Journal of Nuclear and Particle Physics 2015, 5(6): 97-100
DOI: 10.5923/j.jnpp.20150506.01

Theoretical Calculation of Plasma Thermal Energy Using the Solution of Equilibrium Problem

M. Asif

Department of Physics, COMSATS Institute of Information Technology, Lahore, Pakistan

Abstract In this work we presented the plasma thermal energy by using the solution of equilibrium problem with Lithium limiter for circular cross-section HT-7 tokamak. For this, the poloidal beta was obtained by analytical solution of the Grad-Shafranov equation (GSE) and then the plasma thermal energy is calculated. It is observed, the plasma thermal energy obtained from the analytical solution of GSE by using liquid lithium limiter is larger than that using graphite limiter, which shows that the plasma performance was improved.

Keywords Plasma, Thermal Energy, Equilibrium Problem

1. Introduction

In tokamaks the plasma configurations are described in terms of solutions of the Grad-Shafranov equation (GSE) [1-2]. Analytical solutions of the Grad-Shafranov [1-3] equation are very useful for theoretical studies of plasma equilibrium, transport and magnetohydrodynamic (MHD) stability. For tokamak operation, measurements of the poloidal beta β_p, plasma thermal energy and plasma temperature are important. Much more plasma parameters such as plasma confinement time, plasma current density profile, and magnetohydrodynamics (MHD) instabilities can be calculated.

As we know confinement is limited by thermal conduction and convection processes, but radiation is also a source of energy loss. Since maximum energy confinement time is determined by the microscopic behavior of the plasma as collisions, and micro instabilities. Such kind of behavior ultimately leads to macroscopic energy transport which can be either classical or anomalous depending on the processes involved.

In the absence of instabilities the confinement of toroidally symmetric tokamak plasma is determined by Coulomb collisions. Since these phenomena require knowledge of individual particle motion on short length and time scales, they are usually treated by kinetic models, but including only limited geometry because of the complexity of the physics. In the absence of a theoretical understanding of confinement, and given the need to predict the confinement properties of future tokamaks, it has been necessary to resort to empirical methods. The simplest of these is to accumulate data from a number of tokamaks, each operated under a range of conditions, and to use statistical methods to determine the dependence of the confinement time on the parameters involved. This provides scaling expressions which, within some error, allow extrapolation to projected tokamaks [1-15].

Lots of lithium experiments have been carried out in tokamaks for the enhancement of plasma [4-11]. In this work we calculated the the plasma thermal energy by the Simplest Grad–Shafranov Equation (GSE) Solution [12] with Lithium limiter for Circular cross-section HT-7 tokamak. A generalized Grad-Shafranov-type equation [3], has been used. Specific functional forms of plasma internal energy and current are used. GSE is solved by considering linear source functions and fixed boundary conditions. For this, the poloidal beta [13, 14] was obtained from by analytical solution of GSE. Than we can find the plasma thermal energy [15]. It was clearly observed in [14] that the calculated internal inductance and the calculated poloidal beta depend on kind of discharge or plasma current.

It is observed, the plasma thermal energy obtained from the analytical solution of GSE by using liquid lithium limiter is larger than that using graphite limiter, which shows that the plasma performance was improved. Also, the plasma thermal energy was measured using the diamagnetic loop [15].

2. Theoretical Equation

The simplest solution of GSE can be found by assuming that [12],

$$\mu_0(\gamma-1)\frac{\partial u}{\partial \psi} = -A_1,\ F\frac{\partial F}{\partial \psi} = A_2\ , \qquad (1)$$

* Corresponding author:
dr.muha.asif@gmail.com (M. Asif)
Published online at http://journal.sapub.org/jnpp

where A_1 and A_2 are constant. The internal energy in extended Grad-Shafranov equation [3] is a function of ψ. The $u(\psi)$ and $F(\psi)$ are two free functions, while μ_0 and J are the vacuum permeability and plasma current density respectively. This obviously reduces the set of possible current density profile shapes to $J_\phi \propto RA_1 - A_2/R$.

