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Feasibility Analysis of Nano-Lubricants at Conceptual Design Stage Using Digraph and Matrix Approach

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Abstract A procedure based on digraph and matrix method is developed for the evaluation of feasibility of a nano-lubricant at the conceptual design stage. Feasibility analysis parameters of a nano-lubricant are identified and are either referred to as feasibility attributes or nano-lubricant attributes. Consideration of these attributes and their relations are essential in evaluating the feasibility index. This is modelled in terms of feasibility analysis digraph of a nano-lubricant. The digraph is represented by one-to-one matrix for development of a feasibility expression which is based on various attributes. A variable attributes relationship permanent matrix is defined to develop feasibility expression called the variable permanent function which is also useful in comparing two nano-lubricants. Feasibility index of a nano-lubricant is obtained from the permanent of the matrix or from the feasibility expression by substituting the numerical values of attributes and their inter-relations. A higher value of the index implies better feasibility of the nano-lubricant. The ideal value of feasibility index is also obtained from the matrix expression, which is useful in assessing the relative feasibility value of the nano-lubricant alternative. The procedure is useful in selecting nano-lubricant at system conceptual design stage. A step by step procedure for evaluation of feasibility index is also suggested and is illustrated by means of an example.

Keywords Nano-Lubricant, Tribo-element, Digraph

1. Introduction

Tribology, the science of friction and wear has provided spectacular and innovative solutions to the problems of failures occuring in the industry due to poor friction, wear and lubrication properties of materials used in tribo-systems. One of most prominent and effective solution to reduce these failures is obtained by the application of lubricant at the interface between two tribo-elements. This is obtained by applying either liquid or solid lubricant at the interface in a tribo-system. In the recent past, the development of nanomaterials has augmented the performance of these lubricants by many folds in particular for the use of MEMS and NEMS[1,2]. The strategy is based on the feeding of sliding interface with nano-particles of the tribo-active phases dispersed in lubricant base oils or grease to produce the tribo-film without reaction, with the substrate surfaces [3, 4].

The main advantages of the nano-particles is their size in the nanometer range which is well adapted for perfect feeding of the sliding interface and the possibilities of the composite particle synthesis which can combine multiple properties such as friction reduction, anti-wear, anti-corrosion etc[5]. These nano-lubricants not only help in reducing friction and wear by a considerable amount but also play a significant role in conservation of material and energy. It is also evident from literature review that a procedure / methodology is not available for evaluating the performance / index of these nano-lubricants for the use of design and development of tribosystems[4, 5]. In order to reap maximum benefits it is essential to consider evaluation and selection of appropriate lubricant at earlier design stage, in particular at system conceptual design stage for a tribosystem.

In this research work, parameters which influence the performance of nano-lubricants are first of all identified in a systematic and logical manner. On the basis of these parameters and their relationships, a methodology based on digraph and matrix approach, used by many researchers[6-9], is proposed for the evaluation of feasibility index of a nano-lubricant.

2. Feasibility Attributes for a Nano-lubricant

Performance of nano-lubricants depends upon their various physical, chemical, mechanical and tribological properties, e.g. the higher value of co-efficient of friction means a lower performance of the lubricant. Similarly, the higher value of co-efficient of wear implies lower life of tribo-system Environmental compatibility is one of the

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important properties of a nanolubricant. Eco-friendly nano-lubricant is preffered over hazardous lubricant. At operational level one of the typical problems faced by the industry is the recycling and reclamation of lubricant. These features of nano-lubricants are elaborated and discussed as below:

2.1. Friction

Mechanical systems are composed of various tribo-elements. Each tribo-element performs desired function through interaction with one another to achieve the desired output, e.g. in roller bearing, the inner race and balls perform the function of supporting load. The interaction between tribo-elements is always accompanied by friction and higher friction means higher temperature rise, higher wear and loss of energy. The force of friction depends upon the contact area and the applied load. The real area of contact must be minimized to minimize adhesion, friction and wear[10, 11]. The friction between two tribo-elements is generally reduced to a large extent by applying lubricants, viz., grease, viscous oils, etc. The force of friction decides whether the tribo-surfaces shall come in adhesive contact or remain separated by thin, liquid like layer. If the surfaces are separated by one or more molecular layers of fluid and if this cushioning film remains in the liquid like state during sliding, this may ensure that the friction force will be low and the sliding will proceed smoothly[12].

It is a fact that the development of nano-lubricants in the recent years has played a significant role in increasing the efficiency of a lubricant and in turn increased the life of a tribo-element to a large extent. There are various laws that govern the laws of friction in case of this nano lubrication. Squeeze pressure and wetting properties play an important role in defining friction laws for nano-lubrication[13].

Therefore the first attribute for evaluating the feasibility of a nano-lubricant is friction. The calculation of friction can be easily done by having knowledge of co-efficient of friction for the tribo-element—nano-lubricant interface, hence co-efficient of friction becomes our first attribute and is abbreviated as COF.

2.2. Wear

Wear is observed in all mechanical systems due to the interaction between tribo-elements. It has been reported that most of the failures in mechanical system are due to poor friction and wear properties of the materials used for design and fabrication of the tribo-elements[14]. Materials used for design and development of tribo-elements are classified on the basis of wear co-efficient[15]. The wear occurs due to the inter-locking of the asperities of one tribo-element into the asperities of the other e.g. the rubbing of journal and bearing in case of the crankshaft of an IC engine, the rubbing of the cylinder lining and the piston leads to the wearing out of the lining surface and piston both. Friction force shears inter-metallic junctions formed at the regions of the real contact ploughs the surface of the softer material by the asperities of the harder[10]. However wear is reduced to a large extent by applying appropriate lubricants at the interface between two tribo-elements. The performance of a lubricant is determined how best it is able to reduce the wear of the tribo-element. Nano-lubricant has also played a significant role in reducing the wear under severe conditions, as evident from various studies, the development and use of nano-lubricants in recent years has increased the efficiency of the lubricants and in turn increased the life of the tribo-elements to a large extent.

Therefore wear is considered as the second attribute of the Nano-lubricants, to check the feasibility of nano-lubricants at conceptual deign stage. This is calculated from the knowledge of coefficient of wear for the particular nano-lubricant and may be abbreviated as COW.

