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Calculations of Average Numbers of Prompt Neutrons for Actinide Photofission

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Abstract The empirical calculations of the prompt neutrons average number $\bar{\nu}$ for photofission of the ^{232}Th, ^{233}U, ^{234}U, ^{235}U, ^{236}U, ^{238}U, ^{237}Np, ^{239}Pu and ^{241}Am actinides has been done as a function of photon energy, mass and charge of the nuclei, which can be used to evaluate the $\bar{\nu}$ for photo-fission of nuclides for which no or scarce data are available.

Keywords Average number of prompt neutrons, Photofission, Actinides

1. Introduction

The average number of prompt neutrons is one of two (along with the fission cross section) major nuclear-physical parameters required for practical calculations. This value is determined in detail and accurately for neutron-induced reactions for the most nuclides. At the same time, the experimental data and evaluation of the average number of prompt neutrons in the case of photofission are much scarcer, old-fashioned or obtained by indirect mode [1, 2]. With ~~the~~ increasing interest in the methods of nuclear fuel burning and long-lived actinide decontamination the need in precise values of nuclear constants is evident during last years. This is especially true for photonuclear constants. Therefore, the search was focused on a formula that can be used to estimate the value of $\bar{\nu}$ for photofission of arbitrary actinide.

2. General Approach

Let's consider a general approach to modeling the average number of the prompt neutrons in the case of neutron activated fission. This value is a function of three variables: the mass of the compound nucleus A, its charge Z and the incident neutron energy E_n ($\bar{\nu} \equiv \bar{\nu}(A,Z,E_n)$).

In previous studies [3-5] it has been shown that expanding the charge, mass and energy dependence of $\bar{\nu}$ in the form of truncated Taylor series apparently yields a reasonable representation for $\bar{\nu}(A,Z,E_n)$ if the zero-, first- and one second-order cross term are kept in truncation for all isotopes, i. e. it is sufficient to use a linear approximation over all three variables, at least up to the threshold $(\gamma,2nf)$, taking into account the contribution of the even-odd effect in a common form:

$$\bar{\nu}(A,Z,E_n) = \bar{\nu}_0(A,Z) + a(A,Z)(E_n - E_0), \quad (1)$$

where the intercept

$$\bar{\nu}_0(A,Z) = C_1 + C_2(Z - Z_0) + C_3(A - A_0) + C_4 P(A,Z), \quad (2)$$

and slope

$$a(A,Z) = C_5 + C_6(Z - Z_0) + C_7(A - A_0) + C_8 P(A,Z), \quad (3)$$

$$P(A, Z) = 2 - (-1)^{A-Z} - (-1)^{Z}, \quad (4)$$

Z_0, A_0 and E_0 are the values, about which the expansion is to be made [3].

The coefficients C_i in equations (1) – (3) were evaluated by the least-square method [5]. A wide range of existing data for $\bar{\nu}(A,Z,E_n)$ for actinides from ^{232}Th to ^{245}Cm was analyzed and the best simplified parameterization of the slope and intercept was recently found in [6]:

$$\bar{\nu}(A,Z,E_n) = \bar{\nu}_0(Z) + a(A)E_n, \quad (5)$$

$$\bar{\nu}_0(Z) = -22{,}7734 + 0{,}27318\,Z, \quad (6)$$

$$A(A) = -0{,}1636 + 0{,}0013\,A. \quad (7)$$

These calculations were performed in the case of neutron-induced actinide fission without taking into account other reactions.

Generally speaking, all mentioned approaches have to be versatile and suitable for all other cases of fission, including photofission. Therefore, we applied the model (1-4) in the case of photofission, where $(E_n - E_0)$ is replaced by $(E_\gamma - E_s)$, E_γ – photon energy, E_s – nucleon separation energy.

To do this, we take into account the wide experimental data set of photofission $\bar{\nu}(A,Z,E_\gamma)$ in the energy range ~ 8 – 20 MeV for actinides ^{232}Th [7,8], and ~ 5 – 20 MeV for ^{233}U [9, 10], ^{234}U [9, 10], ^{235}U [7, 9], ^{236}U [7, 9], ^{238}U [7, 9], ^{237}Np [9, 10] and ^{239}Pu [9, 10]. We used all these 219 experimental points for our calculation.

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The values of the nucleon separation energy for each actinide were taken from [11, 12] and are presented in Table 1.

