

# Energetic Study of Some States of Debonding in the Composite Structures

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**Abstract** In this paper, a qualitative study is first given to show the influence of the laws of contact, imposed on the constituents interfaces (inclusions-matrix), as well as of type of composites and type of defects or decohesions, on the effective behavior of the “debonded” composite materials. A quantitative analysis is then discussed: we present an energy analysis of some model states of decohesion than may have a composite structure. This energetic study, based on the theory of Francfort and Marigo, consists primarily to reject, for a loading level and a given surface energy (in order of magnitude of  $\varepsilon$  with  $\varepsilon=1/n$  and where  $n$  denotes the number of fibers contained in the structure), the decohesions leading to highest energies of medium. The obtained results of this study are interesting, interesting insofar as they can be used in high tech industry to ensure that of this or that debonding can not appear in such composite pieces, subjected to loading of their use services.

**Keywords** Composite, Homogenization, Matched asymptotic expansion, Surface energy, Energy of medium

## 1. Introduction

For several years, many damage models have been proposed to describe the debonding in the composite structures (see [18] [21] [25] [26] [28]). Among these models, there exist an approach based on fracture mechanics, which deals the decohesion of the constituents interfaces as a cracking problem. It postulates that the debonding, supposedly pre-existing, will spread when the elastic energy restitution rate exactly offsets the surface energy needed for the creation of new debonding areas. These areas are supposed unit, i.e. of the same order of magnitude of  $\varepsilon=1/n$  where  $\varepsilon$  being the parameter characteristic of the microstructure defined by the number of fibers  $n$  contained in the composite structure. It comes to the Griffith's approach [17]. Unfortunately it can not treat the initiation problem of debonding or cracking in general. In starting from its philosophy, but not from its criterion, Francfort and Marigo [9] [14] [15] have developed a new prediction formulation of the initiation and propagation of cracks in sound structures or pre-cracked. By assigning a surface energy thereto, the formulation consist, roughly speaking, to searching, for a given loading level, the cracking leading to the smallest energy of the medium. The energy of the medium being defined as the sum of its elastic energy and its surface energy ( $\mathbf{E}_M=\mathbf{E}_E+\mathbf{E}_S$ ) [10] [11] [22] [23]. For the problem of

debonding in composite materials, it is necessary to introduce the characteristic functions  $\chi^{*\varepsilon}$  characterizing decohesion states of a composite structure [7], which is worth 0 on the bonded part of the interface, noted  $\mathbf{I}^c_\varepsilon$  and which is worth 1 on the debonded complementary part, noted  $\mathbf{I}^d_\varepsilon$ .  $\mathbf{I}_\varepsilon=\mathbf{I}^c_\varepsilon \cup \mathbf{I}^d_\varepsilon$  being the set of all interfaces, bonded and debonded. We also note by  $\Omega^c$  and  $\Omega^d$  the composite structures whose components are bonded respectively debonded.  $\Omega=\Omega^c \cup \Omega^d$  is the domain occupied by the composite structure considered. We also talk of states of cracking or debonding to designate any state of decohesion of the composite structure. The decohesion problem of Francfort and Marigo consist then at finding the fields couple  $(\mathbf{u}^\varepsilon, \chi^\varepsilon)$ , displacement field and state function, minimizing energy of the medium,

$$\begin{aligned} \mathbf{E}_M(\mathbf{u}^{*\varepsilon}, \chi^{*\varepsilon}) &= \mathbf{E}_E(\mathbf{u}^{*\varepsilon}, \chi^{*\varepsilon}) + \mathbf{E}_S(\mathbf{u}^{*\varepsilon}, \chi^{*\varepsilon}) \\ &= \int_{\Omega} \mathbf{W}(\mathbf{x}, \boldsymbol{\varepsilon}(\mathbf{u}^{*\varepsilon})(\mathbf{x}))d\mathbf{x} + \int_{\mathbf{I}_\varepsilon} \mathbf{T}(\mathbf{x})\chi^{*\varepsilon}(\mathbf{x})d\Gamma(\mathbf{x}), \quad (1) \end{aligned}$$

