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Worst Welfare under Supply Function Competition with Sequential Contracting in a Vertical Relationship

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Abstract We compare equilibrium welfares under Cournot, Bertrand, and supply function competitions. Although it is a natural result that equilibrium outcomes under the supply function competition are intermediate between those under Cournot and Bertrand competitions, we show that the supply function competition may yield the smallest social welfare. To obtain this result, we consider a vertical market where an upstream firm sequentially contracts with two downstream firms.

Keywords Supply function competition, Price discrimination, Sequential contracting

1. Introduction

Fierce competition generally brings down price and raises social welfare. Recognizing this, antitrust authorities implement competition policies to intensify market competition. An index that can measure the competition intensity is the type of competition, that is, whether Cournot or Bertrand competition prevails. Previous studies focus on other indicators such as the number of firms and the degree of product differentiation, but regardless of the indicator is employed, tougher competition is desirable for society. Thus, several papers focus on a situation where equilibrium social welfare decreases with the competition intensity (Lahiri and Ono, 1988; Mukherjee and Zhao, 2009; Fanti, 2013). Thus, the literature on industrial organization compares outcomes under Cournot competition with those under Bertrand competition and finds that in many cases, Bertrand competition yields outcomes that are more desirable for society.

For example, a classic study that compares Cournot competition with Bertrand competition is by Singh and Vives (1984). They show that the equilibrium price in Bertrand competition is lower than that in Cournot competition. Studies that followed indicate that this relationship is reversed (e.g., Delbono and Lambertin, 2016a; Häckner, 2000; Zanchettin, 2006).

Some recent studies consider an intermediate competition type between Cournot and Bertrand competitions. One of them is supply function competition (Grossman, 1981; Klemperer and Meyer, 1989). Some recent studies analyze properties of supply function competitions (e.g., Delgado and Moreno, 2004; Ciarreta and Gutierrez-Hita, 2006; Menezes and Quiggin, 2012; Delbono and Lambertini, 2015, 2016b). Under the supply function competition, it is common wisdom that the supply function equilibrium is intermediate between that under Cournot and Bertrand (Delbono and Lambertini, 2015). However, Delbono and Lambertini (2016b) challenge this well-known result. They consider a market with quadratic cost and show that the supply function competition creates the largest social welfare among the competitions.

Following Delbono and Lambertini (2016b), we reconsider a welfare ranking between Cournot, Bertrand, and the supply function competitions. There are some differences between our model and theirs. In particular, we consider a vertically related market where an upstream firm sequentially contracts with two downstream firms. Our formulation of a sequential contract is the same as that of Kim and Sim (2015). In other words, we introduce the supply function competition into the model with the sequential contract presented by Kim and Sim (2015).

Because the supply function competition leads to Cournot and Bertrand equilibria as special cases, considering a supply function competition model suffices to examine whether Bertrand is the best outcome and Cournot, the worst. When comparing the two competition structures with other cases in the supply function competition, we show that Cournot and Bertrand competitions do not lead to the worst outcomes. That is, at parameter values where equilibrium outcomes are not the same as those under Cournot and Bertrand competitions, we obtain the minimum social welfare. This result is quite different from that in Delbono and Lambertini (2016b).

The paper proceeds as follows. The next section presents the model. Section 3 calculates the equilibrium and provides the main results. Section 4 concludes the paper.

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2. The Model

We consider a market with upstream firm U and two asymmetric downstream firms $Di, (i = 1,2)$. To produce one unit of product, each downstream firm must purchase one unit of input from the upstream firm. We assume that the upstream firm can choose different prices for the downstream firms. We denote the wholesale price for downstream firm Di by w_i. Let q_i denote the output of downstream firm Di. We assume that the marginal cost of upstream firm U is zero, that of downstream firm $D1$ is also zero, and that of downstream firm $D2$ is $c_2 (> 0)$. Following the literature on supply function competition, we assume that downstream firm Di uses a linear supply function $q_i = \alpha_i + \beta p$, where p is the price, intercept α is an endogenous variable chosen by each firm, and β is an exogenous parameter.

