Journal of Game Theory 2017, 6(1): 1-6
DOI: 10.5923/j.jgt.20170601.01

Extensions of Bertrand's Differentiated Products Model

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Abstract We consider two extensions of Bertrand's celebrated duopoly and tri-opoly models of differentiated products. One extension consists of generalizing linear production costs to convex ones. For quadratic costs, we obtain the symmetric equilibrium explicitly. The other is a two-period model where the demand in the second period depends on the price in the first period ("reference price") as well. We obtain an explicit solution to the second period problem and characterize the optimal first period price.

Keywords Bertrand, Non-Linear Costs, Reference Prices

1. Introduction

Bertrand's model is one of the classical models of differentiated duopoly and is a forerunner of the Nash equilibrium. In this model demand for each firm increases in the competitor's price while decreasing in its own price [1-3]. In this classical model production costs are usually assumed to be linear. As such is often not the case, (e.g. [4, 5]), we explore duopoly and tri-opoly [6] scenarios with increasing marginal production costs. We also consider a two-period model where a firm's demand in the second period depends on its and its competitors' price in the first period ("reference price"), as well on the second period prices [7, 8]. While such consumer behavior may not be entirely rational, there is ample empirical evidence in the marketing and psychology literature for its prevalence.

For completeness, we first sketch the well-known Bertrand's linear duopoly model of differentiated products [2, 9] and then do so for a tri-opoly (cf. [6, 10]) and we include comparative statics. We show conditions under which a tri-opolist would charge a higher price than a duopolist. We then generalize them to non-linear production costs, where we focus on the case of quadratic costs for which we obtain an explicit solution. Finally, we consider a two-period duopoly with reference prices.

2. The Basic Model

The basic Bertrand duopoly model of differentiated products is

$q_i(p_i,p_j) = \frac{1}{1-\gamma^2}\left[v(1-\gamma) - p_i + \gamma p_j\right], 0 \le \gamma \le 1,$

$i = 1,2,$ where $p_i < v < \frac{p_i}{1-\gamma}$, $i = 1,2,$ and where p_i and p_j are, respectively, the prices set by duopolists *i* and *j*, and q_i the resulting demand of duopolist i (e.g. [2, 9]).

$\frac{\gamma}{1-\gamma^2}$ is the cross effect of p_j on q_i.

$\frac{v}{1+\gamma}$ is the quantity demanded when prices are zero. Thus this quantity is between $\frac{v}{2}$ and v. v and γ are assumed to be identical for both duopolists, so the model is symmetric.

Now, $\frac{dq}{d\gamma} = (2p - v)\gamma^2 + 2(v - p)\gamma + p - v$ is positive if $p < \frac{1}{2}v$ and $\gamma < \frac{p-v-\sqrt{p(v-p)}}{2p-v}$ or if $p > \frac{1}{2}v$ and $> \frac{p-v+\sqrt{p(v-p)}}{2p-v}$.

Note that $i's$ profit is

$\pi_i(p_i,p_j) = (p_i - c)\left(\frac{v}{1+\gamma} - \frac{1}{1-\gamma^2}p_i + \frac{\gamma}{1-\gamma^2}p_j\right)$, where c is the unit production cost, and $c < p_i$ (and thus $c < v$).

$$\frac{d\pi_i}{dp_i} = \frac{1}{1-\gamma^2}\left[v(1-\gamma) - 2p_i + p_j + c\right]$$

$\frac{d^2\pi_i}{dp_i^2} = -\frac{2}{1-\gamma^2} < 0$, so π_i is concave in p_i.

Here the Nash-equilibrium is the solution of

$$p_i = \frac{1}{2}\left[v(1-\gamma) + c + \gamma p_j\right], \quad i = 1,2.$$

The symmetric solution $p_i = p_j \equiv p$ is thus $p^* = \frac{v(1-\gamma)+c}{2-\gamma}$, which increases in v and c and decreases in γ.