among all couples of "admissible" fields  $(\mathbf{u}^{*\varepsilon}, \chi^{*\varepsilon})$  (admissible displacement field and admissible characteristic function).  $\boldsymbol{\varepsilon}(\mathbf{u}^{*\varepsilon})$  and  $\mathbf{W}(\mathbf{x}, \boldsymbol{\varepsilon}(\mathbf{u}^{*\varepsilon}))$  designate respectively, the strain tensor of the displacement field  $\mathbf{u}^{*\varepsilon}$  and the elastic deformation potential associated with this field, at point  $\mathbf{x}$ .  $\mathbf{T}(\mathbf{x})$  denotes the tenacity of interfaces [7] [27]. Thereafter, it will be assumed that there is continuity of the normal displacement field of  $\mathbf{u}^\varepsilon$  on the debonded interfaces. The tenacity is therefore assumed to be constant on all interfaces, bonded and debonded ( $\mathbf{T}(\mathbf{x})=\mathbf{T}$  sur  $\mathbf{I}_\varepsilon=\mathbf{I}^c_\varepsilon \cup \mathbf{I}^d_\varepsilon$ ).

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Published online at <http://journal.sapub.org/mechanics>

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## 2. Elastic Potential and Surface Energy of a Composite with Debonded Constituents

We have shown in [2] and [3] that the potential or the elastic energy (1) takes different forms depending on whether the constituents of the medium are bonded or debonded. In the bonded case the energy of the medium is equal to its elastic energy, in other words, its surface energy is zero (see for example [20]). This is because the states functions  $\chi^{*\varepsilon}$  are zero on all interfaces, since they are all bonded (i.e.  $\mathbf{I}_{\varepsilon}^c = \mathbf{I}_{\varepsilon}$ ). We have then,

$$\begin{aligned} \mathbf{E}_M(\mathbf{u}^{*\varepsilon}, \chi^{*\varepsilon}) &= \mathbf{E}_E(\mathbf{u}^{*\varepsilon}) = \int_{\Omega_e} \mathbf{W}(\mathbf{x}, \boldsymbol{\varepsilon}(\mathbf{u}^{*\varepsilon})(\mathbf{x})) d\mathbf{x} \\ &= \int_{\Omega_e} \mathbf{A}^c(\mathbf{x}) \boldsymbol{\varepsilon}(\mathbf{u}^{*\varepsilon})(\mathbf{x}) \cdot \boldsymbol{\varepsilon}(\mathbf{u}^{*\varepsilon})(\mathbf{x}) d\mathbf{x}. \end{aligned} \quad (2)$$

$\mathbf{A}^c$  is the homogenized elasticity tensor of a composite whose constituents are perfectly bonded. The last equality can be explained by the fact that the displacement field  $\mathbf{u}^{*\varepsilon}$ , unique solution of the energy minimization problem  $\mathbf{E}_M(\mathbf{u}^{*\varepsilon}) = \text{Min}_{\mathbf{u}^{*\varepsilon}}(\mathbf{E}_M(\mathbf{u}^{*\varepsilon}))$  among all kinematically "admissible" fields, is the solution field of the real problem (see [2]). Therefore it is continuous over the entire composite structure  $\Omega_c$ . And since the basic cell associated, denoted  $\mathbf{V}$ , is a connected part of  $\mathbb{R}^2$ , it is approached in first order, by:

$$\begin{aligned} \mathbf{u}^\varepsilon(\mathbf{x}) &\equiv \mathbf{u}^0(\mathbf{x}, \mathbf{x}/\varepsilon) + \mathbf{o}(1) \text{ in } \Omega_c, \\ &= \mathbf{u}(\mathbf{x}) + \mathbf{o}(1) \text{ in } \Omega_c, \end{aligned} \quad (3)$$

where  $\mathbf{x}$  is the macroscopic variable describing the composite structure in  $\Omega_c \subset \mathbb{R}^3$  and  $\mathbf{u}$  the classical displacement field encountered in theory of periodic homogenization (see eg [3]). In the debonded case  $\Omega_d$  (but with contact and perfect relative sliding between constituents), the energy of medium (1) takes an other form. It was shown in [2] that the field in question  $\mathbf{u}^\varepsilon$  is approached by  $\mathbf{u}^0$ , given by:

$$\begin{aligned} \mathbf{u}^\varepsilon(\mathbf{x}) &\equiv \mathbf{u}^0(\mathbf{x}, \mathbf{x}/\varepsilon) + \mathbf{o}(1) \text{ in } \Omega_d \\ &= \mathbf{u}(\mathbf{x}) + \delta(\mathbf{x})\mathbf{e}_3 + \omega(\mathbf{x})\mathbf{e}_3 \wedge \mathbf{x}/\varepsilon + \mathbf{o}(1) \text{ in } \Omega_d. \end{aligned} \quad (4)$$

