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Why the Regulator Should Worry When a Product Could be Retrieved from the Market

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Abstract One resumes the topic already dealt with in the article in the Journal of Game Theory (April 2025): that of the profitable "close down". A multivendor closes an asset, stopping the sales of a product, and makes a gain. If the three prices increase (that of the product retrieved from the market, of course, that of the product sold by the monoproduct firm, and that of the product still sold by the multivendor), it is obvious that the consumers' surplus decreases. It remains that if one of the prices (that of the product "kept") decreases, the consequences of the "close down" are not described. It is demonstrated in this article that the consumers' surplus decreases, in this case. The demonstration uses another model than the one used in the article of the Journal of Game Theory. This other model, which has been sketched in an article in Advances in Emerging Information and Communication Technology (November 2023) shows the three steps of the operation "close down". Therefore, in any case the consumers' surplus decreases. This challenges the regulator. He has to choose between two incommensurable options (1) affordability. The welfare of the consumers is taken into account. The "close down" is prohibited by the regulator (if he can) (2) financing of innovation. More revenue of the firms will be invested in research on new products. The regulator accepts the "close down". This is quickly commented in the conclusion. It appears that the regulator is not an expert, but a strategist.

Keywords Regulation, Differentiated products, Consumers' surplus

1. Introduction

One resumes the topic already dealt with in several articles: the "profitable close down". One supposes that the products sold are substitutable. And the method used is Bertrand competition, with the demands deduced from the consumers' utilities [1] [2]. A multivendor closes an asset, stopping the sales of a product, and makes a gain. This, because the competitor, a monoproduct firm, increases its price as a consequence of the retrieval of a rival product. It allows the multivendor to increase the price of the product it keeps selling, hence a gain which compensates and beyond the loss incurred due to the close down. The strategic effect allows a gain beyond the loss triggered by the direct effect.

Of course there is a condition. Even, the possibility of a "profitable close down" can be considered as a criterion: a criterion of saturated market. It remains that the term "saturated market" has to be interpreted. It is done in the conclusion.

The condition (of a "profitable close down") is obvious. Let us call E_1 the monoproduct firm selling the product 1 and E the multivendor selling the products 2 and 3. The product 3 is retrieved from the market. P_i is the profit from the sales of the product i at the start (it is a Nash equilibrium, with E maximizing $P_2 + P_3$ given the choice of E_1 and E_1 maximizing P_1 given the choice of E). At the end, there is a Nash equilibrium with two products sold, 1 and 2, and the profits are P'_i. The condition is:

$$P'_2 > P_2 + P_3.$$

Of course, E decides the "close down" only if it allows a gain.

Concerning the prices there are two cases:

- The three prices increase. It is obvious that the consumers' surplus decreases.
- One of the prices, that of the product "kept" (the product 2) can decrease [2]. One has to describe, in this case, the consequences of the "profitable close down". It is the goal of this article.

It matters because the fall of a price can occur. Indeed, it should be small. One can deduce that from the declarations of the managers of large firms. In general, everything about the strategy of large firms is confidential. And the communication between the managers and the regulator is part of it. But often the managers appeal to the Opinion because it allows some pressure on the regulator. In these statements, the managers promise that there will be no increase of the prices. They do not dare to announce a decrease of prices (in case of mergers). Hence the pragmatic argument: a decrease of a price is possible (when a profitable

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"close down" occurs), but is should be small. In the paper, one chooses a condition concerning the prices: one of them can decrease a little. And a demonstration is needed to be certain that the consumers' surplus decreases, in this case.

The model used is not the one which has been set out in the article "Evaluation of the monopoly as a stable second best" [2]. This model did not show the steps of the "close down". The model used in this article shows the steps. It has been sketched in "Introducing a vision of regulation more complex than the traditional one" [3].

