Determination of Plasma Position using Poloidal Flux Loops and Comparing with Analytical Solution in IR-T1 Tokamak

A. Salar Elahi[*], M. Ghoranneviss

Plasma Physics Research Center, Science and Research Branch, Islamic Azad University, Tehran, Iran

Abstract   In this contribution we presented comparative measurements of tokamak plasma position. In the first technique, two poloidal flux loop were designed and installed on outer surface of the IR-T1 tokamak chamber, and then the plasma displacement obtained from them. To compare the result obtained using this method analytical solution of the Grad-Shafranov equation based on expansion of free functions as quadratic in flux function is also experimented on IR-T1. Results of the two methods are in good agreement with each other.

Keywords   Tokamak, Plasma Displacement, Poloidal Flux Loop, Grad-Shafranov Equation

1. Introduction

In ohmically heated (low β) tokamaks, radial pressure balance is achieved by the poloidal field, and toroidal force balance is achieved by equality between the external vertical field force (Lorentz force) and outward forces due to toroidal configuration. But, in toroidal force balance problem, the two opposite forces may be not equal and therefore plasma intend to shift inward or outward, which it is dangerous for tokamak plasma. Therefore, plasma equilibrium study is one of the fundamental problems of the magnetically confined plasmas. There are many available experimental methods and analytical solutions of the steady state magnetohydrodynamics (MHD) equations, in particular, the Grad-Shafranov equation for the plasma equilibrium problem. Control of plasma position has important role in plasma confinement and to achieve optimized tokamak plasma operation. Determination of accurate plasma position during confinement time is essential to transport it to a control system based on feedback. Over the years different methods have been developed to analysis the tokamak plasma equilibrium problems [1-54].

In this paper we present two experimental and analytical methods for determination of plasma column center in IR-T1 Tokamak, which it is a small, air core, low β and large aspect ratio tokamak with a circular cross section (see Table 1). Details of the poloidal flux loops technique for determination of Shafranov Shift will present in section 2.

* Corresponding author:
salari_phy@yahoo.com (A. Salar Elahi)

Analytical solution of the Grad-Shafranov equation will present in section 3. Experimental results and comparative between them also will present in section 4. Summary and conclusion will present in section 5.

2. Poloidal Flux Loops Method in Measurement of Plasma Position

Poloidal flux loop is a simple toroidally loop which measure the poloidal magnetic flux and usually array of them use in control and reconstruction of plasma equilibrium states. The magnetic flux passing through such a loop is equal to $2\pi\psi$, where ψ represent to magnetic poloidal flux. In the ohmically heated tokamaks, ohmic coils field is the main fraction of poloidal flux which passing through the flux loop. Therefore to obtain net poloidal flux due to plasma, compensation will require for all excessive flux. Because of large area of the flux loop, the inductive voltage is also large and then it consists of usually one turn. According to relation for frequency response, it is obvious that because of small self inductance, frequency response of flux loop usually is higher than which desired.

Although magnetic probe suitable for measurement of plasma position only in circular cross section plasma and not for elongated one, but the flux loop either in elongated and or circular cross section tokamaks can be used. Therefore we used these two techniques for the IR-T1 tokamak with circular cross section.

The plasma boundary is usually defined by Last Closed Flux Surface (LCFS). In the LCFS poloidal magnetic flux is constant, if we install some flux loops at some distance in vicinity of LCFS, then we can find plasma displacement from difference in poloidal fluxes that received with flux

loops according to Shafranov equation. In the quasi-cylindrical coordinates (r,θ,φ) for the poloidal magnetic flux we have [1].

