International Journal of Optics and Applications 2015, 5(3): 58-64
DOI: 10.5923/j.optics.20150503.02

Analysis of Total Harmonic Distortion in an APD Receiver Circuit

Faramarz E. Seraji

Optical Communication Group, Iran Telecom Research Center, Tehran, Iran

Abstract This paper presents a simple analysis on the nonlinear behavior of a practical APD receiver circuitry, by considering a silicon APD photodetector as an optical receiver. The analysis presents the effects of received input power by the photodetector and the load resistance at the receiving end on the total harmonic distortion (THD), consisting of second- and third harmonics. The behavior of the THD is formulated and graphically illustrated in terms of the input power and load resistance. Typically, for a given circuit, the second HD was found to be -71 dB, for load resistance of $R_L = 100\ \Omega$ and optical input power of $P_a = 1\ \mu\text{W}$. If a high impedance preamplifier is used with $R_L = 10\ \text{k}\Omega$, the second HD will worsen to a value of -11 dB for $P_a = 10\ \mu\text{W}$. The analysis is given in an easy-to-understand manner that can be readily applied to a practical system and would be useful when dealing with analog optical fiber communication systems and optical fiber sensing systems.

Keywords Total harmonic distortions (THD), APD receiver, Load resistance

1. Introduction

In fiber sensor circuits, particularly when operating at a wavelength of 633 nm or 830 nm, a silicon avalanche photo diode (APD) is often used as a detector in the receiver. At higher wavelengths Ge and InGaAs type of APD's with moderate gain are used for detection purposes [1-3].

In analog optical communication links, in interferometric optical fiber sensors, and in analog radio-over-fiber systems, the performance at the receiving end is limited by second order nonlinearity, which is induced by the photodiode acting as the photodetector [4-13]. That is, the degree of nonlinearity of the photodiode is determined by the total harmonic distortion appearing at the output power which in particular includes the second and third order harmonic components [13]. Works have been reported to improve the linearity of photodiodes for the frequency range of 1-2 GHz. [14, 15].

In another attempt, a method was used to improve higher frequency performance of photodiodes by using Mach-Zehnder modulator (MZM) at quadrature bias with balanced detection to nullify the second order nonlinearity of photodiode at high frequencies [16].

In a recent report, yet another method used dual parallel MZM to cancel out the photodiode-induced second harmonic distortion [17]. It was shown that the second harmonic generated by this method was 180 deg. out of phase of that of the photodiode nonlinearity. Recently, the nonlinearity of a commercial photodiode was measured, using three setups of a one-tone heterodyne, two-tone heterodyne and three-tone MZM designs. Mathematically developed data on multiple devices are compared to find under which conditions the measurements by three setups are comparable [18]. A new method is reported theoretically with experimental results to cancel even-order distortion induced by photodiode in microwave photonic links. A single Mach-Zehnder modulator, biased slightly away from the quadrature point, is shown to suppress photodiode second-order intermodulation distortion in excess of 40 dB without affecting the fundamental power [19].

In an another report, a measurement of the nonlinearity responsivity of two commercial photodiodes of types *p-n* Ge and *p-i-n* InGaAs, used in optical fiber power measurements, was presented. The photodiodes in measurements were under high irradiance levels. It was shown that the photodiodes nonlinearities were of the saturation type, which depended on the beam diameter of the radiation source [20]. In newly reported work, a model of simulating photodiode microwave nonlinearities is proposed, which includes the effects of non-uniform absorption in three dimensions and self-heating of the photodiode. The saturated output power and third order output intercept points of two different waveguide photodiodes are simulated, with excellent agreement between measurement and theory [21].

In an early work, a simple method was used to obtain

* Corresponding author:
feseraji@itrc.ac.ir (Faramarz E. Seraji)
Published online at http://journal.sapub.org/optics

harmonic distortion levels of a pin photodiode at different frequencies from the microwave reflection coefficient of the photodiode under *dc* illumination for different incident optical powers [22].

The gain of the APD is a function of the actual bias voltage across the diode, and if the output voltage across the load resistance changes corresponding to a change in the received optical power, the bias voltage of the APD gets modulated leading to a non-linear response of the overall APD receiver. However, when receiving weak optical signals, this non-linearity will be acceptably small [23].