(Note that since for a monopolist with this parametrization $p^*_{(1)} = \frac{1}{2}[v(1-\gamma) + c]$, therefore $p^*_{(1)} \le p^*_{(2)}\ \forall\, \gamma$ and c)

$\Rightarrow q^* = \frac{1}{1+\gamma}(v - p^*) = \frac{v-c}{(1+\gamma)(2-\gamma)}$, which increases in v, decreases in c and increases in γ if $\gamma > \frac{1}{2}$

$\Rightarrow \pi^* = \frac{(c-v)^2(1-\gamma)}{(1+\gamma)(2-\gamma)^2}$, which increases in v and decreases in c and γ; all are intuitive.

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Published online at http://journal.sapub.org/jgt

For example, if $\gamma = \frac{1}{2}$, then $p^* = \frac{1}{3}(v + 2c)$, $q^* = \frac{4}{9}(v - c)$
and $\pi^* = \frac{8}{27}(c - v)^2$.

3. Three Firms

Here the assumption is that

$$q_i(p_i, p_j, p_k) = \frac{1}{1-\gamma^2}\left[v(1-\gamma) - p_i + \gamma p_j + \gamma p_k\right], \quad i = i, j, k,$$

so the effect of other products' prices on the demand of a product is symmetric (cf. [6]). So

$$\pi_i = (p_i - c) \cdot \frac{1}{1-\gamma^2}\left[v(1-\gamma) - p_i + \gamma p_j + \gamma p_k\right]$$

Denoting $p_j + p_k$ by p_{-i}, we have

$$\pi_i = (p_i - c) \cdot \frac{1}{1-\gamma^2}\left[v(1-\gamma) - p_i + \gamma p_{-i}\right]$$

$$\frac{d\pi_i}{dp_i} = \frac{1}{1-\gamma^2}\left[v(1-\gamma) - p_i + \gamma p_{-i} + (p_i - c)(-1)\right]$$

$$= \frac{1}{1-\gamma^2}\left[v(1-\gamma) + c - 2p_i + \gamma p_{-i}\right]$$

$\frac{d^2\pi_i}{dp_i^2} = \frac{2}{1-\gamma^2} < 0$, so π_i is concave.

The optimality condition implies that

$$2p_i = v(1-\gamma) + c + \gamma p_{-i}$$

So the symmetric solution is

$$p = \frac{1}{2}\left[v(1-\gamma) + c + \gamma(2p)\right]$$

and thus

$$p^* = \frac{v(1-\gamma)+c}{2(1-\gamma)} = \frac{1}{2}\left(v + \frac{c}{1-\gamma}\right),$$

which increases in v, c and γ, and thus $p^*_{(3)} \le p^*_{(2)}$ iff $v < \frac{1-c}{1-\gamma}$.

As $q = \frac{1}{1-\gamma^2}\left[v(1-\gamma) - (1-2\gamma)p\right]$,

$\Rightarrow q^* = \frac{1}{1-\gamma^2}\left[v(1-\gamma) - (1-2\gamma) \cdot \frac{1}{2}\left(v + \frac{c}{1-\gamma}\right)\right] = \frac{v(1-\gamma) - c(1-2\gamma)}{2(1+\gamma)(1-\gamma)^2}$,

which increases in v and decreases in c

$$\Rightarrow \pi^* = \frac{-v^2\gamma^3 + 2(v^2 - 2vc + 2c^2)\gamma^2 - 2(v^2 - 2vc + 2c^2)\gamma + (v+c)^2}{(1+\gamma)(2-\gamma)^2}.$$

Here are some relevant comparative statics.

$\frac{d\pi^*}{dv} > 0$, which is intuitive.

$\frac{d\pi^*}{dc} < 0$ if $v < 2c$

or if $v > 3c$ and $\gamma < \frac{v - 2c + \sqrt{3v(v-2c)}}{2(2c-v)}$.

$\frac{d\pi^*}{d\gamma} > 0$ unless γ is very small.

It can be shown that $\pi^*_{(3)} \ge \pi^*_{(2)}$ unless $v > 36c$ (unlikely).

