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Elliptic Flow in Heavy Ion Collisions

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Abstract The study of quark-gluon plasma state in high energy heavy ion collisions is quite complicated as the system is dynamical. But still some detectable signals are present with are very helpful to understand this high energy phenomena. The measurement of elliptic flow is one of those experimentally measured variables. One of the major experimental evidence for the existence of thermalized system is the large anisotropic flow of hadrons. The anisotropic flow is the anisotropy of the particle azimuthal distribution in the momentum space with respect to the reaction plane and supposed to be sensitive to the extent of thermalization of the system immediately after the collision. The various hadron yield with respect to the reaction plane is characterized by Fourier expansion as the thermalised system behave like ideal fluid and the elliptic flow is defined by the second Fourier coefficient $(\boldsymbol{v_2})$. The theoretical calculations matches with the experimental observations well when the event by event fluctuations are considered for the measurement of flow and the Monte Carlo Glauber models are very much helpful to evaluate initial stage parameters.

Keywords Heavy ion collisions, Anisotropic flow, Monte carlo glauber models

1. Introduction

In relativistic heavy ion collisions a large amount of energy is dumped into a very small volume, when the two heavy nuclei collide, and the observation of these collisions at Alternating gradient synchrotron (AGS) at BNL and the super proton synchrotron at CERN have been recorded for different energy ranges and from light to heavy nuclei. These collisions are pictured with various stages in between the initial stage and the end point, with particles observed in the detectors around the collision points, and many detectable signals and experimental observables are recorded at these stages. When the Lorentz contracted nuclei pass through each other, the vacuum left behind is filled with a colour field, indicates the attraction of the two nuclei and the energy of the colour field leads to the production of matter and anti-matter.

The impact parameter roughly defines the nuclei participating in a collision (N_{part}), the number of binary collisions occurring (N_{coll}) and about the distribution of initial energy density in the collision region. The small impact parameter collisions are defined as the central collisions and the collisions with large value of impact parameter are called peripheral collisions.

With the other detectable signals of the initial stage of heavy ion collisions one of the important signal is the **elliptic-flow.** After the initial binary collisions the interacting system reaches to a local thermal equilibrium, and the pressure gradients arise. The pressure gradients are steeper along the direction of impact parameter and lead to anisotropic momentum distribution of particles which is defined as elliptic flow.

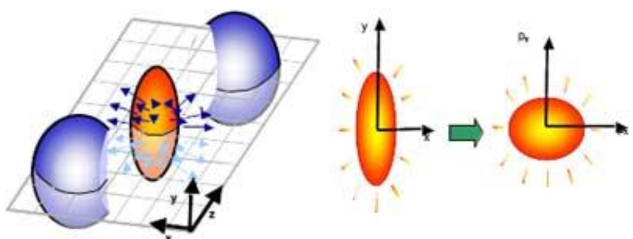

Figure 1. Left: Schematic of the collision zone between two incoming nuclei and x-z is the reaction plane. Right: Initial-state anisotropy in the collision zone converting into final-state elliptic flow, measured as anisotropy in particle momentum

The yield of various hadrons with respect to the reaction plane can be characterized by Fourier expansion, where the different coefficients measure different anisotropies present in the system. The first coefficient is known as the directed flow (v_1), and the second one is defined as elliptic flow (v_2).

Further the partonic matter of system is converted into hadrons as the collision medium expands and cools down. These hadrons initially go through inelastic collisions. As the system keeps cooling down further, below a certain temperature inelastic collisions between hadrons stop and in this state different particle yields is completely defined. This stage is called Chemical freeze-out. Further expansion and cooling of system leads to the elastic collisions of hadrons, and a situation comes, when the produced particle stop colliding called Kinetic freeze-out.

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At different stages of heavy ion collision it is possible to probe the hot and dense medium via different measurable signals. Here in this paper we shall concentrate only on elliptic flow.

Quantitatively the elliptic flow is indicated by the second Fourier coefficient (v_2) of the azimuthal particle distribution relative to the reaction plane (defined by the impact parameter and the beam axis.). The relativistic hydrodynamical models are able to explain and picture the expansion of hot and dense thermalized system very well at low viscosity [1]. The dynamical properties of the system resemble to the liquid rather than a gas. The study of this matter at RHIC energies shows an interesting feature, by the study of azimuthal anisotropy of final state particles, for different particle species.

