

# Comparison between the Magnetic Probes and Analytical Method in Measurement of the Shafranov Shift

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**Abstract** We measured the plasma column shift using array of magnetic pick-up coils (magnetic probes) and compared with analytical solution in the IR-T1 tokamak. In the first technique, four magnetic probes were designed, constructed, and installed on outer surface of the IR-T1 tokamak chamber and then plasma column displacement determined from them. To compare the results obtained using this method analytical solution of the Grad-Shafranov equation based on expansion of free functions as quadratic in flux function is also experimented on IR-T1. Results of the two techniques are in good agreement with each other.

**Keywords** Tokamak, Plasma Column Shift, Magnetic Pick-up Coils, Grad-Shafranov Equation

## 1. Introduction

In the low beta tokamaks, radial pressure balance is achieved by the poloidal field, and toroidal force balance is achieved by the Lorentz force. But, in the second one, plasma may intend to shift inward or outward, which is a very dangerous for tokamak plasma. Therefore, plasma equilibrium study is one of the fundamental problems of the magnetically confined plasmas. There are many available global solutions of the steady state magnetohydrodynamics equations, in particular, the Grad-Shafranov equation. Control of plasma position plays an important role in plasma confinement and to achieve optimized tokamak plasma operation. Accurate determination of the plasma position during confinement time is essential to transport it to a control system based on feedback. Over the years different methods have been developed to analyze the equilibrium problems and determining the plasma column shift[1-10].

In this paper we presented magnetic probes method for measurement of the plasma column shift and comparison with analytical solution of Grad-Shafranov equation in the IR-T1 Tokamak, which is a small, air core, low beta and large aspect ratio tokamak with a circular cross section, (see Table 1). Details of the magnetic probes method for determination of plasma column shift will be discussed in section 2. Analytical solution of the Grad-Shafranov equation will be presented in section 3. Experimental results and comparison between them also will be presented in section 4. Summary and conclusion are also will be

presented in section 5.

## 2. Magnetic Probes Technique for Measurement of the Plasma Column Shift

Because of dependence of the plasma position and plasma current distribution to magnetic field distributions around the plasma, therefore magnetic pickup coils give us information about the plasma position or plasma column shift,  $\Delta R$ . Poloidal and normal magnetic fields distributions around the plasma are[1]:

$$B_{\theta} = \frac{\mu_0 I_p}{2\pi b} - \frac{\mu_0 I_p}{4\pi R_0} \times \left\{ \ln \frac{a}{b} + 1 - \left( \Lambda + \frac{1}{2} \right) \left( \frac{a^2}{b^2} + 1 \right) - \frac{2R_0 \Delta R}{b^2} \right\} \cos \theta, \quad (1)$$

$$B_r = -\frac{\mu_0 I_p}{4\pi R_0} \times \left\{ \ln \frac{a}{b} + \left( \Lambda + \frac{1}{2} \right) \left( \frac{a^2}{b^2} - 1 \right) + \frac{2R_0 \Delta R}{b^2} \right\} \sin \theta. \quad (2)$$

Therefore, by rearranging the above equations the first relation for plasma column shift obtained:

$$\Delta R_{\text{probes}} = \frac{a^2}{4R_0} \left\{ \left( \frac{b^2}{a^2} - 1 \right) - 2 \ln \frac{a}{b} \right\} + \frac{\pi b^2}{2\mu_0 I_p} \left\{ \langle B_{\theta} \rangle \left( 1 - \frac{a^2}{b^2} \right) - \langle B_r \rangle \left( 1 + \frac{a^2}{b^2} \right) \right\}, \quad (3)$$

