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Single Particle Potentials and Three-Body Forces

H. M. M. Mansour

Physics Department, Faculty of Science, Cairo University, Egypt

Abstract The single particle potentials for both asymmetric nuclear matter and pure neutron matter are presented. The Brueckner-Hartree-Fock (BHF) approximation + two body density dependent Skyrme potential which is equivalent to three-body interaction are used. Various modern nucleon-nucleon (NN) potentials are used as follows: CD-Bonn potential, Nijm1 potential, Reid 93 potential and Argonne V_{18} potential are used in the framework of the Brueckner-Hartree-Fock approximation (BHFA).

Keywords Nuclear matter, Neutron matter, Equation of state, Single particle potential, Three body forces

1. Introduction

One of the most challenging aims of nuclear physics and nuclear astrophysics is to study the equation of state (EOS) and single particle (s.p.) properties of asymmetric nuclear matter in a wide density range. At densities around and below the nuclear saturation density the properties of asymmetric nuclear matter and their isospin-asymmetry dependence are closely related to the structure, decay and collective properties of heavy nuclei and neutron-rich nuclei away from the nuclear stability line, such as the radius, the neutron skin thickness and the density distribution. The properties of asymmetric nuclear matter can be predicted by adopting various nuclear many-body approaches, including phenomenological methods and microscopic approaches. In the phenomenological methods such as the Skyrme-Hartree-Fock framework and the relativistic mean field theory, the many-body correlations in nuclear medium have been incorporated implicitly and effectively into the parameters of the adopted effective interactions. Microscopic many-body approaches start from the realistic nucleon-nucleon (NN) interactions which are determined by reproducing the experimental NN phase shifts. It is well known that the nonrelativistic microscopic approaches adopting realistic two-body NN interactions miss the empirical saturation point of nuclear matter, and three-body forces (TBF) are required. In recent years, the EOS and s.p. properties of asymmetric nuclear matter have been investigated extensively within the framework of various microscopic approaches including the Brueckner-Hartree-Fock (BHF) and the extended BHF approaches [1, 2-7], the relativistic Dirac-BHF (DBHF) theory [8-15], the in-medium *T*-matrix and Green function methods [16-27], and the many-body variational approach [28-33]. Up to now, several different kinds of TBF models have been adopted in nuclear microscopic many-body calculations. One is the semi phenomenological TBF such as the Urbana TBF [34]. Another TBF model adopted in the Brueckner theory is the microscopic one [35-37] based on the meson exchange theory for the NN interactions. In the BHF calculation, the TBF contribution has been included by reducing the TBF into an equivalent effective two-body interaction according to the standard and extensively adopted scheme [37]. As an important input for calculations of nuclear structures and simulations of heavy-ion reactions, the single-nucleon potential *Un/p* (k) itself can also be obtained. The results of BHF calculations depend on the choice of single particle potential U(k). The conventional choice, which assumes a single-particle potential U = 0 for single-particle states above the Fermi level, and approximate the energies by the kinetic energy only [15], and U is self-consistent BHF potential for $k < k_F$, while the continuous choice for which U is extended to $k > k_F$, which leads to an enhancement of correlation effects in the medium and tends to predict larger binding energies for nuclear matter and pure neutron matter than the conventional choice.

In a previous work [39], the bulk properties of cold and hot asymmetric nuclear matter were calculated in the framework of (BHF interaction + two-body density dependent Skyrme potential which is equivalent to three body force). In the present work we extend the calculation to present the single particle potentials for the proton and neutron using modern nucleon-nucleon (NN) potentials in the framework of (BHFA).

In the next section we show the model used and in section 3 the results are presented.

* Corresponding author:
mansourhesham@yahoo.com (H. M. M. Mansour)
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2. Single-Particle Potential (BHFA)

The G-matrix is defined by:

$$G(\omega) = V + V\frac{Q}{\omega - H_{\circ} + i\eta}G(\omega) \qquad (2.1)$$

This is known as the Beth-Goldstone equation; here ω is the starting energy which is usually the sum of the single particle energies of the states of the interacting nucleons

$$\omega = e\,(k) + e\,(k') \qquad (2.2)$$

The dashed and wiggly lines denote the bare interaction V and G matrix respectively.