In this paper, a simple analysis on the nonlinear behavior of a practical silicon APD photodetector circuitry is presented, by considering the gain characteristics of the APD photodiode. The analysis of the total harmonic distortion (THD), which is an indication of degree of nonlinearity in an APD photodetector, includes the effects of received input power and the load resistance at the receiving output. The behavior of the THD is formulated and graphically illustrated in terms of the input power and load resistance.

The analysis is given in an easy-to-understand manner that can be readily applied to a practical system and would be useful when dealing with the optical fiber sensor systems and analog optical fiber communication systems.

2. Distortion in APD Receiver

Figure 1 shows a practical circuit diagram of an APD receiver using a silicon APD. The resistor R_x is being used to protect the avalanche photodiode from an accidental break-down, if any, by limiting the maximum possible current flow. The capacitors at the cathode of the APD keeps the voltage at the cathode V_x at *ac* ground. Figure 2 shows the plot of the APD gain $M(v)$ as a function of the bias voltage, obtained experimentally. Around the break-down voltage of the APD, the gain of the receiver increases rapidly.

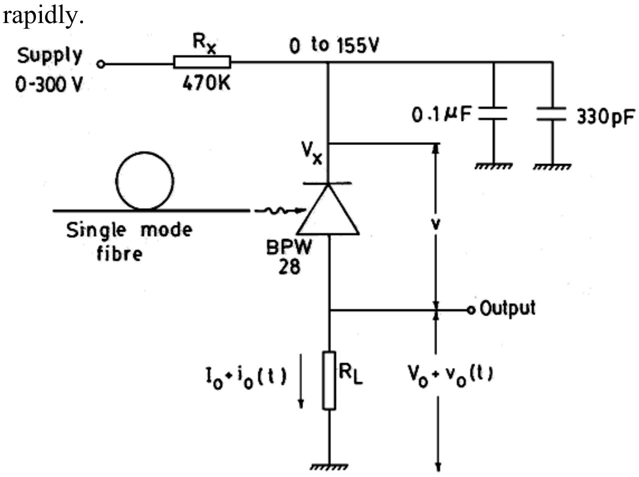

Figure 1. Circuit diagram of the APD receiver

The APD receiver, theoretically, is nonlinear if it is operated at high values of APD gain. The source of nonlinearity arises due to the finite value of the load resistor R_L and the *ac* output voltage $v_0(t)$. For a given *dc* bias voltage V_x at the cathode of the APD (in Fig. 1), the actual voltage (v) across APD changes with the output voltage as $v = V_x - [V_0 + V_0(t)]$, where V_0 is the *dc* voltage across R_L. At a given average optical power level, V_x and V_0 will remain constant but $v_0(t)$ will change with the modulation in the optical power. Since $v_0(t)$ itself is a function of the actual APD bias voltage, distortion results when the output voltage modulates the APD gain. This effect will be worse whenever the load resistance and the optical signal level (and hence the electrical output signal voltage) are high.

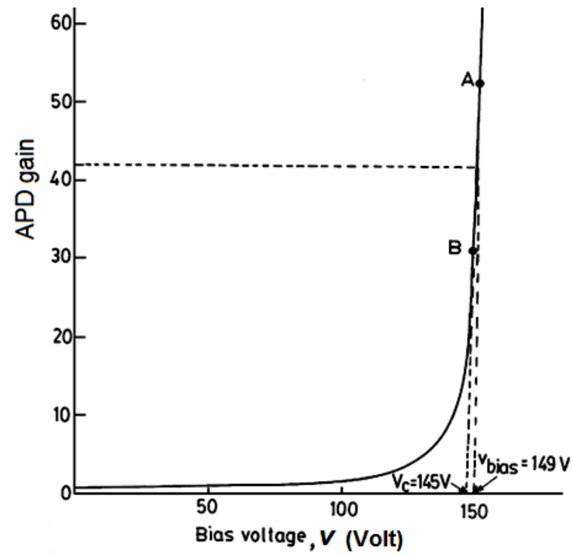

Figure 2. Characteristic curve of the APD gain at 5 kHz as a function of APD bias voltage for a peak *ac* optical input power $P_a = 9.46$ μW

The APD gain characteristic curve of Fig. 2 can be assumed to be linear over a small range of the APD bias voltage and accordingly can be expressed by a straight line equation. This assumption is valid when receiving a small optical signal and helps in simplifying the distortion analysis. For example, the curve AB of Fig. 2 can be expressed as:

$$M = K(v - V_c), \qquad V_B \le v \le V_A \tag{1}$$

where M is the gain of the APD, v is the instantaneous bias voltage across the APD, K is a constant equal to the slope of curve AB, and V_c is the voltage as indicated in Fig. 2, obtained by extending the line passing through points A and B. Equation (1) is valid only for points along the curve AB. The value of K for Fig. 2 was calculated to be about 11.197 V^{-1} and V_c was equal to 145 V.

