Journal of Civil Engineering Research 2017, 7(2): 72-80
DOI: 10.5923/j.jce.20170702.07

Degree of Consolidation of Soils by Vertical Drains under Arbitrary Surcharge and Vacuum Preloading

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Abstract Based on the condition of equal vertical strain, a general solution for the degree of consolidation of soils by vertical drains under both arbitrary surcharge and vacuum preloading, including instantaneous preloading, is proposed, which can take into account the well resistance and the smear effect, radial and vertical direction permeability. Furthermore, an analytical solution for the degree of consolidation under linear time-dependent loading is also presented. The feasibility of the solution is proved by 3-D finite element method. The solution shows that Carrillo's method is not directly used to calculate the degree of consolidation for soils by vertical drains under the condition of equal vertical strain.

Keywords Soils by vertical drains, Arbitrary surcharge and vacuum preloading, Degree of consolidation, Analytical solution

1. Introduction

The vertical drains, in combination with surcharge or/and vacuum preloading, is a good method to improve saturated soft soils. It is necessary to evaluate the consolidation degree of soil for selecting the proper loading rate and estimating its strength change as well as avoiding its shear failure during loading. There are two types of consolidation theory for soils by vertical drains. One is called the equal vertical strain [1, 2], another is called the free strain [3, 4]. The former is perfect in theory but very complicated because of its implicit form. The latter is not perfect in theory but has a simple form. The research results prove that there is only the slight difference for consolidation degree between the equal vertical strain and the free strain [4]. Therefore, the latter is widely applied to analyze the consolidation of soils by vertical drains. The different solutions for consolidation degree of the soils are given according to the different conditions of preloading (surcharge or/and vacuum preloading, instantaneous or/and time–dependent loading), smear effect (whether considering smear effect or not) and permeability (whether considering vertical flow of soils or not) [4-7]. So far, there is not a general solution including the above-mentioned different conditions yet.

Based on the condition of the equal vertical strain, a solution for the degree of consolidation of soils by vertical drains under arbitrary surcharge and vacuum preloading (including instantaneous preloading) is proposed taking into account the well resistance and the smear effect, radial and vertical direction permeability, and an analytical solution for consolidation degree under linear time-dependent loading is also presented. The feasibility of the solutions are proved by 3-D finite element method.

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Published online at http://journal.sapub.org/jce

2. Analysis Model and Its Basic Assumptions

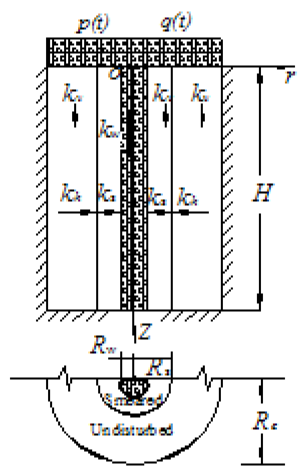


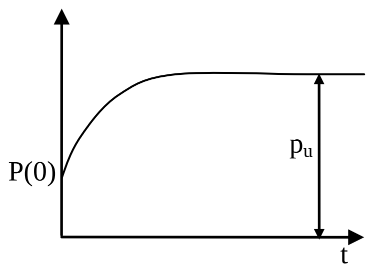
Figure 1. Schematic picture of soil cylinder with vertical drain

The analysis model, a soil cylinder having a vertical drain, is schematically pictured in Fig.1. In the figure, H is the length of drain, R_w, R_s and R_e are the radii of drain, smeared zone and influence zone of drain respectively, k_w is the coefficient of permeability of drain, k_s and k_h are the radial coefficients of permeability of smeared zone and undisturbed zone respectively, k_v is the vertical coefficient of permeability of smeared zone and undisturbed zone, r and z are the radial and vertical coordinates respectively.

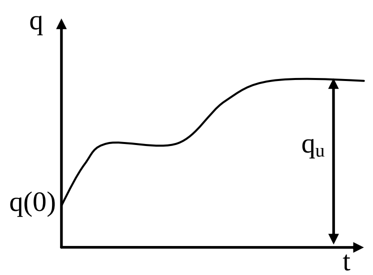
2.1. Basic Assumptions

The basic assumptions in this study are listed as follows:

(1) Barron's hypothesis of the equal vertical strain is valid. Lateral deformation of soil is ignored, and vertical strain is same at the same depth.
(2) Radial flow in drain is neglected, and the total inflow of the pore water through the boundary of the vertical drain is equal to the vertical flow of the pore water within the vertical drain.
(3) Darcy's law is valid.
(4) The bottom of soil is impervious.
(5) The vacuum and surcharge preloadings are schematically pictured in Fig.2. In the figure, p(t) and q(t) denote for vacuum and surcharge preloading respectively, p(0) and q(0) are vacuum and surcharge preloadings at time t=0 respectively, p_u and q_u are ultimate vacuum and surcharge preloadings respectively.