4. Non-Linear Costs Duopoly

Here $C(q_i)$ are the possibly, non-linear costs of duopolist i producing q_i units, with $C' > 0$ and $C'' \ge 0$, $C' < p\ (< v)$

thus $\pi_i = p_i q_i - C(q_i)$

$= p_i \cdot \frac{1}{1-\gamma^2}\left[v(1-\gamma) - p_i + \gamma p_j\right] - C\left\{\frac{1}{1-\gamma^2}\left[v(1-\gamma) - p_i + \gamma p_j\right]\right\}.$

$\frac{d\pi_i}{dp_i} = \frac{1}{1-\gamma^2}\left[v(1-\gamma) - 2p_i + \gamma p_j\right] - C'\left\{\frac{1}{1-\gamma^2}\left[v(1-\gamma) - p_i + \gamma p_j\right]\right\}\left(-\frac{1}{1-\gamma^2}\right);$

$$\frac{d^2\pi_i}{dp_i^2} = \frac{-2}{1-\gamma^2} - C''\{\qquad\qquad\}\left(-\frac{1}{1-\gamma^2}\right)\left(-\frac{1}{1-\gamma^2}\right)$$

$= -\frac{2}{1-\gamma^2} - \left(\frac{1}{1-\gamma^2}\right)^2 C''\{\ \} < 0.$

To obtain an explicit solution we need a specific (family of) function(s) C.

Example

$C(q) = c_1 q + c_2 q^2, c_2 \geq 0,\ i = 1,2$ (e.g. [11]).

$C'(q) = c_1 + 2c_2 q.$ Need $q < \frac{v-c_1}{2c_2}$.

$$\frac{d\pi_i}{dp_i} = \frac{1}{1-\gamma^2}\left[v(1-\gamma) - 2p_i + \gamma p_j\right] - \left\{c_1 + 2c_2 \cdot \frac{1}{1-\gamma^2}\left[v(1+\gamma) - p_i - \gamma p_j\right]\right\}\left(-\frac{1}{1-\gamma^2}\right) = 0\,,$$

so

$$v(1-\gamma) - 2p_i + \gamma p_j + c_1 + \frac{2c_2}{(1-\gamma^2)}\left[v(1-\gamma) - p_i - \gamma p_j\right] = 0\,.$$

Thus

$$v(1-\gamma)(1-\gamma^2) - 2p_i(1-\gamma^2) + \gamma(1-\gamma^2)p_j + c_1(1-\gamma^2) + 2c_2 v(1-\gamma) - 2c_2 p_i - 2c_2\gamma p_j = 0,$$

so

$$p_i[-2(1-\gamma^2) - 2c_2] = p_j[-\gamma(1-\gamma^2) + 2c_2\gamma] - v(1-\gamma)(1-\gamma^2) - c_1(1-\gamma^2) - 2c_2\gamma(1-\gamma),$$

i.e.,

$$[2(1-\gamma^2) + 2c_2]p_i = [\gamma(1-\gamma^2) - 2c_2\gamma]p_j + v(1-\gamma)(1-\gamma^2) + c_1(1-\gamma^2) + 2c_2\gamma(1-\gamma).$$

For the symmetric solution $p_i = p_j \equiv p$

$$p[2(1-\gamma^2) + 2c_2 - \gamma(1-\gamma^2) + 2c_2\gamma] = v(1-\gamma)(1-\gamma^2) + c_1(1-\gamma^2) + 2c_2 v(1-\gamma),$$

so we have:

Proposition 1

For quadratic costs

$p^* = \frac{(1-\gamma)[v(1-\gamma^2) + c_1(1+\gamma) + 2c_2 v]}{(1+\gamma)[(2-\gamma)(1-\gamma) + 2c_2]}$, which increases in v and c_1. $\frac{dp^*}{dc_2} < 0$ if $c_1 > 2\gamma^2 - 3\gamma + 1$ or $\gamma < \frac{c_1}{2v^2 - 3\gamma + 1 - c_1}$.

Under this condition $p^*_{quadratic} < p^*_{linear}$

$\Rightarrow q^* = \frac{(v-c_1)(1-\gamma^2) + 4c_2 v\gamma}{(1+\gamma)^2[(2-\gamma)(1-\gamma) + 2c_2]}$, which increases in v and decreases in c_1. It decreases in c_2 if $v > \frac{c_1}{1 + 5\gamma^2 - 2\gamma^3 - 4\gamma}$.