An expression for the current multiplication factor $M(v)$ has been given as [24]:

$$M(v) = \exp(1.9x) - x\sin(1.25\pi x) \tag{2}$$

where we have:

$$x = \beta\left(\frac{v - V_r}{V_r}\right) \quad (3)$$

$$\beta = \left(\frac{V_r}{V_{bd} - V_r}\right) \quad (4)$$

where v is the instantaneous reverse bias voltage, V_r is the reverse voltage at which avalanche multiplication becomes significant, and V_{bd} is the reverse breakdown voltage of the APD. In this case the slope, K, of the characteristic curve is given by:

$$K = \frac{d\,M(v)}{dv} \quad (5)$$

Then, from Eq. (2) we can write:

$$K = \left(\frac{1}{V_{bd} - V_r}\right)\{1.9\exp(1.9x) - [\sin(1.25\pi x) + 1.25\pi x\cos(1.25\pi x)]\} \quad (6)$$

Figure 3 shows a segment of the characteristic curve of $M(v)$. For a given bias voltage of V_{bias}, the value of K can be evaluated from Eq. (6) along a linear segment PQ shown on the curve in Fig. 3. The value of V_c on the v-axis can then be evaluated by:

$$V_c = v_{bias} - \frac{M(v_{bias})}{K} \quad (7)$$

The empirical expression Eq. (2) is valid for values of x up to 0.8 [24]. More accurate expression for $M(v)$, if any, can be used instead of Eq. (2) without any loss of generality.

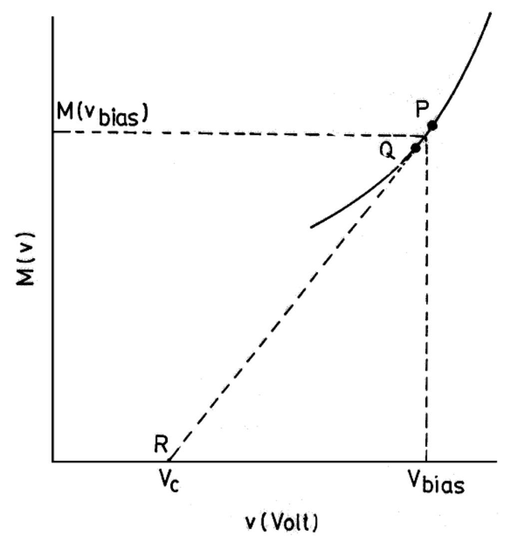

Figure 3. A general characteristics curve of APD for evaluation of K and V_c from the equation of $M(v)$

3. Derivation of Harmonic Distortion in APD

To derive the expression for harmonic distortion in the APD circuit, we will consider the linearized characteristic curve (AB in Fig. 2) over a small range around the operating point of the APD voltage. The expression for the instantaneous *ac* current $i_0(t)$ generated by the APD receiver corresponding to an *ac* input optical power $P_{ac}(t)$ can be written, referring to Eq. (1), as:

$$\begin{aligned} i_o(t) &= K\,S\,P_{ac}(t)(v - V_c) \\ &= K\,S\,P_{ac}(t)[V_x - V_o - i_o(t)R_L - V_c] \end{aligned} \quad (8)$$

where S is the responsivity of the photodiode (with gain equal to unity). After rearranging Eq. (8) we obtain:

$$i_o(t) = \frac{K\,S\,P_{ac}(t)(V_x - V_o - V_c]}{1 + K\,S\,R_L\,P_{ac}(t)} \quad (9)$$

Let us assume a sinewave for the input optical power $P_{ac}(t)$ at an angular frequency ω expressed as:

$$P_{ac}(t) = P_a\cos(\omega t) \quad (10)$$

where P_a is the peak amplitude of the *ac* optical power. The optical power $P_{ac}(t)$ does not contain any *dc* component and is equal to the instantaneous optical power minus the *dc* component of the optical power P_a. Substituting for $P_{ac}(t)$ in Eq. (9), we get:

$$i_o(t) = \frac{K\,S[P_a\cos(\omega t)]\,(V_x - V_o - V_c)}{1 + K\,S\,R_L[P_a\cos(\omega t)]} \quad (11)$$

