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A Displaced Probability Model for the Child Mortality in a Family

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Abstract In Present paper, a displaced probability model has been proposed to study the distribution of families according to the number of child death within the first five years of life. The model involved several parameters, which is estimated with the help of method of maximum likelihood and method of moments and fitted to observed survey data to draw important conclusions.

Keywords Probability model, Child-Mortality, MLE

1. Introduction

Child mortality, also known as under-5 mortality, refers to the death of infants and children under the age of five. In 2011, 6.9 million children under five died, down from 7.6 million in 2010, 8.1 million in 2009, and 12.4 million in 1990. About half of child deaths occur in Sub-Saharan Africa. Reduction of child mortality is the fourth of the United Nations' Millennium Development Goals. Child Mortality Rate is the highest in low-income countries, such as most countries in Sub-Saharan Africa. A child's death is emotionally and physically damaging for the mourning parents. Many deaths in the third world go unnoticed since many poor families cannot afford to register their babies in the government registry.

In demography child mortality are useful as a sensitive index of a nation's health conditions and as guided for the structuring of public health programmes. Child Mortality is interrelated to social, cultural, economic, physiological and other factor. The high rate of infant and child mortality shows a low level development of the health programme and also for the nation's. Infant and Child mortality has been of interest of researchers and demographers because of its apparent relationship with the acceptance of modern contraceptive means (Kabir et al, 1993).

Some effects have been made to estimate the current levels of child mortality by using data available from the different survey and other specific sources. Hill and Devid (1989) have suggested an approach for estimating child mortality from all births which have taken place in last five years before the survey. However, the estimates obtained though this method also suffer from the problem of under reporting Pathak, Pandey, and Mishra, (1991). In this Circumstances, some of the earlier studies about child mortality by using model (Chauhan, 1997; Gavrilova, 1991; Goldblatt, 1989; Heligman and Pollard, 1980; Krishna, 1993; Ronald and Carter, 1992; Thiele, 1972). Initially, Keyfitz (1977) used a hyperbolic function to study the infant and child mortality. Later Arnold (1983) used pareto distribution and Krishnan and Yin (1993) applied finite range model for the same.

The direct measures of mortality being not reliable, the problem may be overcome by the recently developed model building approaches which make it possible to obtain estimates from information other than vital statistics. In present paper a probability model has been proposed to study the distribution of families according to the number of child deaths within the first five years of life.

2. The Model

Let x denote the number of child deaths in a family at the survey point. Then the distribution of x is derived under the following assumption.

1. Only those families are considered in which at least one birth prior to the survey has occurred.
2. At the survey point, a family either has experienced a child loss or not. Let α and $(1-\alpha)$ be the respective proportions.
3. Out of α proportion of families, Let β be the proportion of families in which only one child death has occurred.
4. Remaining $(1-\beta)\alpha$ proportion of families, experiencing multiple child deaths, follows a displaced log distribution with parameter λ according to the

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number of child deaths.

Under these assumptions the probability distribution of X is given by.

$$P[X=0]=(1-\alpha),\ \text{k=0}$$
$$P[X=1]=\alpha\beta,\ \text{k=1}$$
$$P[X=k]=\frac{\alpha(1-\beta)(-1)\lambda^{k}}{(k-1)\ln(1-\lambda)},\ \text{k=2,3,4}\ldots\ldots \quad (2.1)$$

3. Estimation

Method of moment: We shall discuss the method of moments to estimate the three parameters $\alpha,\ \beta,\ \lambda$ and of the probability function (2.1). Let (x, f) be a frequency distribution whose parameters to be estimated from the observed distributions of families according to the number of child deaths. The following estimation technique is employed for estimation for rest of the parameter.

Equating proportions of zeroth cell, first cell frequency and simple mean to their corresponding observed values respectively, which convert into the following equations.

$$\frac{f_0}{f}=1-\alpha \quad (3.1)$$

$$\frac{f_1}{f}=\alpha\beta \quad (3.2)$$

$$\overline{X}=\alpha\beta+\alpha(1-\beta)\left(1-\frac{\lambda}{(1-\lambda)\ln(1-\lambda)}\right) \quad (3.3)$$

Where f_0 = Observed value of zeroth cell frequency
f_1 = Observed value of first cell frequency
f = Total number of observations = $\sum_i f_i$
$\overline{X}$ = Sample mean of the observed values.