$$\psi(r,\theta) = \mu_0 R_0 I_p \left(2 - \ln\frac{8R_0}{r}\right) +$$

$$\frac{\mu_0 I_p}{2}\left[\ln\frac{r}{a} + \left(\Lambda + \frac{1}{2}\right)\left(1 - \frac{a^2}{r^2}\right)\right] r\cos\theta$$

where

$$\Lambda = \beta_p + \frac{l_i}{2} - 1 \tag{1}$$

and where I_p , R_0 , a , β_p , l_i are the plasma current, major and minor plasma radiuses, poloidal beta and internal inductance of the plasma. The relationship between poloidal magnetic flux and plasma displacement is:

$$\Delta R = \frac{b}{2R_0\,\mu_0 I_p \cos\theta_0}\Delta\psi \tag{2}$$

According to Figure (1) the poloidal flux is obtained:
$$\Delta\psi = \psi_{p,out} - \psi_{p,in}$$

where

$$\psi_{p,out} = \psi_{out} + B_{mo}\Delta S_{out}\,, \quad \psi_{p,in} = \psi_{in} + B_{mi}\Delta S_{in} \tag{3}$$

and where ψ_{out} and ψ_{in} are the poloidal flux which obtained with outer and inner flux loops, B_{mo} and B_{mi} are the average magnetic fields between outer and inner flux loops and the plasma surface respectively which can be obtained from magnetic probe, ΔS is the intervening area for each loop defined as: $\Delta S = 2\pi R_{ml}d$ which d is distance between LCFS and each loop and R_{ml} is the distance between the midpoint (d/2) and center of the facility.

In the IR-T1 tokamak two poloidal flux loops designed and installed on outer surface of vacuum chamber in polar angles $\theta_1 = 0$ and $\theta_2 = \pi$, with radiuses $r_1 = 29cm$ and $r_2 = 61cm$ (see Figure (1) and Table (2)).

Table 1.   Parameters of the IR-T1 tokamak

Parameters	Value
Major Radius	45 cm
Minor Radius	12.5 cm
Toroidal Field	$\langle$ 1.0 T
Plasma Current	$\langle$ 40 kA
Discharge Time	$\langle$ 35 ms
Electron Density	$0.7\text{-}1.5\times10^{13}$ cm^{-3}

Table 2.   Design parameters of the poloidal flux loops and magnetic probes

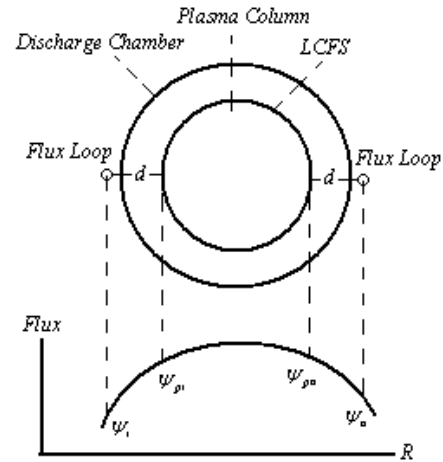| Parameters	Magnetic Probes	Flux Loops
R (Resistivity)	33 Ω	3 Ω 6 Ω
L (Inductance)	1.5mH	1mH 2mH
n (Turns)	500	1
S (Sensitivity)	0.7mV/G	11, 31 mV/G
f (Frequency Response)	22kHz	3kHz 3kHz
Effective nA	0.022 m^2	0.26 m^2 1.17 m^2
d (Wire Diameter)	0.1mm	1mm
$r_{Average}$ (Coil Average Radius)	3mm	290mm 610mm

Figure 1.   Schematic diagrams for positions of the two flux loops on outer surface of IR-T1 tokamak

As mentioned above, the ohmic field is the significant fraction of the poloidal flux which passing through the flux loop, therefore essentially compensation is needed. Compensation is done with all fields discharge without plasma and subtraction them from the total flux that received with flux loop. Experimental result for measurement of plasma position using this method will present in section 4.

3. Analytical Solution of the Grad-Shafranov Equation

For axially symmetric configurations, Maxwell's equations together with the force balance equation from

MHD equations, for stationary and ideally conducting plasmas, reduce to the two-dimensional, nonlinear, elliptic partial differential equation, or Grad-Shafranov equation (GSE) [1]. In some of the work, authors solved the Grad-Shafranov equation by expanding the free functions ($p(\psi)$, and $F(\psi)$) in different order in ψ (flux function). In this section, we regarded quadratic order (which proposed by Guazzotto [3]), and examined on the IR-T1.

