Journal of Nuclear and Particle Physics 2013, 3(3): 29-35
DOI: 10.5923/j.jnpp.20130303.01

Optimization of the Stability Margin for Nuclear Power Reactor Design Models Using Regression Analyses Techniques

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Abstract Multiple regression analysis is applied on twenty-four (24) typical nuclear reactor design models, each having sixteen (16) major design input parameters. An empirical expression for "Safety Factor", Ỳ, as a function of the sixteen major design input parameters is obtained. Further statistical analyses suggest that this empirical expression is acceptable as the calculated values of Ỳ is in good agreement with known typical values. 78.95% of the "Safety Factor", Ỳ, is observed from the sixteen major design input parameters at significant level of 5%. This shows that the regression analyses techniques may be applied as an effective tool for optimization of the stability margin in nuclear power reactor design models.

Keywords Multiple regression analysis, twenty-four (24) typical nuclear reactor design models, sixteen (16) major design input parameters, safety factor, Ỳ, optimization, stability margin in nuclear power reactor design models

1. Introduction

The nuclear power plant concept design process often embraces novel concepts and technologies that carry with them an inherent risk of failure in operation which may be due to their first-time application to energy generation of its kind or to the fact that their design concept are not well studied/understood. Some definitions of "risk" are given by Modarres[1], Molak[2] and Blanchard[3]; however, Wang[4] and Roush[5] define Engineering as "a profession of managing technical Risk." It should be noted that "risk" is a physically measurable quantity. Wang and Roush define risk analysis as the quantification of potential failure. In nuclear industry risk is mostly taken as fear of accident occurring. The development of Ships, Aircrafts, Nuclear Power Plants and other System with risk factor implication pose concerns about their safety and this led to the development of the classical probabilistic risk analysis.

In this work, Ordinary Least Square (OLS) methodology, which is largely used in nuclear industry for modeling safety, is employed. Some related previous works on the application of regression analysis technique include: "Stochastic Modeling of Deterioration in Nuclear Power Plants Components" by Xianxun Yuan[2007], "Polynomial Regression with Derivative Information in Nuclear Reactor Uncertainty Quantification" by M. Anitescu and co-workers [2009], and "Proportional Hazard Regression Models for Point Processes: An Analysis of Nuclear Power Plant Operations in Europe" by RAND[2011]. Others are, "Advanced Power Plant Modeling with Applications to the Advanced Boiling Water Reactor and the Heat Exchanger" by Prasanna Kumar Muralimanohar[2009],"Modeling Water Withdrawal and Consumption for Electricity Generation in the United States" Kenneth Strzepek *et al*,[2012] and Andrew F. Siegel[2002] Practical Statistic McGraw - Hill

This work, "Optimization of the Stability Margin for Nuclear Power Reactor Design Models Using Regression Analyses Techniques", provides an a mathematical expression for predicting "Safety Factor", Ỳ, (dependent variables) given the values of independent variables or input parameters for any nuclear reactor design model. Moreover, the mathematical expression can also be used to determine the contribution of each parameter (independent variables) to the nuclear reactor stability, given the value of dependent variable.

2. Research Approach

A general assessment of twenty four (24) typical nuclear reactor design models, each having sixteen major input parameters was done. The major input parameters, which are the measurable materials or measurable components in the design models of nuclear energy reactor include: Pressure,

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Published online at http://journal.sapub.org/jnpp

Power, Core-inlet enthalpy, Mass flux, Channel coolant diameters, Active core length, Riser diameter, Riser length, Area of the down comer, Diameter of the fuel rod, Number of fuel rods, Volume of materials, Core exist void faction, Number of coolant channels, Safety margin and Safety factor. The twenty four (24) typical nuclear reactor design models are coded: NRDI to NRDXXIV which stands for 'Nuclear Reactor Design I' to 'Nuclear Reactor Design XXIV'. For each of these different design models, a linear regression analysis technique is applied using statistical software, NCSS (Number Cruncher Statistical Software). Furthermore, for the combined twenty four design models, a multiple regression analysis technique is also applied using the NCSS. The results give a model equation for each of the different design models and a general model equation for the combined twenty four designs which can be used to make prediction on the reactor stability in the design concept of each design and the combined twenty four designs. In Table 1a -1d, the values of the 16 major input parameters for the 24 nuclear reactor design models are presented:

Table 1a. NRDI - NRDVI: The Values of 16 Major Input Parameters of 24 Typical Nuclear Reactor Design Models

Parameter	Parameter Symbol	NRD I	NRD II	NRD III	NRD IV	NRD V	NRD VI
Factor of safety	Ỳ	1.4	1.4	1.5	1.45	1.6	1.55
Pressure	X_1	70	69	68	67	70	67
Power	X_2	220	210	219	217	220	200
Core inlet enthalpy	X_3	1204.8	1204.8	1204.8	1210.5	1215.8	552
Mass flux	X_4	1000	800	1000	3000	900	2000
Coolant channel diameter	X_5	0.1076	0.1128	0.1105	0.15	0.1052	0.15
Active core length	X_6	3.82	3.39	6.19	5	4.52	4.5
Riser diameter	X_7	0.1018	0.1063	0.1277	1.15	2.3	3.06
Riser length	X_8	27.5	20	28.3	29	25.5	31.15
Area of the down comer	X_9	0.1007	0.1011	0.1381	0.1389	0.1472	0.241
Diameter of the fuel rod	X_{10}	0.011	0.011	0.011	1.35	0.022	0.022
Number of coolant channels	X_{11}	113	111	112	110	113	112
Number of fuel rods	X_{12}	54	52	53	51	54	53
Volume of material	X_{13}	4.5	3.1138	5.9441	4.5	3.611	4.5
Core exit void fraction	X_{14}	0.8034	0.8025	0.778	0.712	0.645	0.8051
Stability margin	X_{15}	0.092	0.1069	0.1481	266	0.984	0.2039

Table 1b. NRDVII -NRDXII

Parameter	Parameter Symbol	NRD VII	NRD VIII	NRD IX	NRD X	NRD XI	NRD XII
Factor of safety	Y	1.6	1.7	1.8	1.65	1.39	1.42
Pressure	X_1	68	69	66	70	69	67
Power	X_2	210	218	220	219	218	220
Core inlet enthalpy	X_3	552	1214	1217	1215	1208	1209
Mass flux	X_4	1000	3729	2500	2485	1550	1698
Coolant channel diameter	X_5	0.0081	0.14	0.16	0.065	0.115	0.154
Active core length	X_6	3.5	5.451	4.75	3.76	4.55	5.46
Riser diameter	X_7	3.87	1.292	1.18	2.45	1.18	0.175
Riser length	X_8	26.8	33.97	25.6	20.5	27.8	29.5
Area of the down comer	X_9	0.007	0.1481	0.1009	0.1013	0.1389	0.139
Diameter of the fuel rod	X_{10}	0.0081	1.45	0.013	0.015	0.136	0.025
Number of coolant channels	X_{11}	133	111	110	109	111	112
Number of fuel rods	X_{12}	51	52	53	54	51	52
Volume of fuel material	X_{13}	6	11.612	9.34	6.35	5.28	4.7
Core exit void fraction	X_{14}	0.669	0.681	0.735	0.805	0.675	0.734
Stability margin	X_{15}	0.1591	368	0.524	0.195	0.285	0.384