2.3. Material Conservation

Tribo-elements in mechanical systems deteriorate due to the rubbing against each other because the material wears out from the surfaces of the tribo-elements. This wearing out of the material leads to the material losses, e.g. rubbing of piston against the cylinder liner leads to the wearing out of both the cylinder liner and the piston and ultimately leads to the failure. Depletion of material resources has urged a need for developing materials with longevity. At the plant level if the replacement intervals of components and assemblies are increased, impacts on environment are automatically decreased. Nano-lubrication helps in increasing the life of tribo-elements to a large extent by increasing the wear resistance of the materials used for tribo-elements; they have proved excellent due to their anti friction and anti wear properties. Due to the anti wear properties of the nano lubricants these help in material conservation. Hence material conservation becomes our third attribute to judge the feasibility of a nano-lubricant at the conceptual design stage and is abbreviated as MC.

2.4. Thermal Conductivity

It is a well-known fact that machine elements during their operation produce heat. This is because of the rubbing action of tribo-elements against each other. The rubbing of tribo-elements involves friction, which as a result generates heat. This heat is to be transported away from the machine element to increase the life of the machine element. The life of the machine element can also be increased if friction is reduced, which is done by lubrication of the contact surface, e.g. in case of piston cylinder arrangement of an IC engine, friction between cylinder walls and piston is reduced by applying lubricating oil. Moreover choosing a suitable material of the contact surfaces can also do the required job.

With the advancement of technology new methods of removing heat and reducing friction have evolved with time. The use of Nano lubricants is one of the modern methods for the removal of the heat from the tribo elements. Nano particles are excellent at carrying away the heat from the tribo-elements and hence improving the life of the tribo-elements. It has been observed that the particle size reduction caused an increased surface area during chemo-mechanical processing, which increase the heat transfer[16]. Thus thermal conductivity is an important criterion for deciding the feasibility of the nano lubricants at the conceptual design stage and hence serves as the fourth attribute to check the same. It is abbreviated as TC.

2.5. Energy Conservation

It is a fact that machine elements during their operation, have to overcome frictional resistance. Whenever tribo-ele ments rub against each other friction comes into play. A significant amount of energy is used to overcome this frictional resistance, which leads to energy loss and hence reduces the efficiency of the mechanical system, e.g. in case of an internal combustion engine a significant amount of energy is lost as friction power, which reduces the overall efficiency of the engine. Energy losses due to friction between two tribo elements in mechanical systems are quite alarming[17]. The largest friction losses occur in engines, which amount to 18 percent of the engine indicated power in the environmental protection testing mode[18]. It is a fact that by application of certain lubricants this friction power is reduced in magnitude. Moreover the application of nano-lubricants in the field of lubrication has helped to reduce this friction power by a significant amount. Challenges of having smaller co efficient of friction by developing a better combination of contact materials, surface morphology, and lubricant will be constantly required until the impact of friction losses become negligible or small[19].

Thus by using nano-lubricants friction force can be reduced by a certain amount and energy can be conserved, thus an increase in the overall efficiency of the mechanical system. The properties of nano materials will go a long way in the design and development of tribo-elements, which will help in conserving energy. Thus energy conservation, on the basis of energy saving obtained by the reduction of co-efficient of friction through tribological methods, is identified as another i.e. fifth attribute to check the feasibility of the nano lubricants at the conceptual design stage and is abbreviated as EC.

2.6. Environmental Preservation

The present day use of machines has led to number of threats to the environment. Machine elements use lubricants to reduce friction and wear rates at large scale. Though the use of these lubricants has increased the efficiency of these machines but they also impose a great threat to the environment due to the industrial wastes generated by them. Two main aspects have to be considered: saving of resources and reducing the impact on the environment[15].The contaminants and the wastage is a serious concern for environmental experts throughout the globe. In the recent past, solution to the problem is to develop and use biodegradable and eco friendly lubricants[20]. Modern technology has come up with the use of nano-sized particles as lubricants, which have increased the life of the tribo elements and are also biodegradable by nature. Hence the use of nano lubrication is increasing day by day due to their excellent lubricating properties and their bio-degradability.

Thus environmental preservation serves as a factor in governing the feasibility of a nano-lubricant at the conceptual design stage, hence it becomes our sixth attribute for carrying out the feasibility analysis and is abbreviated as EP.

2.7. Economic Viability

As evident from our recent discussions the use of nano lubricants shall increase day by day due to their excellent lubricating properties and eco friendly nature. Each year, more than 4 billion gallons of spent lubricant are generated worldwide. Although they can be considered valuable resources, less than 10 percent of worldwide supply is actually refined into high quality lubricant base stock. The remainder is typically burned as fuel or disposed off using even less environment friendly methods[21]. The use of nano lubricants will only increase if they increase the over all efficiency of the mechanical system and reduce the over all expenditure. This is possible only if they are cost effective. Moreover the recyclability of these nano lubricants adds to this feature. At the service end , one of the main challenges before the tribology researchers, designers, engineers, and end users is disposal/reclamation or reuse of these nan lubricants. Kimura et al.[22] reported that about 30 percent of lubricating oil in advanced countries is wasted, which my partly contaminate the global environment. Hence recyclability reduces the over all cost of the nano lubricant.

This adds to our list of attributes for checking out the feasibility of the nano lubricants at the conceptual design stage, and is abbreviated as EV. In other words this attribute checks whether the nano lubricant chosen is economically viable or not on the basis of best product end-of-life scenario by using digraph and matrix method as done by many researchers for their various investigations[23-25].

3. Modelling of Attributes for a Nano-lubricant at Conceptual Design Stage

The attributes of a nano-lubricant have been identified in the previous section. Each attribute facilitates the *feasibility analysis* through its contributing factors/features. Each attribute possesses distinctive characteristics which help to develop relationship among these attributes, i.e. how one attribute is related to the other in determining the feasibility on the basis of the performance of a nano-lubricant during its entire life-cycle. The relationship among the attributes is called the degree of relationship. The relationship between the attributes varies. It may be taken, as strong to none as two extremes of the degree of relationship. In between this is assumed as medium and weak relationship. These are developed and are also represented in the table 1. This table shows clearly that the degree of relationship varies among attributes. The degree of relationship is represented in the last column of the table 2. The seven attributes of nano-lubricants in general are shown with their serial numbers. The degree of relationship for an attribute among the other attributes is shown against the attribute in its row entry as their serial number.