Method of Maximum Likelihood: The proposed model involves four parameters $\alpha,\beta,\boldsymbol{\lambda}$ to be estimated from the observed distribution of families according to the number of child deaths but it cannot be possible to estimate all these simultaneously by this method, so the value of N has been taken as method of moments. From the population (2.1), the likelihood function L is given by,

$$L=[1-\alpha]^{f_0}[\alpha\beta]^{f_1}\left[\frac{\alpha(1-\beta)(-1)\lambda}{\ln(1-\lambda)}\right]^{f_2}\left[\alpha(1-\beta)\left\{1+\frac{\lambda}{\ln(1-\lambda)}\right\}\right]^{(f-f_0-f_1-f_2)} \quad (3.4)$$

Taking Log both sides we get

$$\log L=f_0\log(1-\alpha)+f_1\log\alpha+f_1\log\beta+f_2\log\alpha+f_2\log(1-\beta)+f_2\log(-1)+f_2\log\lambda-f_2\log\{\ln(1-\lambda)$$
$$+(f-f_0-f_1-f_2)\log\alpha+(f-f_0-f_1-f_2)\log(1-\beta)+(f-f_0-f_1-f_2)\log\{\lambda+\ln(1-\lambda\}- \quad (3.5)$$
$$(f-f_0-f_1-f_2)\log\{\ln(1-\lambda)\}$$

Now, partially differentiating w. r. t. α,β and λ respectively and equating to zero.

$$\frac{\partial}{\partial\alpha}\log L=-\frac{f_0}{(1-\alpha)}+\frac{1}{\alpha}\left[f_1+f_2+(f-f_0-f_1-f_2)\right] \quad (3.6)$$

$$\frac{\partial}{\partial\beta}\log L=\frac{f_1}{\beta}-\frac{1}{1-\beta}\left[(f-f_0-f_1-f_2)+f_2\right] \quad (3.7)$$

$$\frac{\partial}{\partial\lambda}\log L=f_2\left[\frac{1}{\lambda}+\frac{1}{(1-\lambda)\ln(1-\lambda)}\right]+(f-f_0-f_1-f_2)\frac{\lambda}{(1-\lambda)\ln(1-\lambda)\{\lambda+\ln(1-\lambda)\}} \quad (3.8)$$

Equating to zero equation (3.6), (3.7) and (3.8) and solving the estimate of α,β and λ can easily be obtained as:

$$\hat{\alpha}=\frac{(f-f_0)}{f}$$

$$\hat{\beta}=\frac{f_1}{(f-f_0)}$$

$$f_2\left[\frac{1}{\lambda}+\frac{1}{(1-\lambda)\ln(1-\lambda)}\right]+(f-f_0-f_1-f_2)\frac{\lambda}{(1-\lambda)\ln(1-\lambda)\{\lambda+\ln(1-\lambda)\}}=0$$

The second partial derivatives of $Log\ L$ can be obtained as:

$$\frac{\partial^2}{\partial^2\alpha}\log L=-\left[\frac{f_0}{(1-\alpha)^2}+\frac{(f-f_0)}{\alpha^2}\right] \quad (3.9)$$

$$\frac{\partial^2}{\partial^2\beta}\log L = -\left[\frac{f_1}{\beta^2} + \frac{(f - f_0 - f_1)}{(1-\beta)^2}\right] \tag{3.10}$$

$$\frac{\partial^2}{\partial\lambda^2}\log L = -\left[f_2\left(\frac{1}{\lambda^2} + \frac{1-\ln(1-\lambda)}{\{(1-\lambda)\ln(1-\lambda)\}^2}\right) + (f - f_0 - f_1 - f_2)\left(\frac{1-\ln(1-\lambda)}{\{(1-\lambda)\ln(1-\lambda)\}^2} + \frac{\lambda+(1-\lambda)\ln(1-\lambda)}{\{\lambda+\ln(1-\lambda\}^2(1-\lambda)}\right)\right] \tag{3.11}$$