Under this condition $q^*_{quadratic} < q^*_{linear}$

$$\Rightarrow \pi^* = \frac{(1-\gamma)[v(1-\gamma^2) + c_1(1+\gamma) + 2c_2 v]}{(1+\gamma)[(2-\gamma)(1-\gamma) + 2c_2]} \cdot \frac{(v-c_1)(1-\gamma^2) + 4c_2 v\gamma}{(1+\gamma)^2[(2-\gamma)(1-\gamma) + 2c_2]}$$

$$-c_1 \cdot \frac{(v-c_1)(1-\gamma^2) + 4c_2 v\gamma}{(1+\gamma)^2[(2-\gamma)(1-\gamma) + 2c_2]} - c_2\left[\frac{(v-c_1)(1-\gamma^2) + 4c_2 v\gamma}{(1+\gamma)^2[(2-\gamma)(1-\gamma) + 2c_2]}\right]^2$$

i.e.,

$\gamma^6(2c_1 v - 2c_1^2 - v^2) + \gamma^5(-2c_1 v - 2c_1 - 2c_1^2 + 4c_2 v^2 - 6c_1 c_2 v)$

$\gamma^4(-6c_1 v + 2c_1^2 + 2c_1 c_2 - c_1^2 c_2 + 3v^2 + 3c_2 v^2)$

$\gamma^3(-3c_1 v - 8c_2 v^2 + c_1^2 - 8c_1 c_2^2 v + 2c_2^2)$

$\gamma^2(-3v^2 + c_2^2 + 6c_1 c_2 v + 6c_1 v - 4c_1^2 - 6c_1 + 4c_1^2 c_2 - 16c_2^3 v^2 - 2c_2 v^2)$

$\gamma(4c_2 v^2 + 4c_1^2 c_2 + 8c_1 c_2^2 v)$

$$\pi^* = \frac{+v^2 + c_2 v^2 - 4c_1 c_2 v - 2c_1 v + c_1^2 c_2^2}{(1+\gamma)^2[(2-\gamma)(1-\gamma) + 2c_2]}\,.$$

It is possible to demonstrate that if $c_1 = c_{linear}$ then $\pi^*_{quadratic} < \pi^*_{linear}$.

5. With Reference Price(s)

See Güler et al. [8] and Popescu and Wu [7] for suggestions for incorporating reference prices in supply chain models, motivated by Marketing and Psychological studies (e.g. [12, 13] that found it to be prevalent in (consumer) behavior.

Let $p_i^{(1)}, p_j^{(1)}, q_i^{(1)}, q_j^{(1)}$ be the prices and quantities in period 1, and similarly for period 2.

2nd period:

The demand model for the second period(*) is

$$q_i^{(2)} = \frac{1}{1-\gamma^2}\left[v(1-\gamma) - p_i^{(2)} - \alpha_i\left(p_i^{(2)} - p_i^{(1)}\right) + \gamma p_j^{(2)} + \beta_i\left(p_j^{(2)} - p_j^{(1)}\right)\right],\ \ i = 1{,}2,$$

where α_i is the effect of own price change, and β_i, the effect of competitor's price change. We shall assume that $\beta_i < \alpha_i$, $i = 1{,}2$.

Note that if $p_i^{(2)} \geq p_i^{(1)}$ and $p_j^{(2)} \geq p_j^{(1)}$ the contribution of the first reference-price difference is negative, and of the second positive. If $p_i^{(2)} \leq p_i^{(1)}$ and $p_j^{(2)} \leq p_j^{(1)}$, the signs of these contributions reverse, but the expressions remain the same.

Also, $\pi_i^{(2)} = \left(p_i^{(2)} - c_i\right)q_i^{(2)}$ [noise has no effect]. Thus

$$\frac{d\pi_i^{(2)}}{dp_i^{(2)}} = q_i^{(2)} + \left(p_i^{(2)} - c_i\right)\frac{dq_i^{(2)}}{dp_i^{(2)}} = \frac{1}{1-\gamma^2}\left\{v(1-\gamma) - 2p_i^{(2)} - 2\alpha_i p_i^{(2)} + \alpha_i p_i^{(1)} + (\gamma + \beta_i)p_j^{(2)} - \beta_i p_j^{(1)}\right\}$$

