

A First-Order Mass Formula for Quarks in Terms of Constituent Preons

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Abstract Preon masses are predicted using a first-order mass formula. The model assumes there are six preons (D, U, S, C, B, and T) that combine to form the three generations of quarks. The two lightest preon masses (U and D) and two model parameters (δ and ξ) are derived from the $u \rightarrow d + W^+$ quark vertex and an assumed relation between the U and D preons. The remaining preon masses are determined from the model parameters and the s, c, b, and t quark masses.

Keywords First-order mass formula, Quark masses, Exotic Particles

1. Introduction

The concept that fundamental particles have quark substructures [1, 2] was introduced in 1964. Following the success of the quark model, other models postulated the existence of substructures of quarks and other particles (e.g., electrons, neutrinos, and force carrying bosons) [3-11]. These models offer the possibility for a unified theory of elementary particles and include the TCA model [3], a rishon model [4, 5], braded networks [7], a Yang-Mills approach [8], an ABC model [10], and a Knot theory algebra $SL_q(2)$ model for spin $\frac{1}{2}$ particles [11]. [6] provides a general discussion of the various preon models. All preon models do not predict specific mass values. Preon masses are noted if the model predicts a specific value.

The TCA model [3] provides relationships between the masses of the Higgs scalar, weak vector bosons, leptons and quarks or subquarks. These relations suggest the existence of much heavier leptons and/or quarks whose masses approach or exceed the weak vector-boson masses. The existence of heavy subquarks whose pair production threshold is close to the weak vector-boson masses are also postulated in the TCA model. TCA model preon masses are on the order of $W^\pm/\sqrt{3}$ or about $46 \text{ GeV}/c^2$.

The rishon model (RM), developed by Harari [4] and Shupe [5], includes two types of fundamental particles called rishons. Rishons include the T having an electric charge of $e/3$ and V that is electrically neutral. All leptons and 4 quark flavors are three-rishon ordered triplets. These three rishon

groups have total spin $1/2$. Additional discussion of the RM model is provided by Żenczykowski [9].

In [7], particles are defined as excitations of quantum geometry that are represented by braided ribbon networks. These braided ribbons are a generalization of spin networks in two parallel but complimentary models (i.e., the trivalent and tetravalent schemes). The trivalent scheme has been successful in establishing a correspondence between braids and Standard Model particles. The tetravalent model provides a dynamical theory of interactions and propagation of braids that are governed by topological conservation laws. This approach can also be interpreted in terms of preon-like structures.

Bevelacqua [8] formulated a geometric approach based on Koch Polygons and Yang-Mills methodology to provide an order of magnitude estimate for preon masses. Koch polygon preons were used as the candidate Yang-Mills fermions, and the preon masses were estimated to be $< 100 \text{ GeV}/c^2$.

The ABC Preon model [10] includes three preons (A, B, and C), their antiparticles, the photon, and the neutrino. Each preon is assigned a neutrino charge, and the composite particles have a total neutrino charge of zero. Preons are assumed to be bound by a force carried by the neutrino, and the A, B, and C preons have masses of 45.6, 34.8, and 67.9 GeV/c^2 , respectively. The total spin of a composite ABC Preon model particle is the sum of the spins of the constituent preons plus the spin of any binding neutrinos.

A Knot theory algebra approach investigated the properties of a preon model for the substructure of the standard model quarks and leptons [11]. Both local and global group representations for the model particles were provided. $SL_q(2)$ is demonstrated to be applicable to a Knot theory algebra preon model.

The success of these models has been limited, because none of the postulated preons has been observed. Another issue with spin $\frac{1}{2}$ preons derives from the assumed Fermi

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Published online at <http://journal.sapub.org/jnpp>

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statistics that permit the existence of total spin $\frac{1}{2}$ and $\frac{3}{2}$ particles. Although the spin $\frac{1}{2}$ states are usually interpreted to be quarks or other observed fermions, the spin $\frac{3}{2}$ states have not been observed.

Given these issues, this paper proposes a preon model derived from a first-order mass formula. The model results provide specific preon mass values, a possible explanation for the absence of spin $\frac{3}{2}$ particles, and a possible symmetry argument regarding preon interaction characteristics.

The preon masses are derived from quark masses, and the model assumes that six preons (D, U, S, C, B, and T) form the six known quarks (d, u, s, c, b, and t). The proposed approach is consistent with previous preon models in that a number of assumptions must be made to formulate the model. The validity of these assumptions can only be validated if the respective model predictions are observed.

2. Proposed Preon Model

The proposed preon model assumes that each quark is composed of three spin $\frac{1}{2}$ preons. In its ground state, a quark has zero total orbital angular momentum and total angular momentum of $\frac{1}{2}$. Excited states would have higher spin states including the aforementioned $\frac{3}{2}$ states that have not been observed.