(a) Vacuum preloading



(b) Surcharge preloading

Figure 2. Schematic picture of vacuum and surcharge preloading

2.2. Basic Partial Differential Equations of Analysis Model and Their Solutions

Let ε_z, $\bar{\sigma}_z$, $\bar{\sigma}'_z$ and $\bar{u}$ denote for the average strain, average total stress, average effective stress and average excess pore water pressure at the depth z, respectively, and m_v is the coefficient of volume compressibility of soil. According to the above assumptions, the following equations can be got,

$$\varepsilon_z = m_v\bar{\sigma}'_z = m_v(\bar{\sigma}_z - \bar{u}) = m_v[q(t) - \bar{u}] \tag{1}$$

$$\bar{u} = \frac{1}{\pi(R_e^2 - R_w^2)}\int_{R_w}^{R_s} 2\pi u_s r\,dr + \int_{R_s}^{R_e} 2\pi u_e r\,dr \tag{2}$$

where u_s and u_e are the excess pore water pressure within the smeared zone and the undisturbed zone respectively. Taking Derivative of Eq.(1) with respect to t, the following equation is obtained,

$$\frac{\partial \varepsilon_z}{\partial t} = -m_v\frac{\partial \bar{u}}{\partial t} + m_v\frac{dq(t)}{dt} \tag{3}$$

In addition, u_s and $\bar{u}$ should satisfy the following equations:

$$-\frac{k_s}{\gamma_w}\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_s}{\partial r}\right) - \frac{k_v}{\gamma_w}\frac{\partial^2 \bar{u}}{\partial z^2} = \frac{\partial \varepsilon_z}{\partial t} \quad (R_w \le r \le R_s) \tag{4}$$

$$-\frac{k_h}{\gamma_w}\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_e}{\partial r}\right) - \frac{k_v}{\gamma_w}\frac{\partial^2 \bar{u}}{\partial z^2} = \frac{\partial \varepsilon_z}{\partial t} \quad (R_s \le r \le R_e) \tag{5}$$

$$\frac{\partial^2 u_w}{\partial z^2} = -\frac{2k_s}{R_w k_w}\left.\frac{\partial u_s}{\partial r}\right|_{r=R_w} \tag{6}$$

where u_w is the pore pressure in the drain, γ_w is the unit weight of water. The solution conditions for the above Eqs. (1)～(6) are expressed as

Boundary conditions:

(1) $\left.\frac{\partial u_w}{\partial z}\right|_{z=H} = 0$ (The bottom of drain is impervious)

(2) $\left.\frac{\partial u_e}{\partial z}\right|_{r=R_e} = 0$ (The cylindrical surface of soil is impermeable)

(3) $u_w|_{r=R_w} = u_s|_{r=R_w}$ (Continuity of pore water pressure at $r = R_w$)

(4) $u_s|_{r=R_s} = u_e|_{r=R_s}$ (Continuity of pore water pressure at $r = R_s$)

(5) $u_w|_{z=0} = p(t)$ (Pore water pressure at the top of drain is equal to the vacuum preloading)

Initial conditions:

(1) $t = 0$, $u_w|_{z=0} = p(0)$

(2) $t = 0$, $\bar{u}|_{0<z\le H} = q(0)$

Integrating of Eqs.(4) and (5) with the boundary conditions (2), (3) and (4), u_s and u_e can be obtained as follows.

$$u_s = \frac{\gamma_w}{2k_s}\left(R_e^2 \ln\frac{r}{R_w} - \frac{r^2 - R_w^2}{2}\right)\left(\frac{\partial \varepsilon_z}{\partial t} + \frac{k_v}{\gamma_w}\frac{\partial \bar{u}}{\partial z^2}\right) + u_w$$
$$(R_w \le r \le R_s) \tag{7}$$

$$u_e=\left[\frac{\gamma_w}{2k_h}(R_e^2\ln\frac{r}{R_s}-\frac{r^2-R_s^2}{2})+\frac{\gamma_w}{2k_s}(R_e^2\ln\frac{R_s}{R_w}-\frac{R_s^2-R_w^2}{2})\right](\frac{\partial\varepsilon_z}{\partial t}+\frac{k_v}{\gamma_w}\frac{\partial\bar{u}}{\partial z^2})+u_w\ (R_s\le r\le R_e) \tag{8}$$