It is known from the experiments that the higher value of the co-efficient of friction will always result in higher wear of the tribo-surface, thus a higher value of co-efficient of wear, which predicts a strong relationship between the *co-efficient of friction* (COF) results in an increase in the wear of the tribo-surfaces and consequently an increase in *co-efficient of wear* (COW).This shows that the two attributes are directly related to each other. The degree of relationship in this case is hence strong. Similarly increased wear rate (more co-efficient of wear) results in more degradation of the environment; hence the two are also related to each other. It is however mentioned again that the relationship among these attributes needs to be derived by a concurrent engineering team comprising of designers and experts from other fields of nano-tribology.

The modelling of the attributes of nano-lubricants after carrying out their *feasibility analysis* at the conceptual design stage requires the consideration of attributes of nano-lubricants and their relationship. This can be conveniently represented using graph-theoretical concepts, which have been applied in various fields of science and technology (Chen, 1997).

3.1. Feasibility Analysis Digraph

Feasibility Analysis modelling requires consideration of all attributes of nano-lubricants and the degree of relationship among these attributes. This is conveniently represented by a digraph called Feasibility Analysis attributes digraph (NL^g). $NL^g G_d^a=(A^a, E^a)$ for a tribo-element is defined where $\mathbf{A}^{\mathbf{a}}$={ $A_1^a, A_2^a,\ldots, A_5^a$ } is a set of nodes representing seven nano-lubricant attributes (NLA's) and E^a={ $e_{12}^a, e_{13}^a,\ldots$} is a set of edges amoung nodes, e.g. e_{12}^a connects attribute A_1^a to A_2^a. The direction of edge indicates the relationship of A_1^a with A_2^a for feasibility analysis of nanolubricants.

NL^g for a nanolubricant is developed, in general, considering seven identified attributes in section 2 and their degree of relationships. This is shown in Fig 1. The seven nodes represent seven FAAs, e.g. A_1^a (node1) represents COF and A_2^a (node 2) represents COW. The direction of edge from node 1 to node 2 represents the degree of relationship between COF and COW. Similarly, edges in the NL^g are drawn keeping in mind the degree of relationship amoung NLAs. The NL^g shows the attributes of a nano-lubricant visually. For evaluation of a nanolubricant quantitatively, the NL^g is represented by a matrix. This is discussed in section 4.

4. Matrix Representation of a Nano-lubricant Attribute Digraph

Matrix representation of the NL^g for a nano-lubricant consists of all the important nano-lubricant attributes e.g., Co-efficient of Friction (COF), Co-efficient of Wear (COW), Material Conservation (MC), Thermal Conductivity (TC), Energy Conservation (EC), Environmental Preservation (EP) and Economic Viability (EV). In this case, the facilitation among all these attributes is considered to develop the *Feasibility Analysis* expression of the nano-lubricant. The NL^g of the seven attributes is developed based on the discussion in section above and is shown in Fig 1.

Let the NL^g in general, with N nodes be represented by N^{th} order binary matrix[r_{ij}], where r_{ij} represents relationship of i^{th} attribute with j^{th} attribute, otherwise r_{ij}=0, as an attribute can not have relationship with itself. *Feasibility Analysis matrix* for the NL^g as shown in Fig 1, is written as:

$$R=\begin{bmatrix} 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 0 \end{bmatrix} \quad (1)$$

The diagonal elements in the matrix have value 0 and off diagonal elements have value either 0 or 1. This implies that in this matrix only relationship among attributes is considered and value of the attributes is not taken into account. To incorporate this, a new matrix called Nano-lubricant Attributes Relationship Permanent Matrix is defined. A Nano-Lubricant Attributes Relationship Permanent Matrix representing the Nano-Lubricant Attributes Digraph shown in Fig 1 is written as:

$$Q=[AI + R] = \begin{bmatrix} V & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & V & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & V & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & V & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & V & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & V & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & V \end{bmatrix} \quad (2)$$

Where I is identity matrix and V is value of the attributes. It is noted from the matrix expression (2), that all the diagonal elements have been assigned the value of V, i.e., each attribute has equal value in the system. However this is not true in actual practice. Also, the relationship of one attribute with the other attribute (i.e., r_{ij}) may take any value rather than extreme value 0 and 1. Thus there is a need for considering general attribute value (i.e. off diagonal elements r_{ij}) to develop matrix expression characteristic of the Feasibility Analysis of the Nano-lubricant at the conceptual design stage.

These are taken into consideration through a new matrix called Variable Nano-Lubricant Attributes Relationship Permanent Matrix ($VNLA^{per}$). A permanent is a standard matrix operation used in combinatorial mathematics. It is an analog of a determinant where all the signs in the expansion by minors are taken as positive (Jurkat and Ryser, 1966). Let the off diagonal elements of matrix Q be represented by r_{ij} instead of I, where i^{th} attribute is related to j^{th} attribute. Let us also define a diagonal matrix, H, with diagonal elements V_i representing variable value of i^{th} attribute. If an attribute is excellent in a Nano-Lubricant, it is assigned a maximum value. In general, most of the attributes are assigned intermediate values of the interval scale, as attribute may have medium contribution in Feasibility Analysis. The attribute value may be assigned qualitatively or quantitatively.

Variable nano-lubricant Attributes Relationship Permanent Matrix for the digraph shown in figure is given as:

$$\mathbf{Q=[H+F]=}\begin{bmatrix} V_1 & r_{12} & r_{13} & r_{14} & r_{15} & r_{16} & r_{17} \\ r_{21} & V_2 & r_{23} & r_{24} & r_{25} & r_{26} & r_{27} \\ r_{31} & r_{32} & V_3 & r_{34} & r_{35} & r_{36} & r_{37} \\ r_{41} & r_{42} & r_{43} & V_4 & r_{45} & r_{46} & r_{47} \\ r_{51} & r_{52} & r_{53} & r_{54} & V_5 & r_{56} & r_{57} \\ r_{61} & r_{62} & r_{63} & r_{64} & r_{65} & V_6 & r_{67} \\ r_{71} & r_{72} & r_{73} & r_{74} & r_{75} & r_{76} & V_7 \end{bmatrix} \quad (3)$$

It may be noted that any matrix expression (3), represents value of attributes (V_i's) and their relationship (r_{ij}'s) for the given product. Permanent of this matrix (or Per(N)) i.e., $VNLA^{per}$ is called Variable Nano-Lubricant Attributes Relationship Permanent Function, abbreviated as VPF-n. VPF-n is characteristic of the nano-lubricant attributes as it contains number of terms which are its variant.

