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Modelling Seasonal Behaviour of Rainfall in Northeast Nigeria. A State Space Approach

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Abstract The modelling of seasonal behaviour of rainfall has become very important in the wake of the climatic change. Due to the agriculture base of the Northeast Nigeria, it is imperative to explicitly model seasonal behaviour of rainfall in the region for agricultural planning. Hence, this study examines the seasonal behaviour of monthly rainfall data from 1981 to 2013 in Maiduguri and Damaturu areas of Borno and Yobe States respectively, using the state space approach. We employed the local level model with stochastic seasonal and the local level model with deterministic seasonal to modelling of the dynamic features in the data. Our results clearly indicate that the local level model with deterministic seasonal is the parsimonious model between the two state space models considered in this study. This implies that the seasonal patterns of rainfall in the two areas have not significantly changed despite the challenges of global warming and climate change. In addition, the CUSUM test indicates the presence of structural breaks in 1998 and 1990 for Maiduguri and Damaturu respectively. This implies that there was an abrupt change in the rainfall level in 1998 for Maiduguri and in 1990 for Damaturu. We, therefore, recommend that seasonality should be explicitly included in the modelling of rainfall series as the pattern of seasonality could be useful for important decision making. In addition, measures should be put in place to curb human-made activities that are detrimental to the climate since the region is highly vulnerable to the impacts of climate change.

Keywords Rainfall, Climate Change, Kalman filter, Kalman Smoothing, Seasonality, State space model

1. Introduction

Agriculture in North-Eastern region of Nigeria is predominantly dependent on rainfall. Hence, the output from the sector each year is largely determined by the rainfall seasonal pattern, trend and cycles. Weather as we know is dynamic. Similarly, climate fluctuates or changes over time and space. According to Nieuwolt (1982), the non-static behaviour in rainfall patterns are in various magnitudes ranging from variability through fluctuations, trends, and abrupt to gradual changes. Some characteristics of rainfall which include its seasonal and diurnal distribution, intensity, duration, onset, cessation and frequency of rain days all show important variations in respect to time and places. It is therefore important to examine climatic data to ascertain possible trends and changes in the data generating process. Trend could reflect either an increase or decrease in the observed phenomenon and may serve as a good indicator for predicting future occurrence. A fair knowledge of the weather is very necessary, in view of the fact that farmers, power generation (hydroelectric power generation), meteorological stations, military operations, safety and emergency agencies such as National Emergency Management Agency (NEMA) heavily rely on it for their operations, among other socioeconomic activities. The high amounts of rainfall which are particularly received in many elevated area of the tropics, constitute a reliable basis for the construction of many hydro-electric power stations.

There has always been a good deal of interest in the possibility of seasonality in rainfall data. Harvey (1987) pointed out that the analysis of climatic time series is essential for building statistical models to generate synthetic hydrologic records, to forecast hydrologic events, to detect intrinsic stochastic or deterministic characteristics of hydrologic variables. Assessing the seasonal behaviour of rainfall characteristics and trend is a vital for agricultural practice in the North-East states of Nigeria. In addition, the time series analysis of climatic data provides a basis for ascertaining climate change or variability.

The Box-Jenkins SARIMA model has been extensively used to model rainfall patterns in Nigeria, see for example, Gulumbe (2013), Etuk, Moffat and Chims (2013) and Martins et al. (2014). However, the Box-Jenkins approach is based on the elimination of trend and seasonality applying seasonal differencing to the series before fitting an

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appropriate ARMA model to the data. Durbin and Koopman (2001) noted that elimination of trend and seasonality by differencing may pose a problem in applications where knowledge of the seasonal pattern is needed. Fortunately, modern control and system theory has shown that the behaviour of dynamic systems can be conveniently and succinctly described by using the state space models. State space method is more general and is based on the modelling of all the observed features of the data. The different features inherent in a series such as trend, seasonal, cycle, explanatory variables and interventions can be modelled separately before being put together in the state space model. The assumption of stationarity is not needed in state space model.

Although the literature on state space has increased recently, there is however, no empirical application to modelling rainfall pattern in Nigeria. In addition, using the state space approach, the nature of seasonality present in the data will be explicitly determined. It is against these backdrops that this study is being proposed. The rainfall received at the synoptic stations at Maiduguri and Damaturu for thirty-three years period (1981-2013) are analyzed to evaluate the seasonal behaviour and trend in the monthly data rainfall. The research is useful in assessing the seasonal behaviour of rainfall characteristics and the trend in the rainfall data which is a vital requirement for agricultural practice in the North-East states of Nigeria. It is also intended to provide a basis for ascertaining climate change or variability. The rest of this paper is organized as follows. In section 2, state space models are briefly introduced, in section 3, we discuss the modelling methodology employed in this work. The data and the empirical results are presented and discussed in chapter 4. The final section concludes the paper.

2. The Model

2.1. State Space Model

A state space model consists of two equations: the state equation (also called transition or system equation) and the observation equation (also called measurement equation). The observation equation expresses the observed variables (data) as a linear function of the state variable(s) plus noise, while the transition equation describes the evolution of the state variables. The transition equation has the form of a first-order difference equation. The selection of components to include in the state space model is based on the features inherent in the observed time series. For a series that have seasonal patterns, the basic structural model is used, for a strongly trending non-seasonal series; the local trend model is employed. However, for a non-trending series, the local level model is used.

Let y_t denote an $(n\times 1)$ vectors of variables observed at time t and ξ_t be $(r\times 1)$ unobserved state vector. The general state space representation of the dynamics of y is given by:

$$y_t = AX_t + H\xi_t + \varepsilon_t \tag{1}$$

$$\xi_{t+1} = F\xi_t + \eta_t \tag{2}$$

The (n x 1) vector ε_t and the (k x 1) vector η_t are white noise:

$$E(\varepsilon_t \varepsilon'_\tau) = R, \quad \text{for } t = \tau, \quad \text{and 0 otherwise} \tag{3}$$

$$E(\eta_t \eta_\tau) = Q, \text{ for } \quad t = \tau, \quad \text{and 0 otherwise} \tag{4}$$

The disturbances are assumed to be uncorrelated at all lags, that is;

$$E(\varepsilon_t \eta_\tau) = 0 \text{ at all t and } \tau \tag{5}$$

where ξ_t is k x 1 vector of unobserved state variables, H is an n x k matrix that links the observed vector y_t to the unobserved ξ_t, X_t is an r x 1 vector of exogenous or predetermined observed variables, A is a matrix that maps the exogenous variables into the measurement domain, F is the state transition matrix which applies the effect of each state parameter at time t on the system state at time t+1, R is (n x n) and Q is (k x k) matrices of the measurement equation variance and transition equation variance respectively. The R variance matrices play the same role as in the classical regression model, while the Q variance matrices allow the parameters in the state equations to evolve over time. Equation (1) is known as the observation equation and (2) is known as the state equation. The system of (1) through (5) is called state space models. The essential difference between the state space model and the conventional ARIMA model representation is that in the former, the state of nature – analogous to the regression coefficients of the latter – is not assumed to be constant but may change with time.

3. Modelling Methodologies

3.1. State Space Approach: Local Level Model with Seasonality

The local level model or *random walk-plus-noise* model is a simple form of a linear Gaussian state space model for modelling series with no visible trend. The model contains only the level and irregular components; the single state (level) variable follows the random walk:

$$y_t = \mu_t + \varepsilon_t \qquad \varepsilon_t \sim NID(0, \sigma^2_\varepsilon)$$

$$\mu_{t+1} = \mu_t + \eta_t \qquad \eta t \sim NID(0, \sigma^2_\eta) \tag{6}$$

where ε_t and η_t are mutually uncorrelated white-noise processes with variance σ^2_ε and σ^2_η. The interpretation of this model is that μ_t is an (unobservable) local level or mean for the process. The observable y_t is the underlying process mean contaminated with the measurement error ε_t. Although it has a simple form, it provides the basis for the analysis of important real problems in practical time series analysis. It exhibits the characteristics structure of state space models in which there is a series of unobserved values $\mu_{1\ldots}\ \mu_n$ which represents the development over time of the system under study, together with a set of observations $y_{1,\ldots,}\ y_n$. The aim of the analysis is to study the development of the state over time using the observed values $y_{1,\ldots,}$ y_n. However, when a time series consists of daily, monthly, or quarterly observations, the presence of seasonal effects should be taken into consideration. In the state space framework, seasonality can be handled by building the seasonal effects directly into the model. Hence, adding seasonal components to equation above yields,

$$y_t = \mu_t + \gamma_t + \varepsilon_t, \quad \varepsilon_t \sim N(0, \sigma_\varepsilon^2)$$
$$\mu_{t+1} = \mu_t + \eta_t, \quad \eta_t \sim N(0, \sigma_\eta^2)$$
$$\gamma_{t+1} = -\Sigma\gamma_{t+1-j} + \omega_t \quad \omega_t \sim N(0, \sigma_\omega^2) \tag{7}$$

where $t = s$-1,..., n. When the seasonal effect γ_t is not allowed to change over time, we require $\omega_t = 0$ for all $t = s$ -1,..., n. This is done by setting $\sigma_\omega^2 = 0$ and (7) is called the *local level with deterministic seasonal model*. When the seasonal effect γ_t is allowed to vary over time, that is $\sigma_\omega^2 > 0$, the resulting model is called the *local level with stochastic seasonal model*. Since the rainfall data consists of monthly observations, the periodicity of the seasonal is s =12. The stochastic formulation of the seasonal effect in (7) follows from the standard dummy variable methods of modelling seasonal pattern. An alternative way of modelling seasonal effect is by using trigonometric terms at the seasonal frequencies.

State space models are estimated using the Kalman filter. The Kalman filter is a statistical algorithm that enables certain computations to be carried out for a model cast in state space form. However, to obtain a more accurate estimate of the state vector, the smoothing algorithm is performed. Kalman smoothing provides us with a more accurate inference on μ_t, since it uses more information than the filtering. Let Y_{t-1} denote the set of past observations $\{y_1, \ldots, y_{t-1}\}$ and assuming the conditional distribution of μ_t given Y_{t-1} is N(u_t, p_t) where u_t and p_t are to be determined. Assuming that u_t and p_t have been determined, the celebrated Kalman filter equations for updating the above local level model from time $\boldsymbol{t}$ to $t+1$ are given by:

$$u_{t+1} = u_t + k_t \nu_t, \quad p_{t+1} = p_t(1 - k_t) + \sigma_\eta^2, \quad k_t = p_t / f_t, \quad \nu_t = y_t - u_t, \quad f_t = p_t + \sigma_\varepsilon^2, \tag{8}$$

for t = 1, 2, …, n, where ν_t denotes the Kalman filter residual or prediction errors, f_t is its variance and k_t is the Kalman gain. A random walk like μ_t has no *"natural"* level and to handle the initial conditions (u_1, p_1) for the non-stationary model, we employed the exact initial Kalman filter, (Durbin and Koopman, 2001). The Kalman smoothed state ($\hat{\mu}_t$) and smoothed state variance (V_t) can be calculated by the following backward recursions:

$$\hat{\mu}_t = u_t + p_t r_{t-1}, \quad r_{t-1} = f_t^{-1}\nu_t + l_t r_t, \quad l_t = 1 - k_t = \sigma_\varepsilon^2 / f_t, \quad t = n, \ldots, 1$$
$$V_t = p_t - p_t^2 N_{t-1}, \quad N_{t-1} = f_t^{-1} + l_t^2 N_t, \quad t = n, \ldots, 1 \tag{9}$$

with r_n and N_n = 0, for $t = n, \ldots, 1$. The unknown variance parameters in the state space model are estimated by the maximum likelihood estimation via the Kalman filter prediction error decomposition initialized with the exact initial Kalman filter. Harvey and Peters (1990) suggested concentration of the log likelihood when the variance parameters display difficult estimation problems, as this helps to improve the behavior of difficult estimation.

