Journal of Game Theory 2016, 5(2): 42-44
DOI: 10.5923/j.jgt.20160502.02

Riesz Decomposition of Fuzzy Coalition Functions

Murat Beşer

Department of Economics, Agri Ibrahim Cecen University, Agri, Turkey

Abstract In this work, it is shown that when fuzzy measure set which has cone structure is closed, generating and norm normal, it obtains Riesz decomposition. Thus, it is indicated that each fuzzy measure can be stated as the linear combination of non-monotonic fuzzy measures.

Keywords Non-monotonic fuzzy measure, Banach space, Riesz decomposition

1. Introduction

Murofushi [1] had introduced the fuzzy measurement concept that is not monotone by removing monotonousness characteristics defined on fuzzy measurements and examined the features that belong to a special subset that has bounded variation feature of these measurements in his study. In Jang and Kwon's [2] study, they had extended Murofushi's assumptions with the help of the Φ-analysis based on bounded variation that Schramn [3] had presented. Narukawa [4] had examined the features of convergence in Banach space that non-monotonic bounded variation structured measurements had generated. In this study, features of the cone structure created by the fuzzy measurements that had bounded variation feature had been examined and it had been indicated that it had the Riesz decomposition characteristic. Thus, it had been showed that every fuzzy measurement could be expressed as the combination of the non-monotonic characteristics.

2. Non-Monotonic Fuzzy Measure Spaces

Let X be defined as non-empty set and $\mathfrak{P}(X)$ be defined as the measurable space for σ −algebra $(X, \mathfrak{P}(X))$ that is described on this set.

Definition 2.1: If $\mu: \mathfrak{P}(X) \longrightarrow \mathbb{R}_+$ set function which is defined on measurable space $(X, \mathfrak{P}(X))$ provides the following characteristics, it is described as the fuzzy measurement.

- $\mu(\emptyset) = 0$
- $A, B \in \mathfrak{P}(X)$ and $A \subseteq B$ for $\mu(A) \leq \mu(B)$

Definition 2.2: If $\mu: \mathfrak{P}(X) \longrightarrow \mathbb{R}$ set function which is defined on measurable space $(X, \mathfrak{P}(X))$ provides the $\mu(\emptyset) = 0$ characteristic, it is described as the non-monotonic fuzzy measurement.

It is clear that $\mathfrak{P}(X)$ has generated chain of $\emptyset = A_0 \subset A_1 \subset \cdots A_n = A$ under the coverage relation of the subsets belonging to it. Every sequential element of the chain $\{A_j, A_{j+1}\}$ has composed a relation and hereby chain structure has appeared as the combination of the sequential elements.

Definition 2.3: Total variation value $\{A_j, A_{j+1}\}$ that belongs to non-monotonic fuzzy measurement function μ defined on the measurable space $(X, \mathfrak{P}(X))$ is described for a linkage as

$$|\mu|(A) = \sup\{\textstyle\sum_j^n |\mu(A_j) - \mu(A_{j-1})|\} \qquad (1)$$

The decomposition of this expression is as follows:

$$\sup\left\{\left\{\textstyle\sum_{j=1}^n max\{\mu(A_j) - \mu(A_{j-1}), 0\}\right\}\right\} + \sup\left\{\left\{\textstyle\sum_{j=1}^n max\{\mu(A_{j-1}) - \mu(A_j), 0\}\right\}\right\} \qquad (2)$$

$|\mu|(X)$ expression for $A = X$ has been showed with $\|\mu\|$ and it is defined as the bounded variation for $\|\mu\| < \infty$. Let fuzzy measurements set be given as $\mathcal{FM}^+ = \{\eta | \eta: \mathfrak{P}(X) \longrightarrow \mathbb{R}_+\}$. The set of $\mathcal{FM}^+ = \{\nu - \gamma | \nu, \gamma \in \mathcal{FM}^+\}$ has a closed structure for every $\nu, \gamma \in \mathcal{FM}^+$ and it has generated linear space. Although the subset $\mathcal{FM}^+ \subset \mathcal{FM}$ generates the positive cone[1] structure in related linear space. If non-monotonic fuzzy measurement function μ has bounded variation, $\mathcal{FM}$ is the element of the linear space. The opposite of this suggestion is also true. (Aumann and Shapley, 1974:27)

We can describe the norm of $\|\mu\|_* = \inf\left\{k > 0: \left\|\frac{\mu}{k}\right\| \leq 1\right\}$ defined on the linear space of $\mathcal{FM}$[2]. It has been known that the inequality of $\|\mu\| \leq \|\mu\|_*$ is valid for $\|\mu\|_* \leq 1$ [5].