The plan of the article is:

- The model used is set out.
- Demonstration of the result.
- In the Conclusion, one comments on the regulator's choices. He has to choose between two incommensurable options: affordability (the welfare of the consumers is taken into account) or financing innovation (the priority is high revenues for the firms, which will invest them in research on new products). It appears that the regulator is not an expert, but a strategist.

2. Methods and Results

2.1. Setting out the Model Used

To set out the model used, one starts by quoting a passage in the author's article "Introducing a vision of regulation more complex that the traditional one" [3]:

- "p_{i0} is "adapted" to p_{j0} and p_{k0} if: $\partial R_i / \partial p_i (p_{i0}, p_{j0}, p_{k0}) = 0$. That is to say, p_{i0} is the best response of E_i, to p_{j0} and p_{k0}.
- Let us call O p_i p_j [$p_k = p_{k0}$] the axes O p_i p_j if $p_k = p_{k0}$, with the reaction functions R_i and R_j (for $p_k = p_{k0}$). If p_{i0} is "adapted" to p_{j0} and p_{k0}, in O p_i p_j [$p_k = p_{k0}$] the point (p_{i0}, p_{j0}) is on R_i. Or, in O p_i p_k [$p_j = p_{j0}$] the point (p_{i0}, p_{k0}) is on R_i.

Suppose E_1 sells the product 1 and E sells the products 2 and 3. There is a Nash equilibrium (p_{10}, p_{20}, p_{30}). In O p_2 p_3 [$p_1 = p_{10}$], p_3 increases as far as $p_3 = 1$. The representative point moves vertically as far as $p_3 = 1$. The product 3 is no more sold (since one supposes $D_i = 0$ if $p_i = 1$).

One supposes a unique, stable Nash equilibrium when product 3 is no more sold.

In O p_2 p_3 [$p_1 = p_{10}$], the representative point moves on the line $p_3 = 1$ as far as the reaction function R_2 ($p_2 = p'_2$, $p_3 = 1$). When $p_2 = p'_2$, E is "adapted" to $p_1 = p_{10}$ and $p_3 = 1$. On O p_1 p_2 [$p_3 = 1$] the representative point (p_{10}, p'_2) is on R_2. In O p_2 p_3 [$p_1 = p_{10}$], the representative point went to R_2, sliding to the left or to the right:

- If p_2 increased (from p_{20} to p'_2) the best response of E_1 is an increase (because p_2 increased and p_3 also). Therefore, on O p_1 p_2 [$p_3 = 1$], the representative point goes to the Nash equilibrium thanks to increases of p_1 and p_2. At the start, the point is on the lower strand of R_2. The prices p_1 and p_2 increase.
- If p_2 decreased, one does not know the best response of E_1 (to increase or to decrease its price), because p_2 decreased and p_3 increased. If the best response of E_1 is to increase p_1, the reasoning is the same than in the case dealt with. At least the price p_1 increases. The hypothesis of p_1 decreasing is impossible. The representative point would be on the higher strand of R_2. At the start, the profit P_2 is less than Max $P_2 + P_3$ (if $p_1 = p_{10}$). When the point slides alongside R_2 the profit P_2 decreases. Therefore, it cannot be more than Max $P_2 + P_3$ (if $p_1 = p_{10}$).

The three prices increase or at least two (the price of the product which is no longer sold and the price of the product of the competitor of the multiproduct firm).

If the three prices increase, the consumers' surplus decreases. If two only increase, it is not sure".

Therefore, one has to demonstrate that the consumers' surplus decreases, when one of the prices (that of the product still sold by the multivendor) decreases. It is the goal of this article.

Representing the two first steps.

At the start, there is the point P, in O p_2 p_3 [$p_1 = p_{10}$]. One supposes symmetry of the consumers' utilities, therefore of the demands: $D_2 (p_{10}, p_2, p_3) = D_3 (p_{10}, p_3, p_2)$, for instance. There is the same for p_{20} or p_{30} fixed. Therefore, P is on the diagonal, where $P_2 + P_3$ is maximal. Its coordinates are (p_1, p_0, p_0), one calls the quantities D_2 (P) = D_3 (P), x, D_1 (P)= y. And in P, $P_2 = P_3 = x\, p_0$, $P_1 = y\, p_1$, the total profit is $2\, x\, p_0 + y\, p_1$.