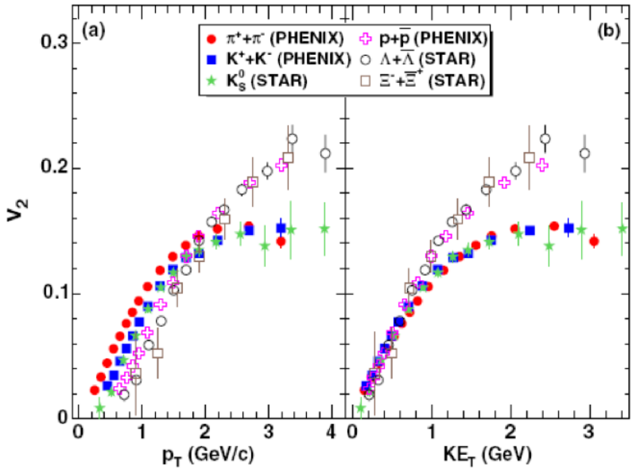

Figure 2. Left: The figure shows a plot, v_2 versus p_T and transverse kinetic energy KE_T. Figure from [19]

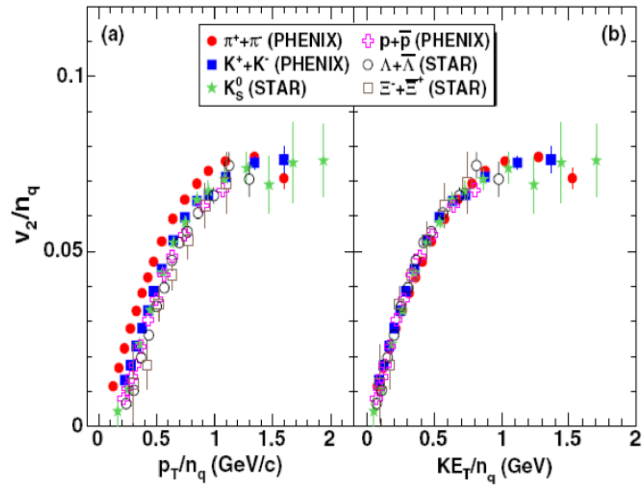

Figure 3. A plot shows the plot of ratios v_2/n_q versus p_T $/n_q$ and KE_T $/n_q$ where n_q is the number of constituent quarks for each type of the particle species considered. Figure from [19]

The figure 1, shows the magnitude of elliptic flow for different particle species, and the figure 2 which indicates the quantities when scaled with the number of constituent quarks like n_q=2 for mesons and n_q=3 for (anti) baryons. At very low energies, the elliptic flow is positive and indicates the in plane momentum anisotropy because the spectator part exits collision region slowly and blocks the in-plane emission from the nuclear overlap zone and the particles emitted from the participant region are bounced out of plane and thus a negative v_2 coefficient. With growing energy the spectators escape faster from the region and the bouncing off-plane dynamics is less dominating. Simultaneously at energies higher than E_{beam} ~ 400 MeV/A, pressure gradients start to develop in-plane contribute positively to v_2. These two things are competing with each other result into a monotonic grow of the elliptic flow coefficient with energy. At E_{beam} ~ 4 GeV/A, v_2 again becomes positive indicates that the pressure gradient developed in plane is dominated over other factors.

This reflects an important feature of flowing thermalized system that it can be best explained in terms of partonic degrees of freedom. The observation of hot and dense matter produced, with the measurements of high transverse momentum particles along with the elliptic flow, indicates the creation of an opaque and strongly interacting partonic matter.

The flow and correlation studies:

To understand the complete theory of heavy ion collisions differential studies of various observables are required, out of which our aim is to study the elliptic flow and two particle correlations, and to develop the deep understanding of the early stages of the heavy ion collisions.

When the elliptic flow is measured in Cu+Cu and Au+Au collisions, at $\sqrt{S_{NN}}$ =200 GeV, as a function of number of participating nucleons, the results can be seen in the figure 4.