As stated above, permanent is a standard matrix function and is used in combinatorial mathematics. Use of this concept in modelling of nano-lubricant at conceptual design will help in representing structural information from combinatorial consideration. This is desirable to associate proper physical meaning to the structural components and their combinations. Moreover using this no negative sign will appear in the expression and hence no information is lost.

VPF-n of matrix, expression (3) is written in sigma form as:

$$\mathbf{PER\,(N)} = \prod_i^7 V_i + \sum_i \sum_j \sum_k \sum_l \sum_m \sum_n \sum_o (r_{ij}\, r_{ji})V_k V_l V_m V_n V_o + \sum_i \sum_j \sum_k \sum_l \sum_m \sum_n \sum_o (r_{ij}\, r_{jk} r_{kl} + r_{lk}\, r_{kj}\, r_{ji}) V_l V_m V_n V_o + \sum_i \sum_j \sum_k \sum_l \sum_m \sum_n \sum_o (r_{ij}\, r_{jk} r_{kl} r_{lm} + r_{ml}\, r_{lk}\, r_{kj}\, r_{ji}) V_m V_n V_o + \sum_i \sum_j \sum_k \sum_l \sum_m \sum_n \sum_o (r_{ij}\, r_{jk} r_{kl}\, r_{lm} r_{mn} + r_{nm} r_{ml} r_{lk}\, r_{kj}\, r_{ji}) V_n V_o + \sum_i \sum_j \sum_k \sum_l \sum_m \sum_n \sum_o (r_{ij} r_{ji}) (r_{kl} r_{lk}) (r_{mn} r_{nm}) V_o + \sum_i \sum_j \sum_k \sum_l \sum_m \sum_n \sum_o (r_{ij} r_{jk} r_{kl} r_{lm} r_{mn} r_{ni} + r_{in} r_{nm} r_{ml} r_{lk} r_{kj} r_{ji}) V_o + \sum_i \sum_j \sum_k \sum_l \sum_m \sum_n \sum_o (r_{ij} r_{ji}) (r_{kl} r_{lm} r_{mn} r_{no}) + (r_{on} r_{nm} r_{ml} r_{lk}) + \sum_i \sum_j \sum_k \sum_l \sum_m \sum_n \sum_o (r_{ij} r_{jk} r_{kl} r_{lm} r_{mn} r_{no} r_{oi} + r_{io} r_{on} r_{nm} r_{ml} r_{lk} r_{kj} r_{ji}) \quad (4)$$

This equation consists of number of terms. These are arranged in eight (i.e., Z+1=8, with Z in this case) groupings in descending order of number of attributes value. The first grouping contains only one term and is a product of seven attribute values, (i.e., V_i V_j V_k V_l V_m V_n V_o). The second grouping is absent as there are no self loops present in the digraph. The third grouping contains number of terms and each term is a multiple of five attribute values, (i.e., V_k V_l V_m V_n V_o) and 2-attribute relationship loop (r_{ij} r_{ji}). Similarly, terms of 4^{th}, 5^{th} and 6^{th} groupings are a multiple (V_o) and 6-attribute relationship loops (i.e., (r_{ij} r_{jk} r_{kl} $r_{lm}r_{mn}$ r_{ni}) or its pair (r_{in} r_{nm} r_{ml} r_{lk} r_{kj} r_{ji}). The 7^{th} and 8^{th} groupings contain two sub groupings. The terms of the first sub grouping consist of one attribute value (V_o) and 3-attribute relationship loops (i.e., (r_{ij} r_{ji}) (r_{kl} r_{lk}) (r_{mn} r_{nm})). The terms of second sub grouping is a multiple of 1-attribute value (i.e. V_o) and 6-attribute relationship loops (i.e., (r_{ij} r_{jk} r_{kl} r_{mn} r_{ni}) or its pair (r_{in} r_{nm} r_{ml} r_{lk} r_{kj} r_{ji}). The terms of 8^{th} sub grouping are also arranged in two sub groupings. The terms of the first sub grouping are a multiple of 2-attribute relationship loops (i.e., r_{ij} r_{ji}) and 4-attribute relationship loops (i.e.,r_{kl} r_{lm} r_{mn} r_{no}) or its pair (i.e., r_{on} r_{nm} r_{lm} r_{lk}). Each term of second sub grouping consist of 7-attribute relationship loop (i.e., r_{ij} r_{jk} r_{kl} r_{lm} r_{mn} r_{no} r_{oi}) or its pair (r_{io} r_{on} r_{nm} r_{ml} r_{lk} r_{kj} r_{ji}). It is once again stated that VPF-n when expanded takes into consideration all terms of the matrix expression and thus no information is lost. This shows that VPF-n is a powerful expression for analyzing Feasibility of a Nano-lubricant.