Theorem 2.1: $\mathcal{FM}(X, \mathfrak{P}(X))$ has generate the norm structure $\|*\|_*$ that is defined on the linear space and $\mathcal{FM}(X, \mathfrak{P}(X))_{\|\ \|_{-*_*}}$ has the Banach space structure.

Proof: (Jang and Kwon, 1997:104,105) and (Schramn, 1985) had described the related proof for $\Phi = \{\phi_n\}$

* Corresponding author:
muratbeser@yahoo.com (Murat Beşer)
Published online at http://journal.sapub.org/jgt

increasing convex functions series as the Φ –bounded variation[3] structure. Privately, if we describe the $\Phi = \{\phi_n\}$ series for every $n \in \mathbb{N}$ as $\phi_n(x) = x$, bounded variation structure is obtained.

It is clear that the cone of $\mathcal{FM}^+$ has the closed cone characteristic according to $\|*\|_*$ norm in the Banach space $\mathcal{FM}(X, \mathfrak{P}(X))_{\| \|_*}$.

Definition 2.4: Let $(, \| \|)$ be Banach space and $\mathcal{K}$ be the closed cone defined in this space. Closed cone $\mathcal{K}$ has the powerful Levi characteristics if every $0 \leq x_n \uparrow$ increasing sequence that is defined in (□, ‖ ‖) provides norm convex feature for $\sup\|x_n\| < \infty$ [6].

Theorem 2.2: Let $\mathcal{FM}(X, \mathfrak{P}(X))_{\| \|_*}$ be given for Banach space and $\mathcal{FM}^+$ be given for closed cone belonging to this space. If cone $\mathcal{FM}^+$ has powerful Levi characteristic, it has also the norm normality feature at the same time.

Definition 2.5: Let V be linear space and $\mathcal{K}$ be the cone in this space. If the equality of $V = \mathcal{K} - \mathcal{K}$ is valid, $\mathcal{K}$ is defined as the generating cone. From the definition, it is clear that $\mathcal{FM}^+$ cone is the generating cone of $\mathcal{FM} = \{\nu - \gamma | \nu, \gamma \in \mathcal{FM}^+\}$ linear space.

Let V be given as the $\geq$ order relation (reflexive, transitive, anti-symmetric) on the linear space. When $x, y \in V$ and $x \geq y$ were given, it is defined as the $(V, \geq)$ ordered vector space if the following conditions are satisfied.

a) $x + z \geq y + z$ for every $z \in V$

b) $\alpha x \geq \alpha y$ for every $\alpha \geq 0$

It is clear that $\mathcal{FM}$ linear space has the ordered vector space characteristic.

Definition 2.6: Let $\nu, y, z \in V_+$[4] be given in the (V, ≥) ordered vector space. If this space has Riesz decomposition characteristic, there are such $x_1, x_2 \in V$ elements for the inequality $\nu \leq y + z$, $\nu = x_1 + x_2$ can be written under the conditions of $0 \leq x_1 \leq y \; and \; 0 \leq x_2 \leq z$.

When data features had been provided for the given two theorems Banach space and $\mathcal{K}$ cone embedded into it in the rest of study, related space has showed that it satisfies the Riesz decomposition characteristic.

Theorem 2.3 [7] Let be Banach space and $\mathcal{K}$ cone that is defined in this space be closed, generating and normal structured. Banach space should provide the following feature to have the Riesz decomposition characteristic.

- Let at most three vectors be for providing the $A, B \in \mathfrak{P}(X)$ and $0 <_{\mathcal{K}} B \leq_{\mathcal{K}} A$ sequence. In this case, the elements $x_\varepsilon \in A, y_\varepsilon \in B$ for every $\varepsilon > 0$ can be found as satisfying the conditions $x_\varepsilon - y_\varepsilon \leq_{\mathcal{K}} A \; and \; B \leq_{\mathcal{K}} x_\varepsilon$ under the assumption of $\|y_\varepsilon\| < \varepsilon$.