V is the bare NN potential, η is infinitesimal small number, $H_{\circ}$ is the unperturbed energy of the intermediate scattering states and Q is the Pauli projection operator, it projects out states with two nucleons above the Fermi level, it is given by:

$$Q\,(k,\ k') = (1 - \Theta_F\,(k))\ (1 - \Theta_F\,(k')) \qquad (2.3)$$

Where Θ_F (k) = 1 for k < k_F and zero otherwise, Θ_F (k) is the occupation probability of a free Fermi gas with a Fermi momentum k_F. Eq.(2.1) sums the ladder-type diagrams depicted in fig.(2.1), where the left-hand side represents the BHFA, it is the sum of the HF contribution, and all the diagrams obtained by adding an arbitrary number of interactions between particles.

In the Brueckner-Goldstone expansion, the average binding energy per nucleon is expanded in a series of terms as the following

$$\frac{E(k)}{A} = \langle K.\hat{E}\rangle + \langle \hat{G}\rangle = \sum_{k}\frac{k^2}{2m} + \frac{1}{2}\sum_{k,k'\langle k_F}\langle kk'|\,G(e(k)+e(k'))\,|kk'\rangle \qquad (2.4)$$

where |kk' > refer to antisymmetrized two-body states. This first order is known as the Brueckner-Hrtree-Fock approximation (BHFA). To completely determine the average binding energy one has to define the single particle potential U(k) which contributes to the single particle energies appearing in the G-matrix elements. The structure of the expression (2.4) suggests choosing the following BHF single particle potential

$$U(k) = \sum_{k'\langle k_F}\langle kk'|G(e(k)+e(k'))|kk'\rangle \qquad (2.5)$$

$$\frac{E(k)}{A} = \sum_{k\langle k_F}\left\{\frac{k^2}{2m} + \frac{1}{2}U(k)\right\}$$

$$= \frac{4}{\rho}\frac{1}{2}\int_0^{k_F}\frac{4\pi k^2}{(2\pi)^3}\left(\frac{k^2}{2m} + e(k)\right)$$

$$= \frac{3k_F^2}{10} + \frac{3}{2k_F^3}\int_0^{k_F} k^2\,dk\,U(k) \qquad (2.6)$$

The G-matrix itself depends on U (k) through the starting energy ω, defined in eq.(2.2) and the lowest order approximation (2.4) along with choice (2.5) for the single particle potential is often known as the lowest order Brueckner theory.

The single particle energy e (k) is defined as

$$e(k) = K.E + U(k) = \frac{\hbar^2 k^2}{2m} + U(k) \qquad (2.7)$$

where K. E is the kinetic energy. The conventional choice for the single particle potential has been to take the BHF potential (eq. (2.5)) for hole states (k < k_F) and zero for particle states (k > k_F).

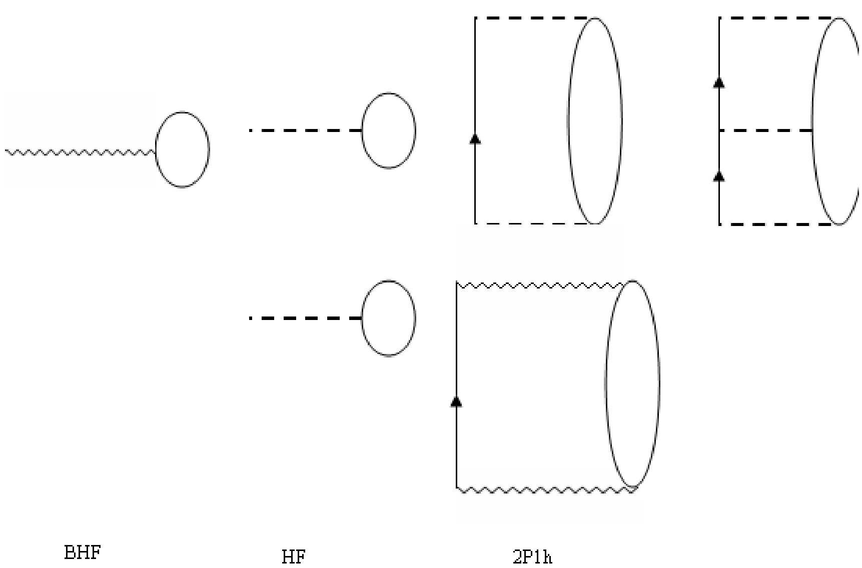

Figure 2.1. Graphical representation of the Bethe-Gold-stone equation

$$U(k)=\begin{cases}\sum_{k'\langle k_F}\langle kk'|G(e(k)+e(k'))|kk'\rangle & k\langle k_F\\ 0 & k\rangle k_F\end{cases} \tag{2.8}$$