Or by rearranging, $i_0(t)$ can be alternatively shown as:

$$i_o(t) = \frac{(V_x - V_o - V_c)\cos(\omega t)/R_L}{(1/K\,S\,R_L\,P_a) + \cos(\omega t)} \quad (12)$$

On multiplying current $i_0(t)$ by the load resistance R_L, the instantaneous *ac* output voltage $v_0(t)$ can be obtained as:

$$v_o(t) = \frac{(V_x - V_o - V_c)\cos(\omega t)}{(1/K\,S\,R_L\,P_a) + \cos(\omega t)} \quad (13)$$

$$v_o(t) = \frac{K_1\cos(\omega t)}{K_2 + \cos(\omega t)} \quad (14)$$

Where we have:

$$K_1 = (V_x - V_o - V_c), \qquad K_2 = \frac{1}{K\,S\,R_L\,P_a} \quad (15)$$

3.1. Fourier Series Expansion of the *ac* Output Voltage of the APD

In general, the Fourier transform of the periodic signal $v_0(t)$ (Eq. 14) in interval $-T/2$ to $+T/2$ can be expressed as:

$$v_o(t) = a_o + 2\sum_{n=1}^{\infty}[a_n \cos(n\omega t) + b_n \sin(n\omega t)] \quad (16)$$

Since $v_0(t)$ is an even function, the coefficient $b_n = 0$; then Eq. (16) simplifies to:

$$v_o(t) = a_o + 2\sum_{n=1}^{\infty}[a_n \cos(n\omega t)] \quad (17)$$

where the constant a_0 is the average value of $v_0(t)$ given by:

$$a_0 = (K_1/T)\int_{-T/2}^{+T/2}\left\{1 - \frac{K_2}{K_2 + \cos(\omega t)}\right\}dt \quad (18)$$

and the coefficient a_n is given as:

$$a_n = (K_1/T)\int_{-T/2}^{+T/2}\left\{1 - \frac{K_2}{K_2 + \cos(\omega t)}\right\}\cos(n\omega t)\,dt \quad (19)$$

By solving the integral in Eq. (18), we obtain:

$$a_o = K_1 - \frac{K_1 K_2}{\sqrt{(K_2^2 - 1)}} \quad (20)$$

For $K_2 >> 1$, one obtains $a_0 = 0$.

To determine the value of a_n, we proceed as follows. From Eq. (19), we can write:

$$a_n = (K_1/T)\int_{-T/2}^{+T/2}\cos(n\omega t)\,dt - (K_1/T)\int_{-T/2}^{+T/2}\left\{\frac{K_2\cos(n\omega t)}{K_2 + \cos(n\omega t)}\right\}dt \quad (21)$$

The value of first term in Eq. (21) in the interval $-T/2$ to $+T/2$ is zero, then we have:

$$a_n = -(K_1/T)\int_{-T/2}^{+T/2}\left\{\frac{K_2\cos(n\omega t)}{K_2 + \cos(n\omega t)}\right\}dt \quad (22)$$

$$a_n = -(K_1/T)\int_{-T/2}^{+T/2}\left\{[1 + \cos(\omega t)/K_2]^{-1}\cos(n\omega t)\right\}dt \quad (23)$$

Assuming $K_2 >> 1$, we can expand the term in the bracket in Eq. (23) as:

$$a_n = -(K_1/T)\int_{-T/2}^{+T/2}\left\{1 - \frac{\cos(\omega t)}{K_2} + \frac{[\cos(\omega t)]^2}{K_2^2} - \ldots + (-1)^m\frac{[\cos(\omega t)]^m}{K_2^m}\right\}\cos(n\omega t)\,dt \quad (24)$$

In the given range of $-T/2$ to $+T/2$, the first term in Eq. (24) is zero. Therefore, a_n can be reduced to:

$$a_n = -(K_1/T)\int_{-T/2}^{+T/2}\left\{-\frac{\cos(\omega t)}{K_2^1} + \frac{[\cos(\omega t)]^2}{K_2^2} - \ldots + (-1)^m\frac{[\cos(\omega t)]^m}{K_2^m}\right\}\cos(n\omega t)\,dt \quad (25)$$