$\mathbf{u}$  is the classical macroscopic field in theory of periodic homogenization. It is defined on the matrix but extended by continuity on the complementary part (fibers or folds).  $\delta(\mathbf{x})$  and  $\omega(\mathbf{x})$  are new scalar fields interpreted as the internal slip and internal rotation of the debonded composite structure. In fact they represent real sliding and rotations of fibers in the microstructure, indexed by the macroscopic variable  $\mathbf{x}$ . They are extended in the matrix by the value 0 and therefore are discontinuous in general on the interfaces. The energy of medium to minimize is then written, in the case of a fibered composite,

$$\mathbf{E}_M(\mathbf{u}^{*\varepsilon}, \chi^{*\varepsilon}) = \mathbf{E}_E(\mathbf{u}^{*\varepsilon}, \chi^{*\varepsilon}) + \mathbf{E}_S(\chi^{*\varepsilon})$$

$$\begin{aligned} &= \int_{\Omega_d} \mathbf{A}^d(\mathbf{x}/\varepsilon) \boldsymbol{\varepsilon}(\mathbf{u}^{*\varepsilon})(\mathbf{x}) \cdot \boldsymbol{\varepsilon}(\mathbf{u}^{*\varepsilon})(\mathbf{x}) d\mathbf{x} \\ &+ \int_{\Omega_d} \mathbf{R} \omega^{*\varepsilon}_{,3}(\mathbf{x}) \omega^{*\varepsilon}_{,3}(\mathbf{x}) d\mathbf{x} + \int_{\Omega_d} \mathbf{K} \delta^{*\varepsilon}_{,3}(\mathbf{x}) \delta^{*\varepsilon}_{,3}(\mathbf{x}) d\mathbf{x} \\ &+ \int_{\Omega_d} \boldsymbol{\Sigma}(\boldsymbol{\varepsilon}(\mathbf{u}^{*\varepsilon})(\mathbf{x})) \delta^{*\varepsilon}_{,3}(\mathbf{x}) + \delta^{*\varepsilon}_{,3}(\mathbf{x}) \boldsymbol{\varepsilon}(\mathbf{u}^{*\varepsilon})(\mathbf{x}) d\mathbf{x} \\ &+ T \int_{\Gamma_d^\varepsilon} \chi^{*\varepsilon}(\mathbf{x}) d\Gamma(\mathbf{x}) \end{aligned} \quad (5)$$

where  $\mathbf{A}^d$ ,  $\boldsymbol{\Sigma}$ ,  $\mathbf{K}$  and  $\mathbf{R}$  are quantities that are defined and interpreted in [2]. They are given by solving 12 elementary problems, on the basis cell  $\mathbf{V}\mathbf{I}$  associated with the debonded part, instead of 6 problems in the bonded case.  $T$  is the tenacity of the debonded interfaces assumed constant as was stated in the introduction.

### Remark 1.

In theory, one speaks of rotation of a solid (deformable or no) but not of a material point. Therefore, there is no loading working in rotation at each point of the structure. We conclude then that the internal rotation is zero, i.e.  $\omega=0$  on  $\Omega_d$ . The translation by cons remains possible, as we have noticed in [2] [3] [4] [5].

Taking into account this remark, the energy of the medium (5) is simplified and it is writing so,