To evaluate the quality of linear approximation (1) – (4), modified for photofission, we calculate $\bar{\nu}$ for each actinide separately. The best possible energy approximation for $\bar{\nu}$ have to be lie within the linear approach [7, 10]:

$$\bar{\nu}\left(E_{\gamma}\right)=\sum_{j=1}^{N}\sum_{i=1}^{j} f_i\left(E_{\gamma}\right)\theta(n), \tag{8}$$

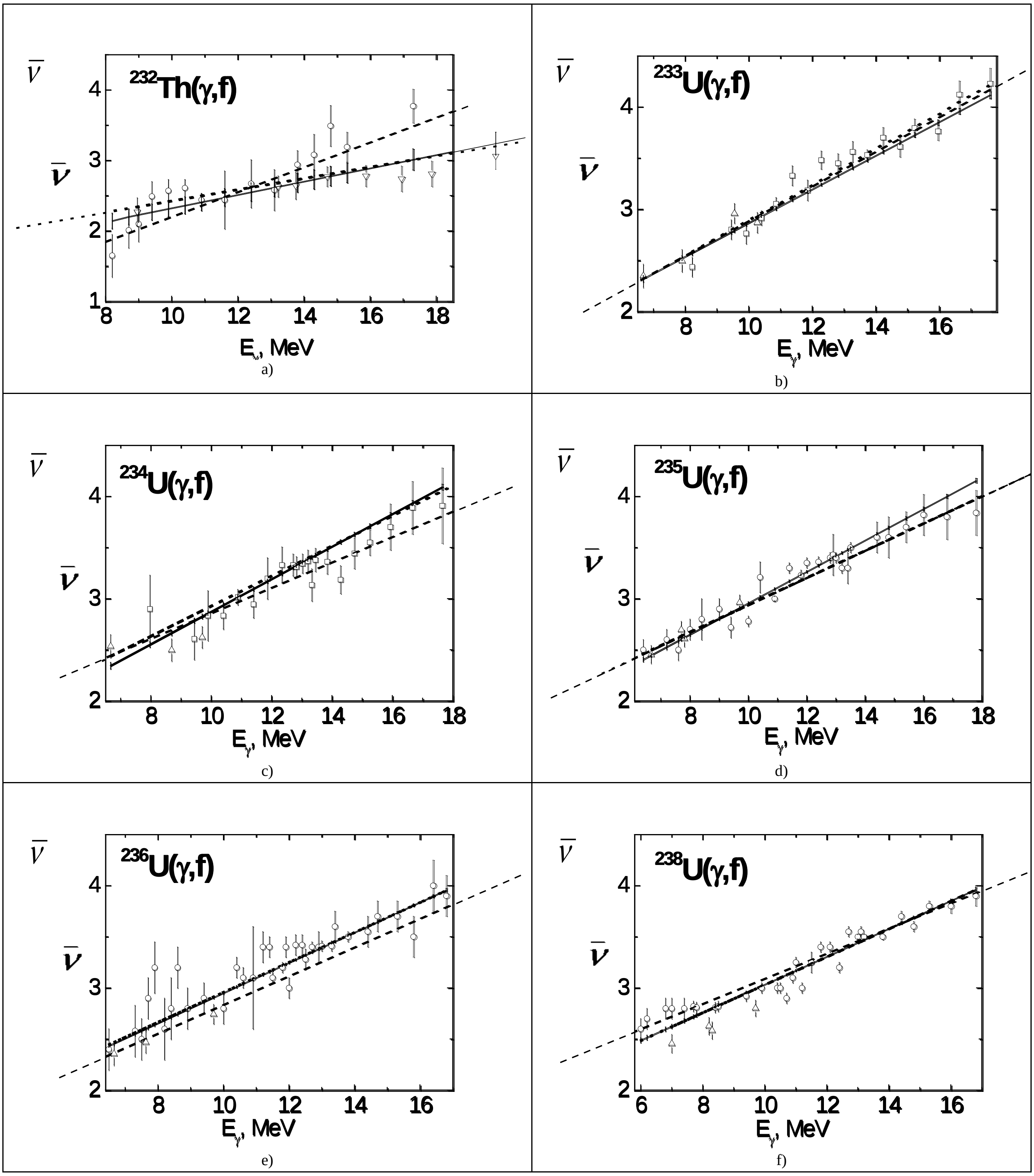

Figure 1. The results of fitting experimental data for average neutron multiplicities using (10)-(12) (solid lines) for ^{232}Th – a), ^{233}U – b), ^{234}U – c), ^{235}U – d), ^{236}U – e): open circles (○) from [7], down triangles (▽) from [8], up triangles (Δ) from [9]. Squares (□) – experimental data from [10]; dash lines – ENDF/B-VII.1 [14]. Dot line – formula (8)

