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Model, Implement and Compare a New Optimal Adaptive Fault Diagnosis Observer with Six Observers

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Abstract This paper presents the design of anew optimal adaptive diagnosis observer (OAD) which is designed for additive fault and disturbance; its gain matrix verifies the proposed Lyapunov conditions. In the presence of disturbance and fault, the performance of the ODA observer is tested using Matlab software by comparing it with six different good linear observers Luenberger Observer (LO), Kalman (Filter) Observer (KO), Unknown Input Observer (UIO), Augmented Robust Observer (ARO), High Gain Observer (HGO) and Sensitive High Gain Observer (SHGO). The assumed disturbance and faults are white noise, coloured noise and non-Gaussian fault while a MIMO DC servomotor has been used as a benchmark in the performance assessments. As the results show, the comparison results of the ODA observer is the best overall in diagnosing fault and disturbance as well asit is the highest instates estimation performance.

Keywords New Optimal Adaptive Diagnosis Observer (OAD), Luenberger Observer (LO), Kalman (Filter) Observer (KO), Unknown Input Observer (UIO), Augmented Robust Observer (ARO), High Gain Observer (HGO), Multiple Inputs Multiple Outputs (MIMO), Sensitive High Gain Observer (SHGO)

1. Introduction

Observers are techniques that are used to estimate and detect the faults of the systems. diagnosis in dynamic systems because of an increase demand for high reliability In the design of observers there are two key elements that should be taken into account, these are as follows 1) the type and size of the fault which is either multiplicative fault (parameter faults) or additive faults (actuator or sensor faults); 2) the disturbance characteristics. Over the past three decades, much attention has been paid to the problem of fault detection and industrial processes. The observers have been formed in design of an integral part of numerous control systems. Luenberger observer was firstly proposed and developed in [1, 2]. The theory of the observer design has been extended by many researchers to include time-variant, discrete, stochastic issues and deterministic continuous time-invariant linear systems. In general, Luenberger Observer possesses a relative simple design that makes it an attractive general design technique [3, 4]. Later, the Luenberger observer was extended to form a Kalman filter [5]. Although the Kalman filter is in use for more than 35 years and has been described in many papers and books, its design is still an area of concern for many researches and studies. It could be argued that the Kalman filter is one of the good observers against a wide range of disturbances [4, 6]. The problem of estimating a state of a dynamical system driven by unknown inputs has been the subject of a large number of studies in the past three decades. An observer that is capable of estimating the state of a linear system with unknown inputs can also be of tremendous use when dealing with the problem of instrument fault detection, since in such systems most actuator faults can be generally modelled as unknown inputs to the system [7, 8]. A new methodology for fault detection and identification subject to plant parameter uncertainties is presented in [7]. A full-order observer procedure was developed for linear systems with unknown inputs using straightforward matrix calculations in [9]. Estimating using a reduced order disturbance de-coupled observer was presented in [10]. A full-order unknown input and output structure is used in order to generate residuals, which can be used to detect fault and isolate on a vertically taking-off and landing aircraft dynamic model in [11]. Designing the unknown input and output observer was reported by considering the unknown constant disturbance of parameters in chaotic systems in [12]. However, when the numbers of sensors and unknown inputs are equal, the observer may not exist. Hence, the unknown input observer method is not always feasible for fault detection. To overcome this drawback, several studies have been developed and implemented for augmented observers by given bounds of plant uncertainty. The fault detection scheme facilitates determining free matrices in the partial state observer, by which the residual function can be

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identified and distinguished for the sensor and actuator faults. An augmented system model that a residual function could be generated according to partial state observers was presented in [13, 14]. The generated residual signals, which disclose the fault, are sensitive to faults while insensitive to uncertainties. To derive residual functions, existence conditions and its design procedure are presented in [15].

Since typically disturbance signals are not affected by the gain parameter; therefore the reasonable high gain observer criterion can be implemented. In addition this observer can not only estimate the angular positions and velocities of the system, but also reject the disturbance [16]. A sensitive high-gain actuator was presented in [17] where faults are sensor and actuator faults, input disturbances, and measurement noises. The sensitive high-gain observer-based identification approach has shown to be suitable for applications of bounded processes [18].

As a result, in section0, owing to the importance of optimal adaptive diagnosis observer (OAD) with state additive fault as well as sensor disturbance, optimal theorems has been designed for the observe. However, the performance criteria are chosen as presented in section 0 as well as a multiple inputs, multiple outputs (MIMO) DC servomotor model, which is considered as a benchmark. In this paper, a DC servo motor is considered as multiple inputs and multiple outputs (MIMO) model. The model is controllable and observable [19]. Moreover, the continuous linear system has been discretized [20] with the sampling time of 0.1 second.

2. Design of Optimal Adaptive Diagnosis Observer (OAD)

A new observer has been proposed based on assumed optimal conditions. The new optimal adaptive diagnosis observer (OAD) has been studied through comparison with six types of linear additive observer $LO, KO, UIO, ARO, HGO, SHGO$. To study the observers, the model of the system (plant) was assumed to be affected by additive fault in the states with a disturbance (noise) present in the measure of the output named sensors faults.

2.1. Model of the System

The system has been assumed to be influenced by additive faults $f_l(k)$ in the states and additive disturbance $f_m(k)$ on the output. The matrices L and M are faults matrices:

$$\begin{aligned} \dot{x}(t) &= Fx(t) + Gu(t) + L_l f_l(t) \\ y(t) &= Cx(t) + Du(t) + M_m f_m(t) \end{aligned} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is a state vector, $u(t) \in \mathbb{R}^m$ represents a control input vector, $y(t) \in \mathbb{R}^p$ is a measurement output vector, and F, G, C and D are known constant matrices [4].

2.2. Design a New Optimal Adaptive Diagnosis Observer (OAD)

A new optimal adaptive observer has been implemented to detect and diagnose an additive fault, a sensor disturbance and estimate the states of the plant in (1) as follows:

$$\begin{aligned} \hat{x}(k+1) &= A\hat{x}(k) + Bu(k) + H\varepsilon(k) + \hat{f}_l(k) \\ \hat{y}(k) &= C\hat{x}(k) + Du(k) \end{aligned} \tag{2}$$

where the gain matrix of the observer is H where it can be found based on the optimal conditions. $\varepsilon(k)$ is assumed to be the residual while $e(k)$ is the state error defined as:

$$\varepsilon(k) = y(k) - \hat{y}(k) \tag{3}$$

$$e(k) = x(k) - \hat{x}(k) \tag{4}$$

By substituting (1) and (3)into(4), the dynamical error can be rewritten as:

$$\begin{aligned} e(k+1) &= \hat{x}(k+1) - x(k+1) \\ &= A\hat{x}(k) + Bu(k) + H(y(k) - \hat{y}(k)) + \hat{f}_l(k) \\ &\quad - (Ax(k) + Bu(k) + Lf_l(k)) \\ &= (A - HC)\hat{x}(k) + Hy(k) + HMf_m(k) \\ &\quad - Ax(k) - Lf_l(k) + \hat{f}_l(k) \\ &= (A - HC)\hat{x}(k) - (A - HC)x(k) \\ &\quad + \left(-Lf_l(k) - HMf_m(k) + \hat{f}_l(k)\right) \\ &= \bar{A}e(k) + \tilde{f}_l(k) + HMf_m(k) \end{aligned} \tag{5}$$

where $\tilde{f}_l(k) = \hat{f}_l(k) - \left(Lf_l(k) + HMf_m(k)\right)$, $\bar{A} = A - HC$. Therefore, the residual can be rewritten to become:

$$\varepsilon(k) = y(k) - C\hat{x}(k) = Ce(k) + Mf_m(k) \tag{6}$$

Faults detection depends on the threshold λ for the system being realized as:

$$\begin{aligned} &\|\varepsilon(k)\| < \lambda \quad \text{no fault occurs} \\ &\|\varepsilon(k)\| \geq \lambda \quad \text{fault has been occured} \end{aligned} \tag{7}$$

Theorem: Assume that the gain matrix H of the adaptive observer in (2) can be obtained such that the following conditions:

$$\begin{aligned} \bar{A}P\bar{A}^T - P &= -Q_1 \\ L^T P - C &= Q_2 \end{aligned} \tag{8}$$

are satisfied, where $P = P^T$, $Q_1 = Q_1^T$, $Q_2 = Q_2^T$ are positive definite and $\bar{A} = A - HC$ is a Hurwitz. The goal of fault diagnosis is to find a diagnostic algorithm for $\hat{f}_l(k)$ and an observer gain vector H such that:

$$\begin{aligned} &\lim_{k\to\infty} \varepsilon(k) \to 0 \\ &\lim_{k\to\infty} \tilde{f}_l(k) \to 0 \end{aligned} \tag{9}$$

where the following adaptive diagnostic algorithm:

$$\begin{aligned} \tilde{f}_l(k) &= \hat{f}_l(k) - \left(Lf_l(k) + HMf_m(k)\right) \\ \hat{f}_l(k+1) &= -\Gamma_1 \hat{f}_l(k) - \Gamma_2 \varepsilon(k) \qquad k > k_f \\ \tilde{f}_l(k+1) &= \hat{f}_l(k+1) \end{aligned} \tag{10}$$

can realize(9). The tuning rate is defined in (10) by $\Gamma_1 = \Gamma_1^T$, $\Gamma_2 = \Gamma_2^T$ which are pre-specified gain matrices and for any $\zeta > 0$, there exists $\eta > 0$, yielding:

$$\|e(k)\| < \zeta \quad \Rightarrow \quad \frac{\|\tilde{f}_l(k)\|}{\|e(k)\|} < \eta_1, \ \frac{\|HMf_m(k)\|}{\|e(k)\|} < \eta_2, \tag{11}$$

Hence, η_1, η_2 should be positive definite matrices.