$$\begin{aligned} \mathbf{E}_M(\mathbf{u}^{*\varepsilon}, \chi^{*\varepsilon}) &= \mathbf{E}_E(\mathbf{u}^{*\varepsilon}, \chi^{*\varepsilon}) + \mathbf{E}_S(\chi^{*\varepsilon}) \\ &= \int_{\Omega_d} \mathbf{A}^d(\mathbf{x}/\varepsilon) \boldsymbol{\varepsilon}(\mathbf{u}^{*\varepsilon})(\mathbf{x}) \cdot \boldsymbol{\varepsilon}(\mathbf{u}^{*\varepsilon})(\mathbf{x}) d\mathbf{x} \\ &+ \int_{\Omega_d} \boldsymbol{\Sigma}(\boldsymbol{\varepsilon}(\mathbf{u}^{*\varepsilon})(\mathbf{x})) \delta^{*\varepsilon}_{,3}(\mathbf{x}) + \delta^{*\varepsilon}_{,3}(\mathbf{x}) \boldsymbol{\varepsilon}(\mathbf{u}^{*\varepsilon})(\mathbf{x}) d\mathbf{x} \\ &+ \int_{\Omega_d} \mathbf{K} \delta^{*\varepsilon}_{,3}(\mathbf{x}) \delta^{*\varepsilon}_{,3}(\mathbf{x}) d\mathbf{x} + T \int_{\Gamma_\varepsilon} \chi^{*\varepsilon}(\mathbf{x}) d\Gamma(\mathbf{x}). \end{aligned} \quad (6)$$

### Remark 2.

The first term of energy (6) is similar to elastic energy of a homogeneous medium whose tensor elasticity is  $\mathbf{A}^d$ , different of  $\mathbf{A}^c$  from the fact that the basic problems depend on the basic cell  $\mathbf{V}$  or  $\mathbf{V}\mathbf{I}$  (see [2]). It can be interpreted also as the homogenized tensor elasticity of a fibered "bonded" composite structure with well chosen rigidities of the constituents fibers-matrix.

## 3. Energy Study of the Medium of a Composite Structure

The reader is aware of the difficulty of the problem, the debonding can pre-exist before application of a loading and the energy of medium can intervene more displacement fields. In adopting always the couple notation, and taking

into account (2) and (6), the problem consist to find the couple of fields  $(\mathbf{U}^\varepsilon=(\mathbf{u}^\varepsilon,\delta^\varepsilon),\chi^\varepsilon)$  minimizing the energy of medium,

$$\begin{aligned} E_M(\mathbf{U}^{*\varepsilon},\chi^{*\varepsilon}) &= E_E(\mathbf{U}^{*\varepsilon},\chi^{*\varepsilon}) + E_S(\chi^{*\varepsilon}) \\ &= \int_{\Omega_c} \mathbf{A}^c(\mathbf{x}/\varepsilon)\boldsymbol{\varepsilon}(\mathbf{U}^{*\varepsilon_1})(\mathbf{x})\cdot\boldsymbol{\varepsilon}(\mathbf{U}^{*\varepsilon_1})(\mathbf{x})d\mathbf{x} + \int_{\Omega_d} \mathbf{A}^d(\mathbf{x}/\varepsilon)\boldsymbol{\varepsilon}(\mathbf{U}^{*\varepsilon_1})(\mathbf{x})\cdot\boldsymbol{\varepsilon}(\mathbf{U}^{*\varepsilon_1})(\mathbf{x})d\mathbf{x} \\ &+ \int_{\Omega_d} \boldsymbol{\Sigma}(\boldsymbol{\varepsilon}(\mathbf{U}^{*\varepsilon_1})(\mathbf{x})\mathbf{U}^{*\varepsilon_{2,3}}(\mathbf{x}) + \mathbf{U}^{*\varepsilon_{2,3}}(\mathbf{x})\boldsymbol{\varepsilon}(\mathbf{U}^{*\varepsilon_1})(\mathbf{x}))d\mathbf{x} \\ &+ \int_{\Omega_d} \mathbf{K}\mathbf{U}^{*\varepsilon_{2,3}}(\mathbf{x})\mathbf{U}^{*\varepsilon_{2,3}}(\mathbf{x})d\mathbf{x} + T \int_{\Gamma_\varepsilon} \chi^{*\varepsilon}(\mathbf{x})d\Gamma(\mathbf{x}), \quad (7) \end{aligned}$$

among all couples of "admissible" fields  $(\mathbf{U}^{*\varepsilon}=(\mathbf{u}^{*\varepsilon},\delta^{*\varepsilon}),\chi^{*\varepsilon})$ .