Substituting Eqs. (7) and (8) into Eq.(2), the average pore water pressure $\bar{u}$ is presented,

$$\bar{u}=\frac{\gamma_w R_e^2 F}{2k_h}(\frac{\partial\varepsilon_z}{\partial t}+\frac{k_v}{\gamma_w}\frac{\partial\bar{u}}{\partial z^2})+u_w \tag{9}$$

here, $F=\frac{n^2}{n^2-1}\left(\ln\frac{n}{s}+\mu_k\ln s-\frac{3}{4}\right)+\frac{s^2}{n^2-1}(1-\mu_k)(1-\frac{s^2}{4n^2})+\frac{\mu_k}{n^2-1}(1-\frac{1}{4n^2})$, $\mu_k=\frac{k_h}{k_s}$, $n=\frac{R_e}{R_w}$, $s=\frac{R_s}{R_w}$.
Substituting Eq. (7) into Eq.(6), the following expression can be obtained,

$$\frac{\partial^2 u_w}{\partial z^2}=-\frac{\gamma_w}{k_w}(n^2-1)(\frac{\partial\varepsilon_z}{\partial t}+\frac{k_v}{\gamma_w}\frac{\partial^2\bar{u}}{\partial z^2}) \tag{10}$$

Combining with Eq.(9) and Eq.(3), and eliminating $\frac{\partial\varepsilon_z}{\partial t}$, the following equation is got,

$$\frac{\partial\bar{u}}{\partial t}=-(\bar{u}-u_w)\lambda+C_v\frac{\partial^2\bar{u}}{\partial z^2}+\frac{dq(t)}{dt} \tag{11}$$

where $\lambda=\frac{2C_h}{R_e^2 F}$, $C_h=\frac{k_h}{\lambda_w m_v}$ and $C_v=\frac{k_v}{\lambda_w m_v}$ are the radial and coefficients of permeability of soil respectively. Combining with Eq.(9) and Eq.(10), and eliminating $\frac{\partial\varepsilon_z}{\partial t}$, the following equation is obtained,

$$\frac{\partial^2 u_w}{\partial z^2}=-(\bar{u}-u_w)\rho^2 \tag{12}$$

where $\rho^2=\frac{2(n^2-1)k_h}{R_e^2 k_w F}$. Combining with Eq.(11) and Eq.(12), and eliminating $(\bar{u}-u_w)$, the following equation is got,

$$\frac{\partial\bar{u}}{\partial t}=\frac{\lambda}{\rho^2}\frac{\partial^2 u_w}{\partial z^2}+C_v\frac{\partial^2\bar{u}}{\partial z^2}+\frac{dq(t)}{dt} \tag{13}$$

Combining with Eq.(12) and Eq.(13),and eliminating $\bar{u}$, the equation about u_w can be expressed as

$$C_v\frac{\partial^4 u_w}{\partial z^4}-\frac{\partial^3 u_w}{\partial z^2\partial t}-\left(\lambda+\rho^2 C_v\right)\frac{\partial^2 u_w}{\partial z^2}+\rho^2\frac{\partial u_w}{\partial t}-\rho^2\frac{dq(t)}{dt}=0 \tag{14}$$

In order to solve inhomogeneous Eq.(14), the following transformation is applied:

$$u_w=u_x+p(t) \tag{15}$$

where u_x is called the transformed variable, $p(t)$ is vacuum preloading. Substituting Eq.(15) into Eq.(14), it can be rewritten as

$$C_v\frac{\partial^4 u_x}{\partial z^4}-\frac{\partial^3 u_x}{\partial z^2\partial t}-(\lambda+\rho^2 C_v)\frac{\partial^2 u_x}{\partial z^2}+\rho^2\frac{\partial u_x}{\partial t}-\rho^2 f(t)=0 \tag{16}$$

in which

$$f(t)=\frac{d[q(t)-p(t)]}{dt} \tag{17}$$

At the same time, the boundary conditions (1) and (5) can be rewritten as

$$(1)' \quad \frac{\partial u_x}{\partial z}\bigg|_{z=H} = 0 \qquad (5)' \quad u_x\big|_{z=0} = 0$$