The GSE is:

$$\Delta^*\psi = -\mu_0 R^2 \frac{dp}{d\psi} - F\frac{dF}{d\psi}, \qquad (4)$$

where

$$\vec{B} = \frac{1}{R}\vec{\nabla}\psi \times \hat{e}_\phi + \frac{F}{R}\hat{e}_\phi,$$

$$\mu_0\vec{J} = \frac{1}{R}\frac{dF}{d\psi}\vec{\nabla}\psi \times \hat{e}_\phi - \frac{1}{R}\Delta^*\psi\,\hat{e}_\phi.$$

If we choose the free functions to be quadratic in ψ as [3]:

$$p = p_0\left(\psi^2/\psi_0^2\right),$$
$$F^2 = R_0^2 B_0^2\left[1 + b_0\left(\psi^2/\psi_0^2\right)\right], \qquad (5)$$

where ψ_0, p_0, b_0 are the values of ψ, p, and F on magnetic surfaces axis, R_0 is the major radius, and B_0 is the vacuum toroidal field.

The Grad-Shafranov equation reduces to:

$$\Delta^*\psi = -\frac{R_0^2 B_0^2}{\psi_0^2}\left(b_0 + \beta_0\frac{R^2}{R_0^2}\right)\psi, \qquad (6)$$

where $\beta_0 = 2\mu_0 p_0/B_0^2$.

With normalizing variables as $R^2/R_0^2 = x$, and $Z/a = y$, Eq. (6) can then be written as:

$$4\varepsilon^2 x\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + (\alpha\,x + \gamma)\psi = 0, \qquad (7)$$

where $\varepsilon = a/R_0$, is the inverse aspect ratio, and

$$\alpha = \left(\frac{aR_0 B_0}{\psi_0}\right)^2\beta_0, \quad \gamma = \left(\frac{aR_0 B_0}{\psi_0}\right)^2 b_0. \qquad (8)$$

The solution of Eq. (7) in cylindrical coordinates (R, Z) can be written as:

$$\psi = \sum_m X_m(\rho)Y_m(y), \qquad (9)$$

where $x = -i\left(\varepsilon/\sqrt{\alpha}\right)\rho$, and for up-down symmetric case, Y_m obtained as:

$$\frac{d^2 Y_m}{dy^2} + k_m^2 Y_m = 0, \qquad (10)$$

$$Y_m(y) = \cos(k_m y).$$

Also $X_m(\rho)$ can be written as:

$$\frac{d^2 X_m}{d\rho^2} + \left[\frac{\lambda_m}{\rho} - \frac{1}{4}\right]X_m = 0, \qquad (11)$$

$$X_m(\rho) = a_m W_{\lambda_m,\mu}(\rho) + b_m M_{\lambda_m,\mu}(\rho),$$

where $\lambda_m = -i\dfrac{\gamma - k_m^2}{4\varepsilon\sqrt{\alpha}}$, $W_{\lambda_m,\mu}(\rho)$, and $M_{\lambda_m,\mu}(\rho)$ are the Whittaker functions, and in this model $\mu = 1/2$.

Guazzotto proposed only three terms for m, and then ψ can be written as:

$$\psi(\rho,y) =$$
$$\sum_{m=1}^{3}\left[a_m W_{\lambda_m,1/2}(\rho) + b_m M_{\lambda_m,1/2}(\rho)\right]\cos(k_m y), \qquad (12)$$

where a_m, b_m, and k_m are nine unknown coefficients which must be determined. The first six of them can be obtained from boundary conditions. The boundary conditions for the points of inner, outer, and top of the plasma cross section are (see Figure (2)):

$$\psi(R_0 + a, 0) = 0, \quad \psi(R_0 - a, 0) = 0,$$
$$\psi(R_0, a) = 0, \quad \frac{\partial\psi}{\partial R}(R_0, a) = 0, \qquad (13)$$

and condition for right convexity on the inboard midplane is:

$$\frac{1}{R_c} = \frac{\dfrac{\partial^2\psi}{\partial Z^2}(R_0 - a, 0)}{\dfrac{\partial\psi}{\partial R}(R_0 - a, 0)} = -\frac{1}{a}, \qquad (14)$$

also two conditions, one defining the location of the magnetic axis (R_{axis}) and the other the normalization for ψ on magnetic axis:

$$\frac{\partial\psi}{\partial R}(R_{axis}, 0) = 0, \quad \frac{\psi(R_{axis}, 0)}{\psi_0} = 1. \qquad (15)$$
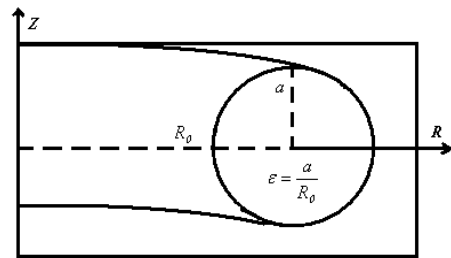

Figure 2. geometry used for the boundary conditions

These are seven boundary conditions for seven unknown coefficients (a_m, b_m, R_{axis}).

But for other three coefficients, by setting $k_1 = 0$ (the simplest solution for GSE independent of Z), and introduce

one approximate value for the γ ($\gamma \approx 2(q/\varepsilon)^2 (\delta B_\phi / B_\phi)$, negative for diamagnetism plasma), and assuming that k_2 be imaginary and k_3 be real, the values of α, k_2, k_3 can be determined by minimizing the error function between traditional plasma shape ($R = R_0 + a\cos\theta$, $Z = a\sin\theta$), and analytical plasma shape. Appropriate error function between them defined as follow:

$$Error \text{ Function} = \frac{\sum_i \sqrt{[R_A(\theta_i) - R_T(\theta_i)]^2 + [Z_A(\theta_i) - Z_T(\theta_i)]^2}}{\sum_i \sqrt{R_T^2(\theta_i) + Z_T^2(\theta_i)}}, \quad (16)$$

where subscripts (A and T) indicates the analytical and traditional plasma shape parameters respectively, and sum in minimum include 3 angles (0, $\pi/2$, π) for our purpose (circular plasma).

In general by minimizing the error function (e.g. by Mathematica) as possible to zero, and finding optimal values for α, k_2, k_3, and also solving seven equations for the boundary conditions (Eqs. (13), (14), (15)), six unknown coefficients (a_m, b_m), moreover R_{axis} can be find. Therefore the magnetic flux surfaces can be plot by substituting these nine coefficients and also input parameters as I_p, α, R_0, and ε in Eq. (12). Moreover the Shafranov shift (ΔR), is also can be obtained by subtracting R_0 from R_{axis}. We repeat 18 times this procedure for different approximate values for γ during time interval of target shot (for example the magnetic flux surfaces at t=15ms correspond to $\gamma \approx -0.32$ shown in Figure (3)), and obtaining time interval of Shafranov shift. Results were shown in Figure (4).

4. Experimental Results and Comparison Between Them

For determination of the plasma position using the first method, we needed for determination of the poloidal magnetic flux around the plasma. Therefore we designed and installed two poloidal flux loops on outer surface of the IR-T1 chamber. Positions of the flux loops were shown in figure (1). Also, plasma current and average vertical field, were obtained from the Rogowski coil and magnetic probe, respectively.

According to the Faraday's law, output of all magnetic diagnostics proportional to derivative of the magnetic flux which passing through them, therefore we needed to integrate the output of the flux loops and magnetic probes after compensating their output.