Now partial derivative of $\frac{\partial \log L}{\partial\alpha}$, $\frac{\partial \log L}{\partial\beta}$ and $\frac{\partial \log L}{\partial\lambda}$ w.r.to β, λ and α respectively we get as:

$$\frac{\partial^2 \log L}{\partial\alpha\partial\beta} = \frac{\partial^2 \log L}{\partial\beta\partial\lambda} = \frac{\partial^2 \log L}{\partial\lambda\partial\alpha} = 0 \tag{3.12}$$

Here,

$$E(f_0) = f(1-\alpha)$$
$$E(f_1) = f\alpha\beta$$
$$E(f_2) = f\alpha(1-\beta)\frac{(-1)\lambda}{\ln\ (1-\lambda)}$$
$$E(f - f_0 - f_1 - f_2) = f.\alpha(1-\beta)\left\{1 + \frac{\lambda}{\ln(1-\lambda)}\right\}$$

Using the above facts, the expected value of the second partial derivatives can be obtained as:

$$\emptyset_{11} = E\left[\frac{-\partial^2 \log L}{\partial\alpha^2}\right]/f = \left[\frac{1}{1-\alpha} + \frac{1}{\alpha}\right] \tag{3.13}$$

$$\emptyset_{22} = E\left[\frac{-\partial^2 \log L}{\partial\beta^2}\right]/f = \alpha\left[\frac{1}{1-\beta} + \frac{1}{\beta}\right] \tag{3.14}$$

$$\emptyset_{33} = E\left[\frac{-\partial^2 \log L}{\partial\lambda^2}\right]/f \tag{3.15}$$

$$= \left(\alpha(1-\beta)\frac{(-1).\lambda}{\ln(1-\lambda)}\right)\left(\frac{1}{\lambda^2} + \frac{1-\ln(1-\lambda)}{\{(1-\lambda)\ln(1-\lambda)\}^2}\right) +$$
$$\left(\alpha(1-\beta).\left\{1 + \frac{\lambda}{\ln(1-\lambda)}\right\}\right)\left(\frac{1-\ln(1-\lambda)}{\{(1-\lambda)\ln(1-\lambda)\}^2} + \frac{\lambda+(1-\lambda)\ln(1-\lambda)}{\{\lambda+\ln(1-\lambda\}^2(1-\lambda)}\right)$$

The covariance between the estimators becomes zero since

$$E\left(\frac{\partial^2 \log L}{\partial\alpha\partial\beta}\right) = E\left(\frac{\partial^2 \log L}{\partial\beta\partial\lambda}\right) = E\left(\frac{\partial^2 \log L}{\partial\alpha\partial\lambda}\right) = 0 \tag{3.16}$$

Thus the asymptotic variances of the estimator can be obtained as:

$$V(\hat{\alpha}) = \frac{1}{f}\left[\frac{1}{\emptyset_{11}}\right],\ V(\hat{\beta}) = \frac{1}{f}\left[\frac{1}{\emptyset_{22}}\right] \text{ and } V(\hat{\lambda}) = \frac{1}{f}\left[\frac{1}{\emptyset_{33}}\right] \tag{3.17}$$

4. Application

The suitability of the proposed model is examined to the sample data collected under a survey entitled "Effect of breastfeeding on fertility in North Rural India" in 1995 and another is entitled "A Demographic survey on fertility and mortality in rural Nepal: A Study of Palpa and Rupandehi Districts" in 2000. On the other hand, one set of data has been taken from a household sample survey in Brazil in 1987 (Sastry,1997).

The parameters of the proposed model have been estimated by the method of moment and method of maximum likelihood. The estimated values of different parameters are given in tables 1 to 3 for the child deaths.

The estimated value of α 0 .3753 and 0.3753 are for India, Nepal, North East Brazil by Method of Moments and MLE respectively. Which shows the proportion of families experienced a child loss, was found slightly higher for India than North East Brazil which is 0.2683 and 0.2683 by Method of Moments and MLE respectively , and in Nepal the value of α is 0.2139 and 0.2683 by Method of Moments and MLE respectively. The estimate of β are (0.5857, 0.7528, .6560) and (0.5855, 0.7527, 0.6560) by Method of Moments and MLE respectively, respectively for India, Nepal, North East Brazil. Which shows the proportion of families having only one child death was found greater for Nepal (0.7527) as compared to other countries and the estimated values for λ are 0.6709, 0.5160 and 0.6509 by the method of moment and 0.6109, 0.5739 and

0.5486 by the maximum likelihood respectively for the above mentioned countries. The graph 1 to 3 shows the deviation of expected values from the observed values in the three types of data in two different methods of estimation techniques. From tables 1 to 3 it is found that the value of χ^2 are insignificant at 5% level of significance for all set of data. The proposed model suitably described the pattern of child mortality for different data in Indian subcontinents.