Proof of the Theorem: Define the Lyapunov function $v(e)$ candidate:

$$v(e(k), \tilde{f}_l(k)) = e^T(k) Pe(k) + \tilde{f}_l^T(k) \Gamma_1^{-1} \tilde{f}_l(k) \tag{12}$$

and assume that $\overline{A}P\overline{A}^T - P = -Q_1$ where $Q_1 > 0$. Then:

$$\begin{aligned} \Delta v(e(k), \tilde{f}_l(k)) &= \frac{1}{2}\left(v\left(e\left(k+1\right)\right) - v\left(e\left(k\right)\right)\right) \\ &= \frac{1}{2}\left(e^T\left(k+1\right)Pe\left(k+1\right) - e^T\left(k\right)Pe\left(k\right)\right) \\ &\quad + \tilde{f}_l^T\left(k\right)\Gamma_2^{-1}\tilde{f}_l\left(k+1\right) \\ &\quad + \tilde{f}_l^T\left(k\right)\Gamma_2^{-1}\hat{f}_l\left(k+1\right) \end{aligned} \tag{13}$$

It can be further expressed as follows:

$$\begin{aligned} \Delta v(e(k), \tilde{f}_l(k)) &= \frac{1}{2}\left[\overline{A}e(k) + \tilde{f}_l(k) + \left(HMf_m(k)\right)\right]^T \\ &\quad P\left[\overline{A}e(k) + \tilde{f}_l(k) + \left(HMf_m(k)\right)\right] \\ &\quad - e^T(k)Pe(k) + \tilde{f}_l^T(k)Ce(k) \\ &\quad + \tilde{f}_l^T(k)\Gamma_2^{-1}\tilde{f}_l(k+1) \\ &= \frac{1}{2}e^T(k)\left[\overline{A}^T P \overline{A} - P\right]e(k) \\ &\quad + \frac{1}{2}\tilde{f}_l^T(k)P\tilde{f}_l(k) + \frac{1}{2}\left(HMf_m(k)\right)^T \\ &\quad + \tilde{f}_l^T(k)\left(L^T P - Q_2\right)e(k) + \tilde{f}_l^T(k)\Gamma_2^{-1}\tilde{f}_l(k+1) \\ &= -\frac{1}{2}e^T(k)Q_1 e(k) + \frac{1}{2}\tilde{f}_l^T(k)P\tilde{f}_l(k) \\ &\quad + \frac{1}{2}\left(HMf_m(k)\right)^T P\left(HMf_m(k)\right) \end{aligned} \tag{14}$$

Through substituting (1), (2) and (8) into (14), it can be shown that:

$$\Delta v(e, \hat{f}_l) \le -\frac{1}{2}\lambda_{\min}(Q_1)\|e(k)\|^2 + \frac{1}{2}\|P\|\|\tilde{f}_l(k)\| + \frac{1}{2}\|P\|\|HMf_m(k)\| \tag{15}$$

If the Raleigh–Ritz inequality is used for the first term and the Cauchy–Schwarz inequality and the index matrix norm for the second and third terms respectively, then the derivate function will be:

$$\Delta v(e, \hat{f}_l) \le \|e(k)\|\left(-\frac{1}{2}\lambda_{\min}(Q_1)\|e(k)\| + \frac{\|P\|\|\tilde{f}_l(k)\|}{2\|e(k)\|} + \frac{\|P\|\|HMf_m(k)\|}{2\|e(k)\|}\right) \tag{16}$$

Assume that for any $\gamma > 0$, no matter how small, there exists $\eta > 0$, yielding:

$$\|e(k)\| < \gamma \quad \Rightarrow \quad \frac{\|\tilde{f}_l(k)\|}{\|e(k)\|} < \eta_1, \ \frac{\|HMf_m(k)\|}{\|e(k)\|} < \eta_2 \tag{17}$$

Therefore, we have obtained as follows:

$$\Delta v(e, \hat{f}_l) \le \|e(k)\|\left(-\frac{1}{2}\lambda_{\min}(Q_1)\|e(k)\| + \left(\eta_1 + \eta_2\right)\|P\|\right) \tag{18}$$

To ensure stability, which means $\Delta v(e, \tilde{f}_l(k)) < 0$, the linear system is asymptotically stable by Lyapunov stability theorem the condition to be satisfied is:

$$\eta_1, \eta_2 < \frac{\lambda_{\min}(Q_1)\|e(k)\|}{2\|P\|} \tag{19}$$

The Raleigh–Ritz inequality and the Cauchy–Schwarz inequality have been used to rewrite the derivate function as:

$$\Delta v(e, \hat{f}_l) \le \begin{bmatrix} -\lambda_{\min}(Q)\|e(k)\| - \eta_2\|P\| & P & P \\ P^T & -\left(\eta_1 + \eta_2\right)^{-1} & 0 \\ P^T & 0 & -\left(\lambda_{\min}(Q)\eta_2\right)^{-1} \end{bmatrix} \tag{20}$$

3. Linear Observers

Six Known Linear Observers to detect and diagnose the fault are studied and demonstrated. The observers differ in model and method of fault dealing which cover most fault detection techniques. Models of the observers will be introduced as follows:

3.1. Luenberger Observer (LO)

The plant shown in (21)and (22) is influenced by additive faults $f_l(k)$ on states and the additive faults $f_m(k)$ on the output. The matrices L and M are faults matrices

$$x(k+1) = Ax(k) + Bu(k) + Lf_l(k) \tag{21}$$

$$y(k) = Cx(k) + Du(k) + Mf_m(k) \tag{22}$$

where L and M are known constant matrices.

The states and output of observer is given by

$$\hat{x}(k+1) = A\hat{x}(k) + Bu(k) + Hr(k) \tag{23}$$

$$\hat{y}(k) = C\hat{x}(k) \tag{24}$$

where H is the gain matrix of the observer, *r(k)* is the residual and *e(k)* is the errors between the plant's states and states of observer.

$$r(k) = y(k) - \hat{y}(k) \tag{25}$$

$$\hat{x}(k+1) = [A - HC]\hat{x}(k) + Bu(k) + Hy(k) \tag{26}$$

Then the equation of the error between the states of plants and observers will be

$$e(k+1) = [A - HC]e(k) + Lf_l(k) - HMf_m(k) \tag{27}$$

and the residual becomes

$$r(k) = y(k) - C\hat{x}(k) = Ce(k) + Mf_m(k) \tag{28}$$

It can be seen that the residual is zero if no faults and disturbances are present. The gain matrix of the observer can be found by pole placement method.

