

# A Grating Optical Filter with Angular Selectivity of the Directional Light Transmission

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**Abstract** An optical filter with thin-film grating layers on both surfaces of the transparent sheet substrate is presented. The gratings are formed by absorptive, reflective, or scattering strips, alternating with directionally transmissive strips. Their relative position on input and output surfaces provides angular selectivity of the light transmission – part of the radiation that has passed through the input gratings is blocked additionally by the output gratings depending on incidence angle. A graphic-analytical calculation method determines the influence of geometric parameters of plane-parallel and coaxial-cylindrical filters on angular characteristics of transmission. At pre-known trajectory of light source relative to the filter pre-adapted angular selective regulation of its light transmission is provided. Architectural glazing to control the transmitted solar radiation without special redirecting devices is the most promising area of application.

**Keywords** Optical filter, Gratings with alternating strips, Graphic-analytical calculation method, Angular selective regulation, Directional light transmission

## 1. Introduction

Classical thin-film multilayer filters are optically homogeneous in each layer of the surface coating. Inhomogeneous coatings with thin microporous scattering films[1], metal films with square holes[2], diffraction gratings[3] and photonic crystals[4] have been used in optical filters. All these surface and bulk inhomogeneities are on the micro- and nanoscales, consequently, the phenomena occurring in them belong to the subject matter of physical optics. Research has been carried out in order to improve the characteristics of filters designed to increase the angular tolerance[5] and, conversely, to apply the angular dependence of the transmission spectrum[6]. A single nanoslit in a metal film flanked by dielectric gratings has been proposed[7]. New possibilities for realization of narrow-band transmission filters have been introduced[8]. Some filters have adaptive properties[9, 10].

In addition to the known matrix and recursive methods for the calculation of the reflection and transmission coefficients, a modular concept has been described[11]. Directional transmission from a single subwavelength slit in a metal film with periodic dielectric bars was analysed numerically[12]. Properties of the fenestration systems with Venetian blinds have been described by a bi-directional transmission distribution function[13]. Regulation of the directional light

transmission of fenestration systems requires the use of additional devices. Smart glass of different types with thin-film coatings[14] and photochromic[15], electrochromic[16] and liquid crystal[17] layers have been used to change the spectral range and intensity of solar radiation. Polarization of the light has been used for invisibility achievement through a window at certain angles of observation[18]. Among all variety there are no optical filters transmitting only the demanded and preliminarily calculated part of incident radiation in different ranges of incidence angles. In this paper, a multilayer grating filter for angular selective regulation of directional light transmission without additional devices is presented. The filters of plane-parallel and coaxial-cylindrical forms are considered. The paper contains:

1. Introduction
2. Design of a plane-parallel filter
3. A graphic-analytical calculation method
4. Dependence of the difference of offsets of refracted beams on the characteristic incidence angle
5. Features of the calculation of a coaxial-cylindrical filter
6. Potential applications of the filter
7. Conclusions

## 2. Design of a Plane-parallel Filter

On both surfaces of a glass substrate the alternating transmissive and absorptive strips of millimetre and submillimetre widths are formed. Absorptive strips can be replaced with reflective or scattering stripes. The gratings formed by alternating strips are fundamentally different from diffraction gratings in size and purpose. Due to the

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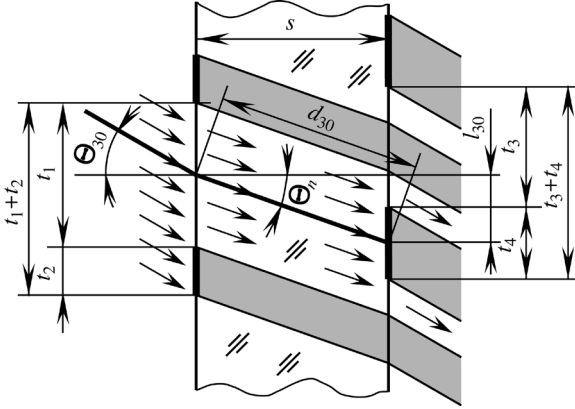
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dimensions of inhomogeneities on the surfaces, in contrast to [1–4], optical properties of the filter can be considered exclusively within geometrical optics. Accordingly, a method of calculation of its characteristics is simplified in comparison with the existing methods [11–13].