We see that the problem is well complex: even if we manage to construct a minimizing sequence, it can do not converge in the right "space", since a sequence of characteristic functions do not necessarily converge towards a characteristic function (representative of a state of debonding). Therefore, the couple of solution field, if it exists can do not be "admissible". So we do not try to solve the problem in all its generality but at given surface energy (fixed). We calculate the elastic energy of the composite structure for different possible cases of decohesions [1] [2] [3] [13], for reject the figure cases, least envigeables, within the meaning of the adopted criterion. To do this, we "class" first, the states of decohesion of the composite structure according to geometrical criteria (volume defects, surface defects, lineal defects or punctual defects). As the surface energy varies only according to the "surface" of total debonding (or "total length" in the case of a fibred composite), we will define, for each type of debonding, the subclasses differentiated by their order greatness, in  $\varepsilon$ , of their elements. We present then first below, some states models of decohesion that may have a composite structure.

### 3.1. Composites with Long Fibers

In the following schemes, to saving space, we will note "ND" to designate "The Number of Decoheion", Likewise "LDOF" to designate "The Length of Decoheion in One Fiber" and finally "TDL" to designate "The Total Decoheion Length".

#### 3.1.1. Volume Defects

1)

<p>Micro-decohesions in the fibred composite</p>	"ND"	$O(1/\varepsilon^3)$
	"LDOF"	$O(\varepsilon)$
	"TDL"	$O(1/\varepsilon^2)$

2)

<p>Micro-decohesions in the fibred composite</p>	"ND"	$O(1/\varepsilon^2)$
	"LDOF"	$O(1)$
	"TDL"	$O(1/\varepsilon^2)$

#### 3.1.2. Surface Defects

3)

<p>Micro-decohesions in the fibred composite</p>	"ND"	$O(1/\varepsilon^2)$
	"LDOF"	$O(\varepsilon)$
	"TDL"	$O(1/\varepsilon)$

4)

<p>Micro-decohesions in the fibred composite</p>	"ND"	$O(1/\varepsilon^2)$
	"LDOF"	$O(\varepsilon)$
	"TDL"	$O(1/\varepsilon)$

#### 3.1.3. Linear Defects

5)

<p>Micro-decohesions in the fibred composite</p>	"ND"	$O(1)$
	"LDOF"	$O(1)$
	"TDL"	$O(1)$

6)

<p>Micro-decohesions in the fibred composite</p>	"ND"	$O(1/\varepsilon)$
	"LDOF"	$O(\varepsilon)$
	"TDL"	$O(1)$

#### 3.1.4. Punctual Defects

7)

<p>Micro-decohesions in the fibred composite</p>	"ND"	$O(1)$
	"LDOF"	$O(\varepsilon)$
	"TDL"	$O(\varepsilon)$

### 3.2. Laminate Composites

#### 3.2.1. Volume Defects

8)

 Micro-decohesions in the laminated composite	“ND”	$O(1/\varepsilon^2)$
	“LDOF”	$O(\varepsilon)$
	“TDL”	$O(1/\varepsilon)$

9)

 Micro-decohesions in the laminated composite	“ND”	$O(1/\varepsilon)$
	“LDOF”	$O(1)$
	“TDL”	$O(1/\varepsilon)$

#### 3.2.2. Surface Defects

10)

 Micro-decohesions in the laminated composite	“ND”	$O(1)$
	“LDOF”	$O(1)$
	“TDL”	$O(1)$

11)

 Micro-decohesions in the laminated composite	“ND”	$O(1/\varepsilon)$
	“LDOF”	$O(\varepsilon)$
	“TDL”	$O(1)$

#### 3.2.3. Linear Defects

12)

 Micro-decohesions in the laminated composite	“ND”	$O(1)$
	“LDOF”	$O(\varepsilon)$
	“TDL”	$O(\varepsilon)$

13)

 Micro-decohesions in the laminated composite	“ND”	$O(1/\varepsilon)$
	“LDOF”	$O(\varepsilon)$
	“TDL”	$O(1)$

#### 3.2.4. Punctual Defects

14)

 Micro-decohesions in the laminated composite	“ND”	$O(1)$
	“LDOF”	$O(\varepsilon)$
	“TDL”	$O(\varepsilon)$