The integrator output V_o is given by:

$$V_o(t) = \frac{r_c}{r_c + r_s} \frac{1}{RC} \int V_i(t)dt = k \int V_i(t)dt, \quad (17)$$

where RC is the integrator time constant, and where V_i is the inductive voltage supplied by the flux loops and each one of the magnetic probes, which were placed around the IR-T1 tokamak vacuum chamber.
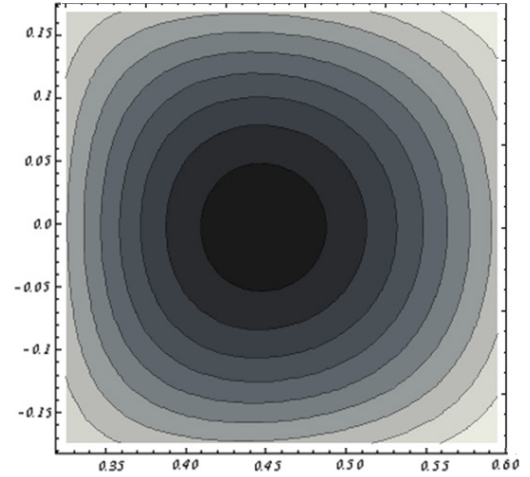

Figure 3. Magnetic Flux Surfaces obtained by the Second Analytical Method at t=15ms in Target Shot correspond to $\gamma \approx -0.32$ on IR-T1 Tokamak, Displacement of the Plasma Column Center observable
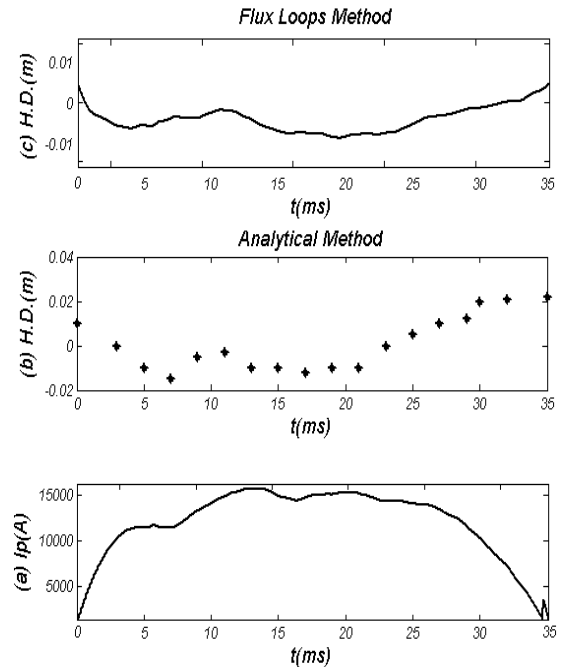

Figure 4. Horizontal Displacement (H.D.), determined by the (c) Flux loop and (b) Analytical methods along the (a) plasma current

We used the two methods to determine the horizontal displacement in IR-T1 as shown in Figure (4). These figures show that two methods give us a same horizontal displacement. Moreover we plot the magnetic flux surfaces or Eq. (12) for plasma parameters at t=15ms in target shot on

IR-T1 tokamak, as we expect there is displacement of plasma column center as shown in Figure (3).

5. Summary and Conclusions

In this paper we determined the Shafranov shift by two experimental and analytical methods in IR-T1 tokamak. In the first method we designed and installed two poloidal flux loops on the outer surface of the IR-T1 tokamak chamber, and then plasma displacement determined from them. To compare the plasma position obtained using this method, the analytical solution of the Grad-Shafranov equation based on expanding of the free functions, quadratic in ψ is also presented, and experimented on IR-T1. Results show that two methods are in good agreement with each other. The acceptable differences between them are because of (1) approximation in measurement of poloidal flux on LCFS, (2) the approximate values chosen for γ, and (3) the errors do not become zero during minimizing the error function.

REFERENCES

[1] A. Salar Elahi et al., IEEE Trans. Plasma Science 38 (2), 181-185, (2010), DOI: 10.1109/TPS.2009.2037965

[2] A. Salar Elahi et al., IEEE Trans. Plasma Science 38 (9), 3163-3167, (2010), DOI: 10.1109/TPS.2010.2066289

[3] M. Emami, M. Ghoranneviss, A. Salar Elahi and A. Rahimi Rad, J. Plasma Phys. 76 (1), 1-8, (2009), DOI: 10.1017/S0022377809008034

[4] A. Salar Elahi et al., Fusion Engineering and Design 85, 724–727, (2010), DOI: 10.1016/j.fusengdes.2010.04.034