Using the separation of variables, and using the boundary conditions $(1)'$ and $(5)'$, the solution for Eq.(16) is

$$u_x = \sum_{m=1}^{\infty} \frac{2\beta_r}{\lambda M}\left(\frac{\rho H}{M}\right)^2 \left[F(m,t) + C_m\right] \exp(-\beta_{rz}\, t)\sin\left(\frac{M}{H} z\right) \tag{18}$$

where C_m is an undetermined coefficient, $\beta_z = C_v\left(\frac{M}{H}\right)^2$, $\beta_r = \frac{\lambda M^2}{\rho^2 H^2 + M^2}$, $d_i = \frac{p(t_i) - p(t_{i-1})}{t_i - t_{i-1}}$, $M = (m - \frac{1}{2})\pi$, $m = 1, 2, \cdots$, and

$$F(m,t) = \int_0^t f(\tau)\exp(\beta_{rz}\tau)\, d\tau \tag{19}$$

Here, $f(t)$ is expressed as Eq.(17). Substituting Eq. (18) into Eq.(15), the solution for u_w is

$$u_w = \sum_{m=1}^{\infty} \frac{2\beta_r}{\lambda M}\left(\frac{\rho H}{M}\right)^2 \left[F(m,t) + C_m\right] \exp(-\beta_{rz}\, t)\sin\left(\frac{M}{H} z\right) + p(t) \tag{20}$$

Substituting Eq. (20) into Eq.(12) and using the initial condition (1) and (2), the solution for $\bar{u}$ is

$$\bar{u} = \sum_{m=1}^{\infty} \frac{2}{M}\left\{ F(m,t) + [q(0) - p(0)]\right\} \exp(-\beta_{rz}\, t)\sin\left(\frac{M}{H} z\right) + p(t) \tag{21}$$

The vertical displacement at the top of soil, S_t, can be determined as follows.

$$S_t = \int_0^H \varepsilon_z\, dz \tag{22}$$

Using Eqs.(1), (21) and (22), S_t is expressed as

$$S_t = m_v H \left\{ \left[q(t) - p(t)\right] - \sum_{m=1}^{\infty} \frac{2}{M^2}\left[F(m,t) + [q(0) - p(0)]\right] \exp(-\beta_{rz} t) \right\} \tag{23}$$

When $t \to \infty$, the ultimate vertical displacement at the top of soil, S_∞, can be got from Eq.(23) as

$$S_\infty = m_v H \left(q_u - p_u\right) \tag{24}$$

where p_u and q_u are the ultimate vacuum and surcharge preloadings respectively (see Fig.2). The overall average degree of consolidation for the whole soil, U_t, is defined as follows.

$$U_t = \frac{S_t}{S_\infty} \tag{25}$$

Substituting Eqs. (23) and (24) into Eq.(25), U_t can be expressed as

$$U_t = \frac{q(t) - p(t)}{q_u - p_u} - \frac{1}{q_u - p_u}\sum_{m=1}^{\infty} \frac{2}{M^2}\left\{F(m,t) + [q(0) - p(0)]\right\} \exp(-\beta_{rz} t) \tag{26}$$

where $F(m,t)$ is expressed as Eq.(19).

3. The Degree of Consolidation for Soil under Linear Multi-Loading

In practice, preloading is exerted step-by-step, and the loading per step can be seen approximately linear (as shown in Fig.3). Let n denote for the total step numbers of loading, for the i-th step $(i = 1 \sim n,\ \ t_i \ge t \ge t_{i-1})$, the vacuum preloading p(t) and surcharge preloading q(t) can be expressed as

$$q(t) = a_i + b_i t \quad p(t) = c_i + d_i t \tag{27}$$

where $b_i = \dfrac{q(t_i) - q(t_{i-1})}{t_i - t_{i-1}}$, $d_i = \dfrac{p(t_i) - p(t_{i-1})}{t_i - t_{i-1}}$, $a_i = q(t_{i-1}) - b_i t_{i-1}$,

$c_i = p(t_{i-1}) - d_i t_{i-1}$. If $i = 1$, then $t_0 = 0$, $p(t_{i-1}) = p(0)$,

$q(t_{i-1}) = q(0)$. Note that when $t_i \ge t \ge t_{i-1}$, Eq.(19) can be expressed as