Preon masses are derived from a first-order mass formula that was based on the work of Zel'dovich and Sakharov [12, 13]. Zel'dovich and Sakharov [12, 13] proposed semiempirical mass formulas that provide predictions for mesons and baryons in terms of effective quark masses. Within this formulation, quark wave functions are assumed to reside in their lowest 1S state. These mass formulas were used as the basis for deriving first-order tetraquark, pentaquark, and hexaquark mass formulae [14-17]. In a similar manner, the first-order mass formula can also be applied to predict preon masses based on quark masses. Using this approach, a quark mass M_i is given by

$$M_i = \delta + \sum_{j=1}^3 m_j^i + \frac{\xi}{3} \left(\begin{array}{l} \frac{m_0^2}{m_1^i m_2^i} \sigma_1 \cdot \sigma_2 \\ + \frac{m_0^2}{m_1^i m_3^i} \sigma_1 \cdot \sigma_3 \\ + \frac{m_0^2}{m_2^i m_3^i} \sigma_2 \cdot \sigma_3 \end{array} \right) \quad (1)$$

where δ and ξ are parameters to be determined, m_j^i labels the i th preon type (i.e., D, U, S, C, B, and T) for the three preons ($j=1, 2,$ and 3) that comprises the quark, m_0 is the average mass of first generation preons (D and U), and σ_k ($k = 1, 2,$ and 3) are the spin vectors for the preons incorporated into the quark. The last term in Eq. (1) represents the spin-spin interaction of the preons, and $\sigma_k \cdot \sigma_l$ is the scalar product of the preon spin vectors. $\sigma_k \cdot \sigma_l$ has the values of $-3/4$ and $+1/4$ for pseudoscalar and vector preon

configurations, respectively. The total angular momentum of the final quark state is obtained by summing the preon spins as described in [14-17].

In formulating the first-order preon mass formula, effective quark masses provided by Griffiths [18] are utilized. The effective masses for d, u, s, c, b, and t quarks are 340, 336, 486, 1550, 4730, and 177000 MeV/c², respectively. These masses are utilized in Eq. (1).

These six quarks are arranged in three generations: [d(-1/3 e), u(+2/3 e)], [s(-1/3 e), c(+2/3 e)], and [b(-1/3 e), t(+2/3 e)] [19]. The three generations are specified by the square brackets, and the quark charges are given within parentheses. Eq. 1 is solved for the preon masses given the aforementioned quark masses.

In a similar manner, the preons are arranged in three generations: [D, U], [S, C], and [B, T]. These six preon types are used in Eq. 1, and are assumed to have the following spin and charge assignments (i.e., D(1/2, -e/9), U(1/2, +2e/9), S(1/2, -e/9), C(1/2, +2e/9), B(1/2, -e/9), T(1/2, +2e/9)).

In order to determine the preon masses, it is necessary to impose a number of assumptions. The validity of these assumptions can only be evaluated if preons are experimentally observed. Accordingly, the following assumptions are made to specify the proposed preon model:

1. The quarks are comprised of preons of the same type. For example, the u quark is defined by 3 U preons and the s quark is comprised of 3 S preons.
2. There are 6 preons types: D, U, S, C, B, and T.
3. The preons are in a relative S state in the mass formula of Eq. 1.
4. The mass (m) of the U quark is derived from the $u \rightarrow d + W^+$ vertex, but the mass of the d quark is ignored in Eq. (2):

$$m_U = m_{W^+} / 3 \quad (2)$$

Ignoring the small d quark mass (0.340 GeV/c²) [18] relative to the W+ mass (80.385 GeV/c²) [19] is insignificant relative to the uncertainty associated with the preon model assumptions and quark mass values.

5. The mass of the D preon is related to the U preon and the u and d quark masses as follows

$$m_D = \left(\frac{M_d}{M_u} \right) m_U \quad (3)$$

6. The δ and ξ parameters are determined from Eq. 1 using the U and D preon masses defined in assumptions 4 and 5.
7. The quantity m_0 is defined in a manner that is analogous to the first-order quark mass formula¹⁴⁻¹⁷:

$$m_0 = \frac{m_U + m_D}{2} \quad (4)$$

8. The S, C, B, and T preon masses are determined from Eq. 1, the U and D preon masses, the masses of the s, c, b, and t quarks, and the δ and ξ parameters.

3. Results and Discussion

Using the approach noted in Section 2.0, the requisite δ and ξ parameters were determined to be -120.79 and $-53.680 \text{ GeV}/c^2$, respectively. The preon masses were derived as noted in the previous section. The resulting D, U, S, C, B, and T masses are 27.114 , 26.954 , 27.894 , 28.802 , 32.748 , and $98.253 \text{ GeV}/c^2$, respectively. These values are of the same order of magnitude as previous preon mass estimates [3, 8, 10].

Using Eq. 1, the masses of representative three preon combinations that form quark (e.g., UUU) and analogue structures (e.g., UUS) are summarized in Table 1. Nearly degenerate states are omitted from Table 1. For example, only the state UUU is presented because states UUD and UDD have similar masses. In addition, all degenerate states of a particular configuration (e.g., UUD, UDU, and DUU) are not provided.