Or, in a compact form, we can show Eq. (25) as follows:

$$a_n = -(K_1/T)\int_{-T/2}^{+T/2}\left\{\sum_{m=1}^{\infty}\frac{(-1)^m}{(K_2)^m}[\cos(\omega t)]^m\right\}\cos(n\omega t)\,dt \quad (26)$$

In Eq. (26), with no loss of generality, we can interchange the position of the summation and the integration operators. Then,

$$a_n = -(K_1/T)\sum_{m=1}^{\infty}\left\{\frac{(-1)^m}{(K_2)^m}\int_{-T/2}^{+T/2}[\cos(\omega t)]^m[\cos(n\omega t)]\,dt\right\} \quad (27)$$

Since the factor $1/K_2^m$ tends to zero for high values of m, by assuming $K_2 >> 1$, the expression for a_n can be approximated by limiting the maximum value of m to 5.

The total harmonic distortion (THD), which indicates the degree of nonlinearity of an APD receiver, can be characterized by an analysis of the output spectral components with an input driven by a pure sinewave. Therefore, the THD in dB and percentage (%) can be defined, respectively, as [25-27]:

$$THD = 10\log\left(\frac{\sum_{n=2}^{\infty} P_n}{\sum P_1}\right), \quad THD(\%) = 100\times\left(\frac{\sum_{n=2}^{\infty} P_n}{\sum P_1}\right) \quad (28)$$

where P_n and P_1 are total signal powers and fundamental signal power, respectively.

Table 1 lists the approximate expressions for the different Fourier coefficients in terms of fundamental, second- and third harmonics components. From these expressions, second- $(HD)_2$ and third harmonic distortion $(HD)_3$ in dB, with respect to the fundamental, can be calculated, respectively, as:

$$(HD)_2 \approx 20\log\left\{\frac{1/(4K_2^2)+1/(4K_2^4)}{1/(2K_2)+3/(8K_2^3)+5/(16K_2^5)}\right\} \text{ dB} \quad (29)$$

$$(HD)_3 \approx 20\log\left\{\frac{1/(8K_2^3)+5/(32K_2^5)}{1/(2K_2)+3/(8K_2^3)+5/(16K_2^5)}\right\} \text{ dB} \quad (30)$$

The value of K_2 can be determined from Eq. (15) for a given values of R_L and P_a. The harmonic distortion will be low, if the value of K_2 is high. In other words, the distortion value will be low, when either the APD receiver's load resistance R_L or the peak optical power P_a is small.

Table 1. Fourier components of the output voltage in the APD receiver circuit

Components	Truncated Series Expression, valid for $K_2 >> 1$
Fundamental	$2a_1 \approx 2K_1\left[\frac{1}{2K_2}+\frac{3}{8K_2^3}+\frac{5}{16K_2^5}\right]\approx\frac{K_1}{K_2}$
Second order	$2a_2 \approx -2K_1\left[\frac{1}{4K_2^2}+\frac{1}{4K_2^4}\right]\approx-\frac{K_1}{2K_2^2}$
Third order	$2a_3 \approx 2K_1\left[\frac{1}{8K_2^3}+\frac{5}{32K_2^5}\right]\approx\frac{K_1}{4K_2^3}$

4. Results of the Distortion Analysis

The harmonic distortion values were computed for load resistance of $R_L = 100\ \Omega$ for different peak values of *ac* optical power P_a. The plot is shown in Fig. 4, at $V_{bias} = 149$ V for which $K_2 \approx 11\ \text{V}^{-1}$ and $M \approx 42$. The second HD was about -71 dB (below fundamental) at $P_a = 1\ \mu$W which increased to -31 dB at $P_a = 100\ \mu$W for $R_L = 100\ \Omega$. The corresponding theoretical value of third HD was -142 dB and -62 dB, respectively. The corresponding THD values are -214 dB for $P_a = 1\ \mu$W and -93 dB for $P_a = 100\ \mu$W.