Diagnostic checking in the state space models are based on the three assumptions concerning the residuals of the analysis. The residuals should satisfy these three properties, in order of importance; independence, homoscedasticity and normality. These assumptions are checked using the following test statistic;

The assumption of *independence* of the residuals can be checked with the Ljung-Box statistic defined as;

$$Q(k) = n(n+2)\sum_{l=1}^{k} \frac{r2}{n-l} \quad \sim \quad \chi^2_{(k-w+1)} \tag{10}$$

For lags $l = 1,\ldots,k$. The number of diffuse initial state values which need to be estimated for the level and seasonal components in (7) corresponds to k, r_l denotes the residual autocorrelation for lag l and w is the number of hyperparameters (i.e. disturbance variances). The second most important assumption is the *homoscedasticity* of the residuals. This is checked using the following test statistic;

$$H(h) = \frac{\sum_{t=n-h+1}^{n} \varepsilon_t^2}{\sum_{t=d+1}^{t=d+h} \varepsilon_t^2} \quad \sim \quad F_{(h,\, h)} \tag{11}$$

where d is the number of diffuse initial elements, and h is the nearest integer to (n- d) /3. The least important assumption is that the residuals are *normally distributed*. This assumption can be checked with the following test statistic;

$$N = n\left(\frac{s}{6} + \frac{(k-3)2}{24}\right) \quad \sim \quad \chi^2_{2df} \tag{12}$$

where s denotes the skewness of the residuals, and k is the kurtosis. In the state space models, the standardized smoothed disturbances are useful for detection of possible outliers and structural breaks. The detection of structural break using the cumulative sum (CUSUM) test is considered in this work.

4. Data and Empirical Results

4.1. The Data

The monthly rainfall data for two North-Eastern areas (Maiduguri and Yobe, see Annexure I) from January, 1981 to December 2013 are examined in this study. The monthly rainfall data measured in millimeter is obtained from the Nigeria Meteorological Agency (NIMET), Lagos state office. The plots of the rainfall series are presented in Figure 1 and Figure 2 for Maiduguri and Damaturu respectively.

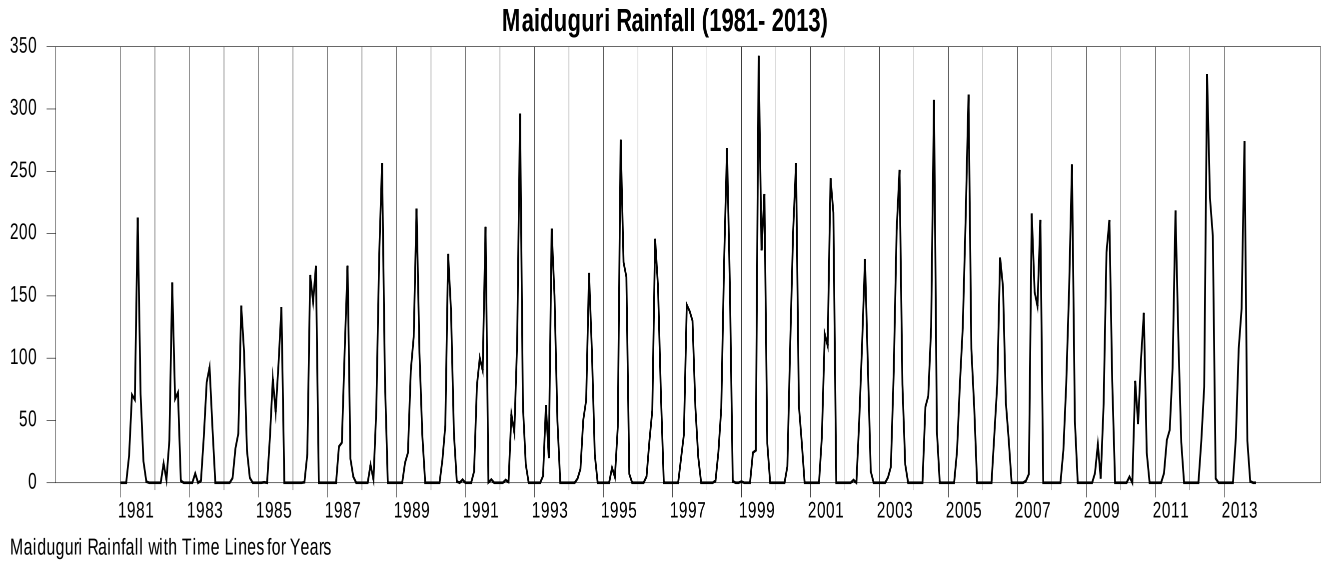


Figure 1. Time Series Plot of Maiduguri Rainfall

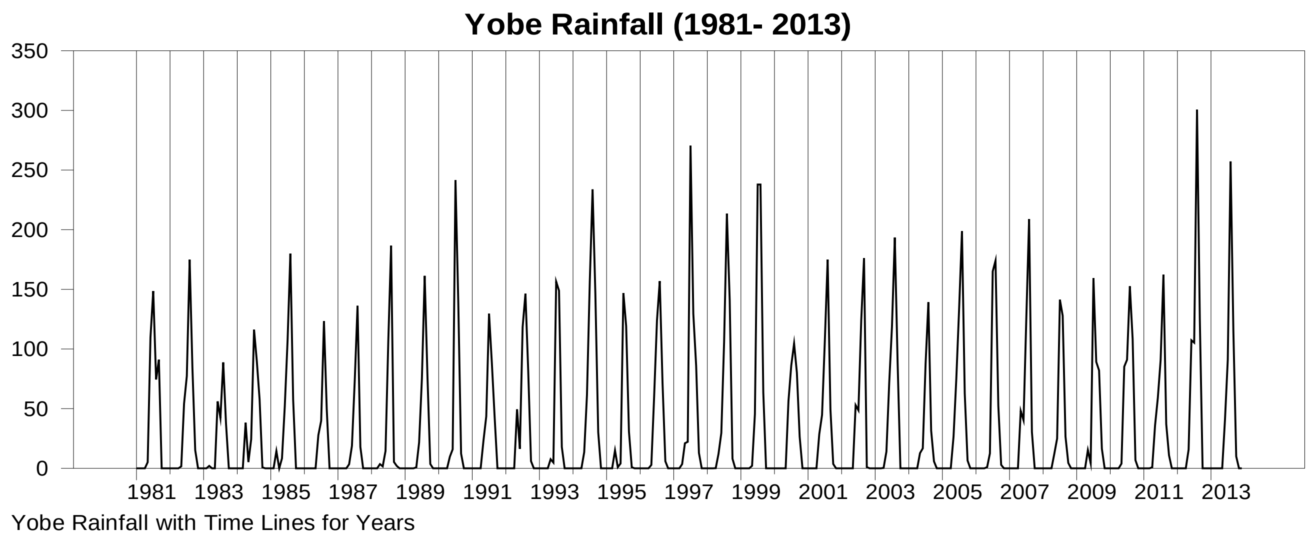


Figure 2. Time Series Plot of Damaturu Rainfall

From figure 1 the rainfall series fluctuates over the years but had the highest spike in 1999 and the lowest in 1983 and figure 2 also indicates that the rainfall had its peak in 2012 and the lowest in 1983. The descriptive statistics of the subsamples of both series are presented in table 1 and 2. Table 1 and 2 indicate that the subsample means and variance are not the same over time, which is an indication that the subsample means and variance are not constant. For example, the subsample mean for Damaturu between 1981:1 to 1991:1 is 28.10 while for 1992:1 to 2001:1 and 2002: 1 to 2013:12 are 37.44 and 36.85 respectively. Hence, the means varies across the samples. Thus, the state space model that accommodates non-stationary features by representing the level of the series as a random walk process, since random walk process is non-stationary, would be appropriate for modelling the series.

Table 1. Descriptive Statistics of Maiduguri Rainfall

Summary Statistics 1981:1-1991:12		Summary Statistics 1992:1-2002:12		Summary Statistics 2003:1-2013:12	
Mean	36.65	Mean	50.34	Mean	53.07
Standard Error	5.12	Standard Error	6.89	Standard Error	7.08
Median	1.30	Median	2.75	Median	2.45
Mode	0.00	Mode	0.00	Mode	0.00
Standard Deviation	58.84	Standard Deviation	79.10	Standard Deviation	81.34
Sample Variance	3461.68	Sample Variance	6257.56	Sample Variance	6616.60
Kurtosis	2.40	Kurtosis	2.03	Kurtosis	1.84
Skewness	1.77	Skewness	1.69	Skewness	1.64
Range	256.50	Range	342.70	Range	328.00
Minimum	0.00	Minimum	0.00	Minimum	0.00
Maximum	256.50	Maximum	342.70	Maximum	328.00
Sum	4838.10	Sum	6644.50	Sum	7005.40
Count	132.00	Count	132.00	Count	132.00

Table 2. Descriptive Statistics of Damaturu Rainfall

Summary Statistics 1981:1-1991:12		Summary Statistics 1992:1-2002:12		Summary Statistics 2003:1-2013:12	
Mean	28.10	Mean	37.44	Mean	36.85
Standard Error	4.29	Standard Error	5.49	Standard Error	5.35
Median	0.00	Median	0.00	Median	0.00
Mode	0.00	Mode	0.00	Mode	0.00
Standard Deviation	49.27	Standard Deviation	63.07	Standard Deviation	61.48
Sample Variance	2427.88	Sample Variance	3978.02	Sample Variance	3780.29
Kurtosis	3.75	Kurtosis	2.57	Kurtosis	3.37
Skewness	2.02	Skewness	1.82	Skewness	1.90
Range	241.60	Range	270.40	Range	300.60
Minimum	0.00	Minimum	0.00	Minimum	0.00
Maximum	241.60	Maximum	270.40	Maximum	300.60
Sum	3709.70	Sum	4941.70	Sum	4864.00
Count	132.00	Count	132.00	Count	132.00

4.2. Estimation Results and Discussion

The selection of components to be included in a state space model is based on the characteristics of the observed series. Since seasonality is normally present in a monthly time series data, we employed the *local level with stochastic seasonal and local level with deterministic seasonal models* described in section 3 to the Maiduguri and Damaturu original series. First, using the data, we estimate the unknown variance parameters (hyperparameters) of the models using maximum likelihood method. This is maximized using the BFGS (Broyden-Fletcher-Goldfarb-Shannon) optimization method. The estimation results for the local level model with stochastic seasonal is presented below.

Table 3. Estimation Variance for Local Level Model with Stochastic Seasonal for Maiduguri Rainfall series (1981:1 – 2013:12)

```
DLM - Estimation by BFGS
Convergence in    51 Iterations. Final criterion was  0.0000000 <=  0.0000100
Monthly Data From 1981:01 To 2013:12
Usable Observations                       396
Rank of Observables                       384
Log Likelihood                       -1954.36

    Variable                         Coeff      Std Error      T-Stat      Signif
*********************************************************************************
1.  SIGSQEPS                       1322.42          97.78       13.52        0.00
2.  SIGSQETA                          0.70           0.64        1.10        0.27
3.  SIGSQOMEGA                        2.07           0.15       13.46        0.00
```

Note: SIGSQEPS, SIGSQETA and SIGSQOMEGA denote estimates σ_ε^2,σ_η^2 and σ_ω^2 respectively.

Table 4. Estimation Variance for Local Level Model with Stochastic Seasonal for Damaturu Rainfall series (1981:1 – 2013:12)

```
DLM - Estimation by BFGS
Convergence in    57 Iterations. Final criterion was  0.0000073 <=  0.0000100
Monthly Data From 1981:01 To 2013:12
Usable Observations                       396
Rank of Observables                       384
Log Likelihood                       -1820.24

    Variable                         Coeff      Std Error      T-Stat      Signif
*********************************************************************************
1.  SIGSQEPS                        625.61          52.06       12.02        0.00
2.  SIGSQETA                          0.33           0.41        0.82        0.41
3.  SIGSQOMEGA                        7.34           0.55       13.39        0.00
```

Note: SIGSQEPS, SIGSQETA and SIGSQOMEGA denote estimates σ_ε^2,σ_η^2 and σ_ω^2 respectively.

The estimation results in table 3 and 4 indicates that the hyperparameters (disturbance variances) of the measurement and seasonal component are highly statistically significant; however, the estimated hyperparameters for the level equations is not statistically significant. This indicates that any seasonal pattern in the observed time series changes over the years. Also, there are 396 observations in both series; however, the estimation is done using only the final 384 observations. This is because 12 diffuse initial state values are estimated (11 for the seasonal components and 1 for the local level components). We proceed into performing the Kalman filtering and kalman smoothing using the estimated hyperparameters. The results of the Kalman filter estimates for Maiduguri rainfall series is presented in figure 3 and 4 while figure 5 and 6 present that of Damaturu rainfall series respectively.