Menezes and Quiggin (2012) and Delbono and Lambertini (2015) show that as β converges to zero, equilibrium outcomes under the supply function competition converge to those under Cournot competition; however, as β diverges to infinity, equilibrium outcomes converge to those under Bertrand competition. Therefore, to compare equilibria under Cournot, Bertrand, and the supply function competitions, it suffices to consider the supply function competition with $\beta \in R_+$. Moreover, as an increase in β moves equilibrium outcomes closer to those in Bertrand competition, we can interpret β as the index of competition intensity.

As downstream firm $D2$ is relatively inefficient, for large c_2, the equilibrium output of $D2$ would take a value of zero. To guarantee a positive outcome, we assume $c_2 < (36 + 24\beta + 6\beta^2)/(52 + 68\beta + 28\beta^2 + 3\beta^3)$.

We assume that the inverse demand function is $p = 1 - q_1 - q_2$. As we assume a linear supply function, substituting $q_i = \alpha_1 + \beta p$ into the inverse demand function and solving it for p, we have

$$p(\alpha_1, \alpha_2) = \frac{1 - \alpha_1 - \alpha_2}{1 + 2\beta}.$$

Substituting this equation into $q_i = \alpha_i + \beta p$, we have the following supply functions:

$$q_1(\alpha_1, \alpha_2) = \alpha_1 + \frac{\beta(1 - \alpha_1 - \alpha_2)}{1 + 2\beta},$$

$$q_2(\alpha_1, \alpha_2) = \alpha_2 + \frac{\beta(1 - \alpha_1 - \alpha_2)}{1 + 2\beta}.$$

Using the above equations, we define the profits of downstream and upstream firms as follows:

$$\pi_{D1} = [p(\alpha_1, \alpha_2) - w_1]q_1(\alpha_1, \alpha_2),$$

$$\pi_{D2} = [p(\alpha_1, \alpha_2) - w_2 - c_2]q_2(\alpha_1, \alpha_2),$$

$$\pi_U = w_1 q_1(\alpha_1, \alpha_2) + w_2 q_2(\alpha_1, \alpha_2).$$

Consumer surplus $CS = [q_1(\alpha_1, \alpha_2) + q_2(\alpha_1, \alpha_2)]^2/2$ and social welfare $SW = CS + \pi_{D1} + \pi_{D2} + \pi_U$.

Following Kim and Sim (2015), we consider the sequential contracts between the upstream and downstream firms. In the first stage, the upstream firm decides wholesale price w_1, and in the second stage, downstream firm $D1$ chooses α_1 in its supply function. After these decisions, in the third stage, the upstream firm offers wholesale price w_2, and in the fourth stage, downstream firm $D2$ determines α_2 in its supply function. We could justify this timing of the game as follows. When the upstream firm already creates a long-term relationship with downstream firm $D1$, the contract may express a commitment effect. Then, given the contract with downstream firm $D1$, the upstream firm and downstream firm $D2$ must negotiate their contract. Hence, a Stackelberg timing structure is a natural assumption here.

We assume complete information. The model is solved by backward induction. Only pure strategies are considered throughout this paper.

3. Calculating Equilibrium and Condition Yielding Worst Welfare

The profit maximization problem of downstream firm $D2$ is

$$\max_{\alpha_2} \; [1 - q_1(\alpha_1, \alpha_2) - q_2(\alpha_1, \alpha_2) - w_2 - c_2]q_2(\alpha_1, \alpha_2).$$

The first-order condition $(\partial\pi_{D2}/\partial\alpha_2 = 0)$ yields

$$\alpha_2^{S4}(\alpha_1, w_2) = \frac{1 - \alpha_1 - (1 + 3\beta + 2\beta^2)c_2 - (1 + 3\beta + 2\beta^2)w_2}{2(1 + \beta)},$$

where superscript $S4$ represents that the outcomes are in stage 4. Substituting this outcome into $q_1(\alpha_1, \alpha_2)$ and $q_2(\alpha_1, \alpha_2)$, we have $q_1^{S4}(\alpha_1, w_2) = q_1[\alpha_1, \alpha_2^{S4}(\alpha_1, w_2)]$ and $q_2^{S4}(\alpha_1, w_2) = q_2[\alpha_1, \alpha_2^{S4}(\alpha_1, w_2)]$.