Figure 1 presents the schematic diagram of a grating filter. On surfaces of a plane-parallel sheet substrate of the thickness  $s$  the transmissive (widths  $t_1$  and  $t_3$ ) and the absorptive ( $t_2$  and  $t_4$ ) parallel strips alternate. Like the period of diffraction gratings, the gratings are characterized by the step (repetition period) of the strips, that is, the total width of the two adjacent strips. The steps of the strips can be either equal (in Figure 1:  $t_1+t_2=t_3+t_4$ ), or one period can be an integer multiply of the other (the multiplicity of steps is  $(t_1+t_2)/(t_3+t_4)$ ). Shift of output gratings of the filter relative to input gratings is characterized by incidence angle of the beam passing through centres of the alternating strips. Characteristic angle  $\Theta_{30}$  is specified in Figure 1 – at incidence angle of  $30^\circ$  the beam passes through the centres of the transmissive strip on the input gratings and the absorptive strip on the output gratings after refraction at the angle  $\Theta_n$ . Offset  $l_{30}$  of the refracted beam on the output surface with respect to the non-refracted beam at normal incidence angle and the length  $d_{30}$  of the path of the refracted beam through the filter glass are indicated.



**Figure 1.** The schematic diagram of light transmission of a grating filter

A part of incident radiation is blocked by the input gratings irrespective of the incidence angle. The output gratings additionally block a part of the radiation which has reached them already depending on the incidence angle when moving band of this radiation relative to non-motile strips on the output gratings (Figure 1). The overall coefficient of light transmission is equal to the ratio of the part of the area of the transmissive strips on the output gratings through which the refracted beams have passed at a given incidence angle, to the total area of the input gratings. This ratio completely depends on the preliminary selection of the geometrical parameters of alternating strips and of characteristic angle. Therefore at a known trajectory of a light source concerning the filter it is possible to calculate in advance its geometrical parameters for providing required values of the coefficient of light transmission at any incidence angles and angular ranges within the limits of

$0^\circ$ – $90^\circ$  [19]. Thus, angular selective regulation of light transmission is pre-adapted to the motion of the light source. Angular selective filtering of the intensity of radiation by this filter distinguishes it from other types of developed optical filters [5–10].

### 3. A graphic-analytical Calculation Method

The coefficient of light transmission is defined by a graphic-analytical calculation method [20]. In calculating the coefficient of light transmission  $\tau$  of the plane-parallel filter for a given incidence angle  $\Theta$  the ratio of the areas considered in Section 2 can be replaced by the ratio of the widths of the strips:

$$\tau = h / (t_1 + t_2), \quad (1)$$

where  $h$  is the width of a part of transmissive strips on the output surface, through which the refracted beams have passed directionally within the limits of a single step of the strips (the width of light transmission). It is defined through an offset function [20] – angular dependence of offset  $l$  of the refracted beam on the output surface with respect to the non-refracted beam at normal incidence angle. This dependence is obtained from a rectangular triangle with catheti  $s$  and  $l$  ( $l_{30}$  in Figure 1) taking into account Snell's law:

$$l = s \sin \Theta / \sqrt{n^2 - \sin^2 \Theta}. \quad (2)$$

We analyze the filter shown in Figure 1 with the geometric parameters:  $s=4$  mm,  $t_1=3$  mm,  $t_2=1$  mm,  $t_3=2.5$  mm,  $t_4=1.5$  mm, characteristic angle  $\Theta_{30}=30^\circ$ , refractive index  $n=1.5$ . The change in the width of light transmission was specified for the four angular ranges with identical characteristics. In the range of incidence angles from  $0^\circ$  to  $14.22^\circ$  the width of light transmission is reduced by equation:

$$h = 0.5t_1 - 0.5t_4 + l_c - l, \quad (3)$$

where  $l_c$  is the offset of refracted beam at the characteristic angle.

An incidence angle  $14.22^\circ$  is the extreme angle. The offset of the refracted beam for this angle is calculated from:

$$l = -0.5t_1 + 0.5t_4 + l_c \quad (4)$$

Value of an incidence angle at the known offset is expressed from the equation (2):

$$\Theta = \arcsin(nl / \sqrt{s^2 + l^2}). \quad (5)$$

By substituting the calculated under the equation (4) value of the offset into this equation the precisely value  $14.22^\circ$  of the extreme angle is determined. With a further increase in the incidence angle the width of light transmission is unchanged and is equal to:

$$h = t_1 - t_4. \quad (6)$$

The width of light transmission is unchanged to following extreme angle of  $45.55^\circ$ , the value of which is defined under the equation (5) through the offset:

$$l = 0.5t_1 - 0.5t_4 + l_c \quad (7)$$

Further the width of light transmission increases in accordance with:

$$h = 0.5t_1 - 0.5t_4 - l_c + l \quad (8)$$

From extreme angle  $68.53^\circ$  defined under the condition:

$$l = -0.5t_1 + t_3 + 0.5t_4 + l_c \quad (9)$$

up to  $90^\circ$  the width of light transmission is again unchanged and is equal to:

$$h = t_3 \quad (10)$$

The angular characteristic of this filter is presented in Figure 2. It is a broken line, but in [20] showed that the slope areas are curvilinear and had the inflection point as a result of a sinusoidal dependence by equation (2).