[5] A. Salar Elahi et al., Phys. Scripta 80, 045501, (2009), DOI: 10.1088/0031-8949/80/04/045501

[6] A. Salar Elahi et al., Phys. Scripta 80, 055502, (2009), DOI: 10.1088/0031-8949/80/05/055502

[7] A. Salar Elahi et al., Phys. Scripta 81 (5), 055501, (2010), DOI: 10.1088/0031-8949/81/05/055501

[8] A. Salar Elahi et al., Phys. Scripta 82, 025502, (2010), DOI: 10.1088/0031-8949/82/02/025502

[9] A. Salar Elahi et al., Phys. Scripta 82 (3), 035502, (2010), DOI: 10.1088/0031-8949/82/03/035502

[10] A. Salar Elahi et al., J. Fusion Energy 28 (4), 346-349, (2009), DOI: 10.1007/s10894-009-9198-x

[11] A. Salar Elahi et al., J. Fusion Energy 28 (4), 416-419, (2009), DOI: 10.1007/s10894-009-9215-0

[12] A. Salar Elahi et al., J. Fusion Energy 28 (4), 408-411, (2009), DOI: 10.1007/s10894-009-9213-2

[13] A. Salar Elahi et al., J. Fusion Energy 28 (4), 412-415, (2009), DOI: 10.1007/s10894-009-9214-1

[14] A. Salar Elahi et al., J. Fusion Energy 28 (4), 394-397, (2009), DOI: 10.1007/s10894-009-9210-5

[15] A. Salar Elahi et al., J. Fusion Energy 28 (4), 404-407, (2009), DOI: 10.1007/s10894-009-9212-3

[16] A. Salar Elahi et al., J. Fusion Energy 28 (4), 390-393, (2009), DOI: 10.1007/s10894-009-9208-z

[17] A. Salar Elahi et al., J. Fusion Energy 28 (4), 385-389, (2009), DOI: 10.1007/s10894-009-9207-0

[18] A. Rahimi Rad, M. Ghoranneviss, M. Emami, and A. Salar Elahi, J. Fusion Energy 28 (4), 420-426, (2009), DOI: 10.1007/s10894-009-9216-z

[19] A. Salar Elahi et al., J. Fusion Energy 29 (1), 1-4, (2010), DOI: 10.1007/s10894-009-9218-x

[20] A. Salar Elahi et al., J. Fusion Energy 29 (1), 22-25, (2010), DOI: 10.1007/s10894-009-9221-2

[21] A. Salar Elahi et al., J. Fusion Energy 29 (1), 29-31, (2010), DOI: 10.1007/s10894-009-9224-z

[22] A. Salar Elahi et al., J. Fusion Energy 29 (1), 26-28, (2010), DOI: 10.1007/s10894-009-9223-0

[23] A. Salar Elahi et al., J. Fusion Energy 29 (1), 32-35, (2010), DOI: 10.1007/s10894-009-9227-9

[24] A. Salar Elahi et al., J. Fusion Energy 29 (1), 36-40, (2010), DOI: 10.1007/s10894-009-9226-x

[25] A. Salar Elahi et al., J. Fusion Energy 29 (1), 62-64, (2010), DOI: 10.1007/s10894-009-9232-z

[26] A. Salar Elahi et al., J. Fusion Energy 29 (1), 76-82, (2010), DOI: 10.1007/s10894-009-9234-x

[27] A. Rahimi Rad, M. Emami, M. Ghoranneviss, A. Salar Elahi, J. Fusion Energy 29 (1), 73-75, (2010), DOI: 10.1007/s10894-009-9236-8

[28] A. Salar Elahi et al., J. Fusion Energy 29 (1), 83-87, (2010), DOI: 10.1007/s10894-009-9235-9

[29] A. Salar Elahi et al., J. Fusion Energy 29 (1), 88-93, (2010), DOI: 10.1007/s10894-009-9237-7

[30] A. Salar Elahi et al., J. Fusion Energy 29 (3), 209-214, (2010), DOI: 10.1007/s10894-009-9260-8