Substituting the above results into the profit of the upstream firm, the maximization problem is

$$\max_{w_2} \; w_1 q_1^{S4}(\alpha_1, w_2) + w_2 q_2^{S4}(\alpha_1, w_2).$$

The first-order condition $(\partial\pi_U/\partial w_2 = 0)$ leads to

$$w_2^{S3}(\alpha_1, w_1) = \frac{1 - (1 + \beta)c_2 + \beta w_1 - \alpha_1}{2(1 + \beta)},$$

where superscript $S3$ represents that the outcomes are in stage 3. Substituting this outcome into $q_1^{S4}(\alpha_1, w_2)$ and $q_2^{S4}(\alpha_1, w_2)$, we have $q_1^{S3}(\alpha_1, w_1) = q_1^{S4}[\alpha_1, w_2^{S3}(\alpha_1, w_1)]$ and $q_2^{S3}(\alpha_1, w_1) = q_2^{S4}[\alpha_1, w_2^{S3}(\alpha_1, w_1)]$.

Substituting the above results into the profit of downstream firm $D1$, the maximization problem is

$$\max_{\alpha_1} [1 - q_1^{S3}(\alpha_1, w_1) - q_2^{S3}(\alpha_1, w_1) - w_1]q_1^{S3}(\alpha_1, w_1).$$

From the first-order condition $(\partial\pi_{D1}/\partial\alpha_1 = 0)$, we have

$$\alpha_1^{S2}(w_1) = \frac{6 - 3\beta + (2 + \beta - \beta^2)c_2 - (8 + 8\beta + 3\beta^2)w_1}{3(4 + \beta)},$$

where superscript $S2$ represents that the outcomes are in stage 2. Substituting this outcome into $w_2^{S3}(\alpha_1, w_1)$, $q_1^{S3}(\alpha_1, w_1)$, and $q_2^{S3}(\alpha_1, w_1)$, we have $w_2^{S2}(w_1) =$

$w_2^{S3}[\alpha_1^{S2}(w_1), w_1]$, $q_1^{S2}(w_1) = q_1^{S3}[\alpha_1^{S2}(w_1), w_1]$, and $q_2^{S2}(w_1) = q_2^{S3}[\alpha_1^{S2}(w_1), w_1]$.

Substituting the above outcomes into the profit of upstream firm, the maximization problem is

$$\max_{w_1} \; w_1 q_1^{S2}(w_1) + w_2^{S2}(w_1) q_2^{S2}(w_1).$$

The first-order condition $(\partial \pi_U / \partial w_1 = 0)$ leads to

$$w_1^* = \frac{(1+\beta)[84 + 48\beta + 9\beta^2 - (-2+\beta)^2 c_2]}{2[88 + \beta(128 + 58\beta + 9\beta^2)]},$$

where superscript $*$ represents that the outcomes are in equilibrium. Summarizing the above results, we obtain the following proposition:

Proposition 1. The equilibrium social welfare is

$$SW^* = \frac{A_2 c^2 + A_1 c + A_0}{8(88 + 128\beta + 58\beta^2 + 9\beta^3)^2},$$

where

$A_2 = 4(4476 + 16736\beta + 25888\beta^2 + 21500\beta^3 + 10364\beta^4 + 2902\beta^5 + 437\beta^6 + 27\beta^7)$,

$A_1 = -72(236 + 780\beta + 1042\beta^2 + 727\beta^3 + 282\beta^4 + 58\beta^5 + 5\beta^6)$, and

$A_0 = 19312 + 56416\beta + 67776\beta^2 + 43192\beta^3 + 15508\beta^4 + 2988\beta^5 + 243\beta^6$.

Differentiating SW^* with respect to β, we have

$$\frac{\partial SW^*}{\partial \beta} = \frac{B_2 c_2^2 + B_1 c_2 + B_0}{2(88 + 128\beta + 58\beta^2 + 9\beta^3)^3},$$

where

$B_2 = 326912 + 1375648\beta + 2522232\beta^2 + 2644000\beta^3 + 1751096\beta^4 + 764604\beta^5 + 222140\beta^6 + 41854\beta^7 + 4698\beta^8 + 243\beta^9$,

$B_1 = -18(8224 + 28800\beta + 43464\beta^2 + 36348\beta^3 + 18034\beta^4 + 5283\beta^5 + 848\beta^6 + 58\beta^7)$,

and

$B_0 = 18(288 + 3152\beta + 7548\beta^2 + 8148\beta^3 + 4718\beta^4 + 1521\beta^5 + 258\beta^6 + 18\beta^7)$.