5. Feasibility Index of a Nano-lubricant

Feasibility Index for a nano-lubricant is a measure of Feasibility of an alternative at system conceptual design stage. This is represented as I_{di}. The nano-lubricant alternative is better, if its feasibility index is higher and vice versa. The index for the nano-lubricant needs to take into account the value of attributes for the nano-lubricant and their degree of inter-relationship among the attributes. VPF-n, developed earlier is a characteristic of the feasibility analysis of a nano-lubricant and this can be used for the evaluation of index as it meets the requirements. Moreover, all terms of the VPF-n are positive. Therefore increase in the value of attribute and their degree of relationship will result in increased value of VPF-n. VPF-n, i.e. permanent of $VNLA^{per}$ is therefore, considered for evaluation of feasibility index of a product. The index can be evaluated if V_i and r_{ij} are assigned quantitative or qualitative values. The values can be assigned on appropriate scale. In this paper, Table 2 used for assigning values to attributes has been proposed. It is suggested that the designer/ user may develop similar scoring tables for the evaluation of criterion. However, methodologies already developed by various researchers may also be used for evaluation of criteria, as is given by (Wani and Gandhi, 1999). An attribute (A_i) is therefore assigned a value on a scale e.g. 0 to 3 based on system design features for the attribute or using the existing scales from the literature. The attribute takes value 3, if the system feature favors feasibility issues of the nano-lubricant to the maximum extent. For example, the value for the attribute COF is given on the basis of table 2. The scale for each attribute is developed on the basis of the understanding of the nano-lubricant features. The degree of inter-relationship r_{ij} is also assigned value 0 to 3 if the degree of interrelationship for attribute i to attribute j is strong, it is assigned value 3. The value of r_{ij} is 0 if the degree of relationship is none. This is given in table. On the basis, appropriate value to r_{ij} is assigned. The values for degree of interrelationship among attribute in fact, depends on the type of analyzing lubricants. If the design alternative under consideration fulfills all the design requirements, the attribute is assigned the highest score value i.e., 3. However if a design alternative fulfills few of the design requirements, the given alternative is assigned the value 2. In case a design alternative does not fulfill any of the requirements, the attribute is assigned a value 0, for a design alternative under consideration. However the suggested score values as per table 2 will be useful for the user to start with. This assessment becomes easier for the team comprising of designers and other experts of life cycle engineers, rather than an individual. It is expected that a team or individual fully conversant with system will have no difficulty in assessment. Numerical value of permanent, i.e. VPF-n becomes a powerful means for feasibility and evaluation of a product as it contains various structure invariant of Nano-lubricant. An index called *Feasibility index*, is defined as numerical value of the permanent. This index is a measure of the feasibility of a particular alternative. Based on the index value the design alternatives are evaluated. The best alternative is the one having the highest index value and is selected. The alternatives are thus evaluated in feasibility index (I_{di}). The ideal value of the Feasibility Index is obtained from VPF-n i.e., expression (4) by taking the value of the diagonal elements to be equal to 3 i.e. the highest score value. The value of the off diagonal elements is obtained from table 2 on the basis of degree of relationship. The Ideal value of Feasibility Index is calculated to be $41.1602 x 10^4$. Comparison of the Feasibility Index value of the system can be relatively made with the ideal value (I_i). This comparison shows to what level Feasibility of the system is achieved of the ideal value. This is obtained as:

$$I_r = I_{di}/I_I \times 100\% \quad (5)$$

Where I_{di} represents the feasibility index of the i^{th} alternative and I_r is the relative index, which represents Feasibility value of the product as percentage of the ideal value of the index. This relative index provides designer qualitative information for improving Feasibility of the nano-lubricant. A scale 0-3 is proposed for assigning value to the attributes and their degree of relationship. The user may select an appropriate scale e.g. 0-5, 0-10 or 0-100. However it is desirable to select a lower scale value to obtain manageable value of index and also to reduce subjectivity. The lower scale is obtained so that there is a limited range of score values available to the designers for uniform evaluation of the design alternative from Feasibility point of view. The final result will not be changed it the user chooses a different scale. The relative Feasibility Index i.e. equation (5) will be useful in this regard.

6. Steps in a Feasibility Analysis and Evaluation Using Digraph and Matrix Approach

The procedure proposed previously for Feasibility Analysis and Evaluation of the index for a Nano-Lubricant is given now. Consider the given number of nano-Lubricants and mark them as (q=1, 2,.......). Study functions and details from feasibility analysis point of view.

1. List the nanolubricants to be considered for feasibility analysis, and name these as q_i , (i=1,2,3…Z).
2. Consider the first nanolubricant (i.e., q=1). Identify various attributes (A_i , i=1, 2, 3………, Z) of the Nanolubricant and also assign values to attributes i.e., V_i , i=1, 2, ...Z
3. Identify the relationship among attributes i.e., in terms of degree of relationship (r_{ij}). Assign value to r_{ij} using Table 2.
4. Develop NL^g for the Nano-Lubricant attributes.
5. Write $VNLA^{per}$. This will be Z X Z matrix with diagonal elements V_i’s and off diagonal elements r_{ij}’s.
6. Derive NL Expression (VPF-n) or permanent function i.e., permanent ($VNLA^{per}$) on the line of expression A=(A_1 , A_2 ,.....)
7. Evaluate the ideal value of Feasibility Index from VPF-n obtained in Step 5 by substituting V_i =3 and r_{ij} as obtained in step 3.
8. Use VPF-n and substitute the value of V_i and r_{ij} obtained in step 2^{nd} and 3^{rd} to evaluate Feasibility Index (I_{di}). Determine also the value of I, using equation (4).
9. Consider the 2^{nd} nanolubricant (i.e., q=2) and repeat step 2 to 5 and 7.
10. Carry out step 8 for all other alternatives i.e., q=3, 4,…..q
11. Calculate the Relative Feasibility Index, I_r.
12. Compare the Feasibility of all alternatives based on step 6 to 10 and identify the best alternative from the best feasible point of view.

7. Example

In this section an example of evaluating and selecting appropriate nano-lubricant for a tribosystem is considered for illustrating the procedure.