This graphic-analytical method of calculation allows only a geometric approach to define the light transmission. It is necessary to correct the results of calculation taking into account angular dependence of the reflection under Fresnel formulas and the absorption by the filter glass under Bouguer-Lambert law.

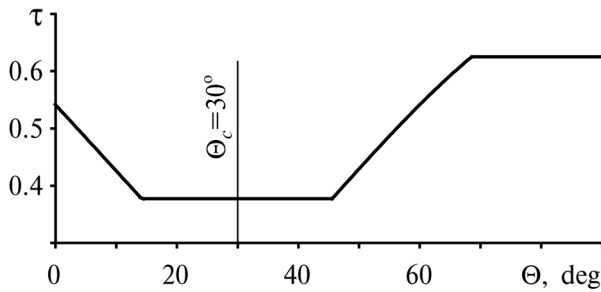


Figure 2. The angular characteristic of the plane-parallel filter

Under Fresnel equations reflection coefficients of the waves polarized perpendicularly and collaterally of a plane of incidence are  $\rho_s = \sin^2(\Theta - \Theta_n) / \sin^2(\Theta + \Theta_n)$  and  $\rho_p = \tan^2(\Theta - \Theta_n) / \tan^2(\Theta + \Theta_n)$ , respectively. At normal incidence the reflection coefficient is equal to  $\rho_0 = (n-1)^2 / (n+1)^2$ , i.e.  $\rho_0 = 0.04$  at  $n = 1.5$ . For linearly polarized light with an azimuth  $\delta$  of oscillations of incident wave the components of a vector of electric field are equal to  $E_p = E \cos \delta$  and  $E_s = E \sin \delta$ . At the azimuth  $\delta = 45^\circ$  will be  $E_p = E_s$ , hence, intensities of the perpendicularly and collaterally polarized components of the incident wave also will be equal. Then the total reflection coefficient is calculated also, as for natural (not polarized) light:  $\rho = 0.5(\rho_s + \rho_p)$ . Taking into account reflection from the input and output surfaces and absorption under Bouguer-Lambert law, transmission coefficient is equal to [21]:

$$\tau = (1 - \rho)^2 \exp(-\alpha d) \quad (11)$$

where  $\alpha$  is a natural absorption coefficient of the glass,  $d$  is a path of the refracted beam through the glass. This path is calculated under the formula obtained from a rectangular triangle with catheti  $s$  and  $l$  ( $l_{30}$  in Figure 1) taking into account the equation (2):

$$d = s \sqrt{1 + \sin^2 \Theta / (n^2 - \sin^2 \Theta)} \quad (12)$$

From the equations (11) and (12) an equation for calculation of the transmission coefficient taking into account reflection and absorption is obtained for the plane-parallel filters [21]:

$$\tau = (1 - \rho)^2 \exp(-\alpha s \sqrt{1 + \sin^2 \Theta / (n^2 - \sin^2 \Theta)}) \quad (13)$$

Corrected by this equation results of the graphic-analytical calculation of 7 filters with different geometric parameters have good consistency with the experimental data [21]. At calculation of the filters with reflective and scattering strips the increase of reflected radiation and the appearance of passing scattered radiation are considered.

#### 4. Dependence of the Difference of Offsets of Refracted Beams on the Characteristic Incidence Angle

Equations (3) and (8) to calculate the areas of decreasing and increasing the width of light transmission only differ in signs in front of the difference of offsets of the refracted beams ( $l_c - l$  and  $l - l_c$ ). Consequently, the value of light transmission coefficient is determined by the difference between the offset at the given and the characteristic incidence angles. Figure 3 shows the dependence of this difference on the incidence angle for the characteristic angles of the filter through every 10 degrees with decreasing and increasing the width of light transmission. The differences of the offsets are shown in units relating to the glass thickness. Refractive index is  $n = 1.5$ .