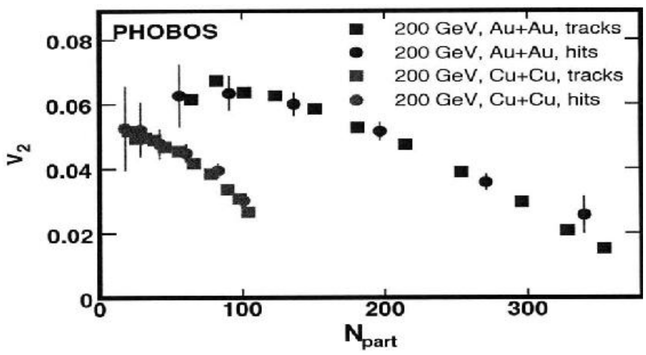

Figure 4. Elliptic flow parameter v_2 as a function of number of participating nucleons in Au+Au (blue) and Cu+Cu (red) collisions at $\sqrt{S_{NN}}$ = 200 GeV [2]

As the elliptic flow is driven by the azimuthal anisotropy in the initial stage of the collision, one expects the small elliptic flow signals for most central Cu+Cu collisions, because of the roughly circular initial geometry. But the observed signals were significantly large [2]. The second interesting thing was the observation of rich structures in angular correlation measurements, in heavy ion collisions [34].

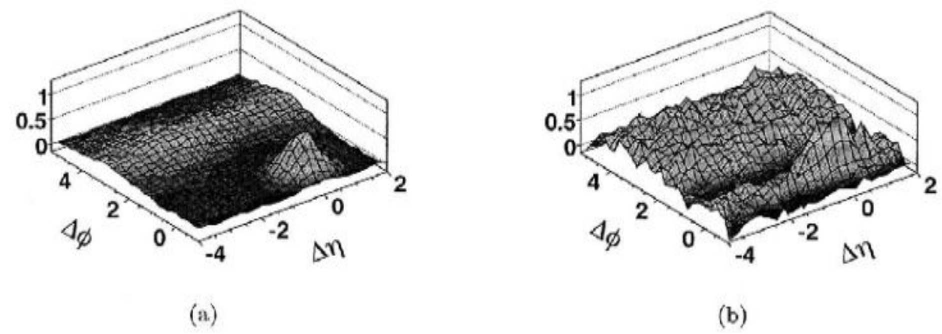

Figure 5. Correlated yield as a function of Δη and $\Delta\phi$ for (a) PYTHIA **p+p** model and **(b) 0-30%** central Au+Au data at $\sqrt{S_{NN}}$ = 200 GeV with respect to a trigger particle with p_T^{trig} > 2.5 GeV/c [3]

The above figure is showing the correlated yield with respect to a trigger particle with p_T^{trig} > 2.5 GeV/c in p+p collisions modeled by PYTHIA, and in most central Au+Au events (0-30%) at $\sqrt{S_{NN}}$= 200 GeV, as a function of pseudo rapidity(Δη) and azimuthal separations ($\Delta\phi$), between particle pairs.

When Au+Au collisions were compared with the p+p system a very rich correlation structure is observed in Au+Au collisions, also an excess yield of correlated particles at $\Delta\phi = 0$ and $\Delta\phi = 120°$ was found for Δη >2. The structure defined as "ridge" or "broad away side" and studied experimentally [3-8] and different theoretical models were used for understanding of their origin [9-15]. But none of the models describe the experimental results successfully [16].

Further the results of observed elliptic flow were supposed to be explained by the consideration of event by event fluctuations in the initial geometry [2]. The anisotropy of the initial geometry can be characterized by the eccentricity of the transverse shape of the initial nuclear overlap region [17].

In Glauber model, even for the most central collisions, the eccentricity of the region is defined by the event by event distribution of the nucleon-nucleon interaction points, is finite and has a large effect of the event by event fluctuations on the elliptic flow. The fluctuations in the initial collision geometry may play a key role to find and understand the source of ridge and broad away side structures in particle correlation measurements.