Theorem 2.4 [8] If is ordered Banach space and $\mathcal{K}$ cone which is defined in this space is closed, generating, norm normal structured cone, it has the following characteristics.

- Banach space has the Riesz decomposition characteristic.
- The dual of Banach space is Riesz space.
- The dual of Banach space has the Riesz decomposition characteristic.

It had been showed that $\mathcal{FM}(X, \mathfrak{P}(X))_{\| \|_*}$ ordered Banach space and $\mathcal{FM}^+$ cone that belongs to this space have closed, generating and norm normal characteristics. With the help of the Theorem 2.3, it is clear that $\mathcal{FM}(X, \mathfrak{P}(X))_{\| \|_*}$ Banach space has Riesz decomposition characteristic so $\mathcal{FM}^+$ cone has it. This case has indicated that every $\nu \in \mathcal{FM}^+$ fuzzy measurement can be expressed with the help of non-monotonic fuzzy measurements.

3. Conclusions

It had been revealed that the cone structure formed by fuzzy measurements which were defined in the ordered Banach space that non-monotonic fuzzy measurements created has the Riesz decomposition characteristic under the closed, generating and norm normal structured feature thus fuzzy measurements can be expressed as the linear combination of the non-monotonic fuzzy measurement.

REFERENCES

[1] C. D. Aliprantis and R. Tourky, "Cones and Duality," American Mathematical Society, 2007.

[2] T. Ando, "On Fundamental Properties of Banach Space with a Cone," Pasific Journal of Mathematics, Vol.12, pp.1163-1169, 1962.

[3] R. J. Aumann & L. S. Shapley, "Values of Non-Atomic Games", Princeton University Press, New Jersey, (1974).

[4] L.C. Jang and J.S. Kwon, "On Non-Monotonic Fuzzy Measures of Φ -Bounded Variation," Fuzzy Sets and Systems, vol. 86, pp. 101-106, 1997.

[5] T. Murofushi, M. Sugeno, & M. Machida," Non-Monotonic Fuzzy Measures and Choquet Integral," Fuzzy Sets and Systems, vol. 64, pp. 73-86, 1994.

[6] Y. Narukawa, T. Murofushi, & M. Sugeno, "Space of Fuzzy Measures and Convergence," Fuzzy Sets and Systems, vol.138, pp. 497-506, 2003.

[7] M. Schramn, "Functions of Φ -Bounded Variations and Riemann-Stieltjes Integration," Transaction of American Mathematical Society, vol. 287, pp. 49-63, 1985.

---

1 The set $\mathcal{K}$ defined on the $\mathcal{V}$ linear space is described as the cone if it provides the following conditions.

- $\mathcal{K} + \mathcal{K} \subseteq \mathcal{K}$
- $\alpha\mathcal{K} \subseteq \mathcal{K}$ for every $\alpha \in \mathbb{R}_+$
- $\mathcal{K} \cap (-\mathcal{K}) = \{0\}$

2 $\|\mu\|_* = \inf\left\{k > 0: \left\|\frac{\mu}{k}\right\| \leq 1\right\}$ norm that is defined on the Banach space $\mathcal{FM}(X, \mathfrak{P}(X))$ is equivalent to the $\sup\left\{\left\{\sum_{j=1}^{n} max\{\mu(A_j) - \mu(A_{j-1}), 0\}\right\}\right\} + \sup\left\{\left\{\sum_{j=1}^{n} max\{\mu(A_{j-1}) - \mu(A_j), 0\}\right\}\right\}$ norm.

3 $\Phi(|\mu|(A)) = \sup\left\{\sum_j^n \phi_j \left|\mu(A_j) - \mu(A_{j-1})\right| \,\middle|\, \emptyset = A_0 \subset A_1 \subset \cdots A_n = A, \{A_j\}_{j=0}^{n} \subset \mathfrak{P}(X)\right\}$

4 $\nu \geq 0$ inequality is valid if $\nu \in V_+$