After a tedious calculation, we obtain the following proposition:

Proposition 2. When we can freely choose a value of β, the equilibrium social welfare is minimized at the following β^*.

$$\beta^* = \begin{cases} \beta^L & if \;\; c_2 \in [0.03825, 0.2755], \\ \beta^H & if \;\; c_2 \in (0.2755, 0.4147], \\ 0 & \text{otherwise}, \end{cases}$$

such that β^L satisfies $c_2 = c_2^L(\beta^L)$ and β^H satisfies $c_2 = c_2^H(\beta^H)$, where

$$c_2^L = \frac{-B_1 - \sqrt{B_1^2 - 4B_2B_0}}{2B_2},$$

$$c_2^H = \frac{-B_1 + \sqrt{B_1^2 - 4B_2B_0}}{2B_2}.$$

Proof.

First, using the discriminant of the numerator for $\partial SW^*/\partial\beta$, we show that the value of β^* that minimizes SW^* does not exist in $\beta \geq 0.2755$. As the sign of denominator is positive, the sign of first derivative is the same as that of numerator. The numerator is the quadratic function of c_2 and the coefficient of c_2^2 is positive $(B_2 > 0)$. Then, if the sign of discriminant $B_1^2 - 4B_2B_0$ is non-positive, we will have $\partial SW^*/\partial\beta \geq 0$. Numerically solving $B_1^2 - 4B_2B_0 \leq 0$ for β, we have $\beta \geq 0.2755$. Hence, the value of β^* that minimizes SW^* does not exist in $\beta \geq 0.2755$.

Next, we consider the case with $\beta < 0.2755$. We will show that the equilibrium social welfare increases with β if $c_2 > 0.4147$ or $c_2 < 0.03825$. As the discriminant takes a positive value, we have two solutions: $c_2 = c_2^L(\beta)$ and $c_2 = c_2^H(\beta)$, which satisfy $\partial SW^*/\partial\beta = 0$. Solving $\partial SW^*/\partial\beta = 0$ for c_2, we have

$$c_2 = \frac{-B_1 - \sqrt{B_1^2 - 4B_2B_0}}{2B_2} \left(= c_2^L(\beta)\right),$$

$$c_2 = \frac{-B_1 + \sqrt{B_1^2 - 4B_2B_0}}{2B_2} \left(= c_2^H(\beta)\right).$$

Then, if $c_2^L(\beta) < c_2 < c_2^H(\beta)$, we have $\partial SW^*/\partial\beta < 0$. Here, from numerical calculation, we show that $c_2^L(\beta) < c_2^H(\beta)$, $\partial c_2^L(\beta)/\partial\beta > 0$, and $\partial c_2^H(\beta)/\partial\beta < 0$. This result implies that the maximum value of c_2 such that $\partial SW^*/\partial\beta < 0$ at some β is derived from $c_2^H(0)$ and the minimum value of c_2 such that $\partial SW^*/\partial\beta < 0$ at some β is obtained from $c_2^L(0)$. That is, $c_2^H(0) = 3(24672 + 1056\sqrt{377})/326912 \approx 0.4147$ and $c_2^L(0) = (74016 - 3168377/326912 \approx 0.03825$. Hence, if $c_2 > 0.4147$ or $c_2 < 0.03825$, then the social welfare is minimized at $\beta = 0$.

Finally, we consider the case with $(c_2, \beta) \in [0.03825, 0.4147] \times [0, 0.2755)$. From numerical calculation, we have $\partial^2 SW^*/\partial\beta^2 > 0$. Hence, SW^* is a strictly convex function of β. Then, the first-order condition $\partial SW^*/\partial\beta = 0$ characterizes the value β^* that minimizes SW^*. That is, by solving the first-order condition for c_2, we have $c_2 = c_2^L(\beta)$ and $c_2 = c_2^H(\beta)$. From the discriminant $B_1^2 - 4B_2B_0 = 0$, we have $c_2^L(\beta) < 0.2755 < c_2^H(\beta)$. Then, given the value of c_2, β^* that minimizes equilibrium social welfare is implicitly determined as follows:

$$\beta^* = \begin{cases} \beta^L & if \;\; c_2 \in [0.03825, 0.2755], \\ \beta^H & if \;\; c_2 \in (0.2755, 0.4147], \end{cases}$$

where β^L satisfies $c_2 = c_2^L(\beta^L)$ and β^H satisfies $c_2 = c_2^H(\beta^H)$. Therefore, we obtain the proposition.