Eccentricity and the elliptic flow:

Anisotropies in the distribution of particle momentum relative to the reaction plane, is defined as the anisotropic collective flow, in the heavy ion collisions. The azimuthal anisotropy in the particle production is characterized by the Fourier transformation with respect to the reaction plane angle ψ_r as-

$$\frac{1}{N}\frac{dN}{d\phi} = \frac{1}{2\pi}\left\{1 + \sum_n 2v_n \cos\left(n(\phi - \psi_r)\right)\right\} \quad (1)$$

The sine terms are excluded from the expansion as the particle production is considered on average symmetric around the reaction plane. Here the second coefficient v_2 is defined as the elliptic flow which appears due to the anisotropy in the initial collision geometry.

The eccentricity in general is quantified as the anisotropy of the collision geometry-

$$\varepsilon = \frac{<y^2-x^2>}{<y^2+x^2>} \quad (2)$$

Here x and y are the transverse coordinates along and perpendicular to the reaction plane respectively. The elliptic flow is caused by the rescattering of the particles produced in the initial nucleon-nucleon collisions. So the elliptic flow at low densities should be proportional to the particle density in the transverse plane [25, 26].

At high densities and vanishingly small mean free path, the elliptic flow signals are supposed to be saturated at a value imposed by hydrodynamical calculations. Also it is expected to be zero for azimuthally symmetric system, and for small anisotropies in the initial geometry the elliptic flow should be proportional to eccentricity (this proportionality was found between elliptic flow and eccentricity well, even for large values of ε) [23].

On the basis of these observations, it is found that the elliptic flow can be understood well by the plot of elliptic flow scaled by eccentricity, v_2/ε as a function of particle density in the transverse plane, $\frac{1}{s}(\frac{dN}{dy})$, where the initial overlap area s and eccentricity ε is taken from Glauber model calculations[26].

The plot for elliptic flow results from AGS, SPS, and RHIC experiments are seen at different collision energies with different projectiles and different centralities lead to the conclusion that the heavy ion collisions satisfy the assumption of hydrodynamical calculations made in the initial state thermalization and interaction near zero mean free path limit (23, 27, 28).

From the hydro-dynamical calculations, which implement finite mean free path, it is observed that the elliptic flow measurements are very sensitive to the viscosity of the system. A large uncertainty in the value of eccentricity is found when different approaches were used to quantify the initial geometry parameters.

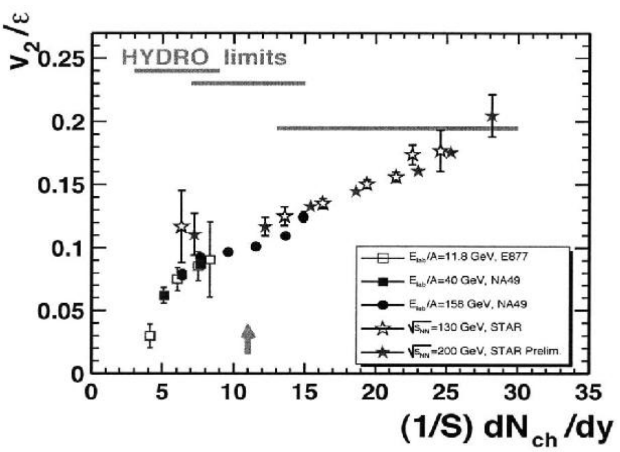

Figure 6. Elliptic flow scaled **by** eccentricity, $^{v_2}/_{\varepsilon}$, as a function of particle density in the transverse plane, ***1/S*(*dN/dy*)**, *for* different collision systems, center of mass energies and centrality ranges [24]

The Glauber modeling for eccentricity calculation:-

The Glauber models are very useful for calculating different geometrical quantities in the initial stage of heavy ion collisions like, the impact parameter, number of participating nucleons and the binary collisions also the initial eccentricity. The modeling is done into two ways; one by Optical Glauber models which consider smooth matter density described by Fermi distribution function in the radial direction and uniform over solid angle, the other type is the Monte carlo based models, assume individual nucleons as stochastically distributed event by event, and the detectable properties are calculated by averaging over multiple events. Both the models provide almost similar results for number of participating nucleons and the impact parameter but give different results in the quantities where event by event fluctuations are significant [21, 22].