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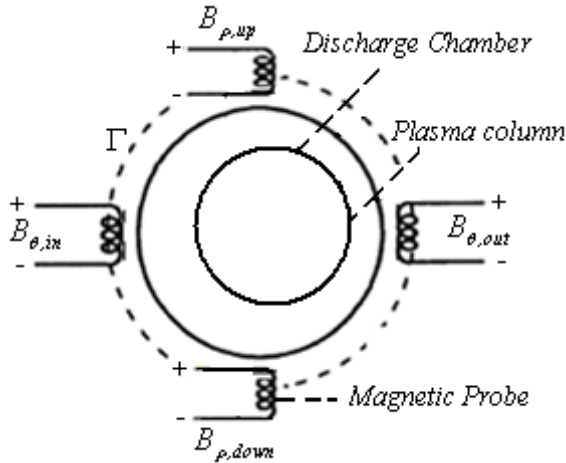
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where we used the quasi-cylindrical coordinates  $(r, \theta, \phi)$ . Equations (1) and (2) accurate for the low beta, large aspect ratio and circular cross section tokamaks as IR-T1, and where:

$$\begin{aligned} \langle B_\theta \rangle &= B_\theta(\theta = 0) - B_\theta(\theta = \pi), \\ \langle B_r \rangle &= B_r(\theta = \frac{\pi}{2}) - B_r(\theta = \frac{3\pi}{2}). \end{aligned} \quad (4)$$

According to this approach, in the IR-T1 tokamak four magnetic probes were designed, constructed, and installed, two magnetic probes were located on the circular contour  $\Gamma$  of the radius  $b = 16.5\text{cm}$  in angles of  $\theta = 0$  and  $\theta = \pi$  to detect the tangential component of the magnetic field  $B_\theta$  and two magnetic probes are also located above,  $\theta = \pi/2$ , and below,  $\theta = 3\pi/2$ , to detect the normal component of the magnetic field  $B_r$ , as shown in Figure (1).

By substituting the poloidal and normal components of the magnetic field which obtained by four magnetic pickup coils (after compensation and integration of their output), in Eq. (3), displacement of the plasma column boundary was determined. Experimental results will be presented in section 4.



**Figure (1).** Positions of the four magnetic probes on the outer surface of the IR-T1 tokamak chamber

### 3. Analytical Solution of the Grad-Shafranov Equation

For axially symmetric configurations, Maxwell's equations together with the force balance equation from MagnetoHydroDynamics (MHD) equations, for stationary and ideally conducting plasmas, reduce to the two-dimensional, nonlinear, elliptic partial differential equation, or Grad-Shafranov equation (GSE)[2]:

$$\Delta^* \psi = -\mu_0 R^2 \frac{dp}{d\psi} - F \frac{dF}{d\psi}, \quad (5)$$

where

$$\begin{aligned} \bar{B} &= \frac{1}{R} \bar{\nabla} \psi \times \hat{e}_\phi + \frac{F}{R} \hat{e}_\phi, \\ \mu_0 \bar{J} &= \frac{1}{R} \frac{dF}{d\psi} \bar{\nabla} \psi \times \hat{e}_\phi - \frac{1}{R} \Delta^* \psi \hat{e}_\phi, \end{aligned} \quad (6)$$

and where  $p(\psi)$  and  $F(\psi)$  are two free functions.

Many authors solved the Grad-Shafranov equation by expanding the free functions on different order in  $\psi$ . In this section, we present quadratic order (which proposed by Guazzotto[3]), and examined on the IR-T1.

If we choose the free functions to be quadratic in  $\psi$  as[3]:

$$\begin{aligned} p &= p_0 (\psi^2 / \psi_0^2), \\ F^2 &= R_0^2 B_0^2 [1 + b_0 (\psi^2 / \psi_0^2)], \end{aligned} \quad (7)$$

where  $\psi_0$ ,  $p_0$ ,  $b_0$  are the values of  $\psi$ ,  $p$ , and  $F$  on magnetic surfaces axis,  $R_0$  is the major radius, and  $B_0$  is the vacuum toroidal field.