3.2. Kalman Observer (State Estimation Observer) (KO)

For the linear system which is represented in (29) and (30) Kalman observer can be formed through the prediction and correction of the states of plant to give

$$x(k+1) = Ax(k) + Bu(k) + Lf_l(k) \tag{29}$$

$$y(k) = Cx(k) + Du(k) + Mf_m(k) \tag{30}$$

Predictive equation is given by

$$\hat{x}\left(k+1 \mid k\right) = A\hat{X}(k \mid k) + Bu(k) \tag{31}$$

and the correction is realized by

$$\hat{x}\left(k+1 \mid k+1\right) = \hat{x}(k+1 \mid k) + k[y(k+1) - C\hat{x}(k+1 \mid k)] \tag{32}$$

where *K* is correction matrix

$$K = P(k)C^T[CP(k)C^T + N]^{-1} \tag{33}$$

$$K(k+1) = P(k+1)C^T[CP(k+1)C^T + N]^{-1} \tag{34}$$

In the above algorithm, the covariance matrix is given by

$$P(k+1) = E\{e(k+1 \mid k)e^T(k+1 \mid k)\} \tag{35}$$

where $\mu > 0$ is the covariance matrix of the estimation error and satisfies the following matrix Riccati equation

$$P(k+1) = AP(k)A^T + BQB^T \tag{36}$$

where the error of states is denoted as

$$e(k) = x(k) - \hat{x}(k) \tag{37}$$

and N is the covariance matrix of the faults on the output

$$N = E\{f_m(k)f_m^{\ T}(k)\} \tag{38}$$

If the prediction (31) is inserted in correction (32) it follows

$$\hat{x}(k+1 \mid k+1) = \left\{ \begin{array}{l} A\hat{x}(k \mid k) + Bu(k) + K(k+1)[y(k+1) \\ -C(A\hat{x}(k \mid k) + Bu(k))] \end{array} \right\} \tag{39}$$

The initial conditions are expressed as:

$$\begin{cases} Q = \mathrm{var}\,iance \\ P = BQB^T \end{cases} \tag{40}$$

3.3. Unknown Input Observer and Robust Fault Detection (UIO)

The plant in (41) and (42) is influenced by additive faults $f_l(k)$ on states and additive faults $f_m(k)$ on output, and then the linear system is represented as

$$x(k+1) = Ax(k) + Bu(k) + Lf_l(k) \tag{41}$$

$$y(k) = Cx(k) + Mf_m(k) \tag{42}$$

To design the observer, let us consider the observer to have the parameters (R , T, K_1 , K_2 and H) in the form of

$$\begin{cases} z(k+1) = Rz(k) + TBu(k) + (k_1 + k_2)y(k) \\ \hat{x}(k) = z(k) + Hy(k) \end{cases} \tag{43}$$

$$\hat{y}(k) = C\hat{x}(k) \tag{44}$$

The residual and the state errors are therefore given by

$$\begin{cases} r(k) = y(k) - \hat{y}(k) \\ e(k) = x(k) - \hat{x}(k) \\ e(k) = (I - HC)x(k) - z(k) - HMf_m(k) \end{cases} \tag{45}$$

Leading to the dynamics of the states error (without $HMf_m(k)$) as

$$\begin{cases} e(k+1) = (A - HCA - K_1C)e(k) \\ \qquad + (A - HCA - K_1C - R)z(k) \\ \qquad + [(A - HCA - K_1C)H - K_2]y(k) \\ \qquad + [(I - HC) - T]Bu(k) \\ \qquad + (I - HC)Lf_l(k) \end{cases} \tag{46}$$

The state error should converge to zero asymptotically when the time ($k \to \infty$), this leads to the following conditions

$$H = L(CL)^T(CL)^{-1}(CL)^T \tag{47}$$

$$T = (I - HC)A \tag{48}$$

$$A_1 = (I - HC)A \tag{49}$$

where

$$\begin{cases} A - HCA - K_1C = 0 \\ (I - HC)L = 0 \\ A - HCA - K_1C - R = 0 \\ (I - HC) - T = 0 \\ (A - HCA - K_1C)H - K_2 = 0 \\ R = A_1 - K_1^{\ T}C \\ K_2 = R\,H \end{cases} \tag{50}$$

At this stage one can find K_1 using the pole placement method by using $(A_1,\ C)$ and assume the observer is stable.

The Eigen values of $M_L(t)$ are the same of the assumed poles. If all Eigen values of R are stable, $e(k)$ will approach to zero asymptotically. Therefore, it is a key to design gains. The assumption is that the matrices $(A\ L)$ is of a full column rank (This condition can ensure that ($A - HCA - K_1C = 0$), so that (A - HCA , C) is an observable pair.

3.4. Augmented Robust Observer (ARO)

Let us consider the plant is a linear system which is represented in (51) and (52) as

$$x(k+1) = Ax(k) + Bu(k) + Lf_l(k) \tag{51}$$

$$y(k) = Cx(k) + Du(k) + Mf_m(k) \tag{52}$$

The second derivate of the abrupt and incipient faults should be zero

$$f(k+2) = 0 \tag{53}$$

By using (51), (52) and (53), the augmented plant is

$$\begin{bmatrix} x(k+1) \\ f(k+1) \\ f(k+2) \end{bmatrix} = \begin{bmatrix} A & 0 & L \\ 0 & 0 & 0 \\ 0 & I & 0 \end{bmatrix} \begin{bmatrix} x(k) \\ f(k+1) \\ f(k) \end{bmatrix} + \begin{bmatrix} B \\ 0 \\ 0 \end{bmatrix} u(k) \tag{54}$$

$$y(k) = \begin{bmatrix} C & 0 & M \end{bmatrix} \begin{bmatrix} x(k) \\ f(k+1) \\ f(k) \end{bmatrix} + Du(k) \tag{55}$$

System in (54)and (55) can be simplified as

$$\bar{x}(k+1) = \bar{A}\bar{x}(k) + \bar{B}u(k) \tag{56}$$

$$\bar{y}(k) = \bar{C}\bar{x}(k) + Du(k) \tag{57}$$

where $\bar{A} = \begin{bmatrix} A & 0 & L \\ 0 & 0 & 0 \\ 0 & I & 0 \end{bmatrix}$, $\bar{B} = \begin{bmatrix} B \\ 0 \\ 0 \end{bmatrix}$ and $\bar{C} = \begin{bmatrix} C & 0 & M \end{bmatrix}$

The stability condition for observer is described in (58). To obtain asymptotically stable observer, a sufficient condition for this is that the pair ($\bar{A}$ and $\bar{C}$).

$$n + 2kk = rank \begin{bmatrix} zI - \bar{A} \\ \bar{C} \end{bmatrix} \tag{58}$$

When *s=0* the condition will be as in (59). Otherwise, it will be as in (60)

$$rank \begin{bmatrix} A & B_f \\ C & D \end{bmatrix} = n + kk \tag{59}$$

$$rank \begin{bmatrix} zI - A \\ C \end{bmatrix} = n \tag{60}$$

$$\hat{x}(k+1) = \bar{A}\hat{x}(k) + \bar{K}(y(k) - Du(k) - \bar{C}\hat{x}(k)) \tag{61}$$

Therefore the dynamics of states error is given by

$$e(k+1) = (\bar{A} - \bar{K}\bar{C})\bar{e}(k) \tag{62}$$

$$e(k+1) = (\bar{A} - \bar{K}\bar{C})e(k) + \bar{N}f(k+2) + \bar{M}f_m(k) \tag{63}$$

$$\bar{N} = \begin{bmatrix} 0 & I_k & 0 \end{bmatrix}^T \tag{64}$$

$$\bar{M} = \begin{bmatrix} M^T & 0 & 0 \end{bmatrix}^T \tag{65}$$

The condition for designing a stable observer is that the pair $(\bar{A}, \bar{C})$ is observable. The observer gain can be found using the poles placement method. Since the relationship in (53) is not always true $f(k+2) \neq 0$ and the bounded disturbance signal not affected by the gain parameter therefore needs to other type of observers like high gain observer.

3.5. High Gain Observer (HGO)

Consider the system as a linear system with faults to be represented as

$$x(k+1) = Ax(k) + Bu(k) + Lf_l(k) \tag{66}$$

$$y(k) = Cx(k) + Mf_m(k) \tag{67}$$

The assumption in (53) is not always true for the system and in this case one can assume that the second derivate of the abrupt and incipient faults should be as

$$f(k+2) \neq 0 \tag{68}$$

Using, (67) and (68), the augmented plant can be given by

$$\begin{bmatrix} x(k) \\ f(k+1) \\ f(k+2) \end{bmatrix} = \begin{bmatrix} A & 0 & L \\ 0 & 0 & 0 \\ 0 & I & 0 \end{bmatrix} \begin{bmatrix} x(k) \\ f(k+1) \\ f(k) \end{bmatrix} + \begin{bmatrix} B \\ 0 \\ 0 \end{bmatrix} u(k) \tag{69}$$

$$y(k) = \begin{bmatrix} C & 0 & M \end{bmatrix} \begin{bmatrix} x(k) \\ \dot{f}(k) \\ f(k) \end{bmatrix} + Du(k) \tag{70}$$

System in (58) and (59) can be simplified as

$$\bar{x}(k+1) = \bar{A}x(k) + \bar{B}u(k) + \bar{N}\ddot{f}(k) + Lf_l(k) \tag{71}$$

$$y(k) = Cx(k) + Du(k) + Mf_m(k) \tag{72}$$

where $\bar{A} = \begin{bmatrix} A & 0 & L \\ 0 & 0 & 0 \\ 0 & I & 0 \end{bmatrix}$, $\bar{B} = \begin{bmatrix} B \\ 0 \\ 0 \end{bmatrix}$, $\bar{C} = \begin{bmatrix} C & 0 & M \end{bmatrix}$,