Table 1. Distribution of observed and expected number of families according to the number of child deaths in Eastern Uttar Pradesh (India)

Number of child dead	Child dead in Eastern Uttar Pradesh		
	Observed number of families	Expected number of families	
		Method of Moment	Method of Maximum Likelihood
0 1 2 3 4 5 6 7	506 178 76 32 8 6 3 10 1	506 178 81.46 24.89 10.13 9.52	506 178 76.0607 25.5145 11.4118
Total	810	810.000	810.0000
α β λ	0 .3753 0 .5857 0.6109		0.3753 0.5855 0.6709
$V(\alpha)$ $V(\beta)$ $V(\lambda)$	0.00029 0.00080 0.00042		
χ^2 $d.f.$	2.8689 3		3.3661 3

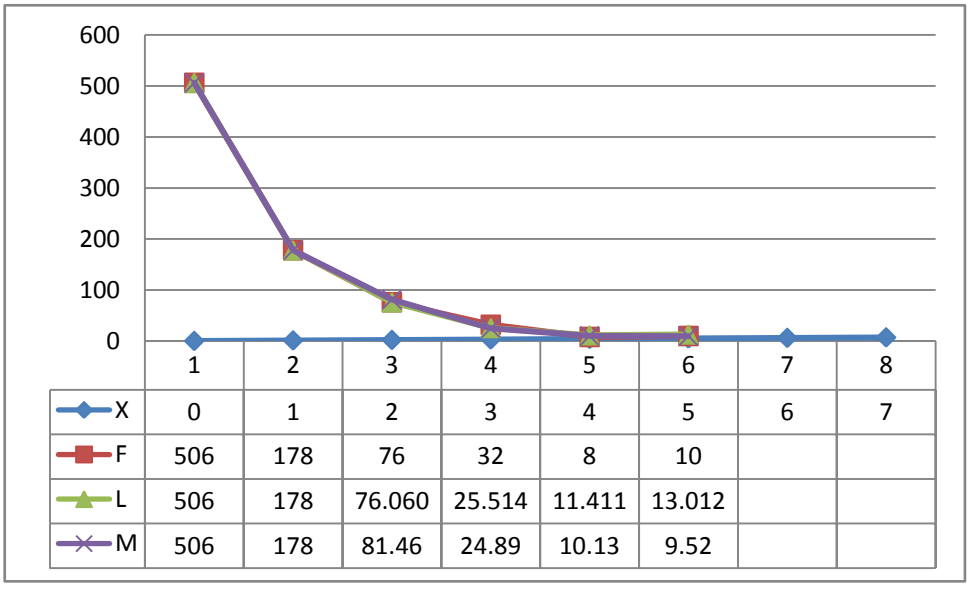


Table 2. Distribution of observed and expected number of families according to the number of child deaths in Nepal

Number of child dead	Child dead in Nepal		
	Observed number of families	Expected number of families	
		Method of Moment	Method of Maximum Likelihood
0 1 2 3 4 5 6 7	669 137 32 6 3 2 7 2 0	669 137 30.23 8.67 6.10	669 137 31.99 8.25 4.74
Total	851	851.0000	851.0000
α β λ		0.2139 0 .7528 0 .5739	0.2138 0.7527 0.5160
$V(\alpha)$ $V(\beta)$ $V(\lambda)$		0.00020 0.00102 0.00118	
χ^2 $d.f.$		1.058 2	1.6860 2