During the first step, p_3 increases from p_0 to 1. The quantities become ($y + \Delta'$, $x + \Delta$, 0), with $\Delta > 0$ and $\Delta' > 0$. The total quantity decreases, since the price p_3 increases: $\Delta + \Delta' - x < 0$. The representative point slides as far as Q (see figure 1).

During the second step, the representative point, slides on $p_3 = 1$, as far as R (p_1, p'_0, 1). The price p_2 decreases from p_0 to p'_0 ($p_1 < p'_0 < p_0$). Here, the quantities are $\mathfrak{D}_1$ and $\mathfrak{D}_2$. The price p'_0 corresponds to Nash equilibrium, N, with two products sold (1 and 2): N (p'_0, p'_0, 1).

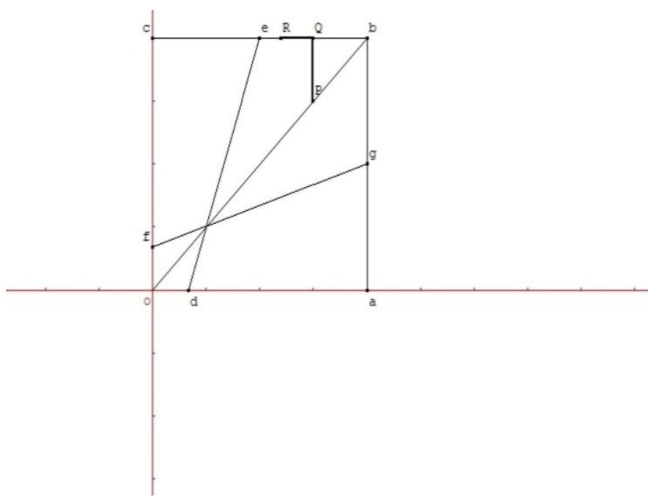

Figure 1. The two first steps are represented by PQ and QR. de: R_1. fg: R_2. The coordinates are for P: (p_1, p_0, p_0), for Q: (p_1, p_0, 1), for R: (p_1, p'_0, 1)

Representing the two last steps.

It is in O p_1 p_2 [$p_3 = 1$]. The second step QR is on the vertical with abscissa p_1. The price p_2 decreases from p_0 to p'_0. Then the third step is horizontal, from R to N (p'_0, p'_0). In N the quantities are $D_1 = D_2 = x'$, the profits are $P_1 = P_2 = x'\ p'_0$. The total profit is 2 x' p'_0.

Notice that the condition "profitable close down" is written: $2\ x\ p_0 < x'\ p'_0$.

The two last steps are represented on Figure 2.

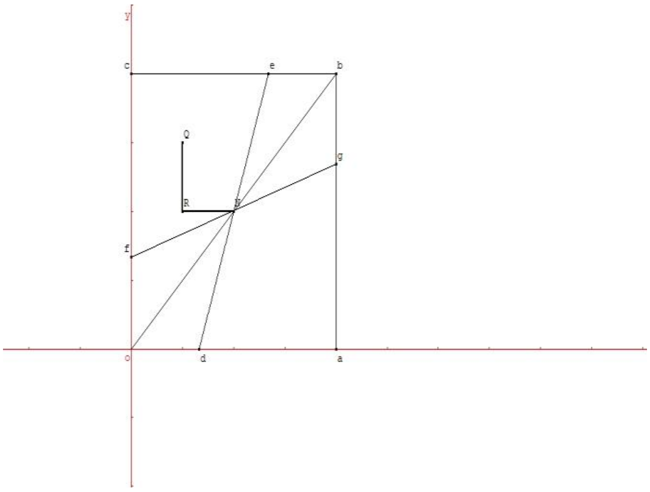

Figure 2. The two last steps are represented by QR and RN. de: R_1. fg: R_2. The prices are, for Q: (p_1, p_0, 1), for R: (p_1, p'_0, 1), for N: (p'_0, p'_0, 1)

2.2. Demonstration of the Result

2.2.1. Study of a Certain Curve

Let us call region A the region limited by $p_1 = 0$, $p_2 = 1$, R_1, and $p_1 = p'_0$.