## 4. Quantitative Analysis of the Obtained Results

If the purpose of this paper is to give, in summary, a few orders of magnitude of the elastic energy corresponding at some states of decohesion presented above, to deduce which states of decohesion should be rejected, in sense of the Francfort and Marigo minimization criterion, we endeavor nevertheless not to lose sight of the generic character of the methods used (Homogenization and matched asymptotic expansions), in computing of macroscopic behavior, that allows analyzing complex situations, in particular taking account, at the macroscopic level, of the presence of micro-defects in the homogenized composite structures. In the few studies conducted i.e., cases of figure numbers 1, 2, 3 and the states of decohesion numbers 9, 10 and 11 (see [2] [3] [6]), we deduce the following points:

### 4.1. Influence of Composites Type

The effective behavior of a composite structure having periodic decohesions differs according to whether it is a laminate composite or a fibered composite. Indeed, the kinematics of the medium is described by two macroscopic displacement fields  $\mathbf{u}$  and  $\delta$  (one field only in the existing literature, see [20]). For a fibered composite, the translation is given by  $\delta(\mathbf{x}) = \delta_3(\mathbf{x})\mathbf{e}_3$  (see [2]). For a laminated composite, it is given by,  $\delta(\mathbf{x}) = \delta_1(\mathbf{x})\mathbf{e}_1 + \delta_2(\mathbf{x})\mathbf{e}_2$  (see [3]). The scalar field  $\delta_3$ , defined on  $\Omega_d$ , is interpreted as the longitudinal relative sliding of the fibers compared to the matrix, the fields  $\delta_1$  and  $\delta_2$ , when at them, are the two components of a planar field representing the slip of steep layers (inclusions) compared to soft layers (matrix).

### 4.2. Influence of Defects Type

The methods used to modeling the damaged composite structures differ according to the type of the latter. For example, to study the cases of figure 1, 2 and 9, we use the periodic homogenization method. Another method, called matched asymptotic expansions [1] [24], was illustrated for the states of debonding numbers 3, 10, and 11. In addition, the effectif behavior of a such composite (with debonded constituents) differs from one case to another. Indeed, for the illustrative case number 1, the effective behavior is similar to a continuous medium whose kinematics is described by a single displacement field, and whose equilibrium is governed by classical three-dimensional equilibrium equations [19]. By cons, for the states of decohesion numbers 2 and 9, the kinematics is described by several displacement fields, and the equilibrium is governed by coupled equations, i.e., three-dimensional equilibrium equations and equations of beam type [2] (in the case of figure number 2, "fibered composite") or plate type [3] (in the case of figure number 9, "laminate composite"). For the cases of figure numbers 5, 11 and 12 (see [1] [6] [24]), although there have micro-defects, they have not influence, in first approximation, on the macroscopic behavior of the

structure. It is not the same for the cases of decohesion numbers 2, 9 and 10, where the influence of the macroscopic debonding is felt from the first order.

#### 4.3. Influence of Contact Laws

The effective behavior of composite materials at debonded constituents depends on the contact law to imposing on the interfaces. For example, for a laminate, the kinematics of the equivalent structure is described by two displacement fields, a displacement field of the matrix  $\mathbf{u}$  and a inner slip field  $\delta$ . If one authorizes the debonded interfaces to sliding without rubbing, while staying in contact, the internal sliding field is written as follow,  $\delta(\mathbf{x})=\delta_1(\mathbf{x})\mathbf{e}_1+\delta_2(\mathbf{x})\mathbf{e}_2$ . But in the case without contact, it is written,  $\delta(\mathbf{x})=\delta_1(\mathbf{x})\mathbf{e}_1+\delta_2(\mathbf{x})\mathbf{e}_2+\delta_3(\mathbf{x})\mathbf{e}_3$ . It is seen well that the macroscopic behavior of such a composite is not the same if one changes the contact law to impose on the debonded interfaces and it was noticed, in the expression (4), for fibred composites.