The Grad-Shafranov equation reduces to:

$$\Delta^* \psi = -\frac{R_0^2 B_0^2}{\psi_0^2} \left( b_0 + \beta_0 \frac{R^2}{R_0^2} \right) \psi, \quad (8)$$

where  $\beta_0 = 2\mu_0 p_0 / B_0^2$ . With normalizing variables as  $R^2 / R_0^2 = x$ , and  $Z / a = y$ , Eq. (8) can then be written as:

$$4\varepsilon^2 x \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + (\alpha x + \gamma) \psi = 0, \quad (9)$$

where  $\varepsilon = a / R_0$ , is the inverse aspect ratio, and

$$\alpha = \left( \frac{a R_0 B_0}{\psi_0} \right)^2 \beta_0, \quad \gamma = \left( \frac{a R_0 B_0}{\psi_0} \right)^2 b_0. \quad (10)$$

The solution of Eq. (9) in cylindrical coordinates  $(R, Z)$  can be written as:

$$\psi = \sum_m X_m(\rho) Y_m(y), \quad (11)$$

where  $x = -i(\varepsilon / \sqrt{\alpha}) \rho$ , and for up-down symmetric case,  $Y_m$  obtained as:

$$\begin{aligned} \frac{d^2 Y_m}{dy^2} + k_m^2 Y_m &= 0, \\ Y_m(y) &= \cos(k_m y). \end{aligned} \quad (12)$$

Also  $X_m(\rho)$  can be written as:

$$\frac{d^2 X_m}{d\rho^2} + \left[ \frac{\lambda_m}{\rho} - \frac{1}{4} \right] X_m = 0, \quad (13)$$

$$X_m(\rho) = a_m W_{\lambda_m, \mu}(\rho) + b_m M_{\lambda_m, \mu}(\rho),$$

where  $\lambda_m = -i \frac{\gamma - k_m^2}{4\varepsilon\sqrt{\alpha}}$ ,  $W_{\lambda_m, \mu}(\rho)$ , and  $M_{\lambda_m, \mu}(\rho)$  are the Whittaker functions, and in this model  $\mu = 1/2$ .

Guazzotto proposed only three terms for  $m$ , and then  $\psi$  can be written as:

$$\psi(\rho, y) = \sum_{m=1}^3 \left[ a_m W_{\lambda_m, 1/2}(\rho) + b_m M_{\lambda_m, 1/2}(\rho) \right] \cos(k_m y), \quad (14)$$

where  $a_m$ ,  $b_m$ , and  $k_m$  are nine unknown coefficients which must be determined. The first six of them can be obtained from boundary conditions. The boundary conditions for the points of inner, outer, and top of the plasma cross section are (see Figure (2))[3]:

$$\begin{aligned} \psi(R_0 + a, 0) &= 0, \quad \psi(R_0 - a, 0) = 0, \\ \psi(R_0, a) &= 0, \quad \frac{\partial \psi}{\partial R}(R_0, a) = 0, \end{aligned} \quad (15)$$

and condition for right convexity on the inboard midplane is:

$$\frac{1}{R_c} = \frac{\frac{\partial^2 \psi}{\partial Z^2}(R_0 - a, 0)}{\frac{\partial \psi}{\partial R}(R_0 - a, 0)} = -\frac{1}{a}, \quad (16)$$

also two conditions, one defining the location of the magnetic axis ( $R_{axis}$ ) and the other the normalization for  $\psi$  on magnetic axis:

$$\frac{\partial \psi}{\partial R}(R_{axis}, 0) = 0, \quad \frac{\psi(R_{axis}, 0)}{\psi_0} = 1. \quad (17)$$

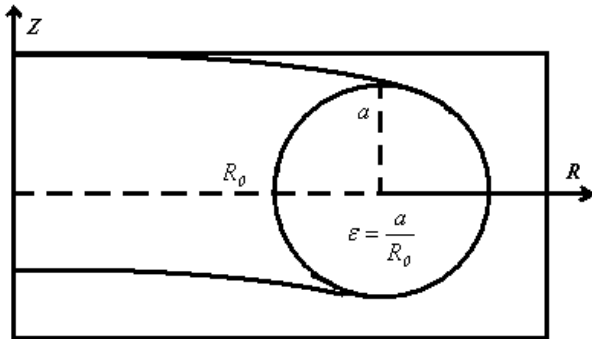


Figure (2). geometry used for the boundary conditions

These are seven boundary conditions for seven unknown coefficients ( $a_m$ ,  $b_m$ ,  $R_{axis}$ ).