The number of significant figures presented in Table 1 is provided to distinguish between the predicted masses. The model does not have this level of accuracy.

The predicted quark and analogue structures are clustered in distinct bands that depend on the mass of their constituent preons. Some states cluster around the known d, u, s, and c quark structures. The t quark is unique and is separated by about $60 \text{ GeV}/c^2$ from other analogue structures. This is attributed to the large mass of the T preon which forms the t quark with its postulated TTT structure.

There are also a number of states that lie between the effective d, u, s, c, b, and t quark masses [18]. These quark analogue structures have not been observed and that fact casts doubt on the model proposed in this paper. Possible explanations for these analogue states are addressed in subsequent discussion.

Table 1. Quark and Quark Analogue States Derived from Three Preons

Preon Configuration			Quark and Quark Analogue State Mass (GeV/c^2)
U	U	U	.33601 ^a
D	D	D	.34000 ^b
U	U	S	.36492
U	D	S	.37066
D	D	S	.37829
U	S	S	.41491
D	S	S	.42696
S	S	S	.48600 ^c
U	U	C	.60245
U	D	C	.61802
D	D	C	.63547
U	S	C	.68535
D	S	C	.70721
S	S	C	.78934
U	C	C	1.00711

Preon Configuration			Quark and Quark Analogue State Mass (GeV/c^2)
D	C	C	1.03880
S	C	C	1.14400
U	U	B	1.35185
U	D	B	1.38035
D	D	B	1.41073
U	S	B	1.47805
D	S	B	1.51284
C	C	C	1.55000 ^d
S	S	B	1.62534
U	C	B	1.86737
D	C	B	1.91198
S	C	B	2.04755
C	C	B	2.52110
U	B	B	2.81652
D	B	B	2.87406
S	B	B	3.04000
C	B	B	3.58110
B	B	B	4.73000 ^e
U	U	T	52.04077
U	D	T	52.15641
D	D	T	52.27394
U	S	T	52.45888
D	S	T	52.58083
S	S	T	52.89809
U	C	T	53.30364
D	C	T	53.43541
S	C	T	53.77575
C	C	T	54.70475
U	B	T	54.85215
D	B	T	54.99685
S	B	T	55.36756
C	B	T	56.36410
B	B	T	58.11236
U	T	T	110.92876
D	T	T	111.16061
S	T	T	111.73609
C	T	T	113.18808
B	T	T	115.53569
T	T	T	177.00000 ^f

^a Configuration corresponds to the first generation u quark.

^b Configuration corresponds to the first generation d quark.

^c Configuration corresponds to the second generation s quark.

^d Configuration corresponds to the second generation c quark.

^e Configuration corresponds to the third generation b quark.

^f Configuration corresponds to the third generation t quark.

4. Model Issues and Possible Explanations for Analogue Structures

Although the model's quark structures consisting of three identical preons are consistent with current observations, the other structures, noted in Table 1, have not been observed. These quark analogue states present a definitive challenge to the validity of the proposed first-order mass formula. The unobserved states suggest that either the proposed mass formula is invalid or preon configurations are governed by a preon quantum chromodynamic (PQCD) interaction that is repulsive for different preon types and attractive only if the preons are the same. PQCD would be an analogue of the QCD interaction and its $SU(3)_C$ symmetry characteristics.

This could suggest that configurations consisting of other than three identical preons are governed by a repulsive PQCD interaction that precludes their existence. Although this postulate could be used to formulate a PQCD interaction model, it is speculative and not based on observed data. The PQCD interaction would be analogous to QCD, but would involve a unique set of interactions with associated symmetry.

The $SU(3)_C$ symmetry for QCD suggests that PQCD could be based on $SU(3)_{PC}$ or a higher order symmetry. A discussion of the explicit nature of PQCD is beyond the scope of this paper, and only general characteristics are inferred.

The lack of observed spin 3/2 states can be attributed to a PQCD interaction that is attractive for spin 1/2 states, but repulsive for spin 3/2 states. This is another assumption inherently contained within the proposed first-order preon mass formula. Its validity and the existence of a PQCD interaction will be determined by future experimental investigations.

5. Conclusions

Preon masses are derived from quark masses using a first-order mass formula. The masses derived from the first-order formula are similar to previous theoretical preon predictions. However, the preons presented in this work only apply to quarks and analogue quark structures and have not been applied to leptons or bosons.

The lack of data supporting preon model predictions remains an objection to the acceptance of these models. However, preon models do suggest physics beyond the Standard Model and their theoretical investigation indicate states that will either confirm or refute their validity. The first-order mass formula preon model could also suggest additional symmetries and provide a rudimentary understanding of interactions governing preons.

ACKNOWLEDGEMENTS

The author acknowledges the assistance of Kevin Fishburne (kevinfishburne@gmail.com) of Eight Virtues

Computers for designing and constructing the LINUX computer system used to perform the calculations summarized in Table 1.

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