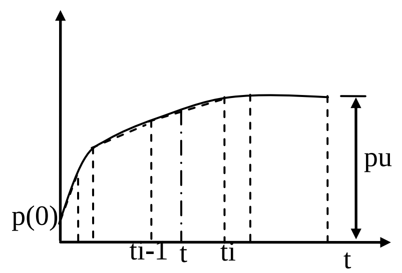
$$F(m,t) = \int_0^t f(\tau)\exp(\beta_{rz}\tau)d\tau = \sum_{k=1}^{i}\int_{t_{k-1}}^{t_k} f(\tau)\exp(\beta_{rz}\tau)d\tau + \int_{t_i}^{t} f(\tau)\exp(\beta_{rz}\tau)d\tau$$

Substituting Eq.(27) into the above equation, $F(m,t)$ can be rewritten as

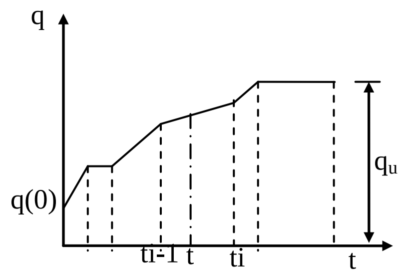
$$F(m,t) = \sum_{k=1}^{i}\frac{b_k - d_k}{\beta_{rz}}\left[\exp(\beta_{rz}t_k) - \exp(\beta_{rz}t_{k-1})\right] + \frac{b_i - d_i}{\beta_{rz}}\left[\exp(\beta_{rz}t) - \exp(\beta_{rz}t_i)\right]$$

Substituting the above equation into Eq.(26), the average degree of consolidation U_t for linear multi-loading can be expressed as

$$U_t = \frac{(a_i + b_i t) - (c_i + d_i t)}{q_u - p_u} - \frac{1}{q_u - p_u}\sum_{m=1}^{\infty}\frac{2}{M^2}\left\{\sum_{k=1}^{i}\frac{b_k - d_k}{\beta_{rz}}\left[\exp\left[\beta_{rz}(t_k - t)\right]\right.\right.$$
$$\left.-\exp\left[\beta_{rz}(t_{k-1} - t)\right]\right] + \frac{b_i - d_i}{\beta_{rz}}\left[1 - \exp\left[\beta_{rz}(t_i - t)\right]\right]$$
$$\left.+\left[q(0) - p(0)\right]\exp(-\beta_{rz}t)\right\} \quad (t_i \ge t \ge t_{i-1}) \tag{28}$$



(a) vacuum preloading



(b) surcharge preloading

Figure 3. Schematic picture for linearized vacuum and surcharge preloadings

4. Discussion

4.1. Relation between Preloading with Degree of Consolidation

From Eq.(28), it is easily seen that the degree of consolidation for soil is dependent on the totality of surcharge and vacuum loading rather than their respective proportion. Especially, if surcharge loading is equal to vacuum loading, that is q(t)=-p(t), they have the same degree consolidation. Obviously, It is right that to convert the vacuum loading into the equivalent amount of surcharge when calculating the degree of consolidation.

Under instantaneous loading, $p(t) = p(0) = a_i = p_u$, $q(t) = q(0) = c_i = q_u$, $b_i = d_i = F(m,t) = 0$, Eq.(28) becomes

$$U_t = 1 - \sum_{m=1}^{\infty} \frac{2}{M^2} \exp(-\beta_{rz} t) = 1 - \sum_{m=1}^{\infty} \frac{2}{M^2} \exp[-(\beta_r + \beta_z) t] \tag{29}$$

The above equation shows that the degree of consolidation under instantaneous loading is independent of the type of preloading.

Additionally, if the vertical flow is ignored, that is $C_v = 0$, $\beta_z = 0$, $\beta_{rz} = \beta_r$, Eq.(29) is identical to Dong Zhi-liang's formulation [5].

If the radial flow is ignored, that is $C_h = 0$, $\beta_r = 0$, $\beta_{rz} = \beta_z$, Eq.(29) degenerates into

$$U_t = 1 - \frac{8}{\pi^2} \sum_{k=1}^{\infty} \frac{1}{k^2} \exp\left(-\frac{k^2 \pi^2}{4} \frac{C_v t}{H^2}\right) \quad (k = 1, 3, 5, \cdots) \tag{30}$$

which is identical to Terzaghi's formulation for one-dimensional consolidation.