As in the figure 1 it can be seen that the shape of the interaction region is strongly dependent on the impact parameter in the non-central collisions and it is elliptic in shape. The initial space anisotropy can be characterized by the eccentricity [29] as-

$$\varepsilon_{RP} = \frac{\sigma_y^2-\sigma_x^2}{\sigma_y^2+\sigma_x^2} \quad (3)$$

$$\text{Also } \varepsilon_{PP} = \sqrt{\frac{(\sigma_y^2-\sigma_x^2)^2+4\sigma_{xy}^2}{\sigma_y^2+\sigma_x^2}} \quad (4)$$

Where $\sigma_x^2 = <x^2> - <x>^2$, $\sigma_y^2 = <y^2> - <y>^2$, and σ_{xy}^2 =<xy>-<x> <y>, are the event-by-event (co-)variances of the participant nucleon distributions projected on the transverse axes, x and y, also RP refers to the reaction plane & PP refers to the participant plane**.** For the calculations in participant plane, the coordinate axes are tilted according to the ellipse formed in the collision region given in the Fig. 7.

Here the plot of impact parameter with eccentricity (for participant plane and reaction plane) is shown in figure 8. It is noticeable that the reaction plane eccentricity is zero at zero impact parameter, as expected for two spherical shaped objects. But the participant eccentricity is having non-zero value even for zero impact parameter collisions. The reason is that ε_{PP} is calculated considering the distribution of nucleons inside the nuclei.

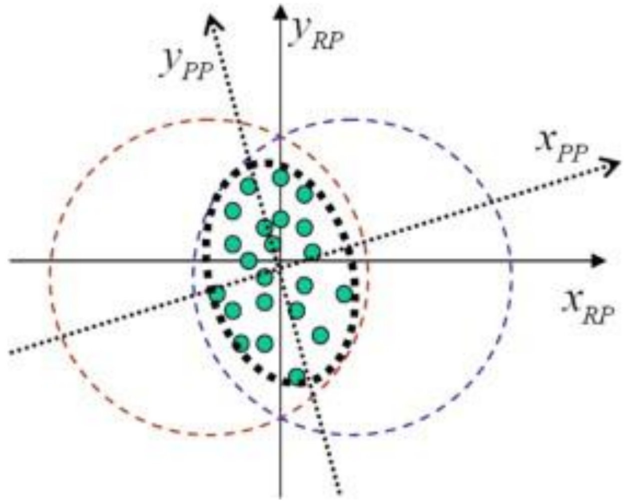

Figure 7. The reaction plane and participant plane in the collision region

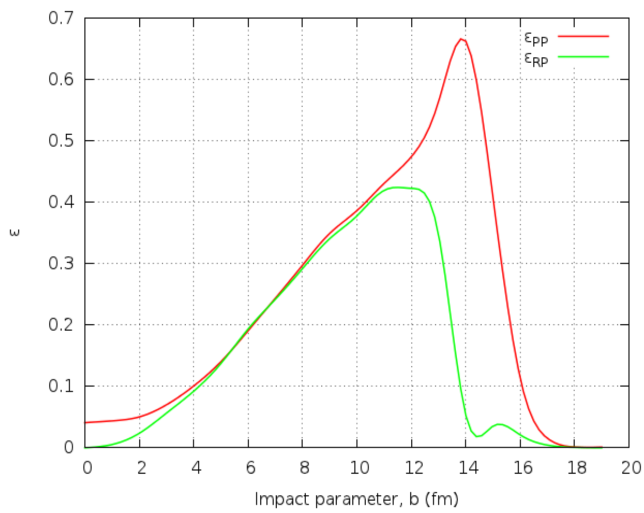

Figure 8. Plot of eccentricity with impact parameter

When the collision energy increases the value of eccentricity decreases (Fig.9). The participant plane eccentricity is dependent on the number of participating nucleons so obviously on their positions. Also the number of participating nucleons changes with energy, so the participant eccentricity is changing too. This variation can be understood with the observation taken for Au+Au collision.