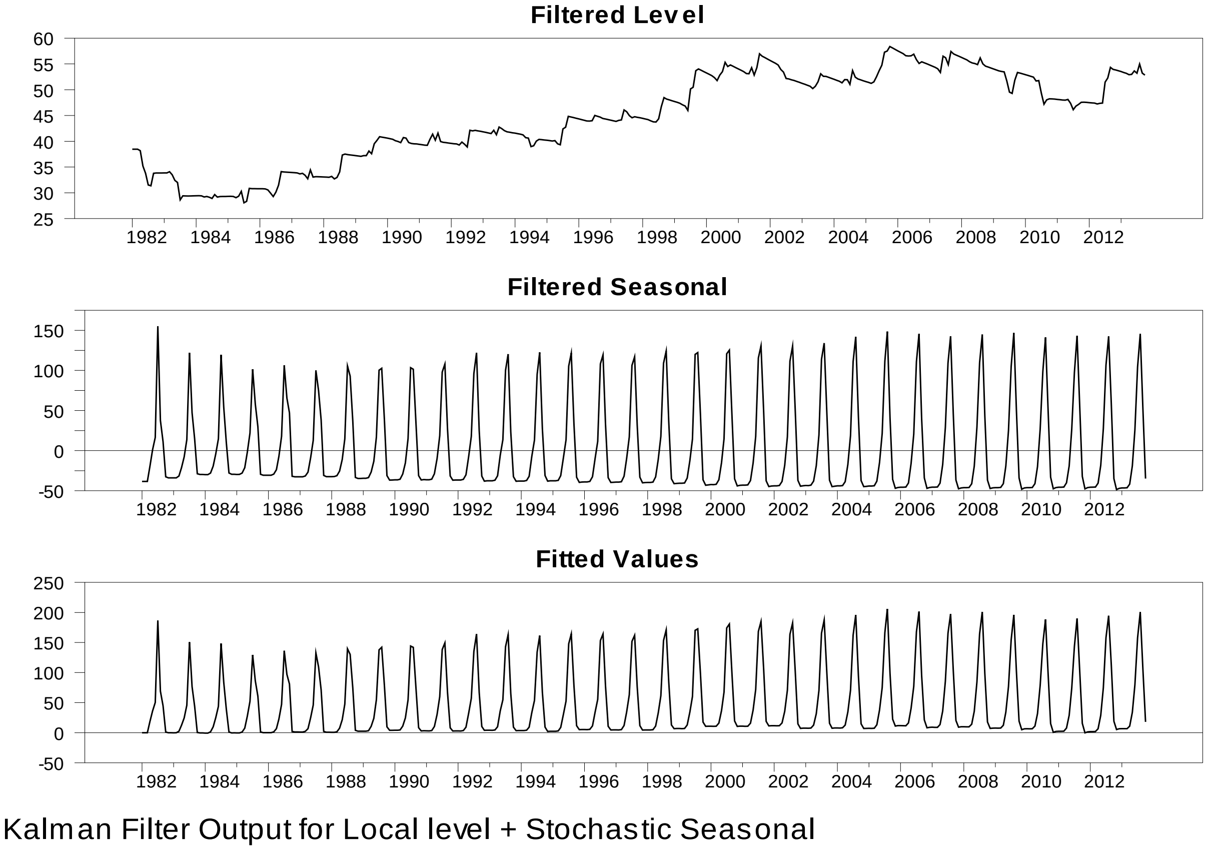


Figure 3. Kalman Filter Output for local level with stochastic seasonal for Maiduguri series

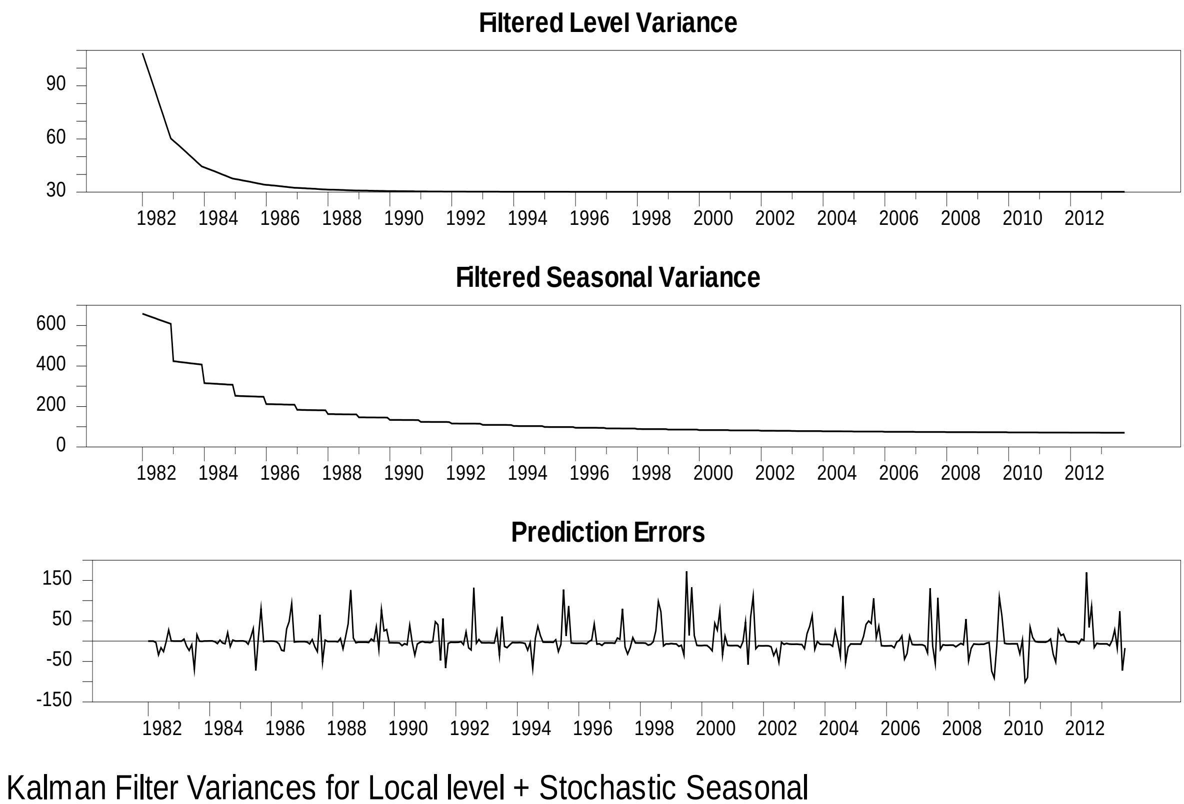


Figure 4. Kalman Filter Output for local level with stochastic seasonal for Maiduguri series

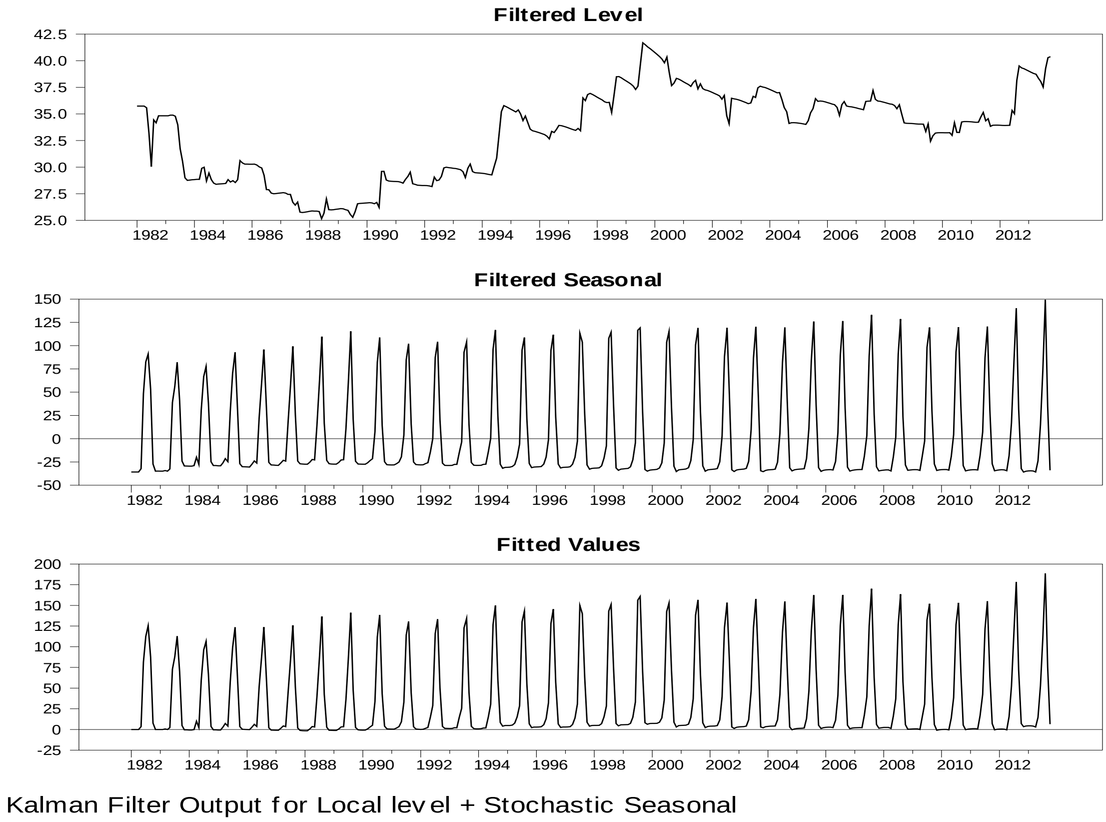


Figure 5. Kalman Filter Output for local level with stochastic seasonal for Damaturu series

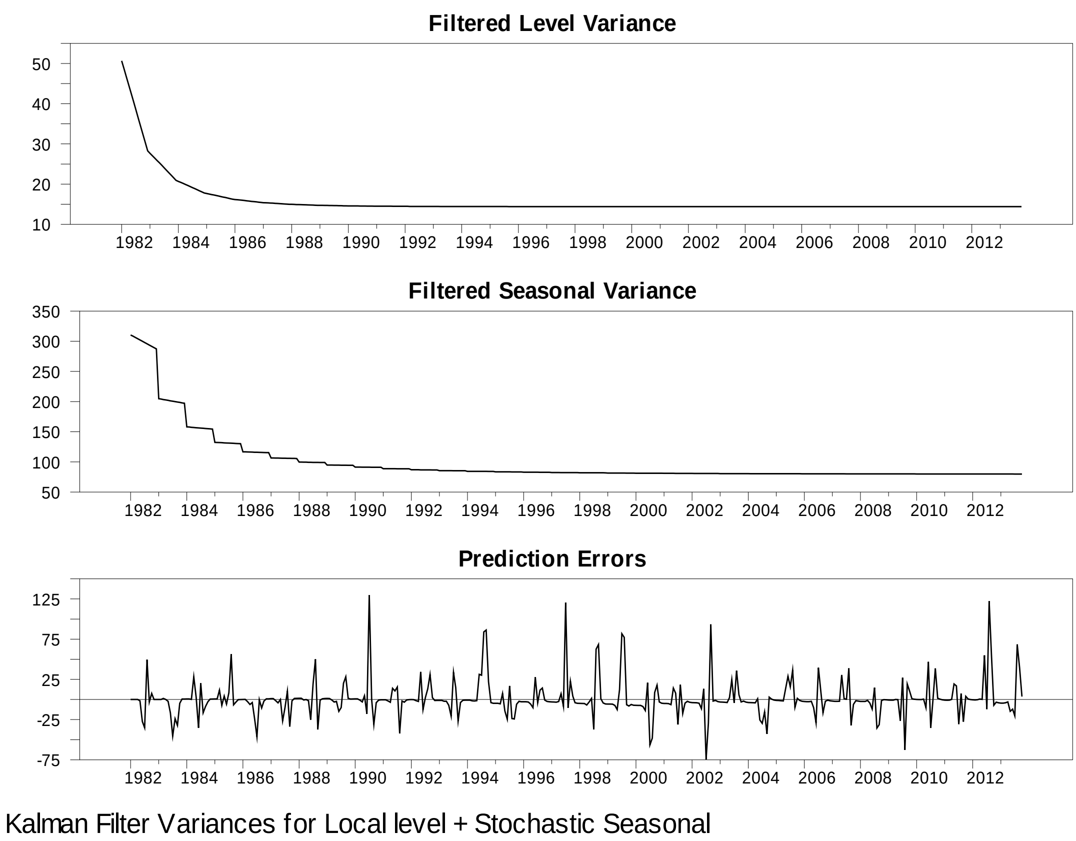


Figure 6. Kalman Filter Output for local level with stochastic seasonal for Damaturu series