There are numerous nanolubricant alternatives available for tribosystems.However only two alternatives are considered here for feasibility analysis evaluation and comparison. These are Molybdenum Disulphide (MoS_2) and Graphite. This completes the first step of the procedure. As per step two of the procedure, various relevant attributes of nano-lubricant are identified and assigned value as discussed in section 2 of the methodology. These attributes are COF, COW, MC, TC, EC, EP and EV. The relationships among these attributes are given in table 2. As per table, values of r_{ij} 's are assigned. These are $\mathbf{r}_{12}$=3, $\mathbf{r}_{13}$=3, $\mathbf{r}_{14}$=0,$\mathbf{r}_{15}$=3, $\mathbf{r}_{16}$=2, $\mathbf{r}_{17}$=0, $\mathbf{r}_{21}$=3, $\mathbf{r}_{23}$=3, $\mathbf{r}_{24}$=1, $\mathbf{r}_{25}$=1, $\mathbf{r}_{26}$=3, $\mathbf{r}_{27}$=3, $\mathbf{r}_{31}$=3, $\mathbf{r}_{32}$=3, $\mathbf{r}_{34}$=2, $\mathbf{r}_{35}$=1, $\mathbf{r}_{36}$=2, $\mathbf{r}_{37}$=2, $\mathbf{r}_{41}$=0, $\mathbf{r}_{42}$=1, $\mathbf{r}_{43}$=2, $\mathbf{r}_{45}$=1, $\mathbf{r}_{46}$=1, $\mathbf{r}_{47}$=0, $\mathbf{r}_{51}$=3, $\mathbf{r}_{52}$=1, $\mathbf{r}_{53}$=1, $\mathbf{r}_{54}$=1, $\mathbf{r}_{56}$=2, $\mathbf{r}_{57}$=0, $\mathbf{r}_{61}$=2, $\mathbf{r}_{62}$=3, $\mathbf{r}_{63}$=2, $\mathbf{r}_{64}$=1, $\mathbf{r}_{65}$=2, $\mathbf{r}_{67}$=3, $\mathbf{r}_{71}$=0, $\mathbf{r}_{72}$=3, $\mathbf{r}_{73}$=2, $\mathbf{r}_{74}$=0, $\mathbf{r}_{75}$=0, $\mathbf{r}_{76}$=3. This helps to model NLAs in terms of NL^g. In this example NL^g remains same, as shown in Fig. 1. This completes step 4 of the procedure. The $VNLA^{per}$ for NL^g is obtained as

$$\begin{bmatrix} V_1 & 3 & 3 & 0 & 3 & 2 & 0 \\ 3 & V_2 & 3 & 1 & 1 & 3 & 3 \\ 3 & 3 & V_3 & 2 & 1 & 2 & 2 \\ 0 & 1 & 2 & V_4 & 1 & 1 & 0 \\ 3 & 1 & 1 & 1 & V_5 & 2 & 0 \\ 2 & 3 & 2 & 1 & 2 & V_6 & 3 \\ 0 & 3 & 2 & 0 & 0 & 3 & V_7 \end{bmatrix} \quad (6)$$

This completes step 5 of the procedure.

The ideal value of the index is obtained by substituting the ideal score values for all attributes (V_i =3) and the value r_{ij} from Table 2, in expression(6) as

$$\begin{bmatrix} 3 & 3 & 3 & 0 & 3 & 2 & 0 \\ 3 & 3 & 3 & 1 & 1 & 3 & 3 \\ 3 & 3 & 3 & 2 & 1 & 2 & 2 \\ 0 & 1 & 2 & 3 & 1 & 1 & 0 \\ 3 & 1 & 1 & 1 & 3 & 2 & 0 \\ 2 & 3 & 2 & 1 & 2 & 3 & 3 \\ 0 & 3 & 2 & 0 & 0 & 3 & 3 \end{bmatrix} \quad (7)$$

I_{ideal} =41.1602x10^4 is obtained from above matrix expression(7). This completes step 6 of the assessment procedure.

As per the procedure, the value of I_{ideal} for the first alternative (i.e. q=1), is evaluated. This is obtained by assigning the values to diagonal elements (V_i 's) on the basis of presence of relevant property value. In this case, the value of attribute V_1 is obtained by finding the value of co-efficient of friction (COF) of MoS_2 which is very less. Hence a highest score value is assigned to V_1 (i.e. 3). Secondly it is found that MoS_2 has a good wear resistance, hence COW is assigned a value V_2 =2. The third attribute MC is direclty related to wear and thermal conductivity, the former is small in case of MoS_2 whereas the latter is also low which is unfavourable and is thus assigned a value of 2. The fourth attribute TC is assigned a score value V_4 =2 as MoS_2 does not have a very good thermal conductivity. Since MoS_2 has a very low value of co-efficient of friction, thus it is at conserving energy, hence an attribute value of 3 is assigned. It is known that MoS_2 is not hazardous at all, thus a score value of 3 is assigned. It is known from the market survey that MoS_2 is a costly nano-lubricant as compared to other nanolubricants available, thus a score value of 1 is assigned. Thus the assigned score values are V_1=3, V_2=2, V_3=2, V_4=2, V_5=3, V_6=3, V_7=1. Thus

$$I_{d1} = \begin{bmatrix} 3 & 3 & 3 & 0 & 3 & 2 & 0 \\ 3 & 2 & 3 & 1 & 1 & 3 & 3 \\ 3 & 3 & 2 & 2 & 1 & 2 & 2 \\ 0 & 1 & 2 & 2 & 1 & 1 & 0 \\ 3 & 1 & 1 & 1 & 3 & 2 & 0 \\ 2 & 3 & 2 & 1 & 2 & 3 & 3 \\ 0 & 3 & 2 & 0 & 0 & 3 & 1 \end{bmatrix} \quad (8)$$

Therefore I_{d1} =21.7368x10^4. The value of I_r in this case is 52.8%. This completes step 7.

The second alternative (Graphite) is considered now and the values of its attributes are assigned on the basis of several aspects as followed in alternative 1. The assigned values are V_1=2, V_2=2, V_3=3, V_4=1, V_5=2, V_6=2, V_7=3.Thus I_{d2} is obtained as

$$\begin{bmatrix} 2 & 3 & 3 & 0 & 3 & 2 & 0 \\ 3 & 2 & 3 & 1 & 1 & 3 & 3 \\ 3 & 3 & 3 & 2 & 1 & 2 & 2 \\ 0 & 1 & 2 & 1 & 1 & 1 & 0 \\ 3 & 1 & 1 & 1 & 2 & 2 & 0 \\ 2 & 3 & 2 & 1 & 2 & 2 & 3 \\ 0 & 3 & 2 & 0 & 0 & 3 & 3 \end{bmatrix} \quad (9)$$

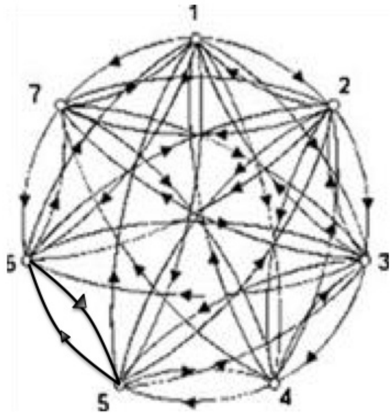
Therefore I_{d2} = 17.8841x10^4. The value of I_r is 43.44%.