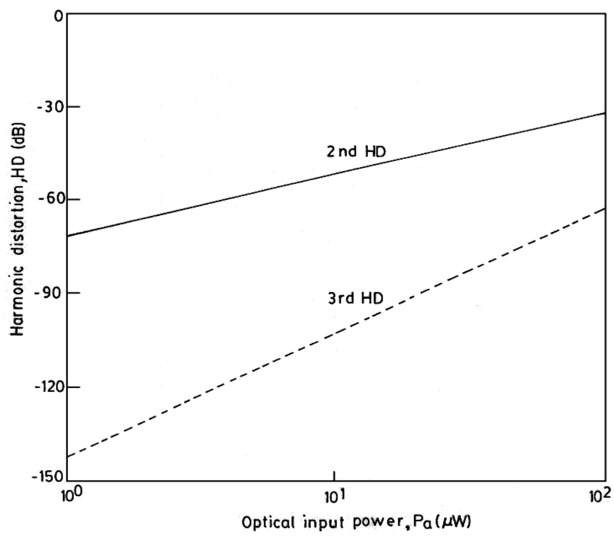

Figure 4. Theoretical harmonic distortion of the APD receiver versus peak optical input power P_a for load resistance $R_L = 100\ \Omega$

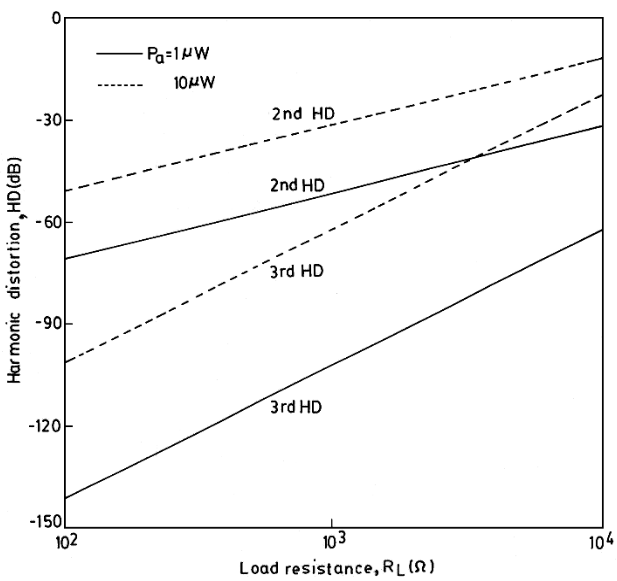

Figure 5. Theoretical harmonic distortion of the APD receiver versus load resistance R_L for peak optical input power $P_a = 1\ \mu$W and $10\ \mu$W

Table 2. Typical values of second- and third harmonic distortion

Load Resistance	$P_a = 1\ \mu$W			$P_a = 10\ \mu$W				
							Ref [28]	
	2nd HD (dB)	3rd HD (dB)	THD (dB)	2nd HD (dB)	3rd HD (dB)	THD (dB)	2nd HD (dB)	3rd HD (dB)
$R_L = 75\ \Omega$	-78	-156	-234	-55	-110	-165	-60	-110
$R_L = 100\ \Omega$	-49	-142	-191	-51	-131	-182	-	-
$R_L = 10\ \text{k}\Omega$	-29	-58	-87	-11	-23	-34	-	-
% Difference in 2nd HD and 3rd HD of the present analysis and Ref [28]							9%	0%

Figure 5 shows the HD as a function of the APD receiver's load resistance for two fixed power levels of $P_a = 1\ \mu$W and $P_a = 10\ \mu$W. At $P_a = 10\ \mu$W, the second HD increases from a value of about -51 dB for $R_L = 100\ \Omega$ to a value of about -11 dB for $R_L = 10\ \text{k}\Omega$.

In Table 2, typical values of second- and third harmonic distortions are listed for two optical powers of $1\ \mu$W and $10\ \mu$W. The results indicate that when the load resistance is lower, the THD is also lower for a specified received optical power.

The theoretical curves of Figs. 4 and 5 are valid for the typical APD circuit shown in Fig. 1, for a case when $M \approx 42$ and $K_2 \approx 11\ \text{V}^{-1}$. For low load resistance, when received power has increased 10 times, the THD only increased to 9 dB, i.e. 4.7% degradation, whereas for high load resistance, the same power increase would cause an increase of 53 dB in the THD, which corresponds to 61% degradation of the THD. That is to say for a high load resistance, a 10 order of magnitude increase in received power, would result in 13 order of magnitude degradation in the THD.

For a general APD characteristic $M(v)$ given by Eq. (2), the value of $M(v)$ and K_2 at different APD bias voltages can be theoretically evaluated, and from Table 1, the harmonic distortion values can be accordingly calculated.