Proposition 2

In the symmetric case

$p^{(2)} = \frac{v(1-\gamma)+c(1+\alpha)+(\alpha-\beta)p^{(1)}}{2(1+\alpha)-\gamma-\beta}$. Note that the denominator is positive since $\alpha \geq \beta$ and $\gamma \leq 1$.

$$\Rightarrow q^{(2)} = \frac{(1+\alpha)[v(1-\gamma) - c(1+\alpha+\gamma+\beta)] + p^{(1)}(\alpha^2 + \alpha - \alpha\beta - \alpha\gamma - 3\beta)}{(1-\gamma^2)[2(1+\alpha) - \gamma - \beta]}$$

$$\Rightarrow \pi^{(2)} = \left[v(1+\alpha)(1-\gamma) - c(1+\alpha)(1+\alpha+\gamma+\beta) + (3\alpha^2 + 3\alpha - 3\alpha\beta - 2\alpha\gamma - \beta)p^{(1)}\right] \times$$
$$\frac{\left[v(1-\gamma) + c(1+\alpha) - (\alpha+\beta)p^{(1)} - 2c(1+\alpha) + c\gamma + \beta c\right]}{(1-\gamma^2)[2(1+\alpha) - \gamma - \beta]^2}$$

As for comparative statics, $p^{(2)}$ is linear in $p^{(1)}$ (decreasing if $\alpha > \beta$). It is increasing in v and c.

$$\frac{dp^{(2)}}{d\gamma} < 0 \text{ iff } v > \frac{c(1+\alpha) + (\alpha-\beta)p^{(1)}}{2\alpha - \beta}\ .$$

$$\frac{dp^{(2)}}{d\alpha} < 0 \text{ iff } p^{(1)} < \frac{c(\beta+\gamma) + 2v(1-\gamma)}{2+\beta-\gamma}$$

$$\frac{dp^{(2)}}{d\beta} < 0 \text{ iff } p^{(1)} > \frac{c(1+\alpha) + v(1-\gamma)}{2+\alpha-\gamma}$$

Or, in different form, if

$$\gamma < \frac{-v - c(1-\alpha) + (2+\alpha)p^{(1)}}{p^{(1)} - v}\ .$$

$\pi^{(2)}$ increases in v and decreases in c_1.

Now, moving to the *first period*,

$$\pi^{(1)}\left(p^{(1)}\right) = \left(p^{(1)} - c\right)q^{(1)}$$
$$= \left(p^{(1)} - c\right)\frac{1}{1-\gamma^2}\left[v(1-\gamma) - (1-\gamma)p^{(1)}\right]$$
$$= \frac{1}{1+\gamma}\left[\left(-p^{(1)}\right)^2 + (v+c)p^{(1)} - vc\right]\ ,$$

(*) For the first period, we still have $q_i\left(p_i^{(1)}, p_j^{(1)}\right)$ as in the basic model.

and

$$\pi_{total}\left(p^{(1)}\right) = \pi^{(1)}\left(p^{(1)}\right) + \pi^{(2)}(p_1) =$$

$$\frac{\left\{\begin{array}{c}\frac{1}{1+\gamma}\left[\left(-p^{(1)}\right)^2 + (v+c)p^{(1)} - vc\right] + \\ \left[v(1+\alpha)(1-\gamma) - c(1+\alpha)(1+\alpha+\gamma+\beta) + (3\alpha^2+3\alpha-3\alpha\beta-2\alpha\gamma-\beta)p^{(1)}\right] \times \\ \left[v(1-\gamma) - (\alpha-\beta)p^{(1)} - c(1+\alpha) + c\gamma + bc\right]\end{array}\right\}}{(1-\gamma^2)[2(1+\alpha)-\gamma-\beta]^2}$$

As the denominator is independent of $p^{(1)}$, and positive, we shall focus on the numerator.