This completes the step 9 of the procedure. It is observed that the Feasibility Index decreases from 21.7368x10^4 to 17.8841x10^4. For two alternatives considered here in this example. The relative index also decreases from 52.8% to 43.4%. The alternative with highest value of I_d = 21.7368x10^4 is considered as the best alternative from feasibility point of view.Therefore in this case, alternative-I, i.e. Molybdenum Disulphide is the best alternative amoung the two. The Feasibility analysis is therefore very vital for the product's performance throughout its operation.

This simple example has been elaborated for the benefit of readers. Although the procedure may appear troublesome and time consuming if performed manually, however it is not so when using a computer.

8. Conclusions

Feasibility analysis of nano-Lubricants is done in terms of a 'feasibility analysis attributes digraph' which considers the feasibility analysis attributes and degree of relationship among one another. The attributes of the nano-lubricants in general have been identified considering all important tribological aspects. The feasibility index (I_{di}) which evaluates the feasibility of nano-lubricant is obtained from VPF-n which in turn is obtained from one-to-one matrix of the digraph. The function is also useful in identification and comparison of product from feasibility analysis point of view. The influence of attributes individually or collectively is obtained using feasibility analysis matrix expression. The relative value of feasibility index (I_r) is also obtained and this is used to compare the relative decrease in the value of feasibility index from ideal value. The major limitation of the proposed approach is that with the increase in number of feasibility attributes, the digraph becomes complex. This problem, however, can be overcome by representing the digraph by a matrix. Another limitation of the proposed method is that evaluation of feasibility index is subjective in nature. The proposed procedure is useful for designers and practicing engineers to compare various alternatives of a nano-lubricant from feasibility point of view.



1. COF
2. COW
3. MC
4. TC
5. EC
6. EP
7. EV

Figure 1. Nanolubricant attributes digraph

Table 1. Nano-Lubricant Attributes

S.No.	Requirements of Nano-lubricants	Tribo-features facilitating requirements of nano-lubricants	Nano-lubricant Attribute
1	Low friction, Low energy loss, high wear resistance, minimum power /energy consumption, minimum environmental hazards, minimum wastage, no carcinogenesis, minimum ecotoxicity, easy reclamation at low cost.	Low Co-efficient of friction, viscosity and viscosity retention, surface roughness, surface hardness, micro-hardness, operating speed, embed ability, conformability, compatibility, less toxicity, eco-friendly and biodegradable.	Co-efficient of Friction (COF)
2	Long life of tribo-system, high wear resistance, low wear, high heat dissipation, minimum environmental hazards, high surface finish, minimum wastage, minimum ecotoxicity, Cost effectiveness.	Low Co-efficient of wear, Low Co-efficient of friction, Higher micro hardness, surface hardness, micro-structure, high thermal conductivity, compatibility, eco-friendly and bio-degradable nano-lubricants.	Co-efficient of Wear (COW)
3	Longevity materials, High wear resistance, minimum power consumption, high heat dissipation, minimum environmental hazards, high surface finish, minimum wastage, dimensional stability, no carcinogenesis, minimum ecotoxicity.	Low Co-efficient of wear, Low Co-efficient of friction, Higher micro hardness, surface hardness, high thermal conductivity, compatibility, embed ability, eco-friendly and bio-degradable materials, Low Coefficient of thermal expansion.	Material Conservation (MC)
4	High heat dissipation capacity, minimum thermal expansion, thermally stable, minimum energy consumption, environmentally safe, should not produce green house gases, low coefficient of friction.	Low co-efficient of thermal expansion, good heat carrying capability, should not suffer thermal strain, environmentally safe, biodegradable, low coefficient of friction.	Thermal Conductivity (TC)
5	Minimum co-efficient of friction, light weight, minimum energy requirement for production, minimum power consumption, minimum energy requirement for marketing, service and disposal/recycling	Good lubrication, better surface coating, low density, bio-friendly materials, light weight,	Energy Conservation (EC)
6	Less emission of gases during manufacture/ processing, minimum wastage, ease of disposal/recycle, minimum energy consumption, ease of production, easy surface coating, bio-degradability, renewability	Contamination, wear, operating temperature, less emission of toxic gases, bio-degradable nano-lubricants.	Environmental preservation (EP)
7	Cost-effective reclamation, easy to recycle/dispose, easy to coat on surface, minimum environmental hazards during recycling, retention of properties	Higher efficient methods of recycling/reclamation, sustainable materials, lower weight, longevity materials, reclamation of lubricants, biodegradable nano-lubricants, etc.	Economic Viability (EV)

Table 2. Nanolubricant attributes and their relationship

S.No.	Feasibility Attribute	Abbreviation	Degree of Relationship among Attributes			
			Strong (r_{ij}=3)	Medium (r_{ij}=2)	Weak (r_{ij}=1)	None (r_{ij}=0)
1.	Co-efficient of Friction	COF	2, 3, 5	6		7,4
2.	Co-efficient of Wear	COW	1, 3,7,6	-	5, 4	-
3.	Material Conservation	MC	1, 2	4, 6, 7	5	-
4.	Thermal Conductivity	TC	-	3	2, 5, 6	1, 7
5.	Energy Conservation	EC	1	6	3, 4, 2	7
6.	Environmental Preservation	EP	7, 2	1, 3, 5	4	-
7.	Economic Viability	EV	6, 2	3	-	1, 5, 4