The results for Cu+Cu and Au+Au collisions when discussed with event-by-event fluctuations and taken into account in the initial eccentricity [2], and the participant eccentricity, was considered to account for these fluctuations, were actually in agreement for the Cu+Cu and Au+Au systems.

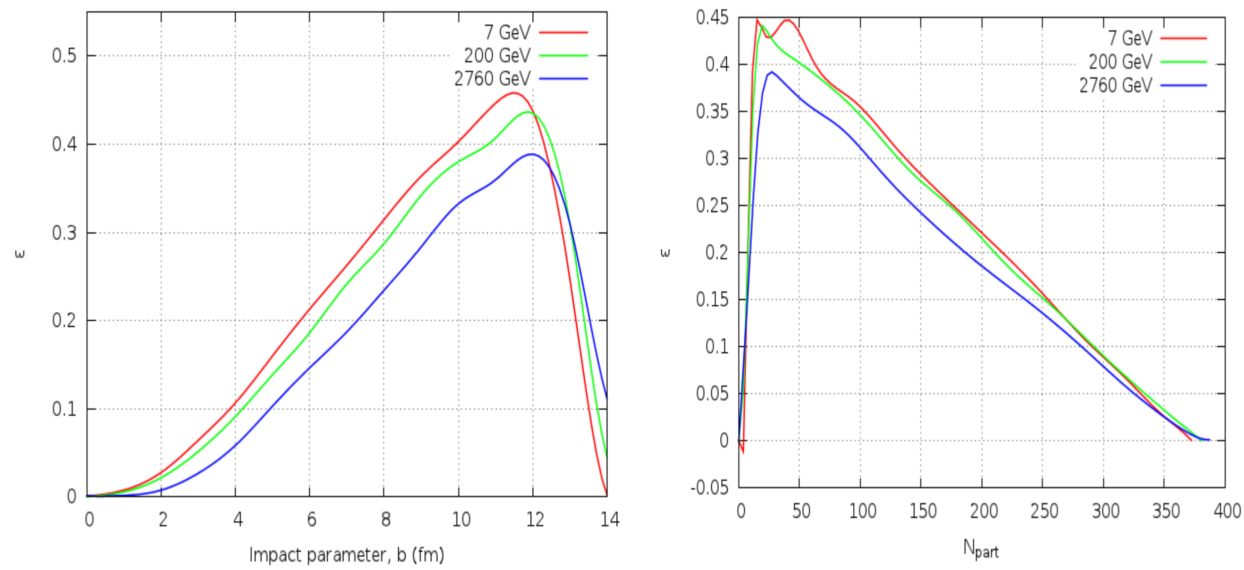

Figure 9. Eccentricity variation for Au + Au collision at different energies (left): with impact parameter, (right): with N_{part}

