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Dependence of Effective Edge Safety Factor on the Energy Confinement Time by Using Equilibrium Problem

M. Asif

Department of Physics, COMSATS Institute of Information Technology, Lahore, Pakistan

Abstract In this work we present the dependence of effective edge safety factor on the energy confinement time with Lithium limiter for circular cross-section HT-7 tokamak. For this purpose, the Shafranov parameter was obtained from analytical solution of GSE. Therefore, effective edge safety factor is calculated. Than we can find the plasma energy confinement time. It is observed, the maximum energy confinement time relate to the low values of effective edge safety factor ($2.6\langle q_{eff}(a)\langle 3$).

Keywords Tokamak, Energy confinement time, Effective edge safety factor

1. Introduction

To achieve the thermonuclear condition (the ignition condition) in a tokamaks [1-10] $(nT\tau_E > 3\times10^{21} m^{-3} keVs)$, it is necessary to confine the plasma for a sufficient long time. The global energy confinement time τ_E, is defined by $\int \frac{3}{2} n(T_e+T_i)\frac{d^3x}{P}$, where n is the plasma density, T_e *and* T_i are the electron and ion plasma temperature and P is the total input power [11-12].

The main purpose of our research work is to understand the theory of confinement in tokomaks. It is necessary to introduce different empirical methods and combine data from different tokamaks which operates in different conditions and by using statistical methods to show the dependence of the confinement time on the different plasma parameters which provides scaling relations for different tokamaks [3-10].

In this paper we present the dependence of effective edge safety factor on the energy confinement time by analytical solution of Grad-Shafranov equation (GSE) [1-2] with Lithium limiter [13] for circular cross-section HT-7 tokamak [8-9]. For this, the Shafranov parameter was obtained from analytical solution of GSE [4, 7]. It is observed, the energy maximum energy confinement time relate to the low values of effective edge safety factor ($2.6\langle q_{eff}(a)\langle 3$).

* Corresponding author:
dr.muha.asif@gmail.com (M. Asif)
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2. Theoretical Equation

For circular cross-section HT-7 tokamak [7-9, 13], which is the ohmically heated tokamak, the Grad–Shafranov equation is solved by formally expanding as follows [5, 7, 12]:

$$\psi(r,\theta) = \psi_0(r) + \psi_1(r)\cos\theta + ..., \quad (1)$$

$$u(\psi) = u_2(\psi_0) + \frac{du_2(\psi_0)}{d\psi_0}\psi_1\cos\theta + ..., \quad (2)$$

$$F(\psi) = RB_\varphi = R_0[B_0 + B_{\varphi 2}(\psi_0) + ...], \quad (3)$$

where R is major radius, $B_0 = const$, is the vacuum toroidal field at $R = R_0$, $B_{\varphi 2}(\psi)$ is a new free function replacing $F(\psi)$. In the first order solution or toroidal force balance approximation and if plasma were surrounded by a perfectly conducting shell located at $r = b$, then first order flux function [5, 7, 12] is:

$$\psi_1(r) = B_{\theta 1}(r)\int_r^b \frac{dx}{xB_{\theta 1}^2(x)} \times \int_0^x \left[2\mu_0(\gamma-1)y^2\frac{du_2(y)}{dy} - yB_{\theta 1}^2(y)\right]dy \quad (4)$$

If there are external coils to produce vertical magnetic field, the boundary condition on the flux function is modified so that we have [12]:

$$\psi(b,\theta) = const. + \psi_v(b,\theta), \quad (5)$$

where $\psi_v(r,\theta) = R_0 B_v r\cos\theta$, is the flux function due to external vertical field coils and therefore the full toroidal correction to ψ is [5, 7, 12]:

$$\psi_1(total) = \psi_{1T}(r)\cos\theta = \left[\psi_1(r) + \left[\frac{bR_0B_v}{B_{\theta 1}(b)}\right]B_{\theta 1}(r)\right]\cos\theta \quad (6)$$

The shift of the plasma column center from the geometrical center of vacuum chamber given by [5, 7, 12]:

$$\Delta R = -\frac{\psi_{1T}(a)}{\psi_0'(a)} = -\frac{\psi_1(a)}{\psi_0'(a)} - \Delta R_v = -\frac{\psi_1(a)}{\psi_0'(a)} - \frac{bB_v}{B_{\theta 1}(b)} \quad (7)$$

where $B_{\theta 1}(b) = \dfrac{\mu_0 I_p}{2\pi b}$.

Therefore, the first relation for plasma position [5, 7, 12] is

$$\Delta R_{Analytical} = \frac{b^2}{2R_0}\times\left[\left(\beta_p + \frac{l_i - 1}{2}\right)\left(1 - \frac{a^2}{b^2}\right) + \ln\frac{b}{a}\right] - \frac{bB_v}{B_{\theta 1}(b)} \quad (8)$$

where β_p is the poloidal beta, l_i is the internal inductance of the plasma, and B_v is the average vertical magnetic field over the vacuum chamber. We can find B_v from saddle sine coil [9] and expression $\beta_p + \dfrac{l_i}{2}$ from magnetic coils measurement [9]:

$$\beta_p + \frac{l_i}{2} = 1 + \ln\frac{a}{b} + \frac{\pi R_0}{\mu_0 I_0}\left(\langle B_\theta\rangle + \langle B_n\rangle\right), \quad (9)$$

where

$$\langle B_\theta\rangle = B_\theta(\theta = 0) - B_\theta(\theta = \pi),$$

$$\langle B_n\rangle = B_n\left(\theta = \frac{\pi}{2}\right) - B_n\left(\theta = \frac{3\pi}{2}\right), \quad (10)$$

We measured these local magnetic fields with magnetic probes [9] at above angles.