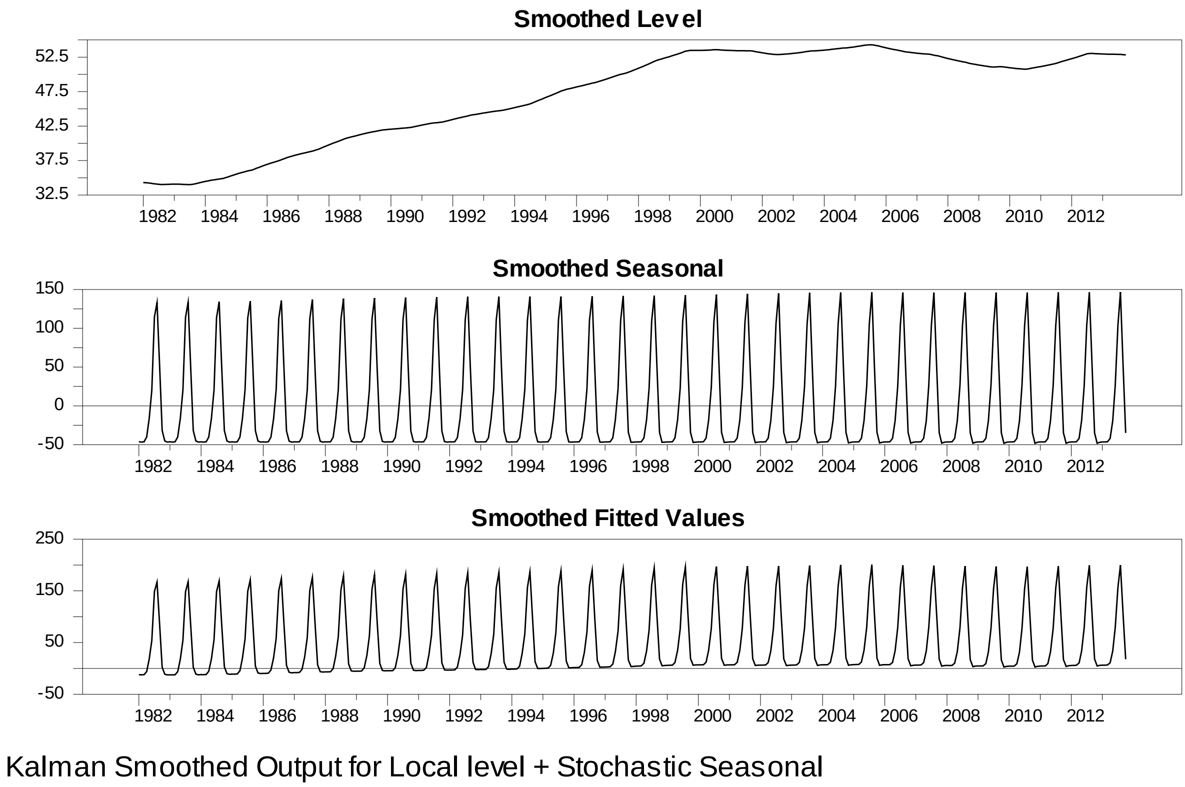


Figure 7. Kalman Smoothed Output for local level with stochastic seasonal for Maiduguri series

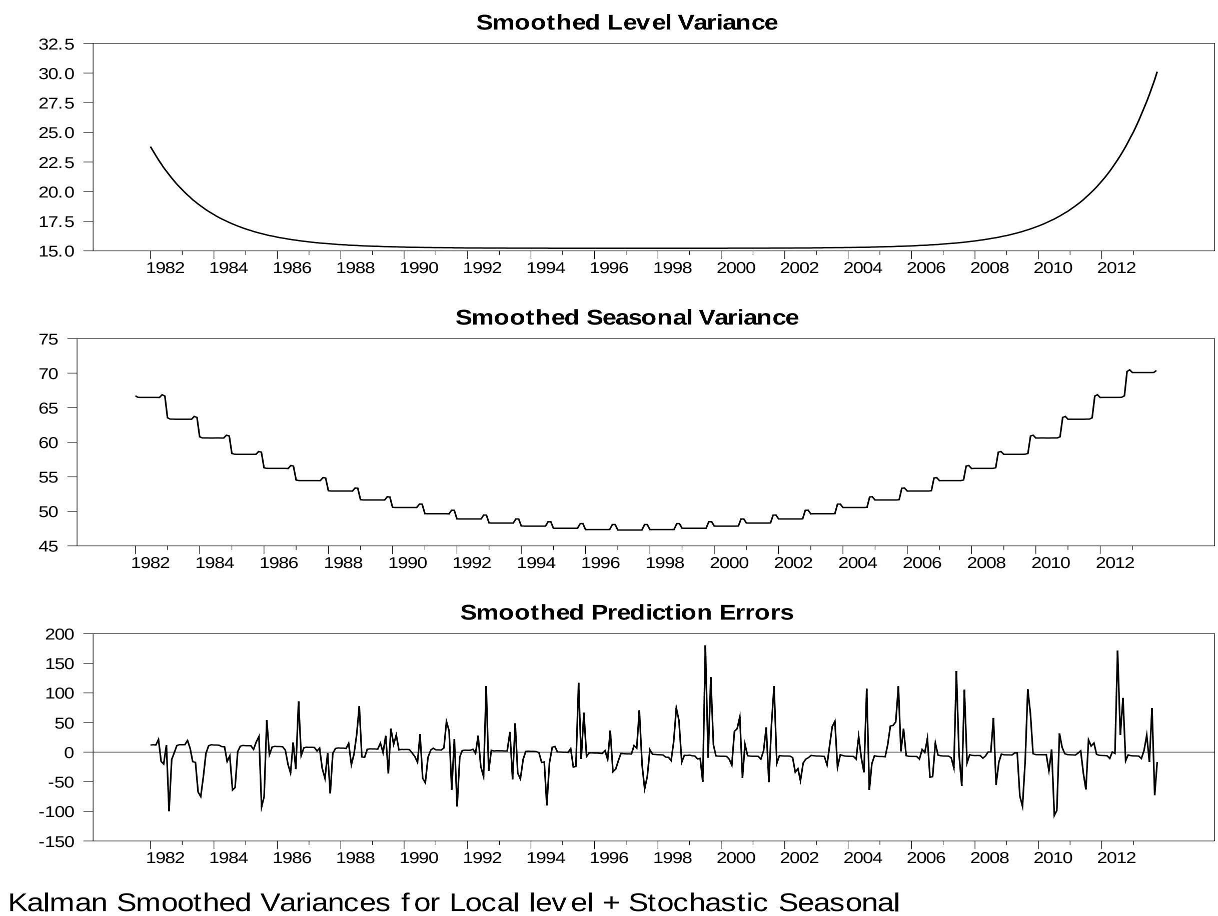


Figure 8. Kalman Smoothed Output for local level with stochastic seasonal for Maiduguri series

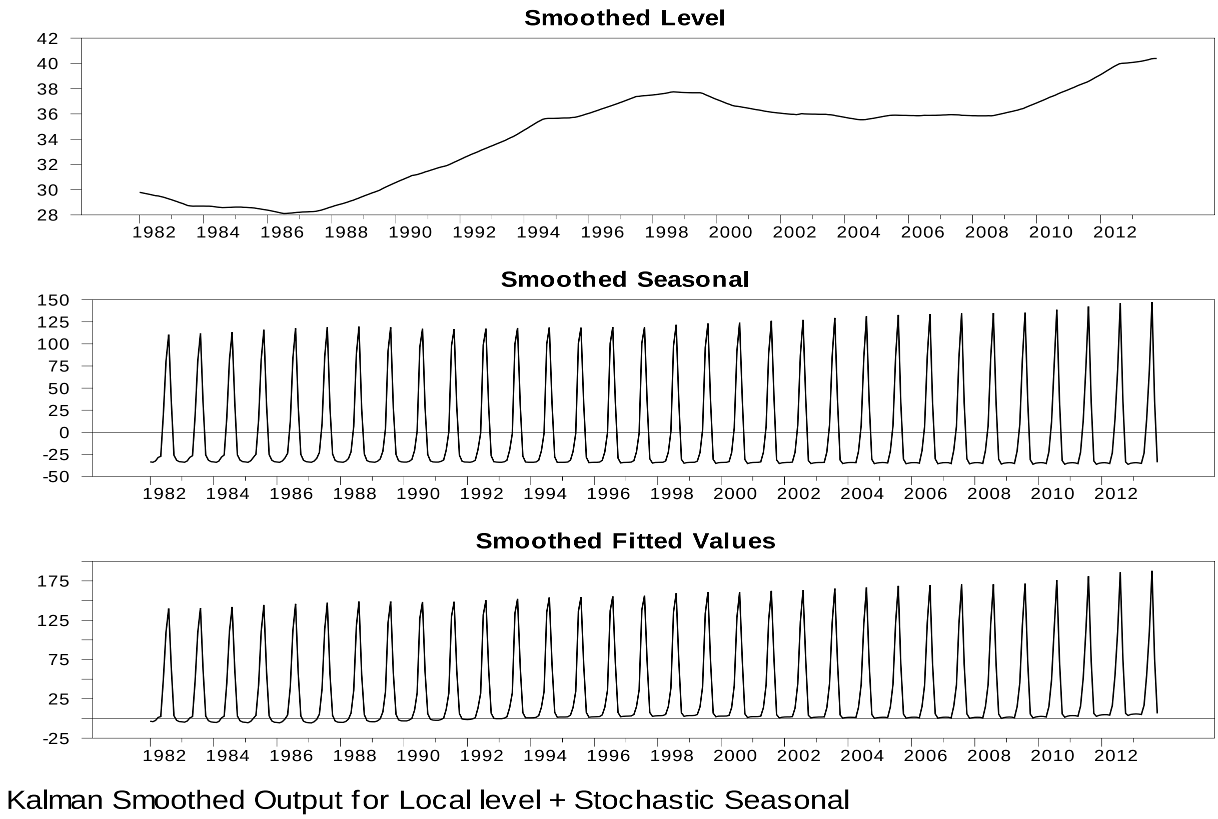


Figure 9. Kalman Smoothed Output for local level with stochastic seasonal for Damaturu series