Table 1c. NRDXIII -NRDXVIII

Parameter	Parameter Symbol	NRD XIII	NRD XIV	NRD XV	NRD XVI	NRD XVII	NRD XVIII
Factor of safety	Y	1.63	1.72	1.46	1.44	1.53	1.76
Pressure	X_1	68	69	70	68.5	69	70
Power	X_2	217	218	220	219	210	218
Core inlet enthalpy	X_3	1216	1215	1214	1200	1214	1213
Mass flux	X_4	3010	2675	1530	1950	878	2120
Coolant channel diameter	X_5	0.161	0.107	0.115	0.104	0.109	0.113
Active core length	X_6	3.85	6.13	4.38	5.25	6.215	4.89
Riser diameter	X_7	0.109	2.65	3.08	3.95	4	2.56
Riser length	X_8	25.8	30.5	32.65	26.9	33.4	32.3
Area of the down comer	X_9	0.1475	0.245	0.081	0.152	0.184	0.151
Diameter of the fuel rod	X_{10}	0.028	0.091	0.147	0.116	0.014	0.018
Number of coolant channels	X_{11}	113	111	110	109	112	113
Number of fuel rods	X_{12}	53	54	52	51	53	54
Volume of fuel material	X_{13}	5.82	3.45	4.75	6.25	8.5	7.35
Core exit void fraction	X_{14}	0.835	0.785	0.654	0.875	0.779	0.684
Stability margin	X_{15}	0.53	0.364	0.782	0.654	0.274	0.269

Table 1d. NRDXIX -NRDXXIV

Parameter	Parameter Symbol	NRD XIX	NRD XX	NRD XXI	NRD XXII	NRD XXIII	NRD XXIV
Factor of safety	Y	1.62	1.48	1.55	1.63	1.72	1.46
Pressure	X_1	67	68	69	70	68	70
Power	X_2	217	220	219	218	220	217
Core inlet enthalpy	X_3	1210	1219	124.8	1204.7	1204.9	1219.7
Mass flux	X_4	2435	1895	1781	1650	2000	1875
Coolant channel diameter	X_5	0.11	0.108	0.124	0.119	0.116	0.108
Active core length	X_6	3.95	5.56	6.23	4.58	5.39	6.21
Riser diameter	X_7	3.31	3.58	2.19	1.35	1.22	1.95
Riser length	X_8	25.8	26.9	31.5	29.3	28.7	32.1
Area of the down comer	X_9	0.115	0.102	0.138	0.154	0.162	0.114
Diameter of the fuel rod	X_{10}	0.021	0.017	0.285	1.28	0.058	0.145
Number of coolant channels	X_{11}	111	110	111	113	112	113
Number of fuel rods	X_{12}	52	53	51	54	51	53
Volume of fuel material	X_{13}	4.39	6.52	10.75	5.35	7.35	9.45
Core exit void fraction	X_{14}	0.784	0.685	0.775	0.834	0.645	0.793
Stability margin	X_{15}	0.358	0.185	0.278	0.198	0.344	0.287

In order to evaluate the models, the following tests were carried out as applicable to multiple regression analysis:

- F-test which is the overall test of the designs
- t-test which is the test of the individual design
- Autocorrelation (whether a present error(s) is/are dependent on the last error(s))
- Testing the significance of regression coefficients, b_i (i.e. the contribution or effect of each design input parameter

on the reactor stability, assuming all other parameters are held constant).

- Check for systematic bias in the forecast (where the average error is zero)
- Normality test.

3. Results and Analyses

The results of the application of multiple regression analysis performed on the sixteen (16) major input parameters of twenty four (24) nuclear reactor design models contained in Table 1a-1b are presented as follows: This regression analysis was carried out with the use of statistical software known as Number Cruncher Statistics Software (NCSS).

3.1. Empirical Expression for Safety Factor, $\grave{Y}$

An empirical expression for the "Safety Factor", $\grave{Y}$, as a function of the stipulated input parameters, X_i: I = 1,2,3,…,15, was obtained as:

$$\grave{Y} = 0.3263 - 0.0583X_1 + 0.0082X_2 + 0.0002X_3 + 0.0001 X_4 - 2.1868X_5 - 0.1144 X_6 + 0.0106 X_7 - 0.0019 X_8 + 2.72284 X_9 + 0.0565X_{10} + 0.0086 X_{11} + 0.0544X_{12} + 0.0419 X_{13} - 0.5770 X_{14} - 0.0002 X_{15} \quad (1)$$

Where 0.3263 is a constant, the X_i's represent major component design parameters of the nuclear power reactor design models as presented in Table 1a-1d.