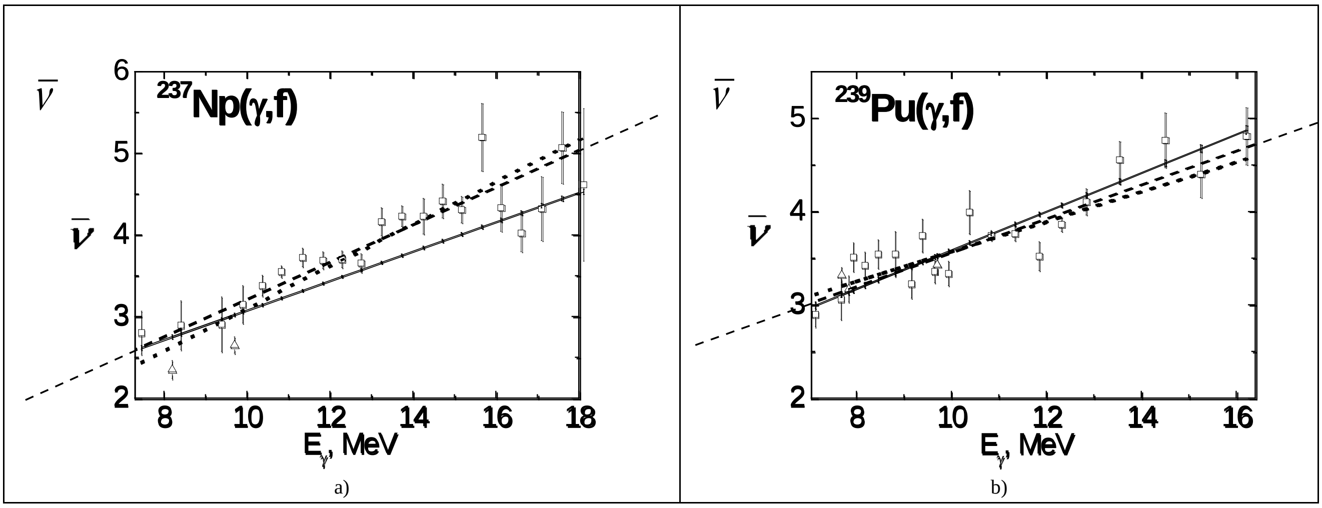

Figure 2. The results of fitting experimental data for average neutron multiplicities using (10)-(12) (solid lines) for ^{237}Np – a) and ^{239}Pu – b): open circles (○) from [7], down triangles (▽) from [8], up triangles (Δ) from [9]. Squares (□) – experimental data from [10]; dash lines – ENDF/B-VII.1 [14]. Dot line – formula (8)

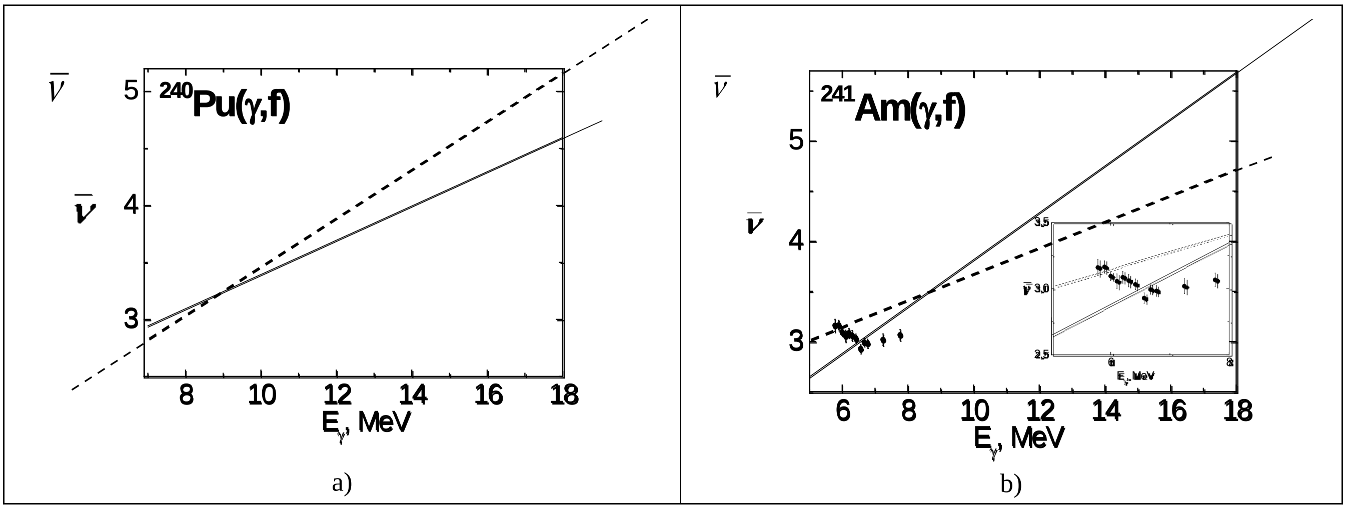

Figure 3. The results of the prediction of average neutron multiplicities for ^{240}Pu – a) and ^{241}Am –b) (solid lines), dash lines – ENDF/B-VII.1 [14], circles (●) – experimental data from [15]