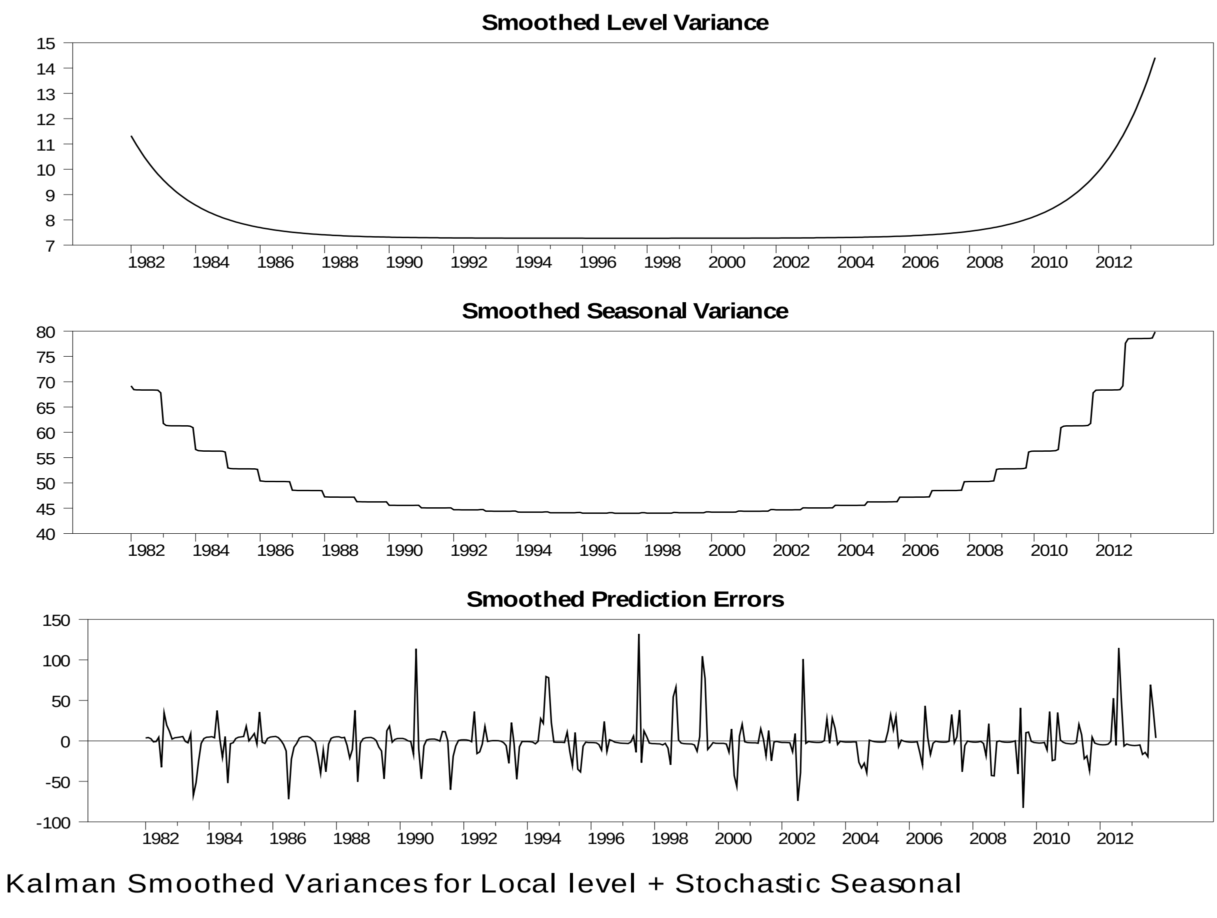


Figure 10. Kalman Smoothed Output for local level with stochastic seasonal for Damaturu series

Figure 3 to 6 present graphically the dynamic evolution of the level, seasonal components and the fitted values obtained by the Kalman filter together with their variances and the prediction errors (residuals). The evolution of the two North-Eastern regions under study is reflected by the estimated level component and is presented in the upper graph of figure 3 and 5. The plots indicate that the period of highest level of rainfall in Maiduguri occurred in the 2006 and the lowest level of rainfall occurred in the 1986. From figure 5, the plot also indicates that the period of the highest level of rainfall in Damaturu occurred in the 2000, while the lowest level of rainfall occurred between 1988 and 1989. One obvious features of figure 4 is that, the Kalman filtered state variances converges to zero value, which implies that there is no variability, while in figure 6, the Kalman filtered state variance converges to a constant value as the sample size increases which empirically confirms that the fitted local level model has a steady state solution.

Figure 7 to 10 present the results of the Kalman smoothing recursion estimates of the level, seasonal and the fitted values of the series together with the state variances of the level, seasonal component and the smoothed prediction errors respectively. On comparing the graphs of the Kalman filtered level and the smoothed level of the two Northeast states in figure 3 to 5 and 7 to 9 respectively, it is obvious that figure 7 to 9 is smoother than that of figure 3 to 5. In addition, figure 7 to 9 reveal that the seasonal pattern in the Maiduguri and Damaturu series have been relatively constant over the years.

In order to detect any systematic and haphazard structural breaks, we employed the cumulative sum (CUSUM) and cumulative sum squares (CUSUMSQ) introduced by Brown et al. (1975) to the residuals of the fitted model. The results of the CUSUM test are displayed in figure 11 and 12. Figure 11 indicates that there was a structural break around 1998 – 1999. Similarly, figure indicates the possibility of a structural break in the year 1990.

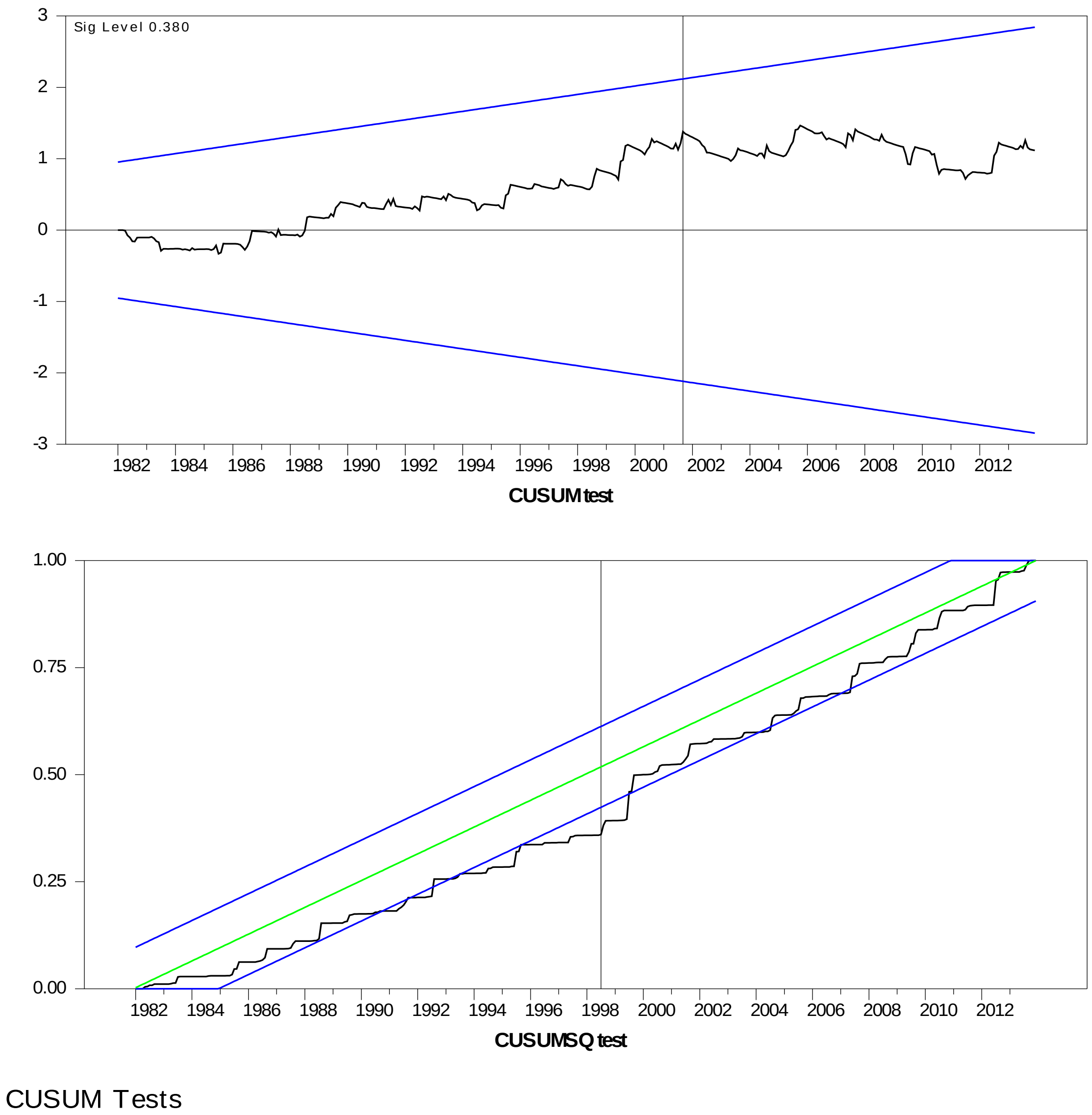
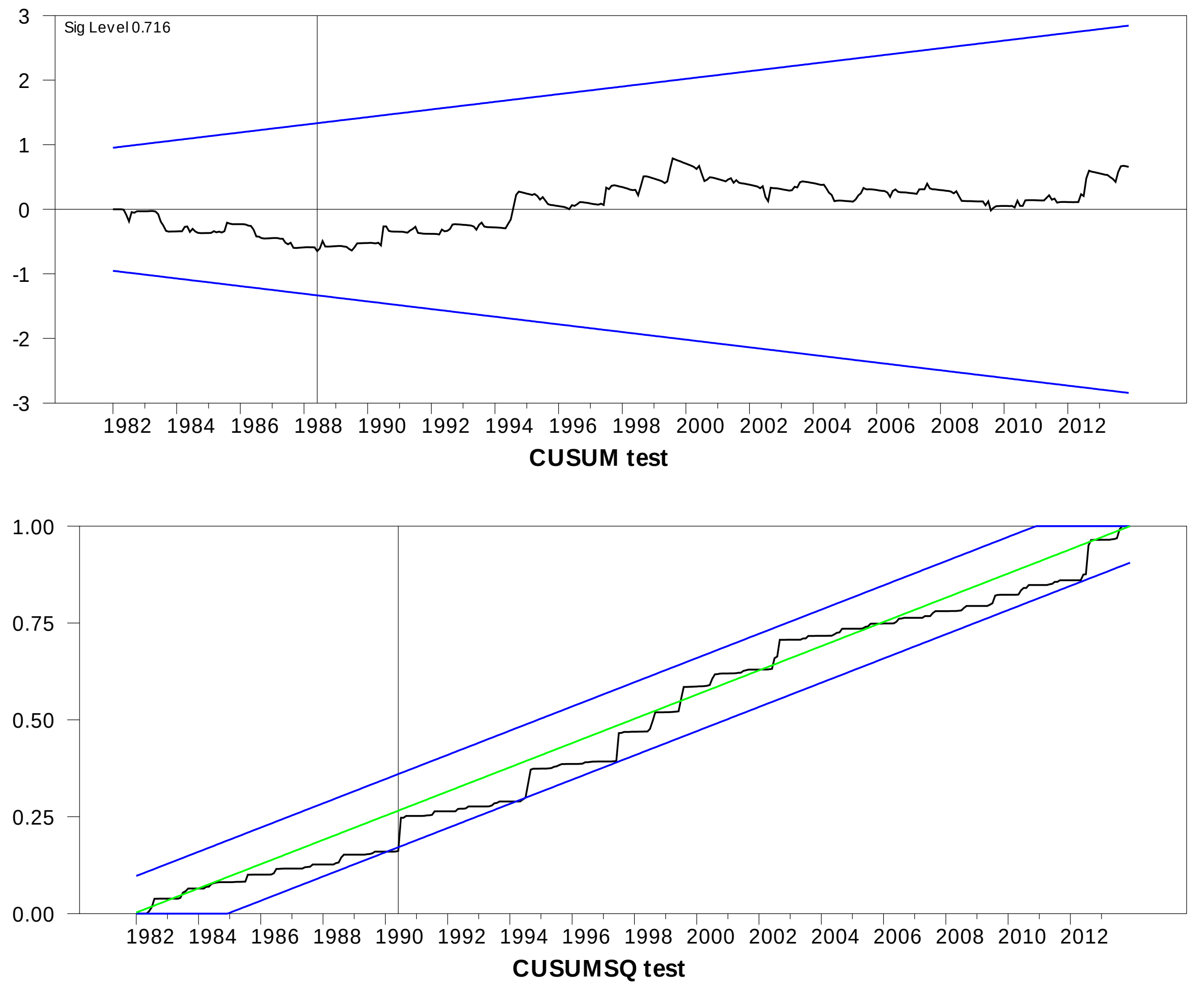


Figure 11. The CUSUM and CUSUMSQ Structural Break test for Maiduguri Rainfall.