Equation (1) is an important design empirical equation which can be used to predict the value of the safety factor, $\grave{Y}$, from given values of X_i, which are the independent variables. Equation (1) can also be used to determine the contribution of each component X_i to reactor stability, given the value of $\grave{Y}$.

3.2. Multiple Regression Report on NRDI to NRDXXIV

Here, F-test and t-test are employed to ascertain the validity of the model expressed mathematically in equation (1).

(I) F-test Result (test of all the design models)

The summary of the F-test report on the twenty four (24) nuclear reactor design models, NRDI to NRDXXIV, is presented in Table 2.

Table 2. Summary of F-test Statistical Data on NRDI to NRDXXIV

Parameter	Value
Dependent Variable	$\grave{Y}$ (Safety Factor)
15(major design input components)	X(Independent Variables)
Coefficient of Determination, (R^2)	0.7895
Coefficient of Variation	0.0603
Mean Square Error, MSE	8.741936×10^{-3}
Square Root of MSE	9.349832×10^{-2}
Average Absolute % Error	2.623
Number of observations, n	24

Table 2 gives the coefficient of determination, R^2, of the model which indicates goodness-of-fit of the regression and also indicates the percentage of the variation in $\grave{Y}$ that could be accounted for by the sixteen (16) X variables. In this work, it is observed that 78.95% of the Safety Factor, $\grave{Y}$, could be accounted for by this sixteen (16) major input parameters, X; while, perhaps 27.05% could be explained by other factors.

Siegel (2002) has shown that R^2 can be used to test the validity of a model. A value of R^2 = 0.7895 or 79% is obtained for the model equation (1) in this work. This is higher than the threshold value of R^2= 0.673 or 67.3% for n=24 and k = 1,2,3,…,15, and promises an acceptable level of validity. Thus this model equation is significant at the given significant level of 5%.

(II) t-test Result (test on the input parameters or independent variables)

Table 3. t-test Statistical Data

Independent variable	Regression Coefficient	Standard Error Sb(i)	t-value to test Ho=B(i)=0	Prob Level	Reject H_o at 5%	Power of test at 5%
X1	-0.0583	0.0399	-1.461	0.1875	No	0.2443
X2	0.0082	0.0082	1.003	0.3492	No	0.1405
X3	0.0002	0.0003	0.795	0.4529	No	0.1062
X4	0.0000	0.0000	0.637	0.5444	No	0.0859
X5	-2.1868	2.3787	-0.919	0.3885	No	0.1257
X6	-0.1144	0.0509	-2.249	0.0593	No	0.4917
X7	0.0106	0.0261	0.406	0.697	No	0.0644
X8	-0.0019	0.0117	-0.164	0.8744	No	0.0523
X9	2.7284	0.7914	3.448	0.0107	Yes	0.8391
X10	0.0565	0.0891	0.634	0.5464	No	0.0855
X11	0.0086	0.0133	0.647	0.5382	No	0.087
X12	0.0544	0.0275	1.981	0.0881	No	0.4016
X13	0.0419	0.0169	2.484	0.0419	Yes	0.571
X14	-0.577	0.3914	-1.474	0.1839	No	0.2479
X15	-0.0002	0.0005	-0.366	0.7254	No	0.0617
Intercept	0.3263	3.7789	0.086	0.9336	No	0.0506

Table 3 below shows the values of the regression coefficients, b(i), and the t-values for every independent variable (input parameters), X_i. This gives the validity or acceptability of each of the input parameters (independent variables).

Table 3 is used to investigate the contribution or the effect of each design input parameter, X_i, on the safety factor of the nuclear reactor design models, assuming all other parameters are held constant,. figure is the graphical representation of the t-value as a function of the major input parameters, X_i: i = 1,2,3,…,15.