Thus introducing a quite large discontinuity in the single particle spectrum at the Fermi surface. However, due to the unphysical discontinuity at the Fermi surface this auxiliary potential cannot be directly related to the average potential felt by a particle or a hole. Moreover, many other interesting properties can be derived such as the momentum distribution and the effective mass which is properly described using a continuous spectrum across the Fermi surface. This was the main motivation which led [38] to the introduction of the continuous choice for the single particle potential thus treating particles and holes in a symmetrical way. The use of the continuous choice potential implies that the G-matrix elements needed in the self-consistent calculation are complex and the prescription advocated is

$$U(k)=\mathrm{Re}\sum_{k'\langle k_F}\langle kk'|G(e(k)+e(k'))|kk'\rangle \tag{2.9}$$

Eqs. (2.1) and (2.7) represents the main equations that we want to solve self-consistently. In order to obtain such a self-consistent solution one often assumes a quadratic dependence of the single-particle energy on the momentum of the nucleon in the form

$$e(k)=\begin{cases}\dfrac{\hbar^2k^2}{2m^*}+\Delta & k\le k_F\\[2ex] \dfrac{\hbar^2k^2}{2m^*} & k\rangle k_F\end{cases} \tag{2.10}$$

where m* is the effective mass of the nucleon and Δ is a constant. Starting with an appropriate choice for the parameters for the effective m* and the constant Δ, one can solve the Bethe-Goldstone equation and evaluate the single-particle energy. The parameters m* and the constant Δ can then be readjusted in such away that the parameterization eq. (2.10) reproduces these two energies. This procedure is then iterated until a self-consistent solution is obtained. The parameterization of eq. (2.10), however, is useful not only to simplify the self-consistent solution of the BHF equations; it also leads to a simplification of the numerical solution of the Bethe-Goldstone equation.

3. Single-Particle Potential (TBF)

In the present work one may introduce a Skyrme effective interaction density dependent term in addition to the BHF single particle potential in the previous section [39].

$$V(\overline{r}_1,\overline{r}_2)=\sum_{i=1}^{4}t_i(1+x_iP_\sigma)\rho^{\alpha_i}\delta(\overline{r}_1-\overline{r}_2) \tag{2.11}$$

This is a two-body density dependent potential which is equivalent to three body interaction. Where t_i and x_i are interaction parameters, P_σ is the spin exchange operator, ρ is the density, $r_1$and r_2 are the position vectors of the particle (1) and particle (2) respectively. α_i = (1/3, 2/3, 1/2 and 1). Using the above potential for asymmetric nuclear matter the following correction for the single particle potential U is used:

$$\Delta U = (3/4)\, t_1\, \rho^{4/3} + (3/4)\, t_2\, \rho^{5/3} \tag{2.12}$$

And for pure neutron matter we obtain the following correction

$$\Delta U = (1/2)\, t_1\rho^{4/3}\, [1 - x_1] + (1/2)\, t_2\, \rho^{5/3}\, [1 - x_2] \tag{2.13}$$

4. Results

After adding TBF corrections to the BHF calculations it was found that among these corrections the best results to obtain the empirical saturation point when α_i = (1/3 and 2/3).The parameters are the same as given in Ref. [39]. Figure (3.1) shows a comparison between the EOS in the framework of the BHFA in (A) and EOS in the framework of BHFA+ two body density dependent Skyrme interaction in (B) by using the CD-Bonn potential [40] and Reid93 potential [41] for conventional Choice.This shows an adjustment of the saturation point location and value by adding the TBF.