Also, one calls:

- region 1: Inside A, $P_1 > P_2$
- region 2: inside A, $P_2 > P_1$.

Region 1 and region 2 are separated by a continuous curve Γ.

Imagine a point on the bisector, below N, going up along the vertical. At the start, $P_1 = P_2$, then the two increase as far as R_2. On R_2, either (1) $P_1 > P_2$, then P_1 increases and P_2 decreases, so there is no point where $P_1 = P_2$ (2) $P_1 < P_2$ and there is a single point where $P_1 = P_2$.

The curve Γ, which passes by (0, 1) and N, is cut by a vertical in zero or one point. [1]

Now imagine a point on $p_1 = p'_0$ above N. Here $P_1 > P_2$. If it slides along the horizontal line towards the left, as P_1 and P_2 decrease, and on $p_1 = 0$, $P_1 = 0$, $P_1 < P_2$, there is one point where $P_1 = P_2$, or an odd number of such points. Therefore, Γ is cut by a horizontal line at least in one point, perhaps in several points.

The shape of the curve Γ could be like in the figure 3, with a "hole" or several "holes".

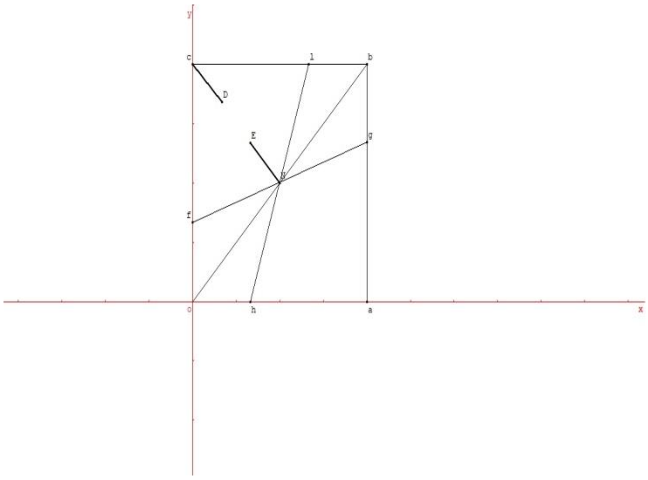

Figure 3. The curve Γ (c D E N) is shown. In this case, there is a "hole". hl: R1. fg: R_2

But it is impossible. One could pass from (0, p'_0) where $P_2 > P_1$ to (p'_0, 1) where $P_1 > P_2$ continuously, without any point where $P_1 = P_2$. Put in other words: the regions 1 and 2 are connected and there is a continuous frontier which separates them. Therefore, there are no "holes".

The curve Γ could be made up of several strands (separating the regions 1 and 2) and one or several arches of R_2.

One demonstrates a lemma.

Lemma. Alongside the curve Γ from (0, 1) to N p_2 / p_1 always decreases.

The demonstration is easy and has been put in the Appendix 1.

A possible path can be described: from (0, 1) to N, one follows a strand of Γ as far as R_2, then an arch of R_2 where $P_1 > P_2$ then possibly an arch or several of Γ with arches of R_2 between them, p_2 / p_1 decreasing from ∞ to 1. It is obvious that on an arch of R_2 concerned, $P_1 > P_2$. Let us call x_1 and x_2 the ends of such an arch. Between the verticals passing by x_1 and x_2, and above R_2, there is no point where $P_1 = P_2$ since there is no point of Γ. Therefore, in x_3 on the arch between x_1 and x_2, $P_1 > P_2$.