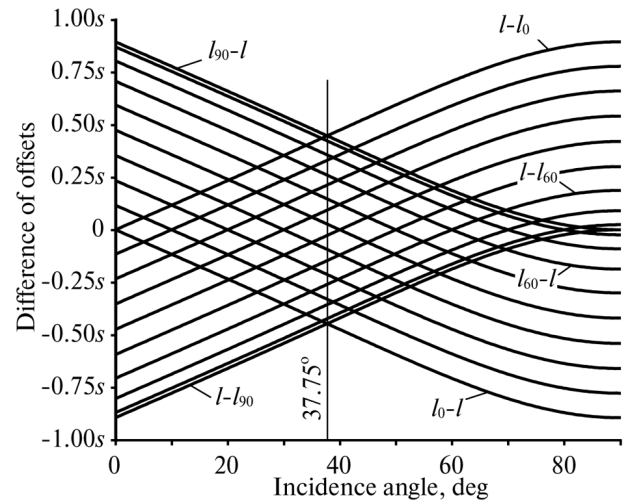


Figure 3. The dependence of difference of the offsets of refracted beams on the incidence angle at different characteristic angles of the filter

The curves are equidistant since for each filter  $l_c = \text{const}$ . Up to the incidence angles approximately  $60^\circ$  the curves are practically linear because of sinusoidal dependence of the offset function on the incidence angle by equation (2) – in this range the sines of angles are changed practically proportionally to these angles. The offset function has an inflection point, the position of which depends on the refractive index of glass and is determined by the second

derivative of the offset function[20]:

$$\frac{d^2 l}{d\Theta^2} = \frac{sn^2 \sin \Theta (-n^2 + \cos(2\Theta) + 2)}{(n^2 - \sin^2 \Theta)^{5/2}}. \quad (12)$$

For  $n=1.5$  the inflection points are at an incidence angle  $37.75^\circ$ . At the incidence angles greater than  $60^\circ$  the sinusoidal dependence are weakened and the curves are become gently sloping. Comparing the curves for different characteristic angles shows that up to the angles of  $60^\circ$  the differences of offsets are almost identical through every  $10^\circ$  – it is also due to the sinusoidal dependence of the offset function. In addition, at large values of characteristic angles the summands of respective offsets become more significant than the summands of offsets at given angles and the differences of offsets become varying slowly (adjacent curves are closer to each other, especially at the characteristic angles of  $80^\circ$  and  $90^\circ$ ).

Figure 3 shows that the difference of offsets is always less than the thickness of the filter glass. The offset value would be equal to the thickness of the glass at a refraction angle of  $45^\circ$  (Figure 1), but even at incidence angle of  $90^\circ$  the refraction angle is only  $41.81^\circ$ .

## 5. Features of the Calculation of a Coaxial-cylindrical Filter

Graphic-analytical method for calculating the characteristics of light transmission is applicable not only for filters with the simplest plane-parallel form. Input and output surfaces of the filter may be non-parallel planar or curvilinear. At a complex movement of a light source and a filter relative to each other (for example, path of the sun relative to the window), the plane of incidence is changed continuously, in contrast to Figure 1. The schemes with non-parallel incident beams are possible in optical systems and lighting devices. In Figure 4 the divergent beams, incident on the filter with curved surfaces are shown. One mutual position of the light source and the filter is selected as the characteristic position. The sizes and configuration of alternating strips of the input gratings are selected in advance. Then traces of these strips on the output surface, formed by refracted beams are defined. Alternating strips of the output gratings (their boundaries shown in Figure 4 by dotted lines) are arranged symmetrically with respect to these traces under the characteristic position of the light source and the filter. To calculate the directional light transmission by equation (1) at different positions of the light source and the filter the corresponding areas are determined.

Figure 5 shows the scheme of calculating filter with coaxial-cylindrical surfaces of radii  $R_1$  and  $R_2$  (filter glass thickness  $s=R_2-R_1$ ). The beams 1 are falling on the input surface at the characteristic angle  $\Theta_c$ , measured from the horizontal. In contrast to the plane-parallel filter, at alternating strips of equal width (the length of the arcs in Figure 5) on the input gratings the widths of strips on the output gratings are different. In addition, the values of the

angles of incidence  $\Theta$  and refraction  $\Theta_n$ , the offsets  $l$  and lengths  $d$  of paths of the refracted beams are increasing also. The filter with transmissive strips of the output gratings coinciding with traces of transmissive strips of the input gratings, has a maximum transmission at characteristic angle. The Figure 5 shows that at incidence of the beams 2 under different angle the light transmission is reduced.