If the plasma is assumed to be up–down symmetric, its shape can be described by four parameters. The equatorial innermost and outermost points, R_i and R_0, and the coordinates of the highest point, (R_t, Z_t) or equivalently, the major radius $R_m = (R_i + R_0)/2$, $R_m \neq R_0$, the minor radius $a = (R_0 - R_i)/2$, the elongation $\kappa_0 = Z_t/a$, and triangularity $\delta = (R_0 - R_t)/2$.

The simplest solution is given by [12]

$$\psi = c_1 + c_2 R^2 + c_3(R^4 - 4R^2 Z^2) + c_4(R^2 \ln(R) - Z^2) - \frac{A_1}{8} R^4 - \frac{A_2}{2} Z^2. \tag{2}$$

For the determination these six coefficients, it is necessary to have six equations. We assume that the internal energy vanishes at the boundary, hence $\psi(R,Z)\big|_b = 0$ [16].

With Eq. (2), the boundary conditions $R = R_0 \pm a$, $Z = 0$ and $R = R_t$, $Z = Z_t$ gives the following equations:

$$\psi(R_i,0) = c_1 + c_2 R_i^{\,2} + c_3 R_i^{\,4} + c_4 R_i^{\,2} \ln(R_i) - \frac{A_1}{8} R_i^{\,4} = 0, \tag{3}$$

$$\psi(R_0,0) = c_1 + c_2 R_0^{\,2} + c_3 R_0^{\,4} + c_4 R_0^{\,2} \ln(R_0) - \frac{A_1}{8} R_0^{\,4} = 0, \tag{4}$$

$$\psi(R_t,Z_t) = c_1 + c_2 R_t^{\,2} + c_3(R_t^{\,4} - 4R_t^{\,2} Z_t^{\,2}) + c_4(R_t^{\,2} \ln(R_t) - Z_t^{\,2}) - \frac{A_1}{8} R_t^{\,4} - \frac{A_2}{2} Z_t^{\,2} = 0 \tag{5}$$

We also assume that the plasma is enclosed in a perfectly conducting toroidal boundary with circular cross section, with radius a, so the normal component of the magnetic field.

$$\frac{1}{R}\frac{\partial \psi(R_t,Z_t)}{\partial R} = 2c_2 + 4c_3(R_t^{\,2} - 2Z_t^{\,2}) + c_4(2\ln(R_t) + 1) - \frac{A_1}{2} R_t^{\,2} = B_z(R_t,Z_t) = 0 \tag{6}$$

The plasma current can be clearly measured by Rogowski coil [17], so the plasma current can be written.

$$2\pi\mu_0 I_p = \iint (RA_1 + A_2/2)\, dRdZ, \tag{7}$$

It is simpler to first solve for a plasma with unit current and unit major radius and use the scaling relations described above to find the final desired equilibrium. However, even in this simplest case only numerical solutions to Eqs. (3-7) have been found. The coefficients can be computed numerically, given a desired plasma description.

We also selected the constraint $\beta_p + l_i/2$, because the parameter can be experimentally deduced using discrete magnetic probes [17], for circular cross section HT-7 tokamak [18],

$$\beta_p + l_i/2 = \frac{\left(\oint dl\right)^2}{(2\pi\mu_0 I_p)^2 \iint RdRdZ} \times \left(2.5A_1 \iint \psi(R,Z) RdRdZ + 0.5A_2 \iint \frac{\psi(R,Z)}{R} dRdZ\right) \tag{8}$$

3. Calculation of Plasma Thermal Energy

The six coefficients c_1, c_2, c_3, c_4, A_1, A_2 can be derived by solving equations [3-8]. So the poloidal magnetic flux ψ is obtained by substituting the six coefficients.