### 5. Qualitative Analysis of the Effective Behavior of Composite Materials

Let us retake the notations to fix ideas, and let us consider a unitary three-dimensional composite structure comprising  $n$  fibers. Let  $\varepsilon=1/n$  the parameter characterizing the microstructure, it also represents the dimensions of micro-defects (or micro-decohesions) that are of the order of  $o(\varepsilon)(2\pi\varepsilon^2$  in surface for a "fibred composite" and  $2\varepsilon^2$  for a "laminate composite") [16]. It is assumed that, when they are present in the microstructure, they are "distributed" periodically, as indicated in the figures above. For the calculate of the orders of magnitude of the surface energies  $E_S$ , we use the following two general rules:

\* For a fibred composite (fibers-matrix):

$$E_S = (\text{Number of damaged fibers}) \times (2\pi\varepsilon^2) \times (\text{Number of defects per fiber})$$

\* For a laminate composite (folds stiffer/soft):

$$E_S = (\text{Number of debonded stiff folds}) \times (2\varepsilon^2) \times (\text{Number of defects per fold})$$

We present then, in the two tables below, the cases of figure studied, i.e., the states of debonding numbers 1, 2 and 3 for a fibred composite and the states of debonding numbers 9, 10 and 11 for a laminate composite. We give then the orders of magnitude, in  $\varepsilon$ , of surface energies and energy restitution elastic (difference between the energy of the "healthy" composite structure and the elastic energy of the "debonded" composite structure) for each figure case studied. These energy restitutions are calculated in [2], [3], [6] and [24].

**Table 1.** Orders of magnitude for fibred composites

Case of Figure Number:	N°1	N°2	N°3
Surface Energy:	$O(1/\varepsilon^2)$	$O(1/\varepsilon^2)$	$O(1/\varepsilon)$
Energy Restitution Elastic:	$O(1)$	$O(1)$	$O(1)$

**Table 2.** Orders of magnitude laminated composites

Case of Figure Number:	N°9	N°10	N°11
Surface Energy:	$O(1/\varepsilon)$	$O(1)$	$O(1)$
Energy Restitution Elastic:	$O(1)$	$O(1)$	$O(\varepsilon)$

### 6. Discussion

When a structure is subjected to an increasing loading, the structure is damaged (a crack appears). For a composite structure there is appearance of a debonding of its constituents. This debonding defines a decohesion state, solution of the energy minimization problem of Francfort and Marigo ( $E_m=E_s+E_e$ ) among all possible fictitious decohesion states. We can say then that all other debonding states are rejected and cannot appear in this composite structure for this loading level. We see that the problem is well complex: we cannot compare all energies of all possible debonding states to find the state of debonding, solution of Francfort and Marigo problem. But we can conclude that a particular state of debonding cannot be a solution (i.e. it cannot appear in the structure) if we find a new state whose energy of medium is smaller. For the states whose energies of surface  $E_s$  are the same (same order of magnitude) we compare their elastic energy  $E_e$ . This is the case for example of the states 1 and 2 and then 10 and 11 of above Tables 1 and 2. The state 10 is rejected in front of the state 11 because the order of magnitude of its elastic energy (so of its energy of medium) is more great. By cons we cannot reject any of the states 1 and 2 because the orders of magnitude of their elastic energy are the same. We must calculate numerically their energy to conclude.

It is therefore very interesting to know the solution of the energy minimization problem of Francfort and Marigo, posed on a composite structure, but it is also interesting to know whether such or such debonding cannot appear in this structure for a given loading level.

For the initiation problem of debonding in healthy composite structures, it is possible to find numerically the solution of this problem, as well as study the propagation of this initiated debonding. Indeed, we first must make an unstructured mesh whose nodes will be "duplicated" so that we can open it to the needs (for calculation of the energy restitution elastic). Then, in the calculations, locate the state of decohesion "initiated" by the criterion of maximum stresses. Considering then all possible local decohesion states, we deduce the initiated decohesion state [8] [12]. We can then repeat the process for studying the propagation of decohesion as well initiated [22] [25].

### 7. Conclusions

The energetic study of the composite structures make in this paper, based on the theory of Francfort and Marigo [9], consist primarily to reject, for a loading level and a given surface energy (in order of magnitude of  $\varepsilon$  with  $\varepsilon=1/n$  and

where  $n$  denotes the number of fibers contained in the structure), the decohesions leading to a highest energies of medium. We can then reject a state of debonding, in doing so, but we can not "keep" some state; keep one state in the sense that it should be compared to another state of decohesion, to finally find the state of decohesion, solution of the minimization problem of Francfort and Marigo [9], because the states of decohesion are very numerous and we can not compare practically all these states. But the obtained results of this study are very interesting because it can be exploited in high tech industry to ensure that of this or that debonding can not appear in such composite pieces, subjected to loading of their use services.

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