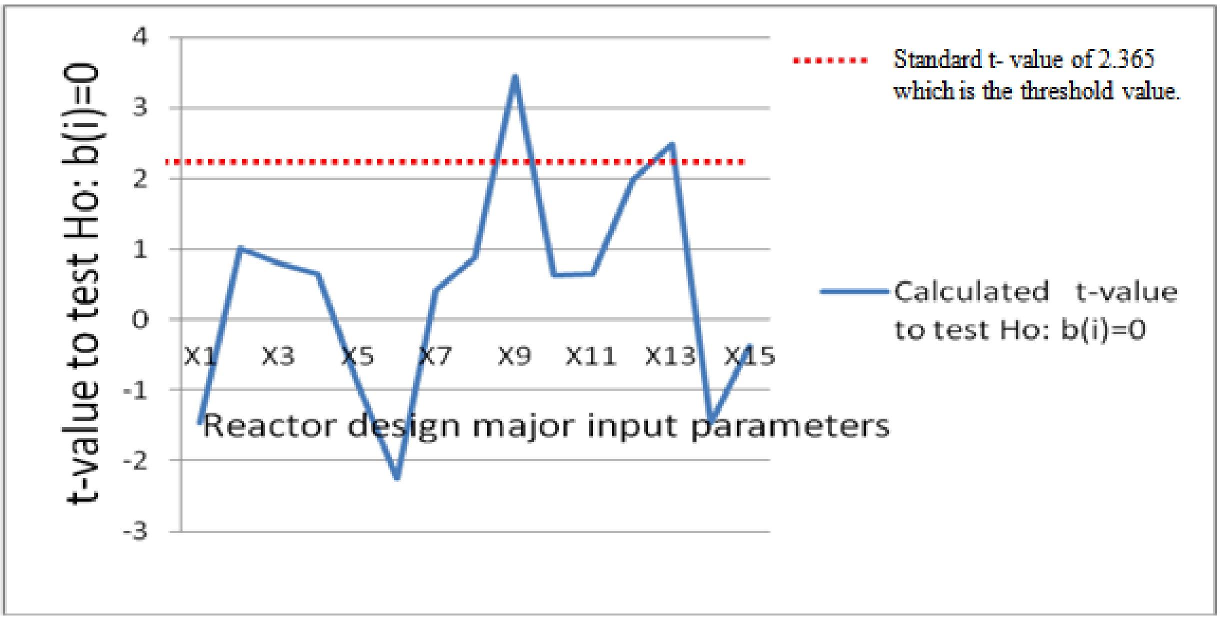

Figure 1. t-value as a function of the reactor design major input parameters

From Table 3 and Figure1, it could be seen that two parameters, the area of down comer, X_9, and the volume of the fuel material, X_{13}, have calculated t-values of 3.448 and 2.484 respectively, each being higher than the t-value of 2.365 which is the threshold value for the acceptability of the developed model. For acceptability, it is required that at least one of the t-values of the input parameters exceeds the threshold value; therefore, the developed model equation is acceptable by this t-test.

Furthermore, the graphical representation of the regression coefficient, b_i, as a function of the major input parameters, Xi: i = 1,2,…,15, is shown in Figure 2,:

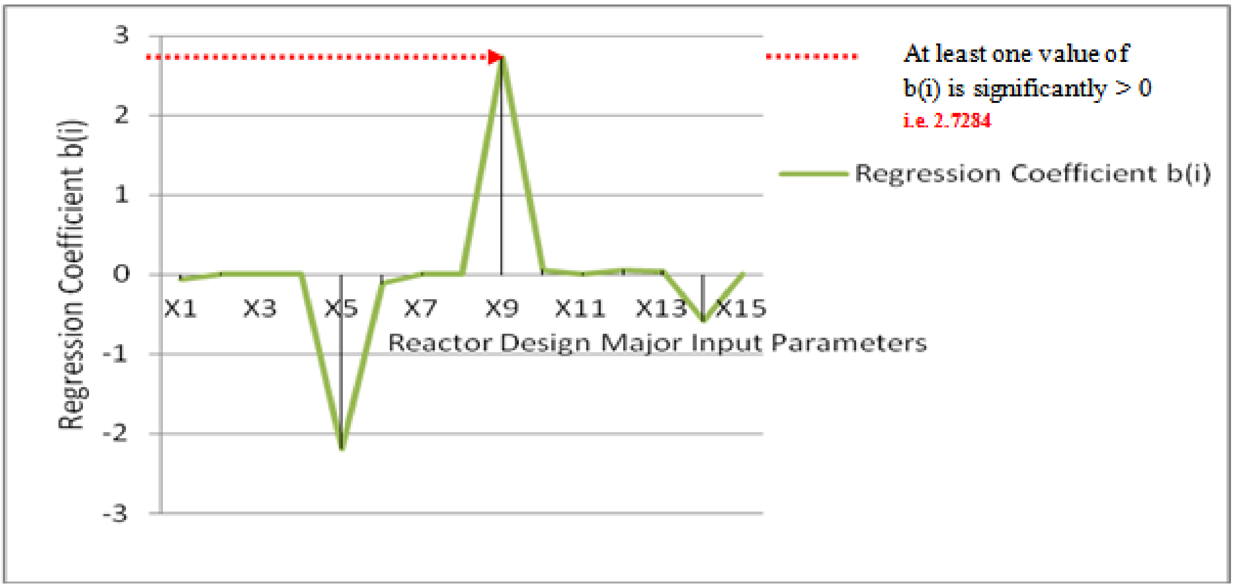

Figure 2. Regression coefficient, b_i, as a function of the reactor design major input parameters

Since at least one value of b_i is not equal to zero (0) i.e since the value of the regression coefficient, b_i, for X_9 is significantly greater than zero, the model could be accepted as being valid. Moreover, it suggests that the major input parameters or components, Xi, are linearly related to the Safety Factor, Ỳ.

4. Comparison of Safety Factor (Ỳ) Values

In Table 4 the typical Safety Factor values as well as the calculated Safety Factor values obtained are presented for comparison.

It could be seen, by inspection of the Table 4, that the calculated values of the Safety Factor, Ỳ, generally agree with those of the typical Safety Factor, Ỳ, values.

The plot in Figure 3 demonstrates clearly the agreement between the typical values of the Safety Factor (Ỳ) for the twenty four nuclear energy reactor design models and the calculated results on Safety Factor (Ỳ).

Figure 3: clearly validates the use of the empirical expression given in Equation (1) for the calculation of Safety Factor, Ỳ.

Table 4. Comparison of Typical and Calculated Safety Factors, Ỳ

NRD	Typical Ỳ	Calculated Ỳ	Standard Error of Predicted	95% Lower Confidence limit of Mean	95% Upper Confidence limit of Mean
I	1.4	1.511	0.073	1.338	1.683
II	1.4	1.356	0.084	1.158	1.554
III	1.5	1.455	0.071	1.286	1.623
IV	1.45	1.45	0.076	1.27	1.63
V	1.6	1.647	0.072	1.478	1.816
VI	1.55	1.557	0.093	1.338	1.777
VII	1.6	1.592	0.093	1.372	1.812
VIII	1.7	1.7	0.085	1.499	1.901
IX	1.65	1.604	0.073	1.431	1.778
X	1.39	1.506	0.059	1.366	1.645
XI	1.42	1.435	0.066	1.278	1.592
XII	1.63	1.61	0.08	1.422	1.798
XIII	1.72	1.67	0.08	1.481	1.859
XIV	1.46	1.373	0.08	1.183	1.564
XV	1.44	1.456	0.086	1.254	1.658
XVI	1.53	1.588	0.078	1.403	1.772
XVII	1.76	1.711	0.056	1.58	1.842
XVIII	1.62	1.604	0.079	1.418	1.79
XIX	1.48	1.55	0.071	1.383	1.718
XX	1.55	1.566	0.093	1.346	1.785
XXI	1.63	1.631	0.092	1.413	1.85
XXII	1.72	1.654	0.074	1.478	1.83
XXIII	1.46	1.434	0.065	1.28	1.588
XXIV	1.39	1.406	0.053	1.266	1.545