According to the definition of the internal inductance [18]

$$l_i = \frac{\left(\oint dl\right)^2 \iint \psi j_\phi dRdZ}{2\pi\mu_0 I_p^2 \iint RdRdZ} \tag{9}$$

We can acquire the internal inductance with Solovev's assumption [18]

$$l_i = \frac{\left(\oint dl\right)^2}{\left(2\pi\mu_0 I_p\right)^2 \iint RdRdZ} \times \left(A_1 \iint \psi(R,Z) RdRdZ + A_2 \iint \frac{\psi(R,Z)}{R} dRdZ \right) \tag{10}$$

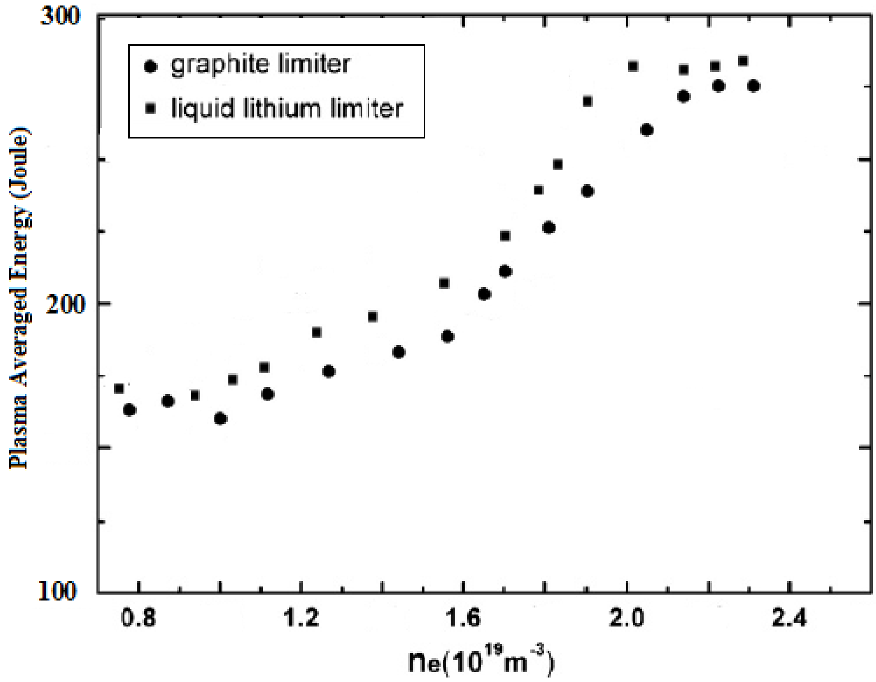

Figure 1. Plasma Averaged Thermal Energy, obtained analytically by the solution of GSE as a function of line averaged electron density

Also the poloidal beta with Solovev's assumption can be determine as follow [18]

$$\beta_p = \frac{\left(\oint dl\right)^2}{\left(2\pi\mu_0 I_p\right)^2 \iint RdRdZ} \left(2A_1 \iint \psi(R,Z) RdRdZ\right) \tag{11}$$

All the double integrals in the above written relations have to be performed over the total cross-section area of the plasma column for circular cross section HT-7 tokamak [17]. We can determine the volume-averaged plasma kinetic pressure $\langle p \rangle$, and then the plasma thermal energy U. $\langle p \rangle$ can be determined directly from the definition of the poloidal beta [18]:

$$\langle p \rangle = \beta_p \frac{B_\theta^2(a)}{2\mu_0} = \mu_0 \frac{I_p^2 \beta_p}{8\pi^2 a^2} \tag{12}$$

where a is the plasma minor radius. For the measurement of the plasma thermal energy, we start from the plasma state equation [15]:

$$\langle p \rangle = \sum_i n_i T_i = \frac{2}{3} \sum_i E_i = \frac{2}{3} E, \tag{13}$$

where subscript 'i' indicates the plasma species i and E indicates the plasma thermal energy density; therefore the plasma thermal energy U is obtained [15]:

$$U = \frac{3}{2}\left(\sum_\alpha n_\alpha T_\alpha\right) V = \frac{3}{2} \langle p \rangle V, \tag{14}$$

where V is the plasma volume.