$\bar{N} = \begin{bmatrix} 0 & I_k & 0 \end{bmatrix}^T$ and $\bar{V} = \begin{bmatrix} L^T & 0 & 0 \end{bmatrix}^T$ The observer is therefore given by

$$\hat{x}(k+1) = A\hat{x}(k) + Bu(k) + Hr(k) \tag{73}$$

$$\hat{y}(k) = C\hat{x}(k) \tag{74}$$

then the state of observer and the dynamic of error will be

$$\hat{x}(k+1) = \overline{A}\hat{x}(k) + \overline{B}u(k) + \overline{K}(y(k) - \overline{D}u(k) - \overline{C}\hat{\overline{x}}(k)) \quad (75)$$

$$r(k+1) = (\overline{A} - \overline{K}\overline{C})r(k) + \overline{N}f(k+2) + \overline{M}f_m(k) \quad (76)$$

To design an again matrix that effects on the disturbance, one needs to design a high gain using Lyapunov function as in following three steps:

1- For continuous system, $\mu > 0$ and μ is less than real parts of all Eigen values of $\overline{A}$. So obtain a stable observer in discrete time model, $\mu < 1$. In our simulation, it has been chosen as $\mu = 0.92$.

2- Find the matrix $\overline{P}$ using discrete Lyapunov function:

$$(\mu I + \overline{A})\overline{P}(\mu I + \overline{A})^T - \overline{P} + \overline{C}^T\overline{C} = 0 \quad (77)$$

3- Calculate the observer gain matrix as

$$\overline{K} = \overline{P}^{-1}\overline{C}^T$$

It can be seen that when μ is increased the matrix P will be decreased. Therefore the observer is robust against the input disturbance and faults.

3.6. Sensitive High Gain Observer (SHGO)

Let us consider the linear system with multiplicative faults in the parameters and additive faults on the states, the model of the system is given as

$$\begin{cases} x(k+1) = (A+\Delta A)x(k) + (B+\Delta B)u(k) + w_i(k) \\ y(k) = x(k) + w_o(k) \end{cases} \quad (78)$$

Let us assume the following

$w_i(k) = Lf_l(k)$, $w_o(k) = Mf_m(k)$, $d(k) = \Delta Ax(k) + \Delta Bu(k)$,

$$\overline{x}(k) = \begin{bmatrix} x(k) \\ d(k) \\ w_i(k) \end{bmatrix}, \ \overline{G} = \begin{bmatrix} I_n \\ 0 \\ 0 \end{bmatrix}, \ \overline{H} = \begin{bmatrix} 0 \\ I_n \\ 0 \end{bmatrix}, \ \overline{N} = \begin{bmatrix} 0 \\ I_n \\ o \end{bmatrix},$$

$$\overline{E} = \begin{bmatrix} I_n & 0 & 0 \\ 0 & I_n & 0 \\ 0 & 0 & 0_{n*n} \end{bmatrix}, \ \overline{A} = \begin{bmatrix} A & I_n & 0 \\ 0 & 0_{n*n} & 0 \\ 0 & 0 & -I_n \end{bmatrix} \text{ and}$$

$$\overline{C} = [I_n \quad 0 \quad -I_n]$$

where n is the number of states. The observer can be modified as

$$\begin{cases} \overline{E}\overline{x}(k+1) = \overline{A}\overline{x}(k) + \overline{B}u(k) + \overline{G}f_l(k) \\ \qquad + \overline{H}d(k+1) + \overline{N}w_o(k) \\ y(k) = \overline{C}\overline{x}(k) \end{cases} \quad (79)$$

where it is assumed that fault ($f_l(k)$, $d(k+1)$ and $f_m(k)$) are bounded. The observer can be further modified to give

$$\begin{cases} \overline{S}\zeta(k+1) = (\overline{A} - \overline{K}\overline{C})\zeta(k) + \overline{B}u(k) - \overline{N}y(k) \\ \hat{\overline{x}}(k) = \zeta(k) + \overline{S}^{-1}Ly(k) \end{cases} \quad (80)$$

where $\overline{S} = \overline{E} + \overline{L}\overline{C}$ and $\overline{K}, \overline{L}$ are the gain matrices.

The following algorithm is implemented to design the gain matrix of observer:

1- Incontinuous time system, choose $\mu > 0$ as a positive number where μ is less than real parts of all Eigen values of $\overline{A}$. For discrete time system, μ is chosen to be less than 1 in order to obtain stable observer.

In the simulation of a DC motor, a discrete time model is considered, therefore $\mu = 0.8$ and $\text{Sensitively} = 3$.

2- The Lyapunov function is used to evaluate the matrix $\overline{P}$

$$(\mu I + \overline{S}^{-1}\overline{A})\overline{P}(\mu I + \overline{S}^{-1}\overline{A})^T - \overline{P} + \overline{C}^T\overline{C} = 0 \quad (81)$$

3- The observer gain matrix can be founded by

$$\overline{K} = \overline{S}\overline{P}^{-1}\overline{C}^T$$

Let us assume the following

$$\overline{L} = \begin{bmatrix} 0 \\ 0 \\ M_m \end{bmatrix} \text{ and } M_s = Sensitively * I_{n*n}$$

where M_s is a non-singular matrix. One can thus consider

$$\overline{S} = \begin{bmatrix} I_n & 0 & 0 \\ 0 & I_n & 0 \\ M_s & 0 & M_s \end{bmatrix} \text{ and } \overline{S}^{-1} = \begin{bmatrix} I_n & 0 & 0 \\ 0 & I_n & 0 \\ -I_n & 0 & M_s^{-1} \end{bmatrix}$$

Then it can be further obtained that

$$\begin{cases} \overline{C}\overline{S}^{-1}\overline{L} = I_n \\ \overline{A}\overline{S}^{-1}\overline{L} = -\overline{N} \end{cases} \quad (82)$$

The dynamic equation of the plant can be expressed as

$$\begin{cases} \overline{S}\overline{x}(k+1) = \overline{A}\overline{x}(k) + \overline{B}u(k) + \overline{G}w_i(k) + \overline{H}d(k+1) \\ \qquad + \overline{N}w_o(k) + \overline{L}y(k+1) \end{cases} \quad (83)$$

The state of observer and the dynamic of states error can be represented in (84) and (85) respectively

$$\overline{S}\hat{\overline{x}}(k+1) = \overline{A}\hat{\overline{x}}(k) + \overline{B}u(k) + \overline{K}(y(k) - \overline{C}\hat{\overline{x}}(k)) + \overline{L}y(k+1) \quad (84)$$

$$e(k+1) = \overline{S}^{-1}[(\overline{A} - \overline{K}\overline{C})e(k) + \overline{G}w_i + \overline{H}d(k+1) + \overline{N}w_o(k)] \quad (85)$$

4. Performance Evaluation

In order to evaluate fault detection, diagnosis and performance, absolute error and relative error criteria are used. Absolute error is the amount of physical error in a prediction, while relative error gives an indication of how good a prediction is relative to the size of the parameter. Root mean squared error (RMSE), mean absolute error (MAE) and variance absolute error (VAE) are used to calculate absolute error. For relative error, mean absolute relative error (MARE) and variance relative error (VRE) are used. The above statistic formulas list as follows

$$RSME = \frac{1}{\mathrm{N}}\sqrt{\sum_{k=1}^{N}\left(\gamma(k)-\hat{\gamma}(k)\right)^2} \tag{86}$$

$$MAE = \frac{1}{N}\sum_{k=1}^{N}\left|\gamma(k)-\hat{\gamma}(k)\right| \tag{87}$$

$$VAE = \frac{1}{N}\sum_{k=1}^{N}\left(\left|\gamma(k)-\hat{\gamma}(k)\right| - MAE\right) \tag{88}$$

$$MARE = \frac{1}{N}\sum_{k=1}^{N}\frac{\left|\gamma(k)-\hat{\gamma}(k)\right|}{\gamma(k)} \tag{89}$$

where $N, \gamma(k), \hat{\gamma}(k)$ represent the number of samples, and the measured and desired values respectively.

