

Really Short Note on Examples of S-convex Functions

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Abstract In this note, we present a few more important scientific remarks regarding the S -convexity phenomenon. This time, we talk about examples. That was one of the first queries Professor Mark Nelson had for us at the ANZIAM meeting that happened this year, in 2017, at the Wollongong University, Information Sciences building. We here talk about a very trivial example. Yet this example will prove a few really old results to be equivocated.

Keywords Noindent Analysis, S -convexity, Example

1. Introduction

We have just presented the new definition for the phenomenon S -convexity at the ANZIAM meeting. Our talk was named *Time to Think of S-convexity* [1]. The new definition is:

Definition 1. A function $f : X \rightarrow \mathfrak{R}$, where $|f(x)| = f(x)$, is told to belong to K_s^2 if the inequality

$$f(\lambda x + (1-\lambda)(x+\delta)) \leq \lambda^s f(x) + (1-\lambda)^s f(x+\delta)$$

holds $\forall \lambda / \lambda \in [0,1]; \forall x / x \in X; s = s_2 / 0 < s_2 \leq 1; X / X \subseteq \mathfrak{R}_+ \wedge X = [a,b]; \forall \delta / 0 < \delta \leq (b-x)$.

Definition 2. A function $f : X \rightarrow \mathfrak{R}$, where $|f(x)| = -f(x)$, is told to belong to K_s^2 if the inequality

$$f(\lambda x + (1-\lambda)(x+\delta)) \leq \lambda^{\frac{1}{s}} f(x) + (1-\lambda)^{\frac{1}{s}} f(x+\delta)$$

holds $\forall \lambda / \lambda \in [0,1]; \forall x / x \in X; s = s_2 / 0 < s_2 \leq 1; X / X \subseteq \mathfrak{R}_+ \wedge X = [a,b]; \forall \delta / 0 < \delta \leq (b-x)$.

Remark 1. If the inequality is obeyed in the supplementary¹ situation by f , then f is said to be S_2 -concave.

This definition includes the essence of s_2 -convexity as presented by Hudzik and Maligranda [2].

In this really short note, we will be proving that $f(x) = \sqrt{x}$ belongs to K_s^2 as we now define it, and, with this, we will be proving that some of the claims from [3] are unfounded.

2. Development

Assume that it is not the case that $f(x) = \sqrt{x}$ is s_2 -convex. That means that it is possible to find at least one value for

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¹ Supplementary here means '>', not '≥'.

a or b or even for δ or x such that it is not true that $(ax + (1-a)(x+\delta))^{0.5} \leq a^s x^{0.5} + (1-a)^s (x+\delta)^{0.5}$. In this case, we know that (A) $(ax + (1-a)(x+\delta))^{0.5} = a^s x^{0.5} + (1-a)^s (x+\delta)^{0.5} + \Delta$, $\Delta > 0$.

a can always assume the value 0. If we replace a with 0 in (A), however, we get that $(x+\delta)^{0.5} = (x+\delta)^{0.5} + \Delta$. The last equation implies that $\Delta = 0$, but this is a contradiction with our assumptions and therefore proves that our assumptions are equivocated. That leads us to the desired result, since the mistake was assuming that $f(x) = \sqrt{x}$ was not part of K_s^2 .

The extract we see below this line came from [3].

Let $s \in (0, 1)$ and $a, b, c \in \mathbb{R}$. We define the function $f: [0, \infty) \rightarrow \mathbb{R}$ as