CUSUM Tests

Figure 12. The CUSUM and CUSUMSQ Structural Break test for Damaturu Rainfall

Table 5. Diagnostics Results for Local Level Model with Stochastic Seasonal for Maiduguri Series

Local Level model + Deterministic Seasonal Diagnostics

	Statistic	Sig. level
Q(15-2)	4.66	0.26
Normality	18.29	0.63
H(128)	1.62	0.69

AIC = 9.89
BIC = 9.92

Table 6. Diagnostics Results for Local Level Model with Stochastic Seasonal for Damaturu Series

Local Level model + Deterministic Seasonal Diagnostics

	Statistic	Sig. level
Q(15-2)	12.33	0.50
Normality	1460.68	0.00
H(128)	1.08	0.67

AIC = 9.21
BIC = 9.24

Table 5 indicates that the diagnostic tests for independence, homoscedasticity, and normality of the residuals for the fitted Maiduguri model are all satisfactory. These tests indicate that the residuals satisfy all of the assumptions of the models since the p-value associated with all the tests are insignificant at the conventional 0.05 significance level. Also, from table 6, the diagnostic tests for independence and homoscedasticity of the residuals are satisfactory for Damaturu fitted model, while that of normality is not satisfactory, which suggests that the residuals satisfy the two most important assumptions of the models as discussed in section 3. However, the residuals are higher at the beginning and end of the sample, as theoretically expected, since the smoothed states are calculated by a backwards recursions. Hence, the uncertainty in the estimation is higher at the beginning of the backward recursions, unlike the Kalman filtering that uses a forward recursion. We now present the results of analysis of Maiduguri and Damaturu rainfall time series with a local level model with a deterministic seasonal.

Table 7. Estimation Results for Local Level Model with Deterministic Seasonal for Maiduguri Rainfall Series (1981:1-2013:12)

```
DLM - Estimation by BFGS
Convergence in    12 Iterations. Final criterion was  0.0000000 <=  0.0000100
Monthly Data From 1981:01 To 2013:12
Usable Observations                          396
Rank of Observables                          384
Log Likelihood                          -1954.62

    Variable                       Coeff      Std Error      T-Stat      Signif
********************************************************************************
1.  SIGSQEPS                     1345.24          97.75       13.76        0.00
2.  SIGSQETA                        0.69           0.61        1.14        0.25
```

Note: SIGSQEPS and SIGSQETA denote estimates σ_ε^2 and σ_η^2 respectively, σ_ω^2 is fixed at zero.

Table 8. Estimation Results for Local Level Model with Deterministic Seasonal for Damaturu Rainfall Series (1981:1-2013:12)

```
DLM - Estimation by BFGS
Convergence in     8 Iterations. Final criterion was  0.0000000 <=  0.0000100
Monthly Data From 1981:01 To 2013:12
Usable Observations                          396
Rank of Observables                          384
Log Likelihood                          -1821.64

    Variable                       Coeff      Std Error      T-Stat      Signif
************************************************************************************
1.  SIGSQEPS                      674.14          48.99       13.76        0.00
2.  SIGSQETA                        0.29           0.37        0.80        0.43
```

Note: SIGSQEPS and SIGSQETA denote estimates σ_ε^2 and σ_η^2 respectively, σ_ω^2 is fixed at zero.

The estimation results display above shows that the estimated hyperparameters (disturbance variances) of the measurement equation is significant while that of the level equation is not significant. Also, there are 396 observations in our series; however, the estimation is done using only the final 384 observations. This is because 12 diffuse initial state values are estimated (11 for the seasonal components and 1 for the local level components).

We perform the Kalman filtering and smoothing based on these two estimates. We present the results of the Kalman filter estimates of the local level model with deterministic seasonal for Maiduguri and Damaturu in figure 13 to 16 and Kalman smoothed estimates of the local level model with deterministic seasonal in figure 16 and 19.

Figures 13 and 16 present the output of the Kalman filtering of the local level model with deterministic seasonality for Maiduguri and Damaturu respectively. The output is very similar to the output of the local level model with stochastic seasonal (this is because the variance is small and insignificant), the evolution of Maiduguri and Damaturu rainfall is reflected by the estimated level component and is presented in the upper graph of figure 13 and 14. The plots also indicates that the period of highest level of rainfall in Maiduguri and Damaturu occurred in 2006 and 2000 respectively, while the lowest level occurred in 1986 and 1988-1989 respectively. Figure 14 also reveals that the Kalman filtered state variances converges to zero, while the filtered seasonal variance converges to a constant value. From figure 16, the filtered level variance converges to constant values as the sample size increases which empirically confirms that the local level model has a steady state solution.

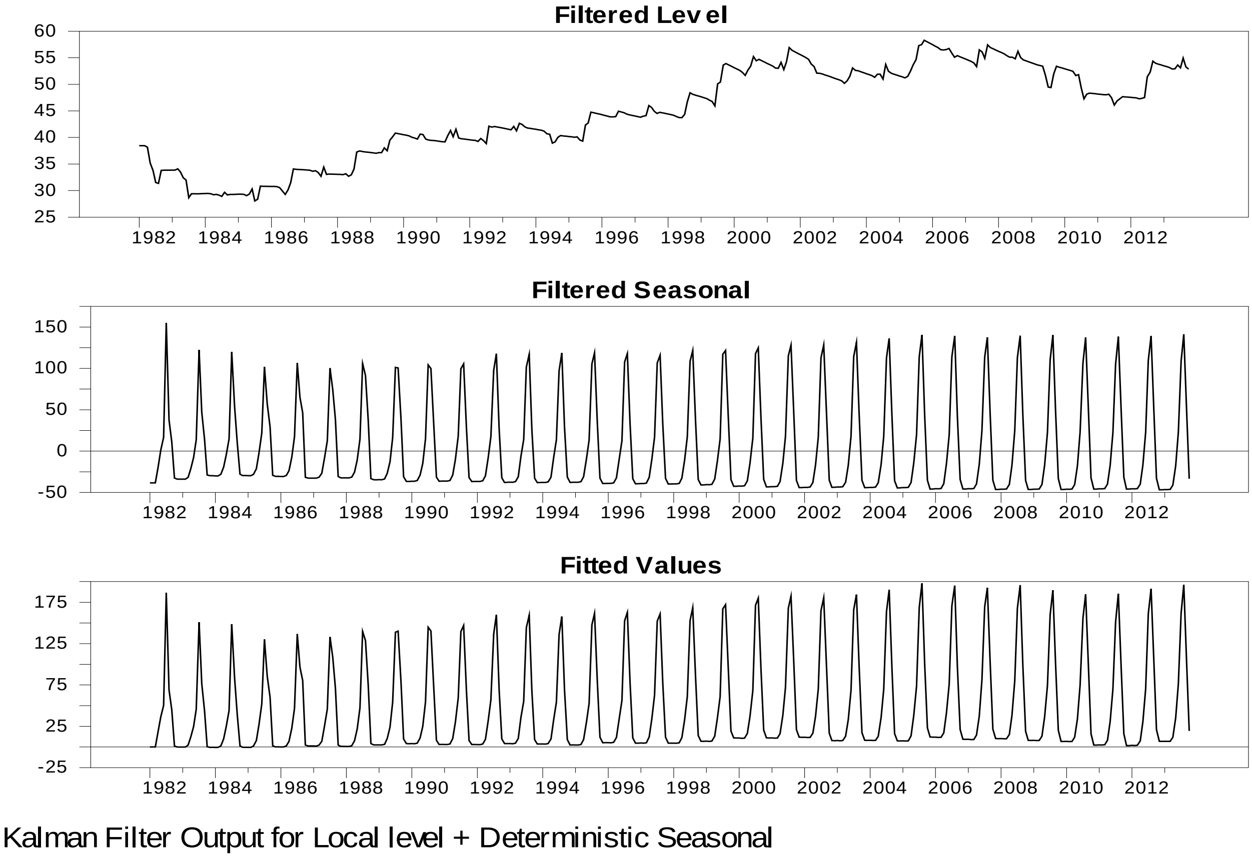


Figure 13. Kalman Filter Output for local level with deterministic seasonal for Maiduguri series

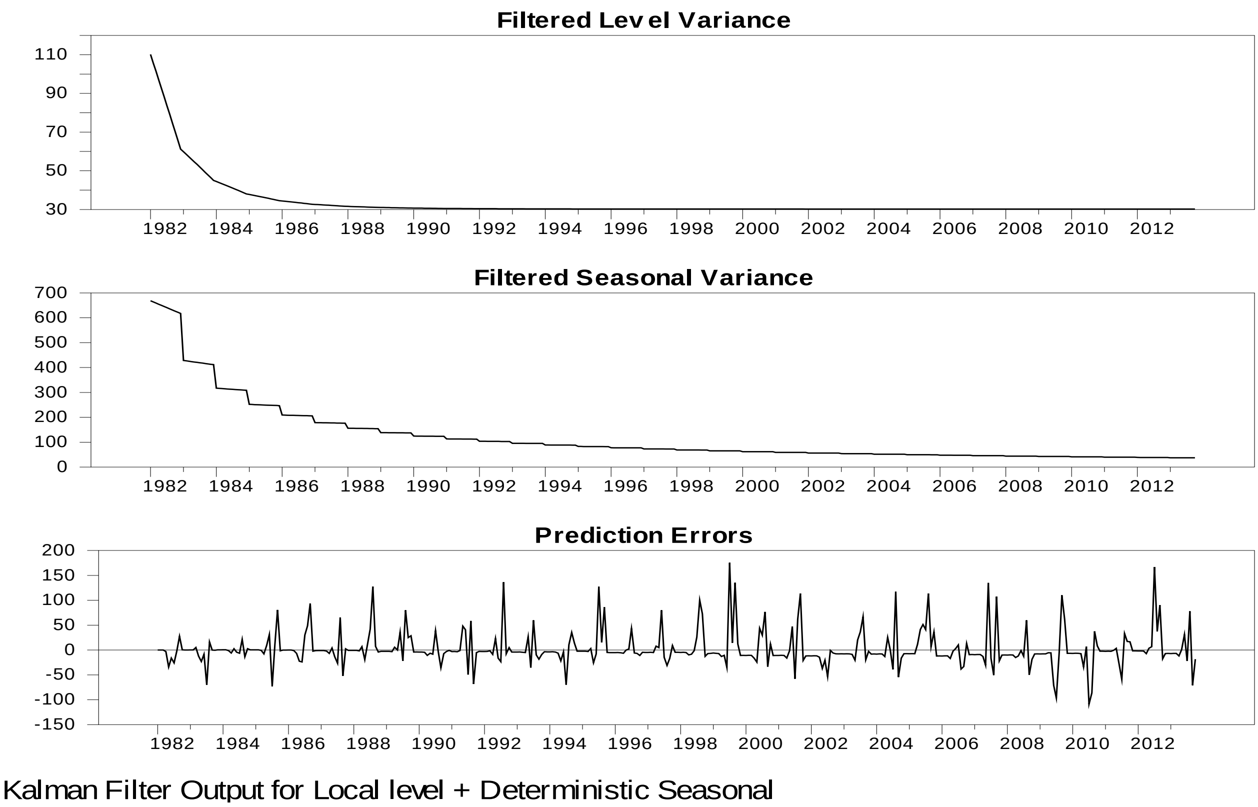


Figure 14. Kalman Filter Output for local level with deterministic seasonal for Maiduguri series