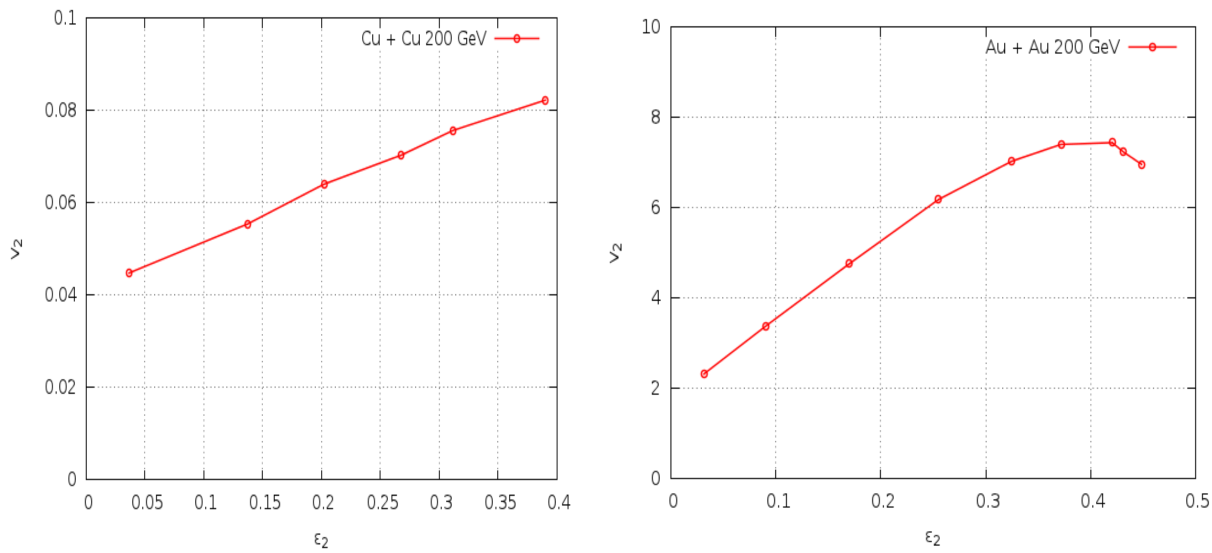

Figure 10. Plot showing the proportionality between elliptic flow and initial spatial eccentricity for Cu + Cu (left) and Au + Au (right) at 200 GeV [31]

It is seen that the initial spatial anisotropy is responsible for the anisotropy in the momentum distribution of the particles which are produced during the collision. The elliptic flow coefficient $v_2$is a measure of this momentum anisotropy and can be expressed as [33]-

$$v_2 =< \frac{p_x^2 - p_y^2}{p_x^2 + p_y^2} > \quad (5)$$

From ideal hydrodynamics [30] it is expected and observed that for small anisotropies as well as for large values of ε, the proportionality is found in between elliptic flow (v_2) and eccentricity.

The Fig.10, is presenting the proportionality relation between the experimentally measured values of v_2 and the spatial anisotropy using Glauber model, for the same centrality as the experimental data [31]. The graph is indicating the transformation from spatial anisotropy to momentum anisotropy clearly for the hot and dense medium created in the heavy-ion collisions. From the observation of the above figure obtained for Au+Au collision it can be guessed that there could be other factors which are affecting the proportionality between the elliptic flow and eccentricity which can dampen this transformation, one of those is viscosity.

When the anisotropy in heavy ion collisions is measured event-by-event the fluctuations may be appeared due to three reasons: statistical fluctuations arise because of the finite number of particles observed, secondly the elliptic flow fluctuations and other may be the many-particle correlations, defined as non-flow correlations. There are analyses methods have been developed to evaluate the statistical fluctuations and non-flow contributions and with the help of these one can calculate elliptic flow fluctuations.

The elliptic flow fluctuations are measured step by step this way, first the dynamic elliptic flow fluctuations i.e. in v_2 are determined by unfolding the statistical fluctuations to azimuthal particle distributions. Then, the by using and calculating the difference in pseudorapidity dependence of flow and non-flow correlations the magnitude of non-flow correlations can be found out [32]. So finally we shall be able to find out the elliptic flow fluctuations by calculating the difference of the contribution of non-flow correlations and the dynamic v_2 fluctuations.

2. Summary & Conclusions

The elliptic flow is an important parameter which may be a tool to draw the information about the evolution of nuclear collision. Quantitatively the elliptic flow coefficient v_2 is a measure of the azimuthal anisotropy of particle momenta. The anisotropy depends on the energy and reflects the reaction dynamics. At different energies the elliptic flow changes. A positive v_2 coefficient means the particles are thrown towards the reaction plane preferably by the momentum azimuthal anisotropy, called the in-plane flow, while when v_2 is negative, when the momentum anisotropy pushes particles preferentially perpendicular to the reaction plane defined as the out-of-plane flow. It is found that there is proportionality between eccentricity of the initial geometry of the collision region and the elliptic flow.

Ideal hydrodynamics (zero viscosity) is not able to explain the v_2 coefficient at all transverse momentum range. Calculations made using hydrodynamics with non-zero shear viscosity η, also the consideration of non-flow effects, and other factors can help to understand the flow fluctuations.

The ultra relativistic nuclear collision program is aimed towards the creation of the QGP – quark-gluon plasma – the deconfined state of quarks and gluons at laboratory level. It is very clear that such a state is required to be appearing in a (local) thermalized system achieved by many re-scatterings per particle during the evolution of the system. It is not clear when such a dynamical thermalization can really occur. An understanding of these phenomena can be achieved by considering elliptic flow.

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