[31] A. Salar Elahi et al., J. Fusion Energy 29 (3), 232-236, (2010), DOI: 10.1007/s10894-009-9264-4

[32] A. Salar Elahi et al., J. Fusion Energy 29 (3), 251-255, (2010), DOI: 10.1007/s10894-009-9267-1

[33] A. Salar Elahi et al., J. Fusion Energy 29 (3), 279-284, (2010), DOI: 10.1007/s10894-010-9275-1

[34] A. Salar Elahi et al., J. Fusion Energy 29 (5), 467-470, (2010), DOI: 10.1007/s10894-010-9307-x

[35] A. Salar Elahi et al., J. Fusion Energy 29 (5), 461-465, (2010), DOI: 10.1007/s10894-010-9305-z

[36] A. Salar Elahi et al., Brazilian J. of Physics 40 (3), 323-326, (2010).

[37] A. Salar Elahi et al., J. Fusion Energy 30 (2), 116-120, (2011), DOI: 10.1007/s10894-010-9359-y

[38] M.R. Ghanbari, M. Ghoranneviss, A. Salar Elahi et al., Phys. Scripta 83, 055501, (2011), DOI: 10.1088/0031-8949/83/05/055501

[39] A. Salar Elahi, J of Fusion Energy 30 (6), 477-480, (2011), 477-480, DOI: 10.1007/s10894-011-9408-1

[40] A. Salar Elahi et al., Fusion Engineering and Design 86, 442–445, (2011), DOI: 10.1016/j.fusengdes.2011.03.121

[41] A. Salar Elahi et al., J of Fusion Energy 31 (2), 191-194, (2011), DOI: 10.1007/s10894-011-9452-x

[42] M.R. Ghanbari, M. Ghoranneviss, A. Salar Elahi and S. Mohammadi, Radiation Effects & Defects in Solids 166 (10), 789–794, (2011), DOI: 10.1080/10420150.2011.610320

[43] A. Salar Elahi et al., IEEE Trans. Plasma Science 40 (3), 892-897, (2012), DOI: 10.1109/TPS.2012.2182990

[44] A. Salar Elahi et al., Accepted for publication in Radiation Effects & Defects in Solids (January 2012), DOI: 10.1080/10420150.2011.650171

[45] Z. Goodarzi, M. Ghoranneviss and A. Salar Elahi, Accepted for the publication in J Fusion Energy (March 2012), DOI: 10.1007/s10894-012-9526-4

[46] M.R. Ghanbari, M. Ghoranneviss, A. Salar Elahi et al., Phys. Scripta 85 (5), 055502, (2012), DOI: 10.1088/0031-8949/85/05/055502

[47] M. Ghoranneviss, A. Salar Elahi, H. Hora, G.H. Miley et al., Accepted for the publication in Laser and Particle Beams (May 2012), DOI: 10.1017/S0263034612000341

[48] M. Ghoranneviss et al., Accepted for the publication in Radiation Effects and Defects in Solids (June 2012)

[49] A. Salar Elahi et al., Accepted for the publication in Radiation Effects and Defects in Solids (June 2012), DOI: 10.1080/10420150.2012.706609

[50] A. Salar Elahi et al., Accepted for the publication in Radiation Effects and Defects in Solids (June 2012), DOI: 10.1080/10420150.2012.706607

[51] K. Mikaili, M. Ghoranneviss, A. Salar Elahi et al., Accepted for the publication in J Fusion Energy (July 2012), DOI: 10.1007/s10894-012-9563-z

[52] A. Salar Elahi et al., Journal of Nuclear and Particle Physics 1(1), (2011), 10-15, DOI: 10.5923/j.jnpp.20110101.03

[53] A. Salar Elahi et al., Journal of Nuclear and Particle Physics 2(2), (2012), 1-5, DOI: 10.5923/j.jnpp.20120202.01

[54] M. Ghasemloo, M. Ghoranneviss, A. Salar Elahi et al., Journal of Nuclear and Particle Physics 2(2), (2012), 22-25, DOI: DOI: 10.5923/j.jnpp.20120202.05