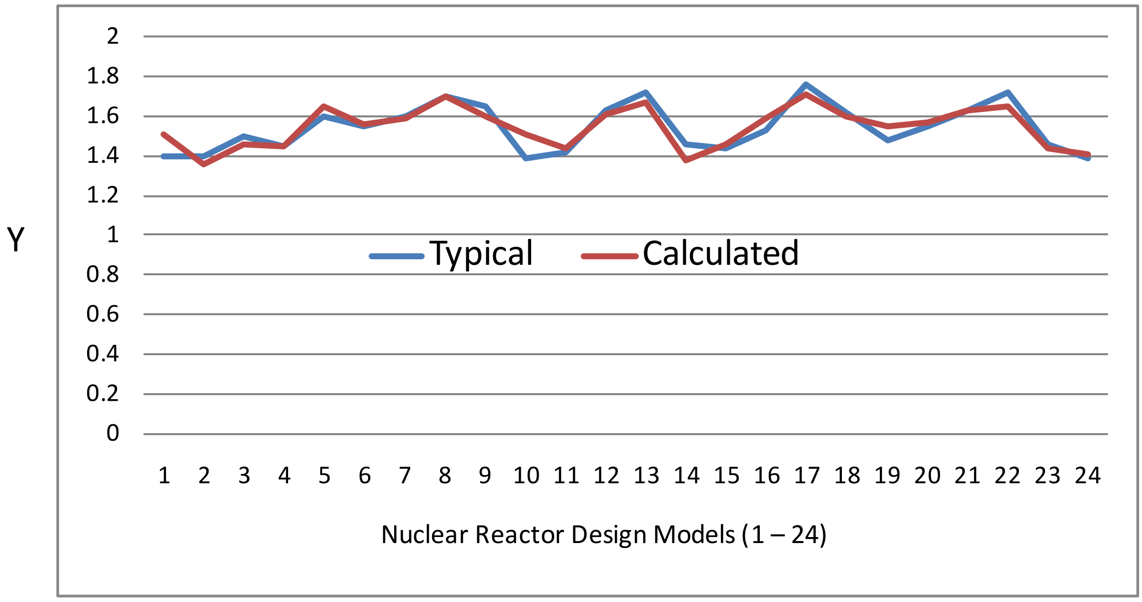

Figure 3. Comparison of typical Safety Factor , Ỳ values with corresponding calculated Safety Factor , Ỳ values

5. Summary / Conclusions

Multiple regression analysis has been applied on twenty-four (24) typical nuclear reactor design models, each having sixteen (16) major design input parameters. This produced an empirical expression for "Safety Factor", Ỳ, as a function of the sixteen major design input parameters. F-test and t-test carried out on this model equation gives a promising level of acceptability or validity with the calculated values of Ỳ being in good agreement when compared to the known typical values. 78.95% or 79% of the "Safety Factor", Ỳ, is observed from the sixteen major design input parameters, X; at significant level of 5%, while, perhaps 27.05% could be explained by other factors.

Also, the empirical formular derived can be applied to any nuclear power reactor design model to test stability of the reactor as well as the contribution of each design component to the stability of the reactor.

This shows that the regression analyses techniques may be applied as an effective tool for optimization of the stability margin in nuclear power reactor design models.

ACKNOWLEDGMENTS

We thank Nigerian Defence Academy (NDA), Kaduna, Nigeria for the material support during the research work.

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