We can depict the result of Proposition 2 in Figure 1. In this figure, there are three regions. In the right area, as we have $q_2^* < 0$, we omit this parameter range; in the shadow area, we have $\partial SW^*/\partial\beta > 0$; in the bottom area, we have $\partial SW^*/\partial\beta < 0$. Hence, given c_2, the boundary between the shadow and bottom areas determines β^* that leads to the minimum social welfare.

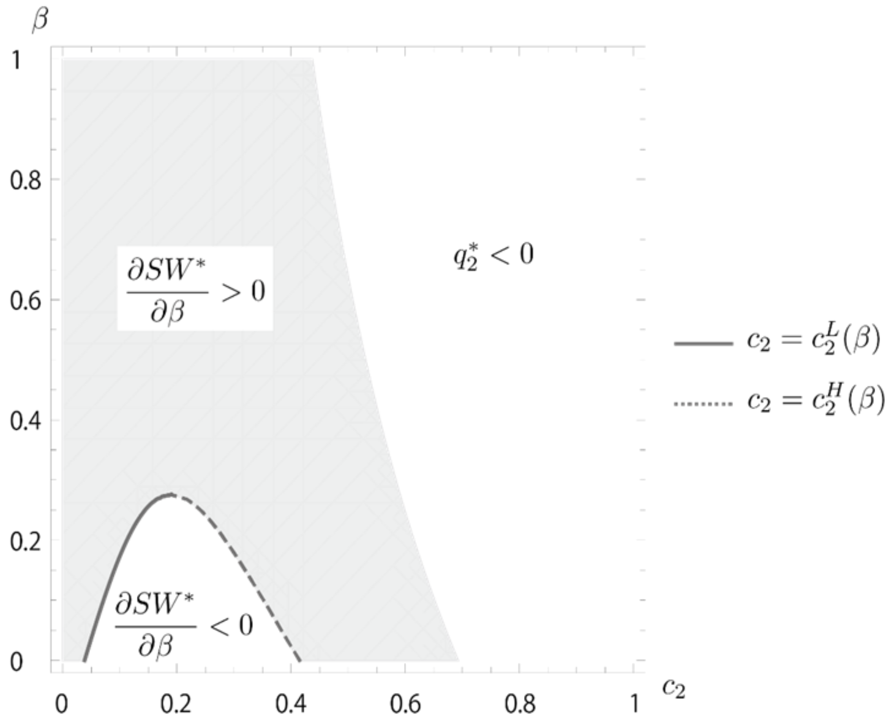

Figure 1. Effect of β on SW^*

This proposition means that if c_2 is in the range $(0.03825, 0.4147)$, the equilibrium social welfare is minimized neither at $\beta = 0$ nor for $\beta \to \infty$. In other words, under the supply function competition except where $\beta = 0$ and $\beta \to \infty$, the equilibrium social welfare is smaller than those under Cournot and Bertrand competitions.

Now, we explain an intuition behind Proposition 2. Since we consider the case of a sequential contract, follower $D2$ faces a residual demand after leader $D1$ decides its output. Thus, the follower behaves less aggressively than in the case with a simultaneous contract. Then, to encourage the follower's production, the upstream firm reduces the wholesale price for follower $D2$. As leader $D1$ knows this action, it reduces its own output. If a decrease in $D1$'s output dominates an increase in $D2$'s output, the total output reduces. Since in this case, an increase in β reduces social welfare and in the other case, the rise of β decreases the social welfare, there exists a positive β such that social welfare is minimized.

4. Conclusions

We consider the supply function competition structure in a vertical market with a sequential contract. We show that if the technological difference between downstream firms is moderate, an intermediate competition intensity yields the minimum social welfare. This result indicates that a competition policy that enhances competition, measured by the type of competition, is not desirable for society.

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