Therefore, according to the above discussion, the poloidal beta [18] was obtained by analytical solution of GSE and then the plasma thermal energy is calculated. It was clearly observed in [14] that the calculated internal inductance and the calculated poloidal beta depend on kind of discharge or plasma current. It is observed from figure, the plasma thermal energy, obtained from the analytical solution of GSE by lithium limiter is larger than that using graphite limiter, which shows that the plasma performance was improved. Also, the plasma thermal energy was measured using the diamagnetic loop [11, 15, 17].

4. Result and Discussion

Here we calculated the plasma thermal energy by the Simplest Grad–Shafranov Equation (GSE) Solution with Lithium limiter for circular cross-section HT-7 tokamak. GSE is solved by considering linear source functions and fixed boundary conditions. For this, the poloidal beta [13, 14, 18] was obtained by analytical solution of GSE and then the plasma thermal energy is calculated.

It was clearly observed in [14] that the calculated internal inductance and the calculated poloidal beta depend on kind of discharge or plasma current. It is observed, the plasma thermal energy obtained from the analytical solution of GSE using liquid lithium limiter is larger than that using graphite limiter, which shows that the plasma performance was improved. These parameters included the poloidal beta and plasma thermal energy can be measured by experimental method based on a diamagnetic effect [11, 15, 17].

REFERENCES

[1] V.D. Shafranov, Sov. Phys. JETP., 6, 545, (1958).; Zh. Eksp. Teor. Fiz., 33,710 (1957) (in Russ.).

[2] L.E. Zakharov, V.D. Shafranov, Sov. Phys. Tech. Phys. 18, 151, (1973).

[3] M. Asif, Magnetohydrodynamics, 47, 11, (2011).

[4] D.K. Mansfield, K.W. Hill, J.D. Strachan, M.G. Bell, S.D. Scott, R. Budny, et al., Phys. Plasmas 3, 1892, (1996).

[5] G.Y. Antar, R.P. Doerner, R. Kaita, R. Majeski, J. Spaleta, T. Munsat, et al., Fusion Eng. Des. 60, 157, (2002).

[6] R. Kaita, R. Majeski, M. Boaz, P. Efthimion, G. Gettelfinger, T. Gray, et al., J. Nucl. Mater. 337, 872, (2005).

[7] R. Majeski, S. Jardin, R. Kaita, T. Gray, P. Marfuta, J. Spaleta, et al., Nucl. Fusion 45, 519, (2005).

[8] V. Pericoli-Ridolfini, M.L. Apicella, G. Mazzitelli, O. Tudisco, R. Zagórski, Plasma Phys. Control. Fusion 49, S123, (2007).

[9] V. Pericoli-Ridolfini, A. Alekseyev, B. Angelini, S.V. Annibaldi, M.L. Apicella, G. Apruzzese, et al., Nucl. Fusion 47, S608, (2007).

[10] S.V. Mirnov, E.A. Azizov, V.A. Evtikhin, V.B. Lazarev, I.E. Lyublinski, A.V. Vertkov, et al., Plasma Phys. Control. Fusion 48, 821, (2006).

[11] J.S. Hu, G.Z. Zuo a , J.G. Li a , N.C. Luo a , L.E. Zakharov b , L. Zhang a, W. Zhang a, P. Xu, Fusion Engineering and Design 85, 930, (2010).

[12] S. B. Zheng, A. J. Wootton, E. R. Solano, Phys. Plasmas 3, 1176, (1996).

[13] M. Asif, Revista Mexicana de Fısica 59, 517, (2013).

[14] A. Rahimi-Rad et al., J. Fusion Energ. 32, 405, (2013).

[15] A. Salar Elahi and M. Ghoranneviss, Journal of Nuclear and Particle Physics 3, 145, (2013).

[16] E. K. Maschke, Plasmas Physics 15, 535, (1972).

[17] B. Shen et al. Review of Scientific Instruments 78, 093501, (2007).

[18] C. V. Atanasiu et al., Phys. Plasmas 11, 3510, (2004).