}_{d_1})$	$V(M_{ed})$	$V(\widehat{M}_{d_1})$
Gamma (1.5, 0.5)				
$n = 5$	2.548382	2.694722	1.42406	1.150496
$n = 11$	2.452031	2.568213	0.648779	0.55312
$n = 31$	2.396914	2.451403	0.22887	0.240564
Gamma (2, 0.5)				
$n = 5$	3.540868	3.688773	2.007508	1.606204
$n = 11$	3.443777	3.55798	0.923177	0.757372
$n = 31$	3.386537	3.439476	0.3292	0.316645
Gamma (1.5, 0.5)				
$n = 5$	4.537728	4.668283	2.585306	2.068874
$n = 11$	4.437642	4.527729	1.192809	0.979008
$n = 31$	4.38161	4.407287	0.428705	0.40072

5. Conclusions

Theoretically and the with the support of empirical and simulation studies, we can say that the developed new parametric relationship for population median i.e. $M_{d_1} = \bar{Y} - \frac{k_{1opt}}{3}\frac{\mu_{30}}{S_y^2}$ is true. Using this new parametric relationship for population median, we can construct the different estimators for population median. This different technique of estimation of population median is very simple and efficient as compared to the existing ones. Further, one more interesting conclusion from this study is that the proposed conventional consistent estimator ($\widehat{M}_{d_1}$) of population median is more stable than the mean per unit estimator of population mean.

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