But for other three coefficients, by setting  $k_1 = 0$  (the simplest solution for GSE independent of  $Z$ ), and introduce one approximate value for the  $\gamma$  ( $\gamma \approx 2(q/\varepsilon)^2 (\delta B_\phi / B_\phi)$ , negative for diamagnetism plasma), and assuming that  $k_2$  be imaginary and  $k_3$  be real, the values of  $\alpha$ ,  $k_2$ ,  $k_3$  can be determined by minimizing the error function between traditional plasma shape ( $R = R_0 + a \cos \theta$ ,  $Z = a \sin \theta$ ), and analytical plasma shape. Appropriate error function between them defined as follow[3]:

Error Function =

$$\frac{\sum_i \sqrt{[R_A(\theta_i) - R_T(\theta_i)]^2 + [Z_A(\theta_i) - Z_T(\theta_i)]^2}}{\sum_i \sqrt{R_T^2(\theta_i) + Z_T^2(\theta_i)}}, \quad (18)$$

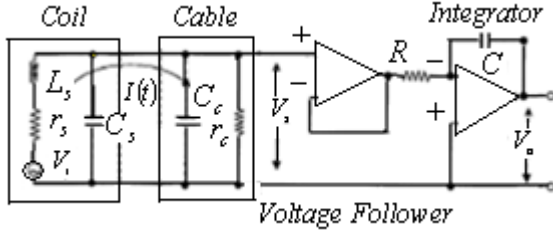
where subscripts ( $A$  and  $T$ ) indicates the analytical and traditional plasma shape parameters respectively, and sum in minimum include 3 angles ( $0, \pi/2, \pi$ ) for our purpose (circular plasma).

In general by minimizing the error function (e.g. by Mathematica) as possible to zero, and finding optimal values for  $\alpha$ ,  $k_2$ ,  $k_3$ , and also solving seven equations for the boundary conditions (Eqs. (15), (16), and (17)), six unknown coefficients ( $a_m$ ,  $b_m$ ), moreover  $R_{axis}$  can be find. Therefore the magnetic flux surfaces can be plot by substituting these nine coefficients and also input parameters as  $I_p$ ,  $\alpha$ ,  $R_0$ , and  $\varepsilon$  in Eq. (14). Moreover the Shafranov shift ( $\Delta R$ ), is also can be obtained by subtracting  $R_0$  from  $R_{axis}$ . We repeat 18 times this procedure for different approximate values for  $\gamma$  during time interval of target shot (for example the magnetic flux surfaces at  $t=15\text{ms}$  correspond to  $\gamma \approx -0.32$  shown in Figure (5)), and obtaining time interval of Shafranov shift. Results were shown in Figure (4).

## 4. Experimental Results and Comparison between Them

For determination of the Shafranov shift using the magnetic probes method, we needed for determination of the magnetic field distribution around the plasma. Therefore we designed, and constructed a four magnetic pickup coils, and installed them on outer surface of the IR-T1 chamber. Positions of the four magnetic probes on the outer surface of the IR-T1 chamber were shown in figure (1). Also, plasma current obtained from the Rogowski coil.

The electric circuit used for the measurement of the induced voltage that is supplied by the magnetic probes is showed in Figure (3):



**Figure (3).** Block diagram of the circuit for magnetic probe

In the diagram,  $I(t)$  is the current in the coil circuit and  $V_s(t)$  is the coaxial cable output signal. The application of Kirchhoff's voltage law for this circuit yields the following equation:

$$\frac{dI(t)}{dt} + \frac{(r_s + r_c)}{L_s} I(t) = \frac{V_i(t)}{L_s}, \quad (19)$$

where,  $r_s$  is the probe resistivity,  $r_c$  is the cable resistivity and  $L_s$  probe self-inductance. So, the integrator output  $V_o$  for steady state current given by