5. Case Study and Results

5.1. General Model of Continuous Linear DC Servomotor

A DC servomotor is a second order system with multiple inputs and multiple outputs. It has power of $P = 550$ *watts* and speed of $n = 2500$ *rpm*, and the motor has two pairs of brushes and two pole pairs. The model has been obtained according to the parameters of armature resistance, armature inductance, magnetic flux, voltage drop factor, inertia constant and viscous friction. The input signals are the armature voltage $U_A(t)$, which has been represented in simulations codes as a step function, and the torque load $M^*_L(t)$, which is assumed equal to 0.1. The measured output signals are the armature current $I_A(t)$ and the speed of motor $\omega(t)$. The values of the parameters were identified by the well-known least square estimation in the continuous time domain as follows [4]:

Armature resistance (R_a) =1.52 Ω,

Armature inductance (L_a) = 6.82*10^{-3}Ω *s*

Magnetic flux (Ψ) = 0.33 *V s*

Inertia constant (J) = 0.0192 *kg m*^{2}

Viscous friction (M_{F1}) = 0.36*10^{-3}*N m s*

Dry friction (M_{FO}) = 0.11*N m*

Voltage drop factor (K_B) = 2.2*10^{-3}*V s /A*

The continuous time model of a DC motor as a state-space form is thus obtained as:

$$\begin{bmatrix} \dot{I}_A(t) \\ \dot{\omega}(t) \end{bmatrix} = \begin{bmatrix} -R_a/L_a & -\Psi/La \\ \Psi/J & -M_{F1}/J \end{bmatrix}\begin{bmatrix} I_A(t) \\ \omega(t) \end{bmatrix} + \begin{bmatrix} 1/L_a & 0 \\ 0 & -1/J \end{bmatrix}\begin{bmatrix} U_A(t) \\ M_L(t) \end{bmatrix} \tag{90}$$

$$\begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix} = \begin{bmatrix} I_A(t) \\ \omega(t) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} I_A(t) \\ \omega(t) \end{bmatrix} + \begin{bmatrix} 0 & 0 \end{bmatrix}\begin{bmatrix} U_A(t) \\ M_L(t) \end{bmatrix}$$

5.2. Comparison with an Additive Fault and Disturbance

Matlab code has been used to implement the plant (DC servomotor) and the seven discrete time observers LO, KO, UIO, ARO, HGO, $SHGO$ and ODA with a 0.1 second sample time, where the additive faults have been proposed on the states after 10 seconds and their faults matrices are assumed as:

$$L = \begin{bmatrix} 1.9 & 0.2 \\ 0.19 & -0.2 \end{bmatrix},\ M = \begin{bmatrix} 0.1 & 2 \\ -0.2 & 0.3 \end{bmatrix}$$

The gain matrix of the observers LO, ARO and UIO can be evaluated using the pole placement method. Therefore the poles of discrete observers are chosen as $((-0.1,\ -0.2), (-0.2,\ -0.3), (-0.09,\ +0.267, -0.4, +0.4, -0.6, -0.7))$ respectively. Moreover, the tuning parameter for HGO is chosen as $\mu = 0.77$ whereas for $SHGO$ the parameter is $\mu = 0.8$ and the Sensitively $=3$ while the gain matrices for HGO are as follows:

$$H = \begin{bmatrix} 0.1127 & -1.4705 & 0 & 0 & 0.3054 & -0.2157 \\ 0.0408 & 0.2872 & 0 & 0 & -0.0534 & 0.1677 \\ 0.0082 & 0.0216 & 0 & 0 & -0.0035 & 0.0229 \\ -0.0746 & -0.0953 & 0 & 0 & 0.0116 & -0.1779 \\ -0.0160 & -0.0307 & 0 & 0 & 0.0045 & -0.0412 \\ 0.2628 & 0.7982 & 0 & 0 & -0.1334 & 0.7650 \end{bmatrix}$$

Furthermore, the gain matrix for $SHGO$ is obtained as follows:

$$H = \begin{bmatrix} 0.3480 & 0.1993 & 0.0024 & -0.1270 & -2.5842 & -0.1816 \\ 0.2889 & 0.0282 & -0.0613 & -0.0666 & -0.1816 & -2.3289 \end{bmatrix}^T$$

However, the parameters for the adaptive diagnosis, which will verify the conditions in (8), are evaluated as follows:

$$H = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.15 \end{bmatrix}, P = -1\times10^{-3}\begin{bmatrix} 0.4 & 5.4 \\ 5.4 & 73.6 \end{bmatrix},$$

$$Q = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, Q_2 = \begin{bmatrix} -1.0019 & -0.0250 \\ -0.0010 & -0.9863 \end{bmatrix}$$

Whereas for the adaptive fault diagnosis, the parameters are:

$$\Gamma_1 = \begin{bmatrix} 0.27 & 0 \\ 0 & 0.23 \end{bmatrix}, \Gamma_2 = \begin{bmatrix} 1.02 & 0 \\ 0 & 1.02 \end{bmatrix}$$

To study the observers' activity, three types of fault and disturbance are applied to the system: white noise (random with zero mean), coloured noise (randomly with mean) and non-Gaussian noise (randomly sinusoidal noise). The effectiveness of each observer design is tested through comparing the method of the gain matrix design, according to the performance criteria in0. Moreover, Table 1 to Table 3 include the performance values of the first output of the observer, while Table 4 to Table 6 show the performance for the second output of the observers.

Table 1. Performance of first o/p (with white noise)

Observers type	RMSE	MAE	VAE	MARE
OAD	0.120267	2.167208	3.996138	116.0595
LO	0.100324	1.808617	2.777941	15.57127
KMF	0.064204	1.161176	1.129071	4.344264
ARO	0.168401	3.028527	7.87E+00	4.861938
HGO	0.124362	2.239697	4.278707	8.56527
SHGO	0.157476	3.055033	5.570723	2.940003
UIO	0.144559	2.596694	5.816543	3.398448

Table 2. Performance of first o/p(with coloured noise)

Observers type	RMSE	MAE	VAE	MARE
OAD	0.019092	0.427659	0.036173	0.703466
LO	0.027731	0.621314	0.076139	1.644972
KMF	0.019989	0.44799	0.039443	0.765377
ARO	0.019183	0.428832	3.73E-02	0.708286
HGO	0.015208	0.340669	0.022952	0.478834
SHGO	0.070687	1.720015	0.044504	2.421445
UIO	0.001547	0.011916	0.001296	0.012322

Table 3. Performance of first o/p (with non-Gaussian noise)

Observers type	RMSE	MAE	VAE	MARE
OAD	0.030709	0.558601	0.254729	2.805453
LO	0.038196	0.699717	0.387232	1.836299
KMF	0.027236	0.50024	0.195565	4.269008
ARO	0.031101	0.557936	2.70E-01	3.401119
HGO	0.025261	0.456212	0.175393	2.136725
SHGO	0.07254	1.61624	0.550228	6.047357
UIO	0.01916	0.34644	0.100604	1.878128

Table 4. Performance of second o/p (with white noise)

Observers type	RMSE	MAE	VAE	MARE
OAD	0.01402	0.262031	0.049466	0.182056
LO	0.021872	0.394609	0.131785	0.330156
KMF	0.064207	1.162532	1.126176	2.96229
ARO	0.006047	0.107936	1.03E-02	0.067895
HGO	0.012038	0.218522	0.039348	0.152632
SHGO	0.016857	0.313578	0.072445	0.244939
UIO	0.016857	0.313578	0.072445	0.244939

Table 5. Performance of second o/p (with coloured noise)

Observers type	RMSE	MAE	VAE	MARE
OAD	0.001256	0.013841	0.000756	0.009868
LO	0.004594	0.102102	0.002261	0.064947
KMF	0.02	0.449346	0.038483	0.376284
ARO	0.000382	0.008505	1.52E-05	0.004933
HGO	0.001191	0.009353	0.000765	0.007247
SHGO	0.004108	0.035616	0.008875	0.049325
UIO	0.004108	0.035616	0.008875	0.049325

Table 6. Performance of second o/p (with non-Gaussian noise)

Observers type	RMSE	MAE	VAE	MARE
OAD	0.002807	0.049004	0.002333	0.030822
LO	0.006269	0.116427	0.010063	0.074227
KMF	0.027243	0.501596	0.194464	0.637918
ARO	0.001369	0.02422	5.39E-04	0.014252
HGO	0.00297	0.047205	0.002383	0.029615
SHGO	0.00508	0.070803	0.010494	0.070461
UIO	0.00508	0.070803	0.010494	0.070461