For sake of comparison, to the best of our knowledge a newer than the Ref. [28] was not traced to have reported an experimental replica of the present analysis. The then experimental results showed that for an optical input power of 10 μW, the second order (HD) and third order harmonic distortions were found to be -60 dB and -118 dB, respectively. The biasing voltage was 155 V, the gain was 76, and the load resistance was 75 Ω.

In the present analysis, for a gain of 42, biasing voltage of 145 V, and input power of 10 μW, the second order HD and third order HD are found to be -55 dB and -110 dB, respectively, for the same load resistance at 75 Ω. In Table 2, it shown that the percentage differences between 2nd HDs of the present analysis and Ref [28] is 9% and for 3rd HDs is nil.

5. Conclusions

In this paper, a simple model is proposed to compute the total harmonic distortion in an APD receiver circuit. The model is explained, by using the gain characteristic curve of an experimental circuit. For making the approach more general, a procedure is given so that the whole analysis could be based on an analytical equation of the gain $M(v)$. Direct expressions, for the second and third harmonic distortions in dB (with reference to the fundamental), are derived.

It is shown that when input power increases, the 2nd HD and 3rd HD would increase, and at a given input power, the 2nd HD is higher than 3rd HD. Typically, for the given circuit, the THD was found to be -191 dB, for $R_L = 100\ \Omega$ and $P_a = 1\ \mu$W. If a high impedance preamplifier is used with $R_L = 10\ \text{k}\Omega$, the THD will become worse to a value of -34 dB for $P_a = 10\ \mu$W. The results of the analysis indicates that for low THD, the load resistance and the received optical power at the output end should be at low level. In a given comparison shows that the obtained results almost tally with a reported experimental results.

The analysis presented here will be useful for analog optical fiber transmission systems and optical sensor applications.

REFERENCES

[1] H. Melchior, M.B. Fisher, F.R. Arama, "Photodetectors for Optical Communication Systems", IEEE Proc., Vol. 58, No. 10, pp. 1466-1486, Oct 1970.

[2] D. H. Newman and S. Ritchie, "Sources and Detectors for Optical Fibre Communications Applications: The First 20 Years", IEE Proc., Vol. 133, Pt. J, No. 3, June 1986.

[3] Meredith Nicole Draa, *High power high linearity waveguide photodiodes: measurement, modeling, and characterization for analog optical links,* PhD Dissertation, University of California, San Diego, 2010.

[4] R. R. Hayesand, D. L. Persechini, "Nonlinearity of p-i-n photodetectors", IEEE Photon. Technol. Lett., 5, No. 1, pp.

70–72, 1993.

[5] K. J. Williams, L. T. Nichols, and R. D. Esman, “Photodetector nonlinearity limitations on a high-dynamic range 3 ghz fiber optic link,” J. Lightwave Technol., Vol. 16, No. 2, pp. 192–199, 1998.

[6] Hsu-Feng Chou, Anand Ramaswamy, Darko Zibar, Leif A. Johansson, John E. Bowers, Mark Rodwell, and Larry A. Coldren, “Highly Linear Coherent Receiver With Feedback”, IEEE Photon. Technol. Lett., Vol. 19, No. 12, pp. 940-942, 2007.

[7] Chun-Ting Lin, Po-Tsung Shih, Jason (Jyehong) Chen, Wen-Qiang Xue, Peng-Chun Peng, and Sien Chi, “Optical Millimeter-Wave Signal Generation Using Frequency Quadrupling Technique and No Optical Filtering”, IEEE Photon. Technol. Lett., Vol. 20, No. 12, pp. 1027-1029, 2008.

[8] H. Jiang, D. S. Shin, G. L. Li, T. A. Vang, D. C. Scott, and P. K. L. Yu, “The Frequency Behavior of the Third-Order Intercept Point in a Waveguide Photodiode”, ”, IEEE Photon. Technol. Lett., Vol. 12, No. 5, pp. 540-542, 2000.

[9] Guo-qing Wang, Tuan-wei Xu, and Fang Li, “PGC Demodulation Technique With High Stability and Low Harmonic Distortion”, IEEE Photon. Technol. Lett., Vol. 24, No. 23, pp. 2093-2096, 2012.

[10] Josep Prat, María C. Santos, and Mireia Omella, “Square Root Module to Combat Dispersion-Induced Nonlinear Distortion in Radio-Over-Fiber Systems”, IEEE Photon. Technol. Lett., Vol. 18, No. 18, pp. 1928-1930, 2006.