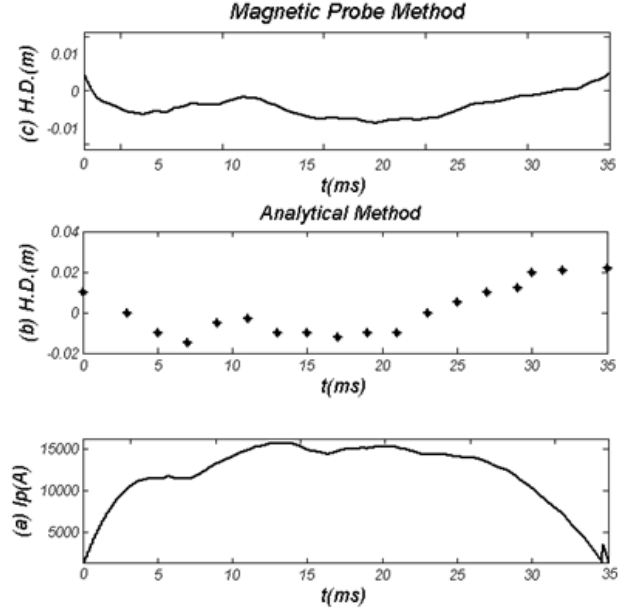
$$V_o(t) = \frac{r_c}{r_c + r_s} \frac{1}{RC} \int V_i(t) dt = k \int V_i(t) dt, \quad (20)$$

where  $RC$  is the integrator time constant, and where  $V_i$  is the inductive voltage supplied by each one of the magnetic probes.

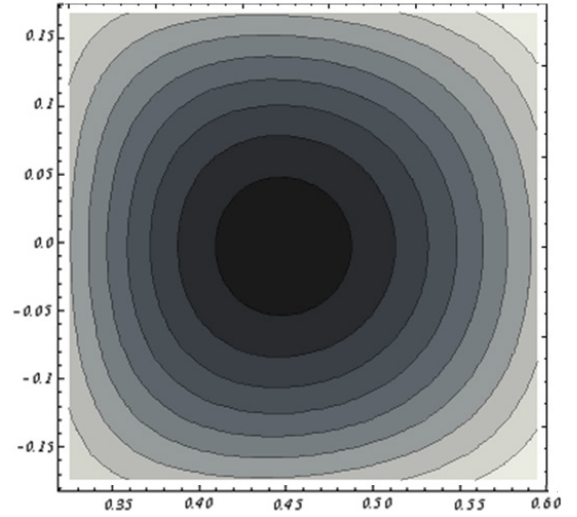
We used the two methods to determine the Shafranov shift in IR-T1 tokamak, and results presented in Figure (4). These figures show that results of two methods are in good agreement with each other. Moreover we plot the magnetic flux surfaces for plasma parameters at  $t=15$ ms in target shot on IR-T1 tokamak, as we expect there is displacement of plasma column center as shown in Figure (5).

**Table 1.** Parameters of the IR-T1 Tokamak

Parameters	Value
Major Radius	45 cm
Minor Radius	12.5 cm
Toroidal Field	$\langle 1.0$ T
Plasma Current	$\langle 40$ kA
Discharge Time	$\langle 35$ ms
Electron Density	$0.7-1.5 \times 10^{13} \text{ cm}^{-3}$



**Figure (4).** (a) Plasma current, (b) Horizontal Displacement (H.D.) determined by the Analytical method, and (c) H.D. obtained by the magnetic probes method along the plasma current



**Figure (5).** Magnetic Flux Surfaces obtained by the Second Analytical Method at  $t=15$ ms in Target Shot correspond to  $\gamma \approx -0.32$  on IR-T1 Tokamak, Displacement of the Plasma Column Center observable

## 5. Summary and Conclusions

In this paper we presented magnetic probes technique for measurement of the Shafranov shift and comparing with analytical technique in the IR-T1 tokamak. In the first technique, four magnetic pickup coils were designed, constructed, and installed on outer surface of the IR-T1 tokamak chamber and then, plasma displacement measured

from them. To compare the results measured using this method analytical solution of the Grad-Shafranov equation based on expansion of free functions as quadratic in flux function is also experimented on the IR-T1. Results of the two techniques are in good agreement with each other. The acceptable differences between them are because of (1) approximation in measurements of magnetic fields distribution because of discrete probes measurements, (2) the approximate values chosen for  $\gamma$ , and (3) the errors do not become zero during minimizing the error function.

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