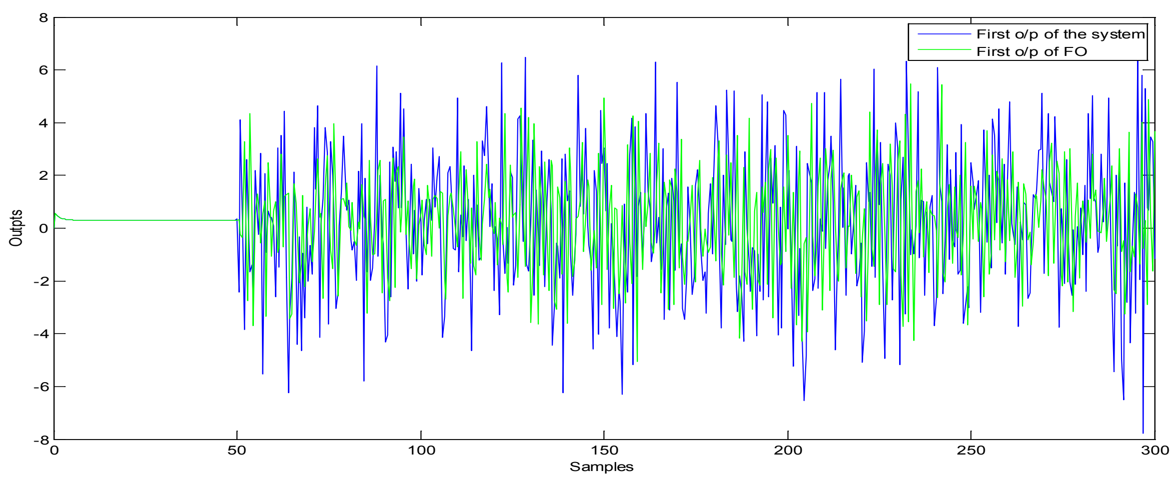

Figure 1. First o/p ODA observer (with white noise)

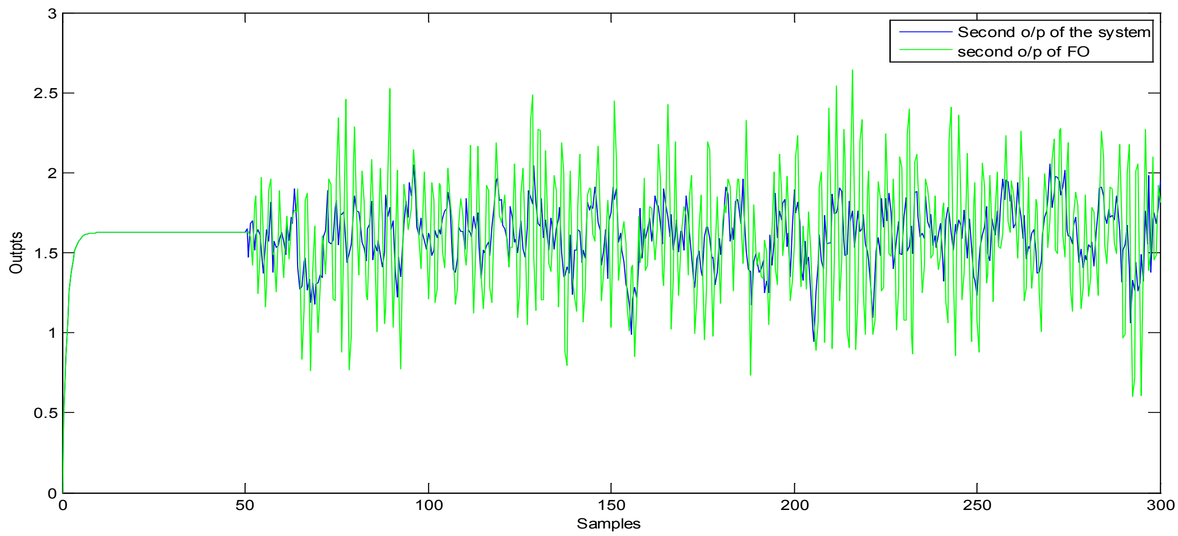

Figure 2. Second o/p of ODA observer (with white noise)

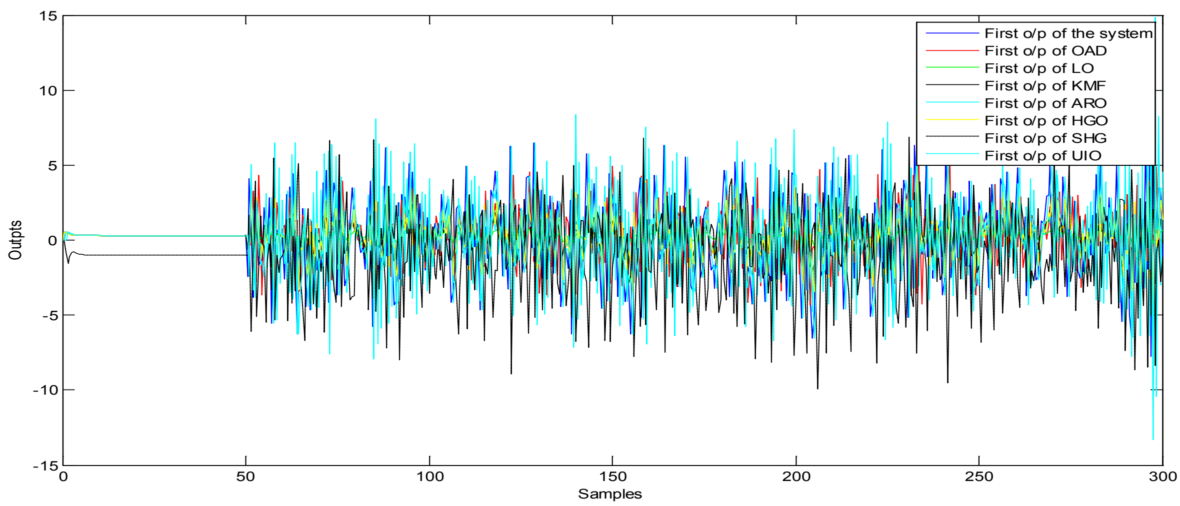

Figure 3. First o/p of additive fault observers (with white noise)

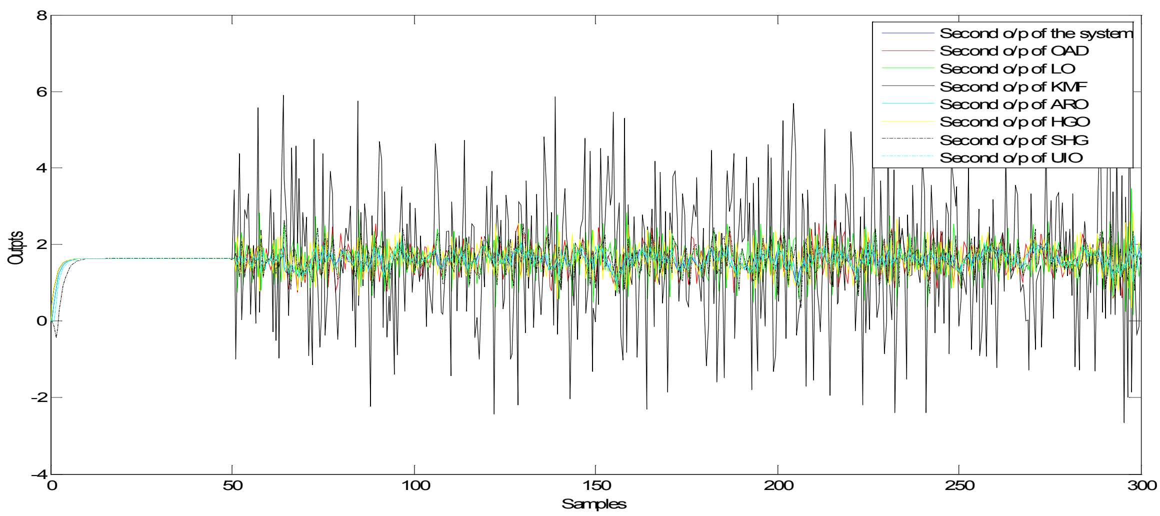

Figure 4. Second o/p additive fault observers (with white noise)

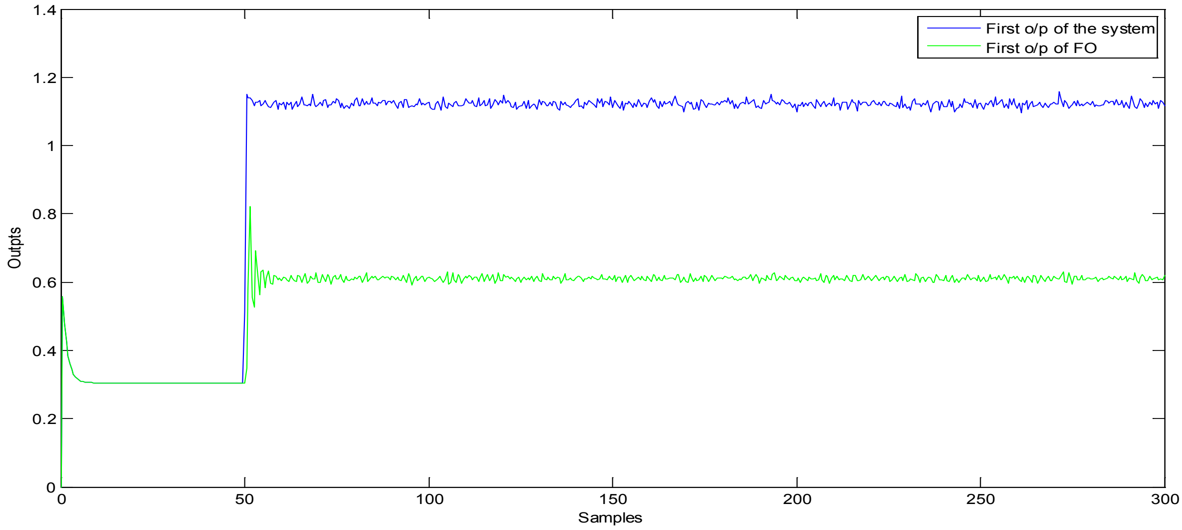

Figure 5. First o/p of ODA observer (with coloured noise)