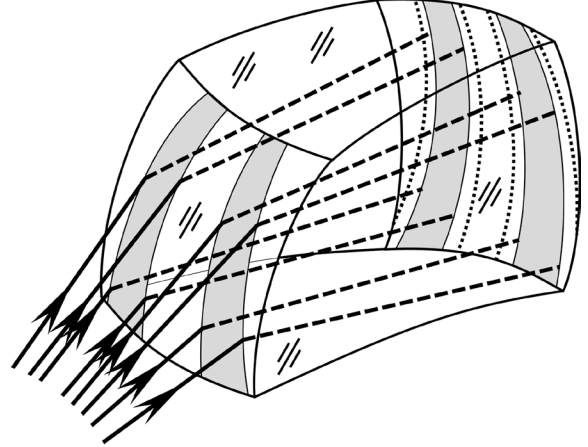


Figure 4. The scheme of determining traces of alternating strips on the output surface of the filter

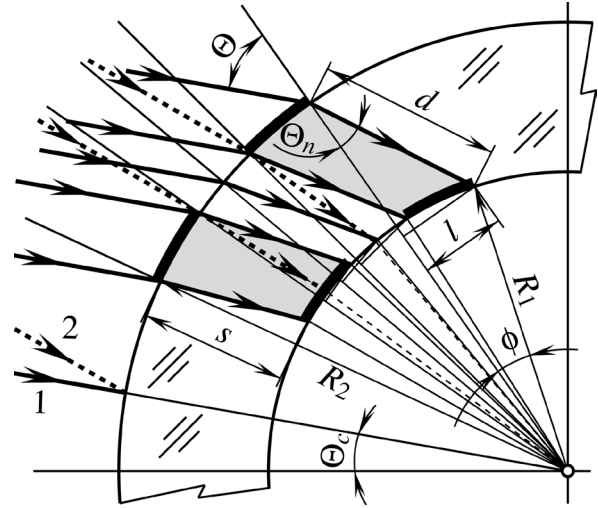


Figure 5. The scheme for calculating a coaxial-cylindrical filter:  $\phi$  – central angle,  $R_2$  and  $R_1$  – radii of input and output cylindrical surfaces

When analyzing Figure 5, an equation for calculating the length of the path of the refracted beam through the cylindrical filter glass is obtained:

$$d = 0.5 \left( 2R_2 \cos \left( \arcsin \left( \frac{\sin \Theta}{n} \right) \right) - \sqrt{\left( 2R_2 \cos \left( \arcsin \left( \frac{\sin \Theta}{n} \right) \right) \right)^2 - 4(R_2^2 - R_1^2)} \right) \quad (13)$$

The offset of refracted beam is given by:

$$l = \frac{d \sin \Theta}{n}. \quad (14)$$

When calculating the light transmission by equation (1),

the ratio of the areas can be replaced by the ratio of the corresponding lengths of the arcs of the input and output surfaces. The arc length  $L$  is calculated by the known equation:

$$L = \phi R, \quad (15)$$

where the central angle  $\phi$  is expressed in radians,  $R$  is the radius of the circle ( $R_2$  and  $R_1$  at the input and output gratings).

Equations (13)–(15) are also valid for calculating parameters of the filter with concentric spherical surfaces (in the planes of incidence).

## 6. Potential Applications of the Filter

Application of the considered method presents opportunities for the creation of a novel family of filters for many different purposes. In particular, such a filter would be useful for architectural glazing to control the light transmitted into the room in a way that depends on the incidence angle without the use of additional redirecting devices. The alternating strips adapted to the trajectory of the sun relative to the specified window (for example, in the hottest season of the year) may be used on the surfaces of window glazing. Compared with sun-blinds and other devices, the filter does not require manual or automatic control, is easy to use in windows with bent and inclined surfaces, and creates the possibility of dividing the window area into zones with different characteristics of light transmission.

Except the calculation of protection from the sun at known latitude of district for a given window with a known azimuth and parameters of surrounding buildings, the complex calculation for providing invisibility from the windows of opposing buildings, and also reduction of thermal losses with long-wavelength radiance from the heating devices is possible. For such bilateral regulation of light and heat transmission it is possible to apply more than two gratings with different optical and geometrical parameters.

When necessary the angular selective regulation of light transmission, the use of the filter in other areas such as vehicles, production of optical systems, lighting equipment and eyeglasses is possible.

## 7. Conclusions

In this paper, a novel grating optical filter for the angular selective regulation of light transmission and a graphic-analytical method for calculating its parameters are presented. The regularities of calculation for the plane-parallel and cylindrical-coaxial filters are obtained that need to be taken into account to achieve the desired angular characteristics of regulation.

Preliminary selection of the geometric parameters of alternating strips of filter gratings allows to pre-adapt the regulation of light transmission at the pre-known trajectory

of the light source. This approach opens up possibilities for a novel family of the filters with wide applicability in various fields, particularly in architectural glazing.

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