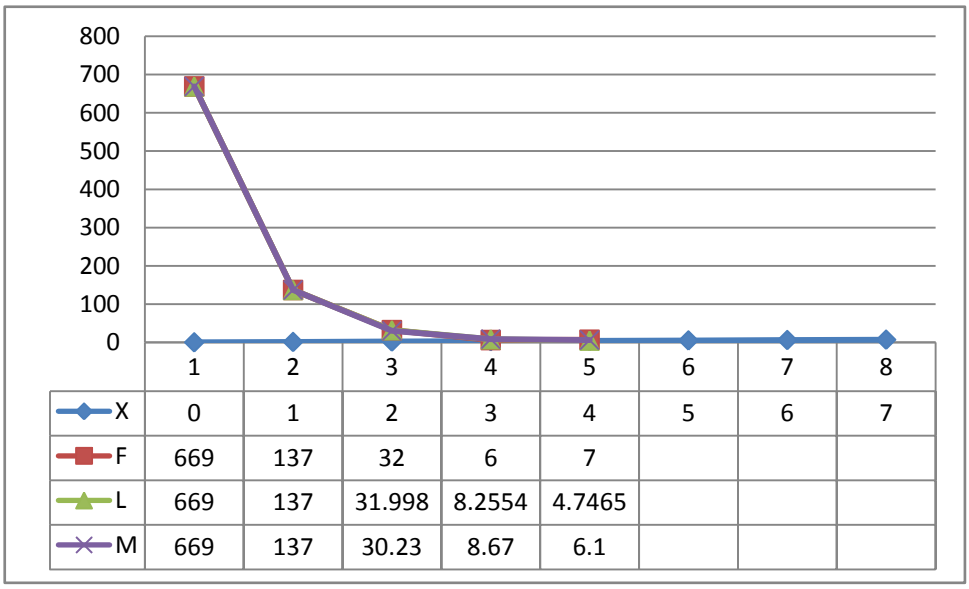
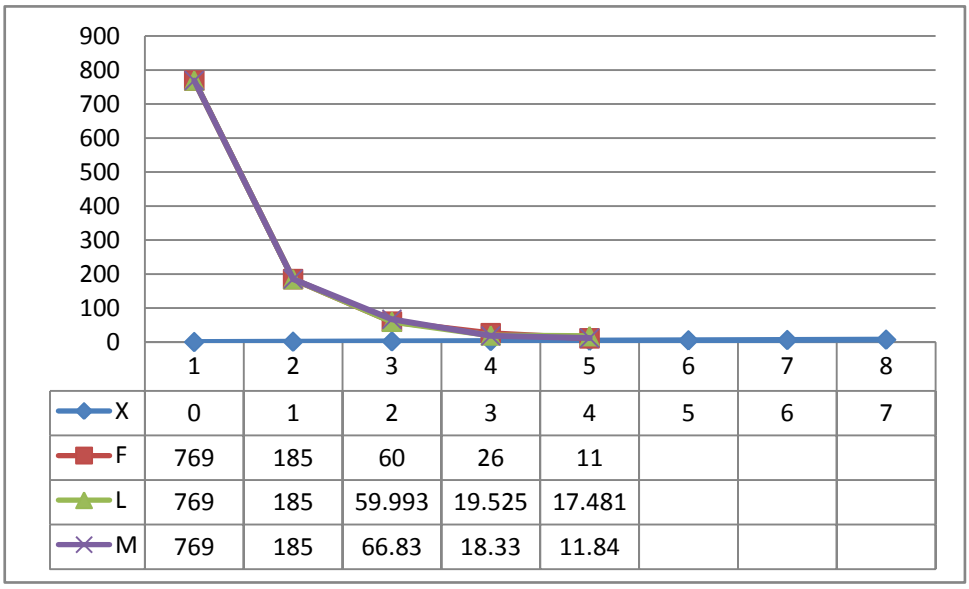


Table 3. Distribution of observed and expected number of families according to the number of child deaths in North East Brazil

Number of child dead	Child dead in North East Brazil		
	Observed number of families	Expected number of families	
		Method of Moment	Method of Maximum Likelihood
0 1 2 3 4 5 6 7	769 185 60 26 9 1 11 1 0	769 185 66.83 18.33 11.84	769 185 59.99381 19.52499 17.4812
Total	1051	1051.0000	1051.0000
α β λ		0.2683 0.6560 0 .5486	0.2683 0.6560 0 .6509
$V(\alpha)$ $V(\beta)$ $V(\lambda)$		0.00019 0.00080 0.00055	
χ^2 $d.f.$		3.9669 2	4.5502 2



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