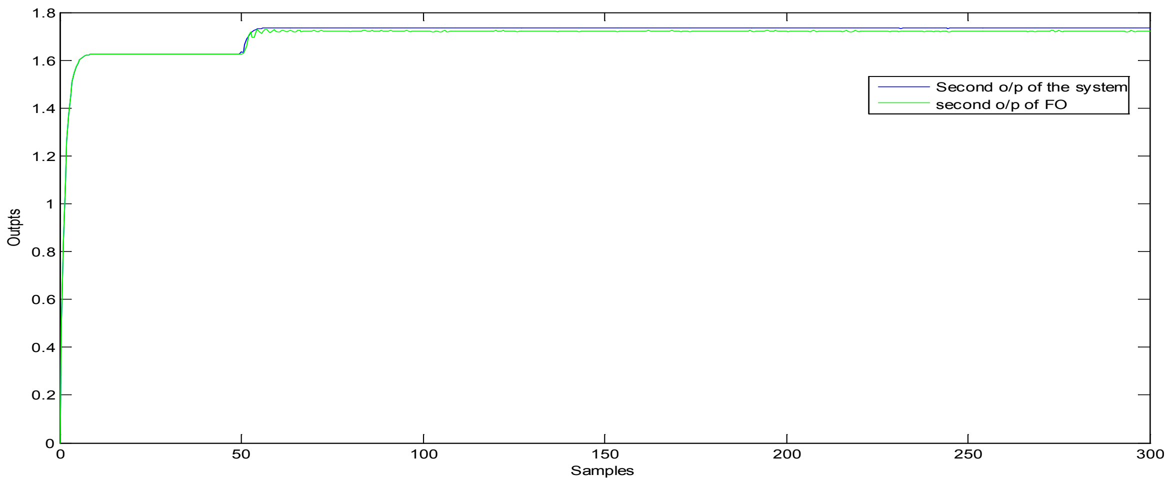

Figure 6. Second o/p of ODA observer (with coloured noise)

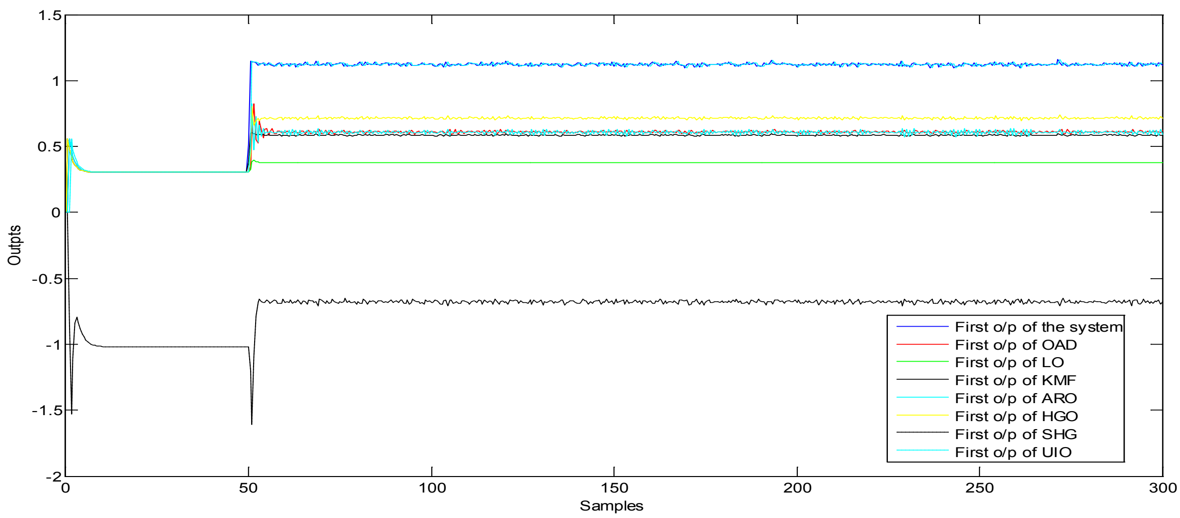

Figure 7. First o/p of ODA observer (with coloured noise)

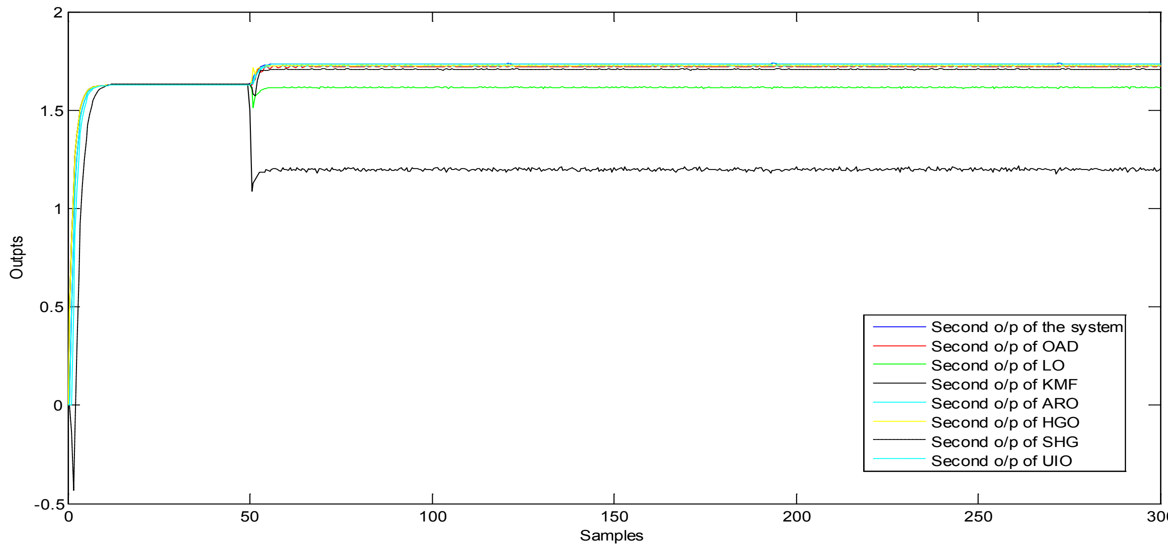

Figure 8. Second o/p additive fault observers (with coloured noise)

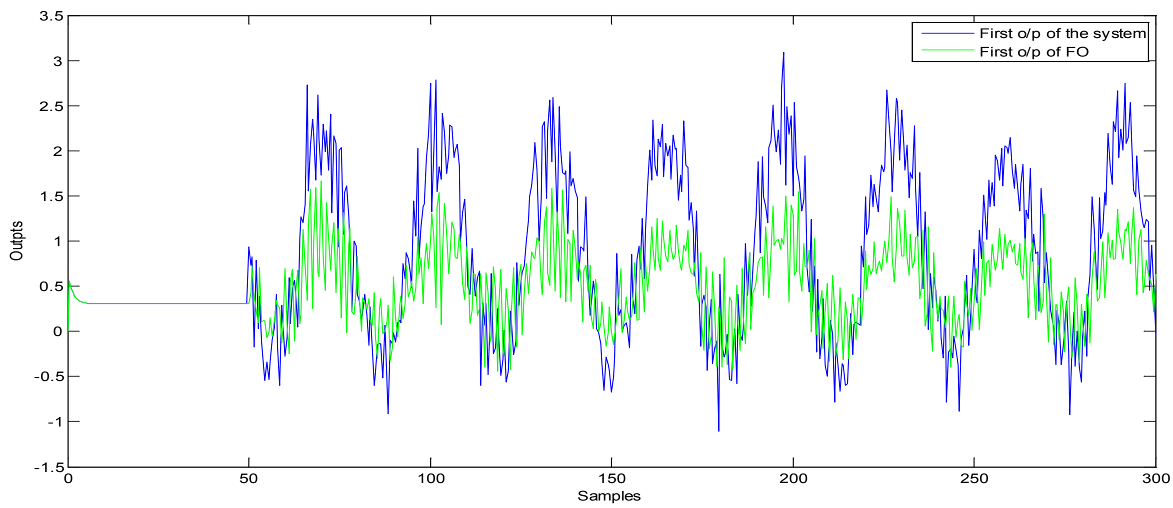

Figure 9. First o/pof ODA observer (with non-Gaussian noise)