[11] K. J. Williams, R. D. Esman, and M. Dagenais, “Effects of high space-charge fields on the response of microwave photodetectors,” IEEE Photon. Technol. Lett., Vol. 6, No. 5, pp. 639–641, 1994.

[12] K. J. Williams, R. D. Esman, and M. Dagenais, “Nonlinearities in PIN microwave photodetectors,” J. Lightwave Technol., Vol. 14, No. 1, pp. 84–96, 1996.

[13] V. J. Urick, F. Bucholtz, J. D. McKinney, P. S. Devgan, A. L. Campillo, J. L. Dexter, and K. J. Williams, “Long-haul analog photonics, ”IEEE J. Lightwave Technol., Vol. 29, No. 8, pp. 1182-1205, 2011.

[14] A. S. Hastings, D. A. Tulchinsky, and K. J. Williams, “Photodetector nonlinearities due to voltage-dependent responsivity”, IEEE Photon. Technol. Lett., Vol. 21, No. 21, pp. 1642–1644, 2009.

[15] J. D. McKinney, D. E. Leaird, A. S. Hastings, A. M. Weiner, and K. J. Williams, "Optical Comb Source and High-Resolution Optical Filtering for Measurement of Photodiode Harmonic Distortion, "J. Lightwave. Technol., Vol. 28, No. 8, pp. 1228-1235, 2010.

[16] A. S. Hastings, V. Urick, C. Sunderman, J. Diehl, J. McKinney, D. Tulchinsky, P. Devgan, and K. Williams, “Suppression of even-order photodiode nonlinearities in multioctave photonic links”, J. Lightwave Technol., Vol. 26, No. 15, pp. 2557–2562, 2008.

[17] Preetpaul S. Devgan, Alexander S. Hastings, Vincent J. Urick, and Keith J. Williams, “Cancellation of photodiode-induced second harmonic distortion using single side band modulation from a dual parallel Mach-Zehnder”, Opt. Express, Vol. 20, No. 24, pp. 27163-27173, 2012.

[18] Meredith N. Draa, Alexander S. Hastings, and Keith J. Williams, “Comparison of photodiode nonlinearity measurement systems”, Opt. Express, Vol. 19, No. 13, pp. 12635-12645, 2011.

[19] Vincent J. Urick, Meredith N. Hutchinson, Joseph M. Singley, Jason D. McKinney, and Keith J. Williams, “Suppression of even-order photodiode distortions via predistortion linearization with a bias-shifted Mach-Zehnder modulator”, Opt. Express, Vol. 21, No. 12, pp. 14368-14376, 2013.

[20] M. López, J. C. Molina, H. Hofer, A. Sperling, and S. Kück, “Measurement of the Nonlinearity of Ge-and InGaAs-Photodiodes at High Irradiance Levels”, *MĀPAN* - J Metrology Soc. India, Vol. 25, No. 1, pp. 47-52, 2010.

[21] Molly Piels, Anand Ramaswamy, and John E. Bowers, “Nonlinear modeling of waveguide photodetectors” Opt. Express, Vol. 21, No. 13, pp. 15634-15644, 2013.

[22] H. Jiang and P. K. L. Yu, “Equivalent Circuit Analysis of Harmonic Distortions in Photodiode”, IEEE Photon. Technol. Lett., Vol. 10, No. 11, pp. 1608-1610, 1998.

[23] T. Ozeki and E. H. Hara, “Measurement of Non-Linear Distortion of Photodiode", Electron. Lett., Vol. 12, No.3, pp. 80-81, 1976.

[24] M.T. Abuelma'atti, "Theory of Avalanche Diode Harmonic Optoelectronic Mixer", IEE Proc., Vol. 135, Pt. J, No. 2, 1988.

[25] Eduard Sackinger, *Broadband Circuits* for *Optical Fiber Communication*, John Wiley & Sons, Inc., Hoboken, New Jersey, 2005.

[26] Mohammad Azadeh, *Fiber Optics Engineering*, Springer Science+Business Media, LLC, New York, USA, 2009.

[27] Avigdor Brillant, *Digital and analog fiber optic communications for CATV and FTTx applications,* SPIE Press, Washington, USA, 2008.

[28] T. Ozeki and E. H. Hara, “Measurement of Nonlinear Distortion in Photodiode”, Electron. Lett., Vol. 12, No. 3, pp. 80-81, 1976.