Table 1. Nucleon separation energy

Actinide number	A	Z	E_s , MeV	References
1	232	90	6.438	[11]
2	233	92	5.760	[11]
3	234	92	6.844	[11]
4	235	92	5.298	[11]
5	236	92	6.545	[11]
6	237	93	6.580	[11]
7	238	92	6.152	[11]
8	239	94	5.647	[11]
9	240	94	6.534	[11]
10	241	95	6.647	[12]

i, j – actinide number, $n=i-j$, $N=8$, $\theta(n)$ - step function

$$\theta(n)=\begin{cases}0 & n<0\\ 1 & n\geq 0\end{cases},$$

$$f\left(E_\gamma\right)=x_i+y_i(E_\gamma-E_s),$$

for $E_i^{\min}\leq E_\gamma\leq E_i^{\max}$,

where $E_i^{\min}$ and $E_i^{\max}$ are the lower and upper bound of photon energy interval for respective *i*-th actinide, $E_\gamma-E_s>0$.

The common quality of the description of the data set for each actinide by (8) is evaluated by [13]:

$$\left\langle\chi^2/dof\right\rangle=\frac{\sum_{i=1}^{N}\frac{\chi_i^2}{n_{data_j}-m_{par}}}{N}, \qquad (9)$$

where each *i*-th interval out of total of N intervals contains *n* data points and two parameters give a resulting combination to χ_i^2 in fitting eq. (8) with result $<\chi^2/dof>$ = 2.1 (dot line in Fig. 1, 2). Consequently it is naturally that we have to build the most general formula describing the average prompt neutrons for photofission depending on the photon energy of all actinides, which is close to the best results of local linear calculation. As the initial formula, we chose that similar to (1):

$$\bar{\nu}(A,Z,E_\gamma) = \bar{\nu}_0(A,Z) + a(A,Z)(E_\gamma - E_s(A,Z)), \quad (10)$$

where the slope $\bar{\nu}_0(A,Z)$ and the intercept $a(A,Z)$ are chosen as (2) – (3).

The results are given in Fig. 1, 2 (solid line). Coefficients C_i were calculated by least-square method [14]. The final formula for calculating the average number of prompt neutrons for photofission of actinides is:

$$\bar{\nu}_0(A,Z)=(1{,}97 \pm 0{,}05) + (0{,}165 \pm 0{,}028)(Z-90) + (0{,}0341 \pm 0{,}0093)(A-232) - (0{,}0853 \pm 0{,}0094)\cdot P(A,Z), \quad (11)$$

$$a(A,Z)=(0{,}0963\pm0{,}75\cdot10^{-2})+(0{,}0371\pm0{,}43\cdot10^{-2})(Z-90)- (0{,}566 \pm 0{,}138)\cdot10^{-2}\cdot(A-232). \quad (12)$$

The quality of the description of the experimental data in this case: $\chi^2/dof = 2.5$.

This value is in accord with the best linear approximation for each actinide.

We also checked the quality of the recommended values from ENDF/B-VII-1 [15] (dash lines in Fig. 1, 2.) for the same data set. All the approaches are in accord with each other at least up to the threshold *(γ,2nf)*.

Also using formulas (10) – (12) we have calculated the photon energy dependence of $\bar{\nu}$ in the case of the ^{240}Pu and ^{241}Am [16] isotopes (see Fig. 3).

3. Conclusions

The empirical formula for estimation of the average number of prompt neutrons $\bar{\nu}$ as a linear function of the $(E_\gamma - E_s(A,Z))$ of actinides photofission was obtained. The formula contains 7 free parameters taking into account the even-odd effect. The quality of description of $\bar{\nu}$ by formulae (10) – (12) ($\chi^2/dof = 2.5$) is comparable (see Fig.1.) with that of a local description for each actinide separately ($\chi^2/dof = 2.1$). All this gives us enough reason to use the resulting formula to calculate the average number of prompt neutrons $\bar{\nu}$ for any actinide photofission.

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