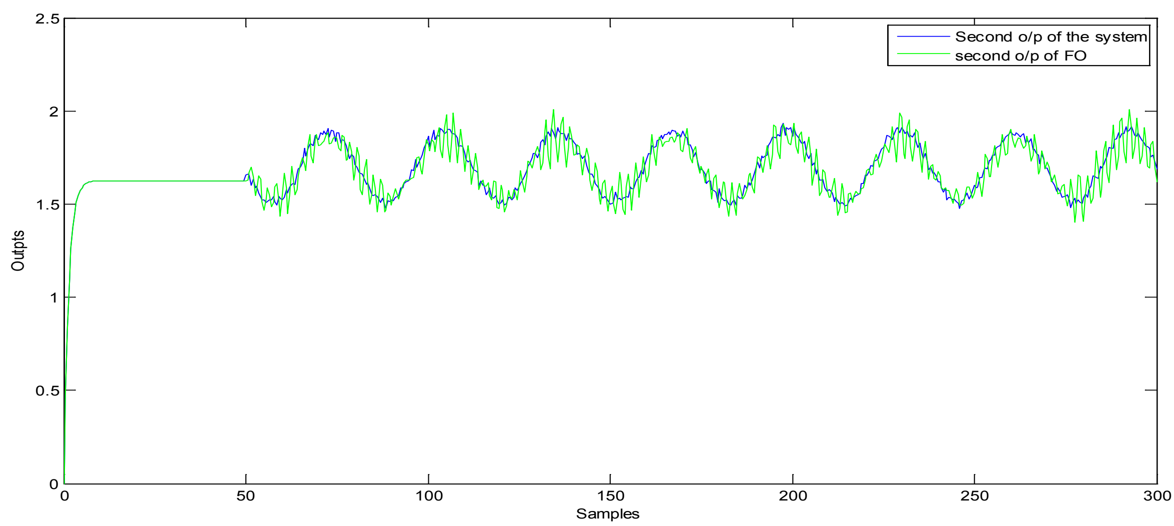

Figure 10. Second o/p of ODA observer (with non-Gaussian noise)

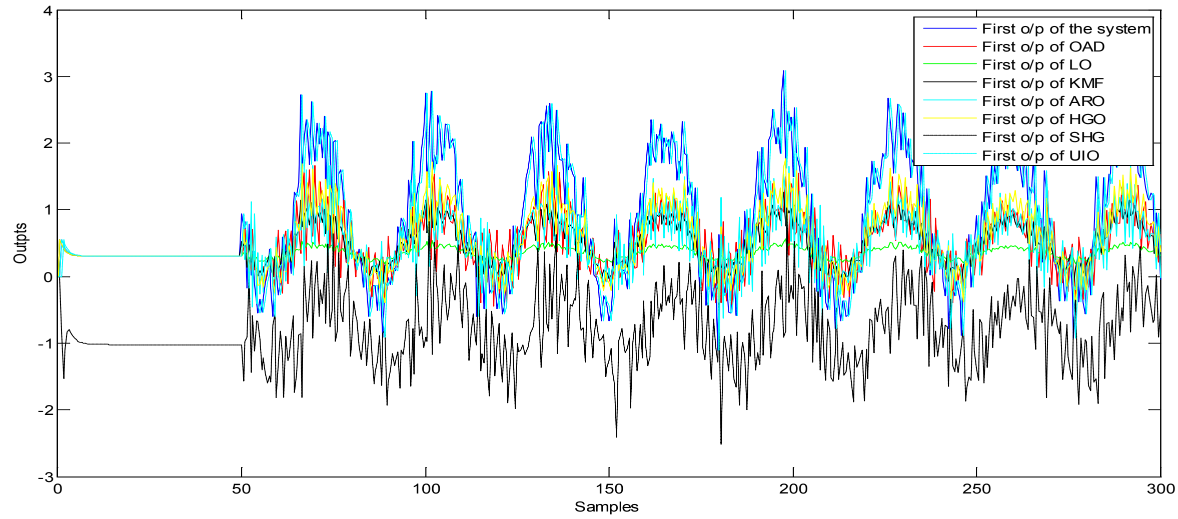

Figure 11. First o/p of observers (with non-Gaussian noise)

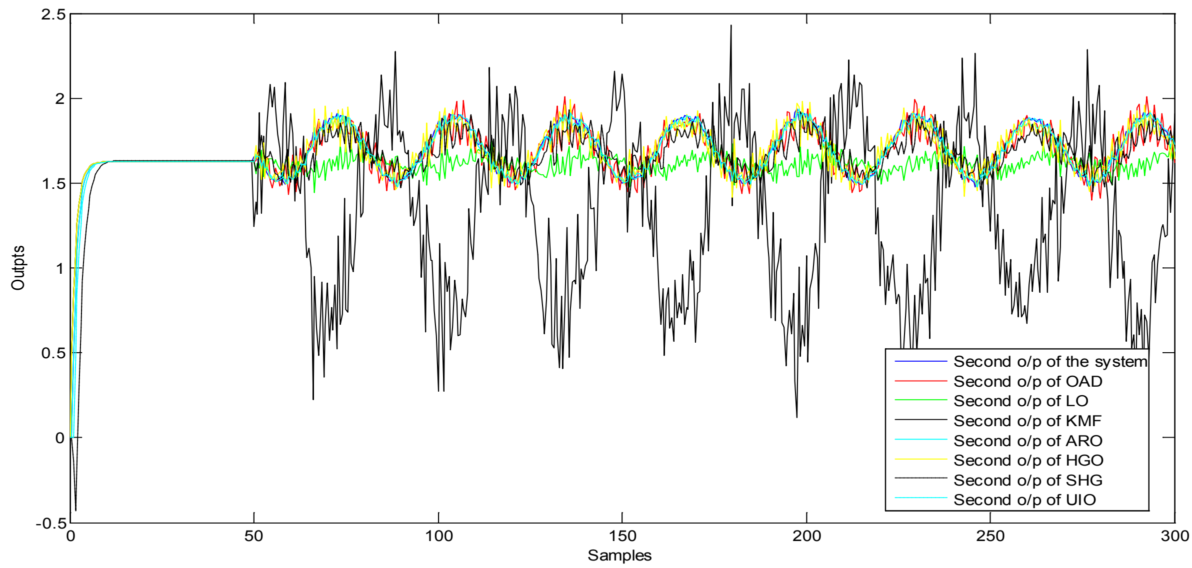

Figure 12. Second o/p of observers (with non-Gaussian noise)

Furthermore, Figure 1, 5and 9 show the first output of the observers and Figure 2, 6 and 10 show the second output of the observers. The comparisons are also clear in Figure 3, 7 and 11 for the first outputs while Figure 4, 8 and 12show the second outputs. The results of the comparisons according to the three types of fault and disturbance show that the new proposed observer OAD can more than compete with the other six good observers $(LO,\ KO,\ UIO,\ ARO,\ HGO, SHGO)$; it has the highest performance.

6. Summary

This paper presents the design of anew optimal adaptive diagnosis observer (ODA) which is designed for additive fault and disturbance; its gain matrix verifies the proposed Lyapunov conditions. In fact, the strategy of a design for dynamic estimated fault for the observer depends on the two positive pre-specified matrices: one tunes the estimated fault and the second matrix tunes the residual between the plant and the observer. In addition, in the presence of disturbance and fault, the performance of the ODA observer is tested through comparing it with six different good linear observers Luenberger Observer (LO), Kalman (Filter) Observer (KO), Unknown Input Observer (UIO), Augmented Robust Observer (ARO), High Gain Observer (HGO), and Sensitive High Gain Observer (SHGO).

The assumed disturbance and faults are white noise, coloured noise and non-Gaussian fault. A MIMO DC servomotor has been used as a benchmark in the performance assessments. While the considered criteria of performance are root mean squared error (RMSE), mean absolute error (MAE) and variance absolute error (VAE) and mean absolute relative error (MARE).

However, the comparison results of the ODA with the six other good observers show that it is the best overall where it has a high ability to detect and diagnose different fault and disturbance type as well as it is the best in states estimation performance.

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