What matters for us is the shape of the curve above $p_2 = p'_0$. The shape is shown in the figure 3. There is a strand going down from (0, 1) to $p_2 = p'_0$ then, possibly one arch or several arches. Between the arches, there are segments of $p_2 = p'_0$.

Also, the slope of Γ at N is 1.

The slope of Γ is – A / B with:

$A = \partial P_1 – P_2 / \partial p_1 = D_1 + p_1 \partial P_1 / \partial p_1 – p2 \partial D2 / \partial p1$

$B = \partial P_1 – P_2 / \partial p_2 = p_1 \partial D_1 / \partial p_2 – D_2 – p_2 \partial D_2 / \partial p_2$

In N, $A = D_1 – p'_0 \partial D_2 / \partial p_1$, $B = p'_0 \partial D_1 / \partial p_2 – D_2$, and $D_1 = D_2$, $\partial D_1 / \partial p_2 = \partial D_2 / \partial p_1$.

Therefore, at some point Γ drops below $p_2 = p'_0$, to reach the bisector. Either Γ is tangent to the bisector at N. The last part of Γ is a curve between R_2 and the bisector, tangent to the bisector at N. Either the last part of Γ is a segment on the bisector, an end of which is N.

A numerical example is set out in the Appendix 2. In this example, Γ is made up of a strand, from (0, 1) to some point

[1] In (0, 1) $P_2 = 0$ because one supposes D_2 (p_1, 1) = 0 for any p_1.

on the bisector, then a segment of the bisector, as far as N (see Figure 4).

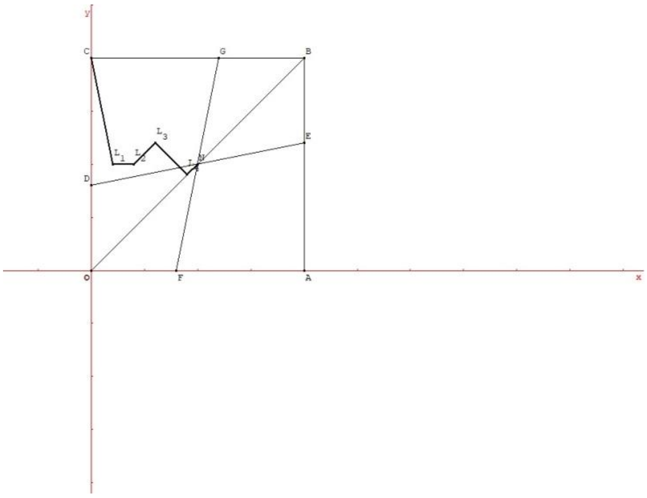

Figure 4. An example of curve Γ is shown: it is C L1 L2 L3 L4 N. First, there is a decreasing strand C L1, then a segment L1 L2 part of $p_2 = p'_0$, then an arch and the curve drops below $p_2 = p'_0$, reaches the bisector, the last part being a segment of the bisector, L4 N. DE: R_2. FG: R_1

But we are not done with the study of the curve Γ.

Now we demonstrate an interesting lemma.

Lemma. Alongside Γ, a point where p_1 p_2 is maximal is impossible.

One uses a proof by contradiction.

One supposes such a point, β. It is a point where the slope of Γ is negative (at this point Γ is tangent to the hyperbola (p_1 p_2 = Constant). Let us call H (Y) the hyperbola (p_1 p_2 = Constant) passing by the point Y, and CS (Y) the contour line for the function consumers' surplus, passing by Y.

One considers a point α, above β, on Γ, as near from β than one wants.

The curve Γ is the locus of $P_1 = P_2$, separating the two regions, the region 1 where $P_1 > P_2$ and the region 2 where $P_1 < P_2$. In the region 1, at any point the slope of CS is lower than that of H. And in the region 2, at any point the slope of CS is higher than that of H.