$\mathbf{r}_{12}=3, \mathbf{r}_{13}=3, \mathbf{r}_{14}=0, \mathbf{r}_{15}=3, \mathbf{r}_{16}=2, \mathbf{r}_{17}=0$
$\mathbf{r}_{21}=3, \mathbf{r}_{23}=3, \mathbf{r}_{24}=1, \mathbf{r}_{25}=1, \mathbf{r}_{26}=3, \mathbf{r}_{27}=3$
$\mathbf{r}_{31}=3, \mathbf{r}_{32}=3, \mathbf{r}_{34}=2, \mathbf{r}_{35}=1, \mathbf{r}_{36}=2, \mathbf{r}_{37}=2$
$\mathbf{r}_{41}=0, \mathbf{r}_{42}=1, \mathbf{r}_{43}=2, \mathbf{r}_{45}=1, \mathbf{r}_{46}=1, \mathbf{r}_{47}=0$
$\mathbf{r}_{51}=3, \mathbf{r}_{52}=1, \mathbf{r}_{53}=1, \mathbf{r}_{54}=1, \mathbf{r}_{56}=2, \mathbf{r}_{57}=0$
$\mathbf{r}_{61}=2, \mathbf{r}_{62}=3, \mathbf{r}_{63}=2, \mathbf{r}_{64}=1, \mathbf{r}_{65}=2, \mathbf{r}_{67}=3$
$\mathbf{r}_{71}=0, \mathbf{r}_{72}=3, \mathbf{r}_{73}=2, \mathbf{r}_{74}=0, \mathbf{r}_{75}=0, \mathbf{r}_{76}=3$

REFERENCES

[1] Qiu, S., Zhou, Z., Dong, J., Chen, G. 1999. Preparation of Ni nanoparticles and evaluation of their tribological properties as potential additives in oils. J.Tribol, 123(3): 441-443

[2] Liu, W., Chen, S. 2000. An investigation of tribological behaviour of surface modified ZnS nanoparticles in liquid paraffin. Wear, 238(2): 120-124

[3] Tenne, R., Homyonfer, M., Feldman, Y.1998. Nanoparticles of layered compounds with hollow cage structure, Chem. Mater, 10(11): 3225-3228

[4] Thomas, P., Delbe, K., Himmel, D., Mansot, J.L., Guerin, K., Dubois, M., Delabarre, C., Hamwi, A. 2006.Tribological properties of low temperature graphite fluorides. Influence of structure on the lubricating performances. Journal of Physics and Chemistry of Solids, 67: (n° 5-6), 1095-1099

[5] Mansot, J.L., Bercion, Y., Romana, L., Martin, J.M. 2009. Nanolubrication. Brazilian Journal of Physics, 39:186-197

[6] Paramasivam, V., Senthil, V. 2009. Analysis and evaluation of product design through design aspects using digraph and matrix approach. International Journal on Interactive Design and Manufacturing, 3(1): 13-23

[7] Anand, A., Wani, M.F. 2010. Product life-cycle modeling and evaluation at the conceptual design stage: A digraph and matrix approach. Journal of Mechanical Design, Transactions of the ASME, 132(9): 0910101-0910109

[8] Rao, R.V. , Padmanabhan, K.K. 2010. Selection of best product end-of-life scenario using digraph and matrix methods. Journal of Engineering Design, 21(4): 455-472

[9] Ankush Anand, Mohd. Farooq Wani. Design Evaluation of Mechanical Brakes at System Conceptual Design Stage: A MADM Approach. Proceedings of the 2012 International Conference on Industrial Engineering and Operations Management Istanbul, Turkey, July 3 – 6, 2012

[10] Bowden, F.P., Tabor, D. The Friction and Lubrication of Solids Parts I & II. Ist and 2nd ed. Clarendon: Oxford;1950 and 1964

[11] Blackman, G.S., Mate, C.M., Philpott, M.R.1990. Interaction forces of a sharp tungsten tip with molecular films on silicon surfaces. Phys. Rev.Lett, 65(18): 2270-2273

[12] Yoshizawa, H., Chen, Y.L., Isrealachvili, Y.L. 1993. Nanotribology: Friction, wear and lubrication at the atomic scale. Science, 259:1305-1308

[13] Buzio, R., Boragno, C., Valbusa, U. 2006. Friction laws for lubricated nanocontacts. J.Chem.Phys, 125, 094708

[14] Quinn, T.F.J.1984. The role of wear in failure of common tribo-systems. Wear, 100(2): 399-436

[15] Bartz, W.J. 1994. Long-life and life-time lubrication possibilities and limitations. Lubr. Engng, 49(7): 518-524

[16] Zhang, Wenyang, Demydov, Dmytro, Jahan, Muhammad, P., Mistry, Kuldeep, Erdemir, Ali, Malshe, Ajay P. 2012. Fundamental understanding of the tribological and thermal behaviour of Ag-MoS_2 nano-particle-based multi-component lubricating system. Wear : 9-16

[17] Dake, L.S., Russell, J.A., Debrodt, D.C. 1986. A review of DOE ECUT tribology surveys. ASME. J.Tribol, 108(3): 447-501

[18] Glaeser, W.A., Erickson, R.C., Dufrance, K.F., Kannel, J.W.1992. Tribology: the science of combating wear. Lubr.Engng, 48: 867-871

[19] Kato, K., Ito, K. 2005.Modern tribology in life cycle assessment.Tribol. Interf. Engng. Ser, 48: 495-506

[20] Wani, M.F., Anand, A. 2010. Life cycle assessment modeling and life-cycle assessment evaluation of a tribo-element. J.Tribol, 244:1209-1220

[21] Paul, K.T. A low cost environmentally benign waste lubricant recycling/refining technology. EPA Project report 1997

[22] Kimura, Y., Tanaka, M., Enomoto, Y. 1996. Tribology research in Japan- a state of the art report. Proc. IMechE. Part

J: J.Engineering ofTribology, 210(3): 159-172

[23] Jangra, K., Grover, S., Chan, F.T.S. Aggarwal, A. 2011. Digraph and matrix method to evaluate the machinability of tungsten carbide composite with wire EDM. International Journal of Advanced Manufacturing Technology, 56(9-12): 959-974

[24] Aju Kumar, V.N., Gandhi, O.P. 2011. Quantification of human error in maintenance using graph theory and matrix approach. Quality and Reliability Engineering International, 27 (8): 1145-1172

[25] Hetmańczyk, M.P. 2013. The reliability model of AC-asynchronous drive based on the multilevel prognosis system supported by matrices and digraphs methods. Diffusion and Defect Data Pt.B: Solid State Phenomena, 199: 85-90