Therefore the effective edge safety factor at the plasma edge is given by [14]:

$$q_{eff} = \frac{2\pi a^2 B_\phi}{\mu_0 R_0 I_p}\left(1 + \varepsilon^2\left(1 + \frac{(\Lambda + 1)^2}{2}\right)\right) \quad (11)$$

From this relation, using the value of Shafranov parameter obtained from analytical solution of GSE [7], we plotted time history of the effective edge safety factor in target shot for circular cross section HT-7 tokamak, as shown in Figure 1.

3. Experimental Work

Before we derive the relation for the energy confinement time, we must determine the volume-averaged plasma kinetic pressure $\langle p\rangle$, and then the plasma thermal energy *U*. $\langle p\rangle$ can be determined directly from the definition of the poloidal beta [4]:

$$\langle p\rangle = \beta_p \frac{B_\theta^2(a)}{2\mu_0} = \mu_0 \frac{I_p^2 \beta_p}{8\pi^2 a^2} \quad (12)$$

where *a* is the plasma minor radius. For the measurement of the plasma thermal energy, we start from the plasma state equation [4]:

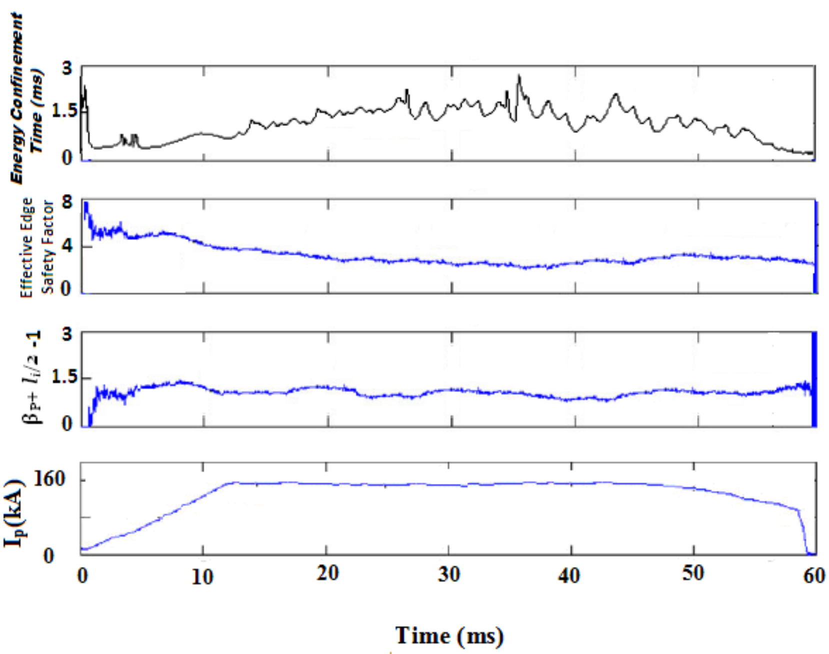

Figure 1. The dependence of effective edge safety factor on energy confinement time, obtained analytically by solution of GSE

$$\langle p \rangle = \sum_i n_i T_i = \frac{2}{3}\sum_i E_i = \frac{2}{3}E, \tag{13}$$

where subscript '*i*' indicates the plasma species *i* and *E* indicates the plasma thermal energy density; therefore the plasma thermal energy *U* and the plasma temperature are obtained [4]:

$$U = \frac{3}{2}\left(\sum_\alpha n_\alpha T_\alpha\right)V = \frac{3}{2}\langle p \rangle V, \tag{14}$$

where *V* is the plasma volume. The plasma-specific resistance in the steady state plasma can be written as [4]

$$\rho_p = \frac{1}{\sigma_p} = \frac{A}{l}R_p = \frac{a^2}{2R_0}\frac{V_R}{I_p} \tag{15}$$

where σ_p is the plasma conductivity, R_p is the plasma resistance and V_R is the resistive component of the loop voltage (the poloidal flux loop).

But the most important of these measurements is determining the plasma thermal energy confinement time, which is defined by [4]

$$\frac{dU}{dt} = P_{Ohmic} - \frac{U}{\tau_E}, \tag{16}$$

where τ_E is the plasma energy confinement time and P_{Ohmic} is the rate of input heating power. Rearranging equation (16), the ohmic heating power is [4]

$$P_{Ohmic} = V_R I_p - \frac{d}{dt}\left(\frac{1}{2}LI_p^2\right), \tag{17}$$

If the plasma is in thermal equilibrium ($L' = 0$ and $I' = 0$), then from equations (14) and (16), we have

$$P_{Ohmic} = V_R I_p = R_p I_p^2 = \frac{U}{\tau_E}, \tag{18}$$

$$\tau_E = \frac{3}{8}\mu_0 R_0 \frac{I_p \beta_p}{V_R} = \frac{3}{8}\mu_0 R_0 \frac{\beta_p}{R_p}, \tag{19}$$

Also if $\frac{dU}{dt}$ is not negligible, then from equations (14) and (16), we have

$$\frac{1}{\tau_E} = \frac{8}{3}\frac{R_p}{\mu_0 R_0 \beta_p} - \frac{2I'}{I} - \frac{\beta_p'}{\beta_p}, \tag{20}$$

Therefore, according to the above discussion, the Shafranov parameter was obtained from by analytical solution of GSE. It is observed, the maximum energy confinement time relate to the low values of effective edge safety factor ($2.6\langle q_{eff}(a)\langle 3$).

4. Result and Discussion

The dependence of effective edge safety factor on the energy confinement time have been studied by analytical solution of Grad-Shafranov equation (GSE) with Lithium limiter [13], for circular cross-section HT-7 tokamak. It is observed, the maximum energy confinement time relate to the low values of effective edge safety factor ($2.6\langle q_{eff}(a)\langle 3$).

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