The reader can easily check that:

- at α, where the tangent to CS and H has a lower slope than that of Γ, H (α) "envelops" CS (α): the W strand of CS (α) is above H (α), the E strand of CS (α) is also above H (α). If Γ is concave or convex at α, does not matter.
- At β, the W strand of CS (β) is above H (β), but the E strand of CS (β) is below H (β). It does not matter if at β, Γ is concave, convex, or if it is an inflection point.

That is impossible. If one supposes continuity, the E strand of CS cannot pass from above to below H, the distance between α and β being arbitrarily small. The strands of CS and H cannot coincide, since they are the contour lines of two different functions.

The argument can be presented in another way. Suppose an observer linked to a mobile frame: when α slides toward β, the observer for which the tangent to H or CS is fixed (it is the axis of the abscissas), sees H (α) and CS (α) above the horizontal axis. The two curves are convex. The they are tangent to the horizontal axis at the origin of the axes. The two curves become distorted, but H (α) remains below CS (α). And all of a sudden, at β, the strand of CS which is on the right is below H. That is impossible.

2.2.2. Demonstration of the Result

First, the condition which is chosen. It is: p_0 $p_1 < p'^2_0$. In other words, the decrease of the price p_2 (that is to say $p_0 - p'_0$) is small enough, so p_0 $p_1 < p'^2_0$. One could have chosen $p_1 + p_2 < 2$ p'_0. In this case the demonstration is straightforward: the slope of the segment Q N is more than – 1, so the slope of the curves CS being less than – 1, the consumers' surplus decreases along the segment Q N (from Q to N). But the condition chosen is more "general": $p_1 + p_2 < 2\ p'_0 \Rightarrow p_0\ p_1 < p'^2_0$.

Consider H (N). It cuts p_2 = p0 on the right to Q. And H (N) does not cut Γ, since in any point of Γ the quantity p_1 p_2 is less than p'02. In fact, at N, the quantity p_1 p_2 is equal to p'^2_0, Γ and H (N) are tangent, the slope of the tangent being – 1. H (N) is on the right to Γ.

The path along which the consumers' surplus decreases, from Q to N is obvious: Q → point of intersection of H (N) and $p_2 = p_0$ → H (N) → N. The curve H (N) is in the region 1 so the consumers' surplus decreases when a point on H (N) moves to the right.

3. Conclusions

We have made three hypotheses:

- The demand functions (deduced from the customers' utilities) are "regular" (twice continuously derivable). The curves (profits …) which are concerned are also "regular". The response functions exist and are increasing.
- The utilities, therefore the demands are symmetrical.
- There is the condition p_0 $p_1 < p'^2_0$. The decrease of the price of the product "kept" is small enough, so this condition is fulfilled.

Given these hypotheses, the model which is presented is an "experience of thought". But it stimulates the thought about the phenomenon "profitable close down".

The regulator has to possible choices (1) He takes into account the consumers' welfare, and, since it decreases if the "close down" occurs, he prohibits it and (2) He can favor the financing of innovation and let the "close down" occur. When the three prices increase, the total profit increases. It is easy to demonstrate. As the "close down" is supposed profitable, it is enough if the monovendor makes a gain. This is the case. The order of the steps does not matter. Therefore, suppose that p_3 (the price of the product no more sold), then p_2 (the price of the product "kept") increase, first. The profit of the monovendor increases, because its price does not change, and the demand for its product increases. When p_1 increases, it is to reach the Nash equilibrium, and again the profit P_1 of the monovendor, increases.

One can suppose that the total profit increases also when the price p_2 of the product "kept" decreases just a little. So, more money can be invested in R and D, innovation and upgrading the products sold.

The two choices are incommensurable. The choice which is made depends on the outcome of the social play. There are what the French sociologist Tarde calls 'logical duels [4]. Some are ended, other are ongoing.