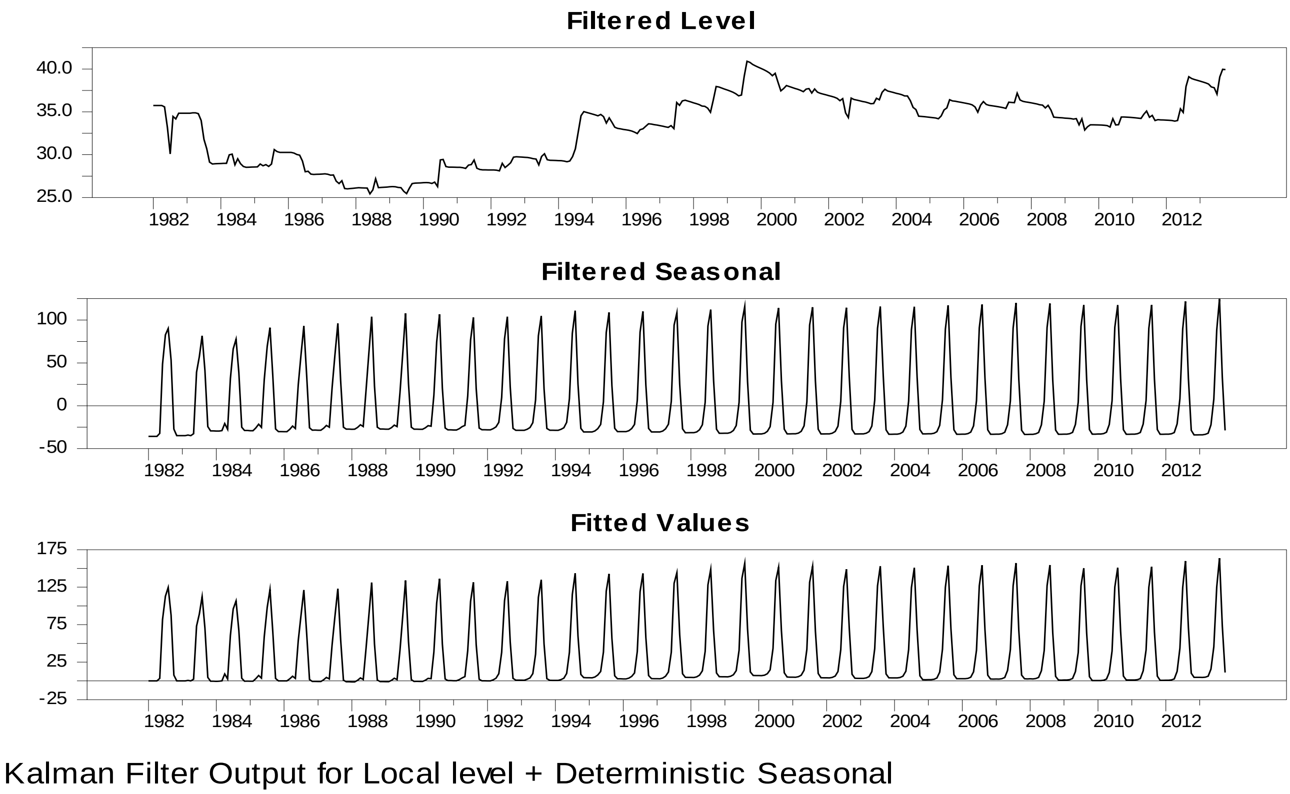


Figure 15. Kalman Filter Output for local level with deterministic seasonal for Damaturu series

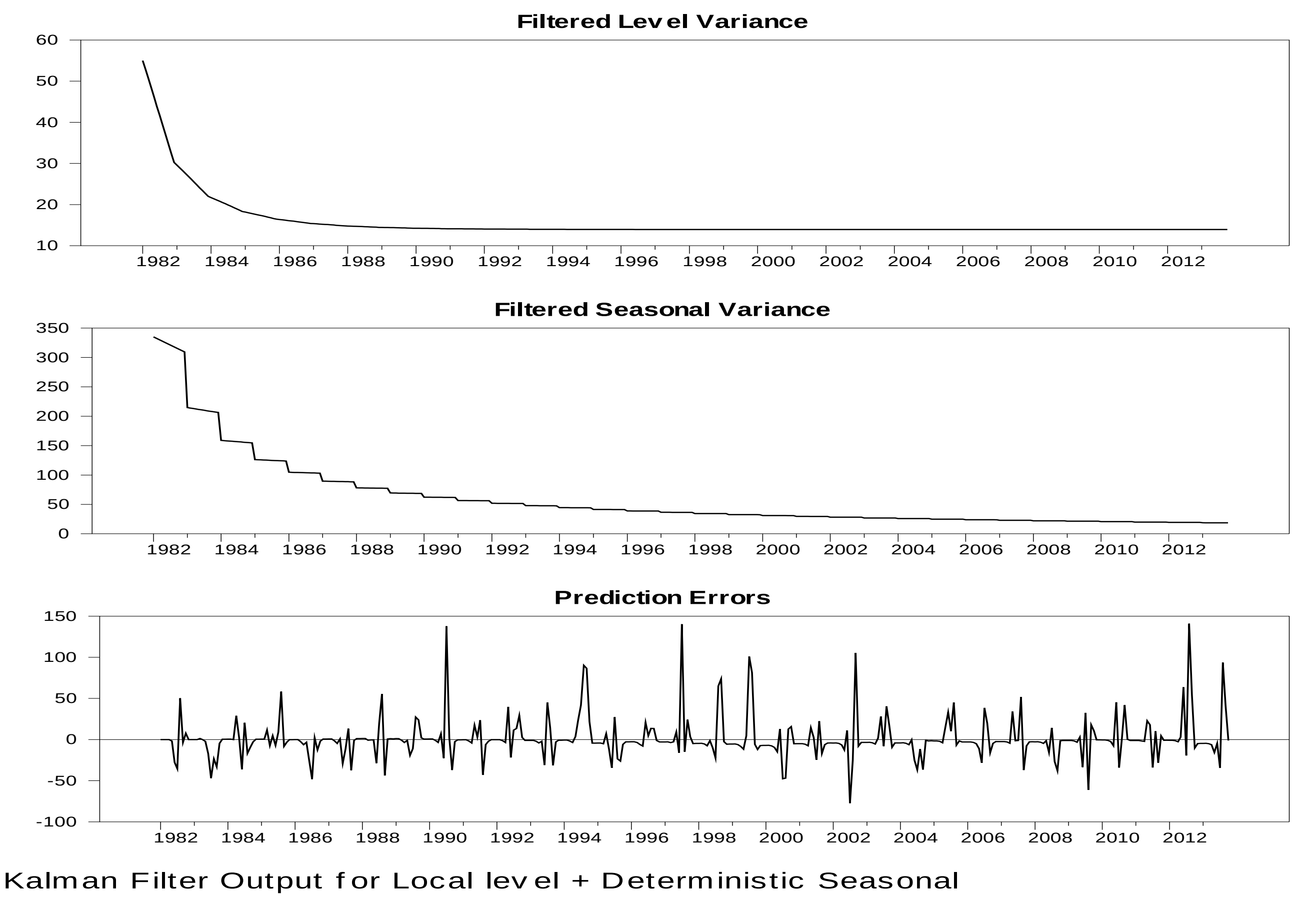


Figure 16. Kalman Filter Output for local level with deterministic seasonal for Damaturu series

Figures 17 to 20 present the output of the Kalman smoothing recursion estimates of the level, seasonal and the fitted values of the series together with the state variances of the level and seasonal component and the smoothed prediction errors for Maiduguri and Damaturu series. Comparing the graphs of the Kalman filtered level and the smoothed level in figure 13 and 15 and figure 17and 19, we see that the graph in figure 17 and 19 is smoother than that of figure 13 and15 for both states. However, the constant seasonal pattern in the series is clearly apparent in figure 18 and 20.

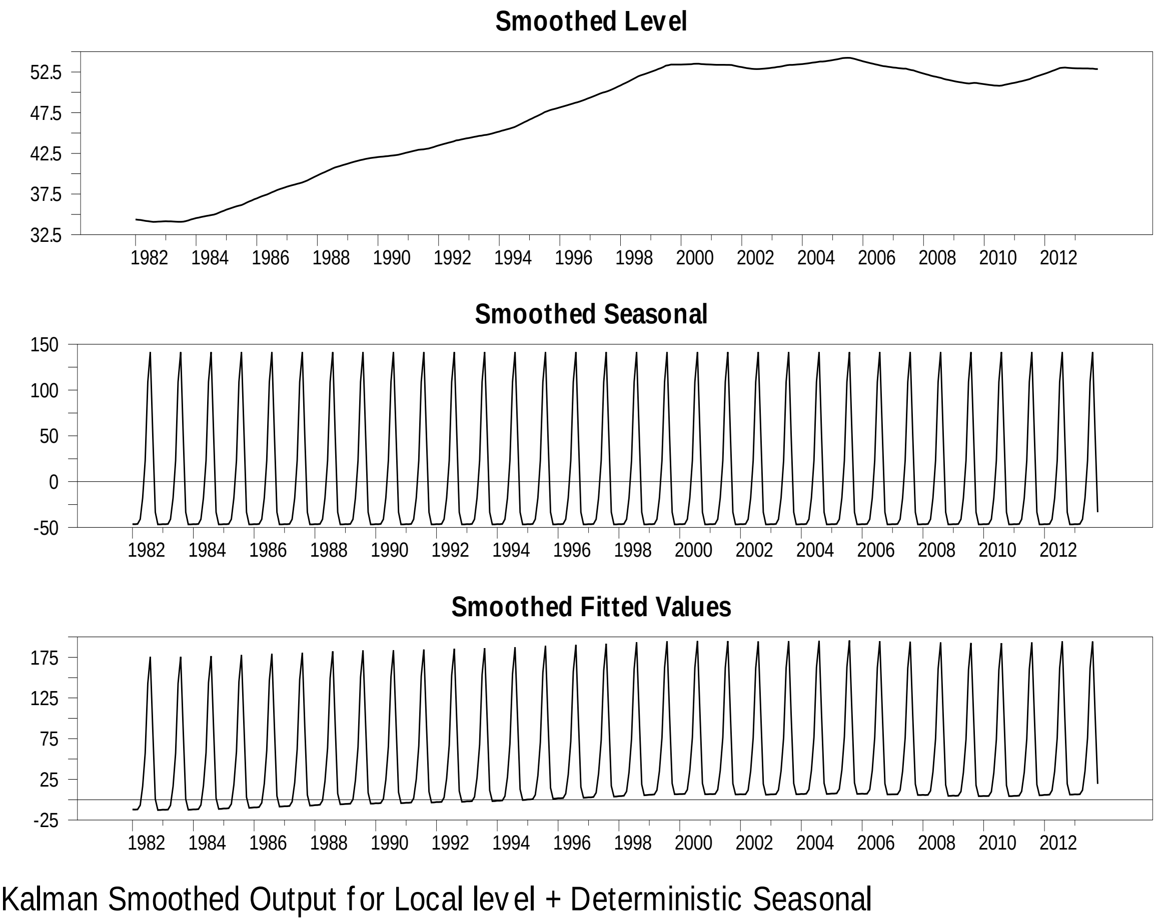


Figure 17. Kalman Smoothed Output for local level with deterministic seasonal for Maiduguri series

Table 9. Diagnostics Results for Local Level Model with Deterministic Seasonal for Maiduguri Rainfall Series

Local Level model + Deterministic Seasonal Diagnostics

	Statistic	Sig. level
Q(15-1)	5.22	0.32
Normality	69.63	0.06
H(128)	0.62	0.65

AIC = 9.88
BIC = 9.90

Table 10. Diagnostics Results for Local Level Model with Deterministic Seasonal for Damaturu Rainfall Series

Local Level model + Deterministic Seasonal Diagnostics

	Statistic	Sig. level
Q(15-1)	12.37	0.58
Normality	1765.44	0.00
H(128)	1.13	0.49

AIC = 9.21
BIC = 9.23

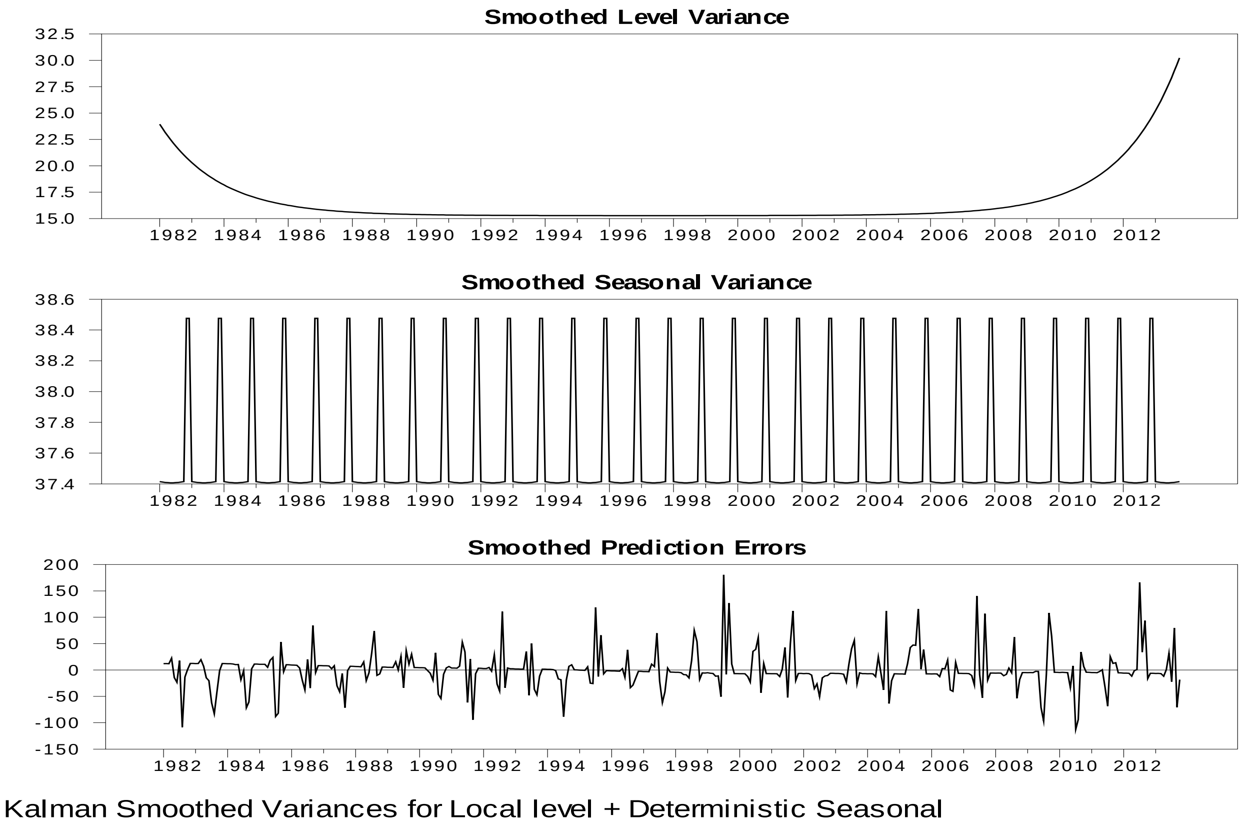


Figure 18. Kalman Smoothed Output for local level with deterministic seasonal for Maiduguri series