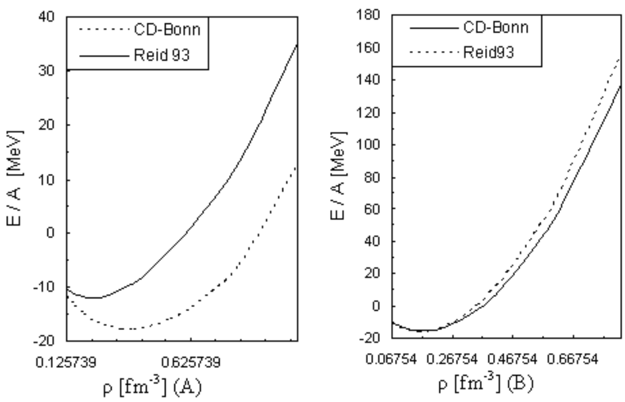

Figure 3.1. The EOS in the framework of the BHFA in (A) and EOS in the framework of BHFA+ two body density dependent Skyrme interaction (which is equivalent to three body force) in (B) by using the CD-Bonn potential and Reid93 potential for conventional Choice

The single particle potential of e.g. a proton U is defined together with kinetic energy as the energy required to remove this proton from the nuclear system leaving a hole in the state. The results of BHF calculations depend on the choice of the single particle potential U (k) in the "standard or conventional" choice, U = 0 for k > k_F and U is the self-consistent BHF potential for k < k_F, the alternative "continuous" for which U is again the self-consistent BHF potential. The single particle potential for asymmetric nuclear matter is calculated using the CD-Bonn potential, the $\mathrm{Argonnev}_{18}$ [42], the Nijm1 [41] potential and the Reid 93 potential for conventional and continuous choices. The dependence of the single particle potential on the momentum k for asymme-tric nuclear matter at $k_F = 1.333\ fm^{-1}$ for various potentials for conventional and continuous choices are shown in figure (3.2). The results for all potentials are approximately similar in the conventional choice as well as the continuous choice. The single particle potential increases with increasing the momentum k (U is directly proportional to k).

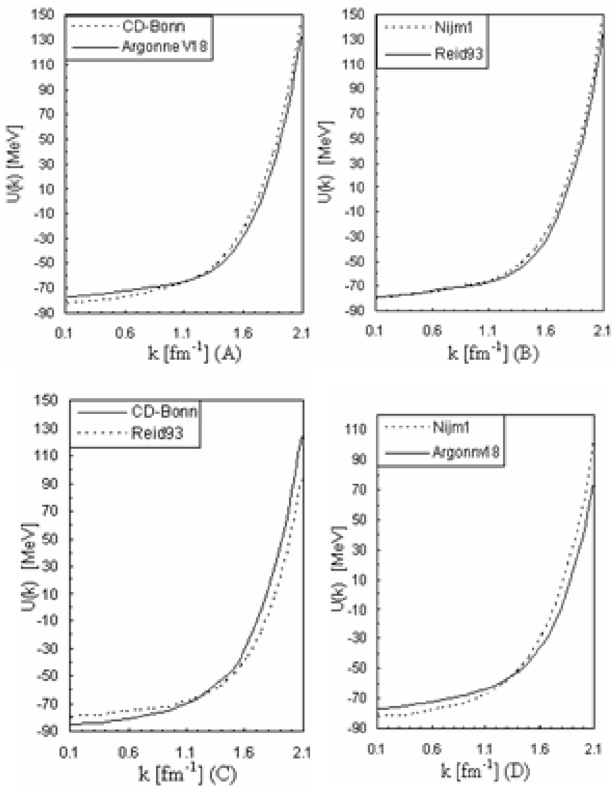

Figure 3.2. The single particle potential for asymmetric nuclear as a function of momentum k at ($k_F = 1.333\ fm^{-1}$) for different potentials (A and B) for conventional choice, (C and D) for continuous choice

In figure (3.3) the dependence of the single particle potential on the momentum k for the pure neutron matter at $k_F = 1.333\ fm^{-1}$ is plotted for various potentials for conventional choice the CD-Bonn potential (A), the Nijm1 potential (B), the Reid 93 potential(C) and the Argonne v_{18} potential (D).In figure (3.4) we get the same as above but for continuous choice. It is observed that the results for all potentials are approximately similar in the conventional choice and the continuous choice the results for different potentials have the same behavior. The single particle potential increases with increasing the momentum k. In conclusion we claim that a simple and analytic term for the three –body potential has corrected the deficiency in the BHF calculation in the sense that we get the right value for the energy at the saturation point, the symmetry energy increases with increasing the density and good agreement with other theoretical works for the physical quantities of relevance to the nuclear and neutron matter as shown in previous works [39, 40-46]. For completeness we present here the calculation for the single-particle potential.