Let us give examples of such "logical duels":

- Net neutrality. This "logical duel" has ended. Affordability has been definitively chosen. The providers of standard Internet services (operators, CDNs, Content Distribution Networks, …) are competitors. To choose affordability is justified when networked services are concerned: the utility of a customer increases with the availability of the service for the other customers. It is preferable if a large quantity of services are sold, thanks to affordable prices.
- Medicaments. The spokespersons of Big Pharma praise the profits of the firms in the pharmaceutical sector: they allow more research on new treatments, or medicaments (bio drugs …). They disapprove the manufacturing and the sale of generic medicaments. From this point of view, if the prices are low because many differentiated products are available for the consumers, it is only a "windfall" for the consumers.

A well-known advocate of affordability is Christensen. After him, a real innovation involves a breakthrough, utility for the users, and an affordable price [5] [6]. He gives as examples the personal computer, and the mobile phone. One could add Net neutrality.

It appears that the regulator is not an expert, he is a strategist.

Indeed, the array of his choices is wider than just affordability / financing of innovation. For instance, if a firm is threatened to disappear or to be dismantled, it can be saved if a competitor buys it. But there is a risk: the bought firm is managed by a multivendor, and this has consequences on the investment. In France, recently, in the sector of the sale of furniture, the n°2 was Conforama and the n°3 was But. Conforama met great difficulties and was saved when But bought it. After the merger, the n°2 is But and the n°3 is Conforama [7]. Notice that the model presented by the author in the Journal of Game Theory [2] shows this: when an independent brand is bought then integrated in a multivendor, how this brand is managed changes.

Appendix 1

One demonstrates this lemma:

Lemma. Along Γ, from (0, 1) to N, the quantity p_2 / p_1 decreases.

The lemma is not used in the demonstration of the result which is the goal of this article. But it helps to know the shape of the curve Γ in a better way.

Let us call:

$A = \partial P_1 - P_2 / \partial p_1 = D_1 + p_1 \partial P_1 / \partial p_1 - p_2 \partial D_2 / \partial p_1$

$B = \partial P_1 - P_2 / \partial p_2 = p_1 \partial D_1 / \partial p_2 - D_2 - p_2 \partial D_2 / \partial p_2$. $B > 0$.

The quantity p_2 / p_1 cannot have an extremum on Γ:

$A\, d p_1 + B\, d p_2 = 0$, and:

$- p_2\, d p_1 + p_1\, d p_2$, simultaneously, is impossible.

It is enough to demonstrate that: $A\, p_1 + B\, p_2 = O$ is impossible.

This expression is written:

$p_1 [D_1 + p_1 \partial D_1 / \partial p_1 - p_2 \partial D_2 / \partial p_1] + p_2 [p_1 \partial D_1 / \partial p_2 - D_2 - p_2 \partial D_2 / \partial p_2]$

Given $P_1 = P_2$ on Γ, and $\partial D_1 / \partial p_2 = \partial D_2 / \partial p_1$, this expression is: $p_1^2 \partial D_1 / \partial p_1 - p_2^2 \partial D_2 / \partial p_2$.

This expression is positive.

$p_1 \partial D_1 / \partial p_1 > - D_1 \Rightarrow p_1^2 \partial D_1 / \partial p_1 > - p_1 D_1 = - P_1$. A minorant is $- P_1$.

$p_2 \partial D_2 / \partial p_2 < - D_2 \Rightarrow p_2^2 \partial D_2 / \partial p_2 < - p_2 D_2 = - P_2 \Rightarrow - p_2^2 \partial D_2 / \partial p_2 > P_2$. A minorant is P_2.

A minorant of the expression is $- P_1 + P_2 = 0$. Therefore, the expression is positive.

Appendix 2

One presents a numerical example of curve Γ. In this particular case, the consumers' utilities are defined: $d (p_1, p_2) = 1$, there is a homogeneous density of utility.

We shall calculate the demands, the profits, the coordinates of the Nash equilibrium, the equation of Γ and check the properties of the curve Γ.