$$f(t) = \begin{cases} a, & t = 0, \\ bt^s + c, & t > 0. \end{cases}$$

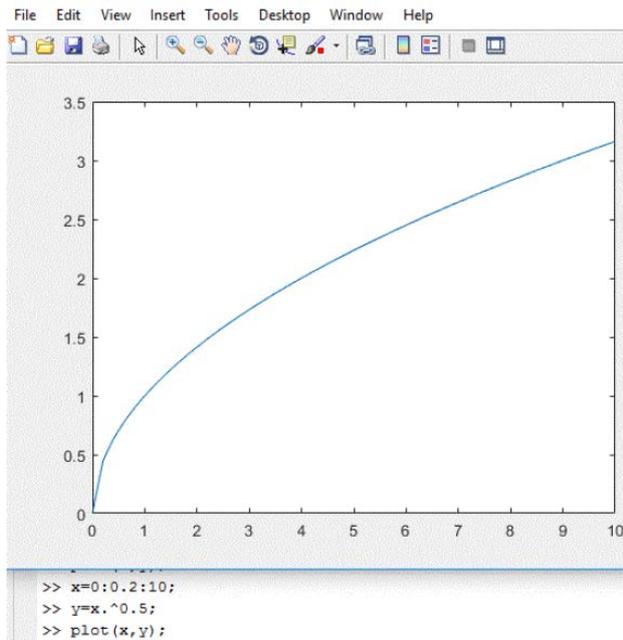
It can be easily checked that

(i) if $b \geq 0$ and $0 \leq c \leq a$, then $f \in K_s^2$,

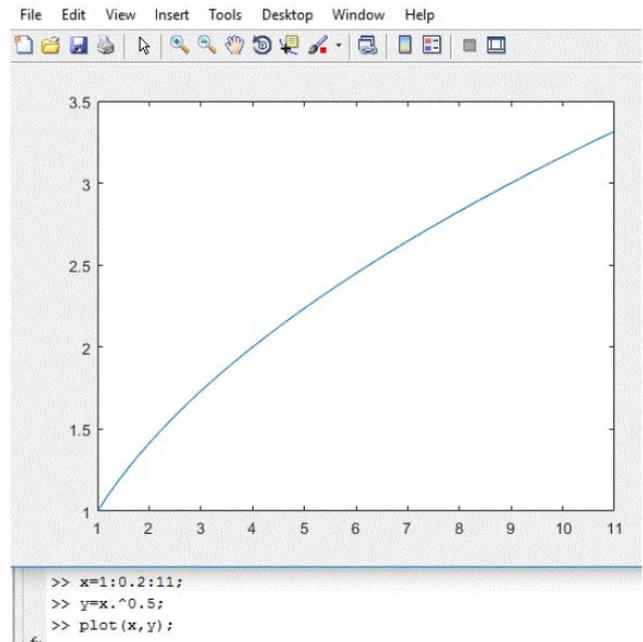
(ii) if $b > 0$ and $c < 0$, then $f \notin K_s^2$

Figure 1. Hudzik and Maligranda's theorem to generate examples

We can tell, from the graphs below, that moving one unit down won't make the graph escape S_2 -convexity: It is just a matter of saying that x is now greater than 1. In this case, $g(x) = x^{0.5} - 1$ does belong to K_s^2 , but, according to the Figure 1, that could not be true ($b > 0$, since $b = 1$, and $c < 0$, since $c = -1$).



MATLAB, $g(x) = x^{0.5}$



MATLAB, $g(x) = x^{0.5} - 1$

One could allege that [3] talks about the domain starting from zero, and, in our counter-example, it would start from one. In this case, consider $j(x) = \sqrt{x+2}$.

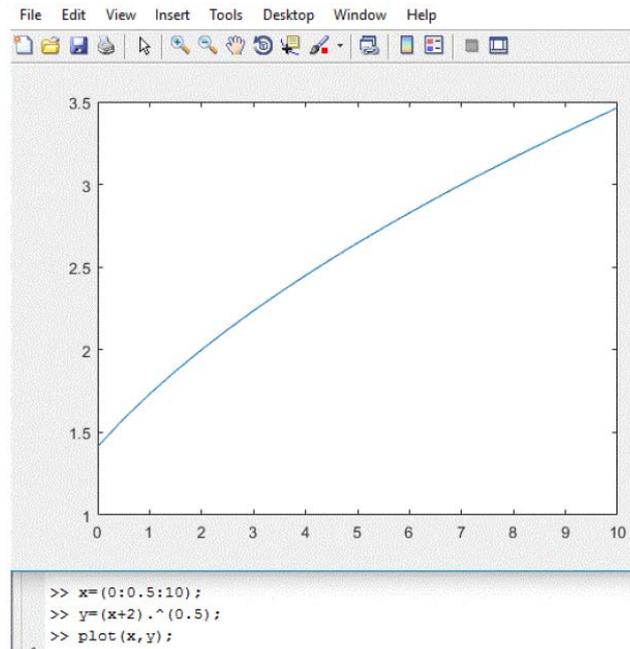


Figure 4. $(x+2)^{0.5}$

The proof that it is S_2 -convex is analogous. See: Assume that it is not the case that $j(x) = \sqrt{x+2}$ is S_2 -convex. That means that it is possible to find at least one value for a or b or even for δ or x such that it is not true that $((a(x) + (1-a)(x+\delta)) + 2)^{0.5} \leq a^s(x+2)^{0.5} + (1-a)^s(x+\delta+2)^{0.5}$. In this case, we know that (A) $((a(x) + (1-a)(x+\delta)) + 2)^{0.5} = a^s(x+2)^{0.5} + (1-a)^s(x+\delta+2)^{0.5} + \Delta$, $\Delta > 0$. a can always assume the value 0. If we replace a with 0 in (A), however, we get that $(x+\delta+2)^{0.5} = (x+\delta+2)^{0.5} + \Delta$. The last equation implies that $\Delta = 0$, but this is a contradiction with our assumptions, and therefore proves that our assumptions are equivocated. That leads us to the desired result, since the mistake was assuming that $j(x) = \sqrt{x+2}$ was not part of K_s^2 .

3. Conclusions

In this paper, we have provided a very trivial example of S_2 -convex function. In fact, we provided a family of trivial examples of S_2 -convex functions (change the constant, values above and below 0). That should make Professor Mark Nelson like our S -convexity more, but it should also make Hudzik, Maligranda, Dragomir, and a few other professors a bit more upset, since we have proven that one of the theorems of Hudzik, supposed to generate S -convex functions, cannot really be trusted. We have provided an entire family of trivial counter-examples to it here. Now the definition of S -convexity is settled through [4], so that we can write more about examples. Next, we will be analysing the work already done in terms of examples.

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