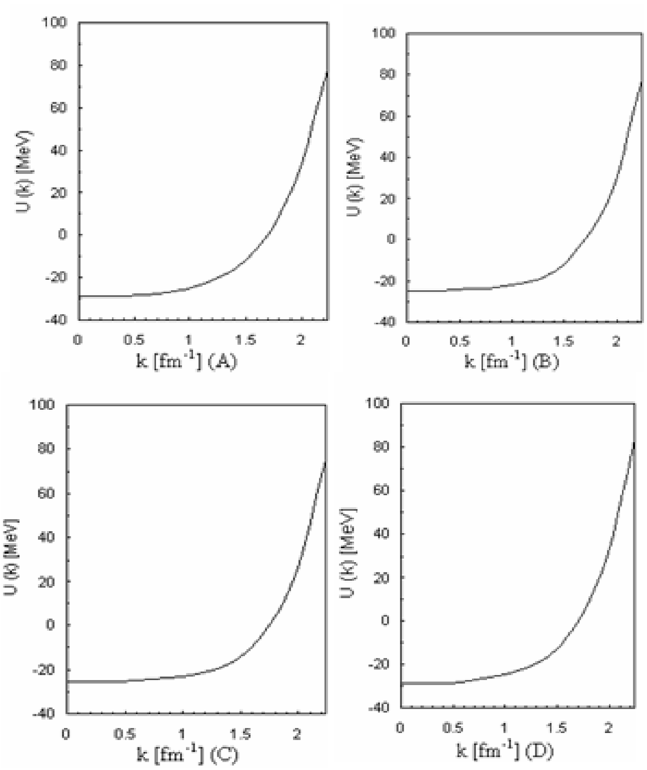

Figure 3.3. The single particle potential for the pure neutron matter in [Me V] as a function of momentum k at ($k_F = 1.333\ fm^{-1}$) for continuous choice of different potentials

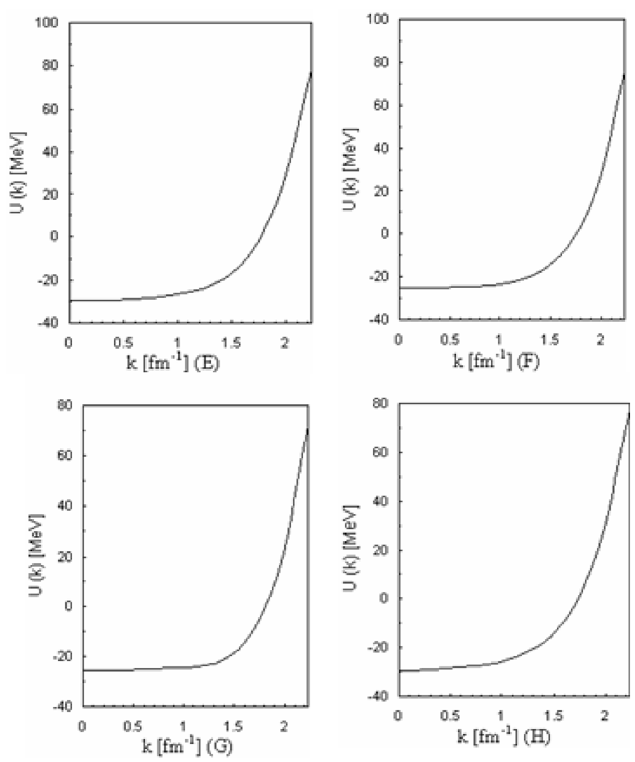

Figure 3.4. The single particle potential for the pure neutron matter in [Me V] as a function of momentum k at ($k_F = 1.333\ fm^{-1}$) for continuous choice of different potentials

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