The demands $D_1 (p_1, p_2)$ and $D_2 (p_1, p_2)$ are easy to calculate, since they correspond to the area ($u_1 - p_1 > u_2 - p_2$ and $u_1 - p_1 > 0$ for D_1, and $u_2 - p_2 > u_1 - p_1$ and $u_2 - p_2 > 0$, for D_2). Therefore:

$D_1 = 1 / 2 - p_2^2 / 2 + p_2 - p_1$

$D_2 = 1 / 2 + p_2^2 / 2 - p_1 p_2 + p_1 - p_2$

The profits are written:

$P_1 = p_1 [1 / 2 - p_2^2 / 2 + p_2 - p_1]$

$P_2 = p_2 [1 / 2 + p_2^2 / 2 - p_1 p_2 + p_1 - p_2]$

For the Nash equilibrium:

$+ 1 = 0$

$\partial P_2 / \partial p_2 = 1 / 2 + 3 p_2^2 / 2 - 2 p_1 p_2 + p_1 - 2 p_2 = 0.$

The coordinates of N are $(\sqrt{2} - 1, \sqrt{2} - 1)$ or (0, 41, 0, 41).

The curve R_2 has for equation $\partial P_2 / \partial p_2 = 0$, is increasing (positive slope) and goes from (0, 1 / 3) to N.

The curve R_1 has for equation $\partial P_1 / \partial p_1 = 0$, is increasing (positive slope) and goes from N to (1 / 2, 1). (See figure 5).

The equation of Γ is $P_1 = P_2$:

$p_1 [1 / 2 - p_2^2 / 2 + p_2 - p_1] - p_2 [1 / 2 + p_2^2 / 2 - p_1 p_2 + p_1 - p_2]$.

Obviously, the equation holds if $p_1 = p_2$, therefore

$(p_2 - p_1)$ is a factor:

$(p_2 - p_1) [p_1 + p_2 - 1 / 2 - p_2^2 / 2] = 0.$

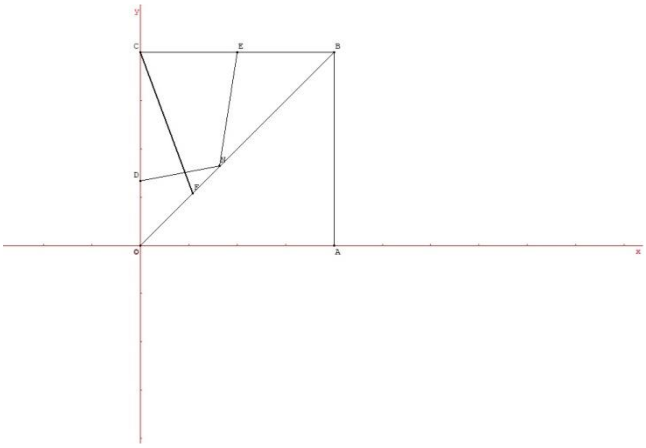

Figure 5. The curve Γ is shown: it is an arch of parabola going from C to F, then the bisector from F to N. The response function R_2 is DN. The response function R_1 is NE

Γ is made up of two parts:

- An arch of parabola $p_2^2 - 2\ p_1 - 2\ p_2 + 1 = 0$. The summit is at (0, 1). The curve is decreasing, cutting the bisector at (0, 27, 0, 27), below N.
- The bisector from (0, 27, 0, 27) to N.

It remains to check this property of Γ: $p_1\ p_2$ reaches the value $(\sqrt{2} - 1)^2$ only at N.

The equation of the parabola is $(p_2 - 1)^2 / 2 = p_1$ and one supposes that $p_1 = (\sqrt{2} - 1)^2 / p_2$. There is no root of this equation: $(p_2 - 1)^2 / 2 = (\sqrt{2} - 1)^2 / p_2$.

The equation of the third degree:

$p_2^3 - 2\ p_2^2 + p_2 - 2\ (\sqrt{2} - 1)^2 = 0$ has no root between 0 and 1.

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