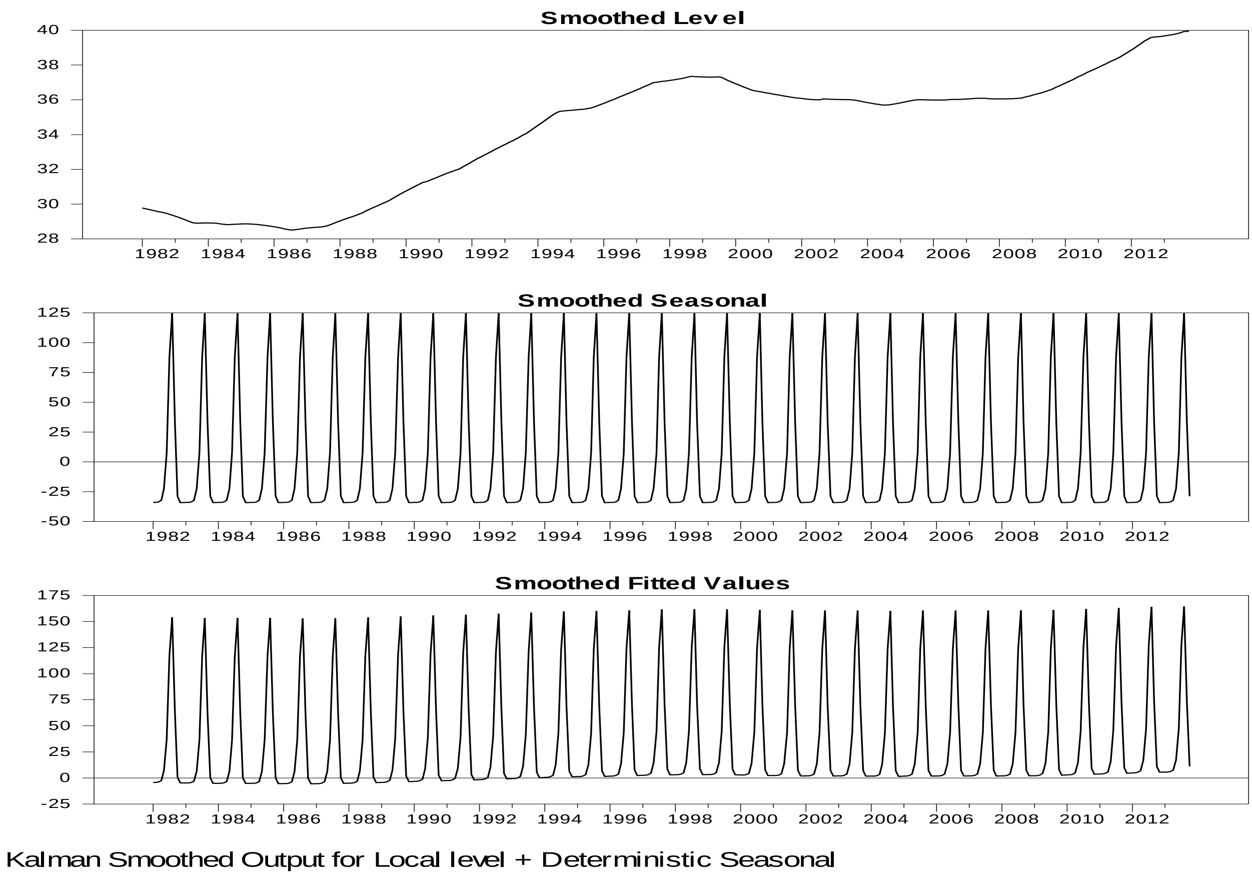


Figure 19. Kalman Smoothed Output for local level with deterministic seasonal for Damaturu series

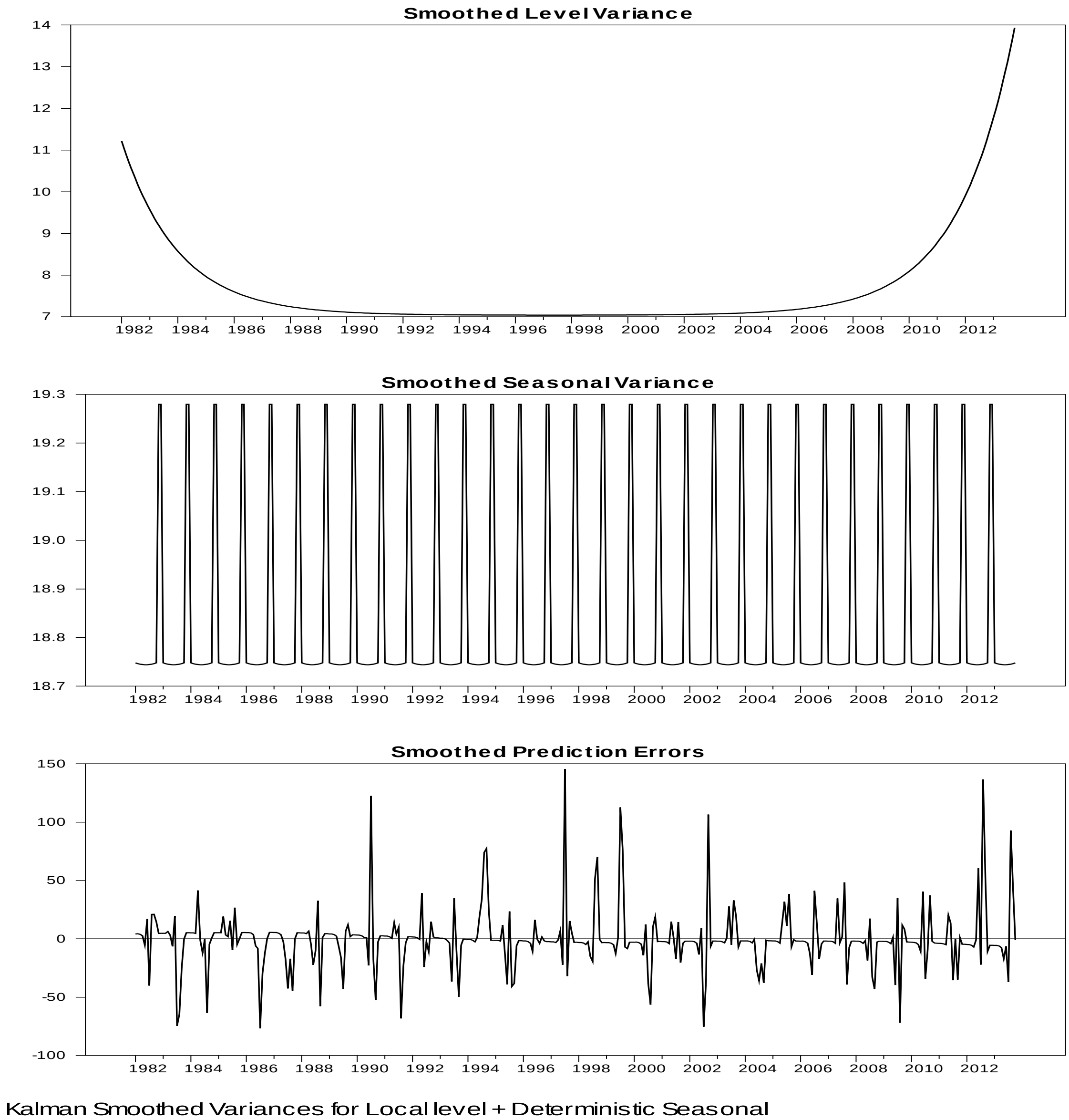


Figure 20. Kalman Smoothed Output for local level with deterministic seasonal for Damaturu series

From table 8 and 10, the results of the diagnostic tests for the residuals of the local level model with deterministic seasonal are again satisfactory except for the normality of Damaturu. From table 9, the assumption of independence, homoscedasticity and normality are satisfied at the conventional level while table 10 indicates that the assumption of normality is not satisfied at conventional significance level.

From the two competing model that is; local level stochastic and local level deterministic model, the Akaike Information Criterion (AIC) and the Bayesian Information Criterion (SIC) of the two models are examined to obtain the parsimonious model. Using the information criteria approach, models that yield smaller values for the criterion are preferred, and regarded as best fitting model. From table 5 and 6, the AIC and BIC for the local level model with stochastic seasonal are 9.89, 9.21 and 9.92, 9.24, for Maiduguri and Damaturu respectively, while the AIC and BIC for the local level model with deterministic seasonal displayed in table 9 and 10 are 9.88, 9.21 and 9.90, 9.23 for Maiduguri and Damaturu respectively. Hence, the local level model with deterministic seasonal is slightly better than the model with stochastic seasonal component. In addition, the log-likelihood values of the two models for the two states are almost identical -1954.62, -1820.29 and -1954.36, -1821.65 for the local level model with stochastic seasonal and the local level model with deterministic seasonal respectively. Hence, the improved fit of the local level model with deterministic model can completely be attributed to its greater parsimony. Commandeur and Koopman (2007) pointed out that, in state space modelling, a small and insignificant state disturbance variance indicates that the corresponding state component may as well be treated as a deterministic effect, resulting in a more parsimonious model. Therefore, the local level model with deterministic seasonal is able to model the dynamic features in the Maiduguri and Damaturu rainfall time series.

5. Conclusions

The variability in elements of climate such as rainfall and temperature exposes the country to the negative impact of climate change such as erratic rainfall, rise in temperature, desertification, low agricultural yield, drying up of water bodies, and sea level rise.

This paper analysed the seasonal pattern of rainfall in Maiduguri and Damaturu in the state space framework. We employ the local level model with deterministic and stochastic seasonal to modelling the monthly rainfall of the two states. The AIC and BIC of the two state space models suggest that the local level model with deterministic seasonal provide a better fit to the data than the model with stochastic seasonal. The detection of deterministic seasonal in the two states implies that the rainfall fall patterns have little variability or indication of climate change as regards its rainfall element. In addition, the CUSUM test indicates the presence of structural breaks in 1998 and 1990 for Maiduguri and Damaturu respectively. This implies that there was abrupt change in the rainfall level in 1998 for Maiduguri area and in 1990 for Damaturu area. We, therefore, recommend that seasonality should be explicitly included in the modelling of seasonal time series data as the pattern of seasonality could be useful for important decision making. These techniques can be adopted to the analysis of time series drawn from other domains. In addition, measures should be put in place to curb human-made activities that are detrimental to the climate since the region is highly vulnerable to the impacts of climate change. All computations performed in this study are done using the RATS econometric time series software.

Annexure I. Geographical Map of the Study Area

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