It can be shown that

$$\text{numerator} = \left(3\alpha\beta^2 + 2\alpha^2\gamma + 2\alpha\beta\gamma + \beta^2 - 3\alpha^3 - 3\alpha^2 - 2\alpha\beta + \tfrac{1}{1+\gamma}\right)\left(p^{(1)}\right)^2$$
$$(-\beta v + \beta v\gamma + 2\beta c - 6v\alpha\gamma - 7v\alpha\beta - 4v\alpha^2 - 5v\alpha^2\gamma + 5v\alpha + 7v\alpha\beta\gamma - 4c\alpha^2 + 4c\alpha^3 - 3c\alpha$$
$$+10c\alpha\beta + 8c\alpha^2\beta + 6c\alpha^2\gamma + 6c\alpha\gamma - 2c\gamma^2 - 2c\alpha\beta^2 - 4c\alpha\beta\gamma + \frac{v+c}{1+\gamma})p^{(1)}$$
$$+constant\ (independent\ of\ p^{(1)})$$

Thus

$$\frac{d\pi_{total}}{dp^{(1)}} \approx \frac{v+c}{1+\gamma} - \beta v + \beta v\gamma + 2\beta c - 6v\alpha\gamma - 6v\alpha\beta$$
$$-4v\alpha^2 - 5v\alpha^2\gamma + 5v\alpha + 7v\alpha\beta\gamma - 4c\alpha^2$$
$$+4c\alpha^3 - 3c\alpha + 10c\alpha\beta + 8c\alpha^2\beta + 6c\alpha^2\gamma$$
$$+6c\gamma - 2c\gamma^2 - 2c\alpha\beta^2 - 4c\alpha\beta\gamma$$
$$+2\left(\frac{1}{1+\gamma} - 3\alpha^3 - 3\alpha^2 - 2\alpha\beta + 3\alpha\beta^2 + 2\alpha^2\gamma + 2\alpha\beta\gamma + \beta^2\right)p^{(1)} = 0\,,$$

where "≈" means "has the same sign as".

So

$$p^{(1)*} = \frac{\begin{array}{c}v[1-(1+\gamma)(-\beta+\beta\gamma-6\alpha\gamma+6\alpha\beta-4\alpha^2-5\alpha^2\gamma+5\alpha+7\alpha\beta\gamma)] \\ +c[1-(1+\gamma)(2\beta-4\alpha^2+4\alpha^3-3\alpha+10\alpha\beta+8\alpha^2\beta+6\alpha^2\gamma+6\alpha\gamma-2\gamma^2-2\alpha^2)]\end{array}}{2(1+\gamma)(-1+3\alpha^3+3\alpha^2+2\alpha\beta-3\alpha\beta^2-2\alpha^2\gamma-2\alpha\beta\gamma-\beta^2)}$$

Since $p^{(1)^*}$ can be written as $\frac{vA+cB}{D}$, and, for a setting without reference prices,

$$p^* = \frac{v(1-\gamma)+C}{2-\gamma},$$

sufficient conditions for $p^{(1)^*} < p^*$ are $A < 1-\gamma, B < 1$, $D > 2-\gamma$. These conditions hold with some restrictions on the parameters.

6. Concluding Remarks

We generalized the basic Bertrand model of differentiated duopoly in three ways: 1. Extension from two firms to three (tri-opoly). 2. Extension from linear to non-linear (convex) production costs (with two and three firms). 3. Extension to a two-period model where demand in the second depends also on the price in the first (reference price). Note that the term "reference effects" is used in some literature when the price of a product influences one's attitude to the price of another (e.g., [14]) which is a totally different meaning than our temporal "reference effect".

The tri-opolists are shown to charge, under certain conditions, higher price than the duopolists (who charges a higher price than a monopolist). For non-linear costs, we used a quadratic function. We provide complete comparative statics.

Our reference price model assumes that demand in the second period depends on one's own price difference and competitor's price difference between periods one and two. We find the optimal price and profit in the second period (as a function of the price in the first), then add to it the profit in the first period (which is also a function of the price in that period) and then optimize the sum over that price.

One possible extensions is to embed non-linear production costs in the reference price model.

One other possible generalization of the Bertrand model and its extensions is to assume a non-linear demand model, like

$$q_i(p_i, p_j) = \alpha_i - p_i^{\gamma} + b_i p_i^{\delta}, \alpha_i > 0,$$
$$0 < b_i < 1, \qquad i = 1,2, \qquad \gamma, \delta \geq 0$$

(e.g. [15] and references therein).

Another direction is the extension of the reference price model to more than two periods (for a continuous time model where demand always also depends on the price at time zero, see Fibich et al. [16]).

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