4.2. The Degree of Consolidation under Vacuum Preloading

If vacuum loading is only considered, that is $q(t) = 0$, $a_i = b_i = 0$, Eq.(28) degenerates into

$$U_t = \frac{(c_i + d_i t)}{p_u} - \frac{1}{p_u} \sum_{m=1}^{\infty} \frac{2}{M^2} \left\{ \sum_{k=1}^{i} \frac{d_k}{\beta_{rz}} \left[\exp[\beta_{rz}(t_k - t)] - \exp[\beta_{rz}(t_{k-1} - t)] \right] \right.$$
$$\left. + \frac{d_i}{\beta_{rz}} \left[1 - \exp[\beta_{rz}(t_i - t)] \right] + p(0) \exp(-\beta_{rz} t) \right\} (t_i \ge t \ge t_{i-1}) \tag{31}$$

Further, if vertical flow is neglected, Eq.(31) is identical to Yin Jing's formulation [6].

4.3. The Degree of Consolidation under Surcharge Preloading

If surcharge loading is only considered, that is $p(t) = 0$, $c_i = d_i = 0$, Eq.(28) degenerates into

$$U_t = \frac{(a_i + b_i t)}{q_u} - \frac{1}{q_u} \sum_{m=1}^{\infty} \frac{2}{M^2} \left\{ \sum_{k=1}^{i} \frac{b_k}{\beta_{rz}} \left[\exp[\beta_{rz}(t_k - t)] - \exp[\beta_{rz}(t_{k-1} - t)] \right] \right.$$
$$\left. + \frac{b_i}{\beta_{rz}} \left[1 - \exp[\beta_{rz}(t_i - t)] \right] + q(0) \exp(-\beta_{rz} t) \right\} (t_i \ge t \ge t_{i-1}) \tag{32}$$

which is identical to Xiao-Wu Tang's formulation [7].

Especially, if loading is instantaneous, that is $a_i = q(t) = q(0) = q_u$, $b_i = 0$, Eq.(32) is same as to Eq.(29). Besides, if vertical flow is ignored, that is $C_v = 0$, $\beta_z = 0$, $\beta_{rz} = \beta_r$, Eq.(32) becomes

$$U_t = 1 - \sum_{m=1}^{\infty} \frac{2}{M^2} \exp(-\beta_r t) \tag{33}$$

which is identical to Xie Kang-he's formula [4].

4.4. The Feasibility of Carrilo's Method Used to Calculate the Degree of Consolidation

When both radial and vertical flow are considered, the overall average degree of consolidation of soils, U_{rz}, can be determined according to Carrilo's method.

$$U_{rz} = 1 - (1 - U_r)(1 - U_z)$$

Where U_r and U_z are the overall degree of consolidation for radial and vertical flow respectively. It is usually considered that Carrilo's method is valid under time-independent loading (i.e., instantaneous loading). Under this condition, let $\beta_z=0$ and $\beta_r=0$ in Eq.(29) respectively, U_t in the equation denotes for the degree of consolidation without vertical and without vertical flow U_r and U_z respectively. Obviously

$$(1-U_r)(1-U_z)=\left[\sum_{m=1}^{\infty}\frac{2}{M^2}\exp(-\beta_r t)\right]\cdot\left[\sum_{m=1}^{\infty}\frac{2}{M^2}\exp(-\beta_z t)\right]\neq\sum_{m=1}^{\infty}\frac{2}{M^2}\exp(-\beta_{rz} t)=1-U_{rz}$$

It is concluded that Carrillo's method is not directly used to calculate the degree of consolidation for soils by vertical drains under the condition of equal vertical strain because the average pore pressure $\bar{u}=\bar{u}(z,t)$ is adopted in Eqs.(4) and (5) according to the assumption of equal vertical strain.

5. Validation of the Solution Used 3-D Finite Element Method

Here the 3-D FEM is applied to verify Eq.(26). The FEM equations for consolidation take the form of

$$\begin{bmatrix} K_e & K_c \\ K_c^T & -\theta\Delta t K_s \end{bmatrix}$$

$$\begin{Bmatrix}\Delta\delta \\ u_{t+\Delta t}\end{Bmatrix}^e=\begin{Bmatrix} R_F(t+\Delta t)-K_e\delta_t \\ \Delta t R_q(t+\Delta t)-\Delta t(\theta-1)K_s u_t\end{Bmatrix}^e \quad (34)$$

Here, $\{\Delta\delta\}^e=\{\delta_{t+\Delta t}\}^e-\{\delta_t\}^e$ is the increments of nodal displacements in a element, $\{\delta_t\}^e$ and $\{u_t\}^e$ are the nodal displacements and nodal pore pressures at t respectively, $\{\delta_{t+\Delta t}\}^e$ and $\{u_{t+\Delta t}\}^e$ are the nodal displacements and pore pressures at $(t+\Delta t)$ respectively, Δt is time interval, $\{R_F(t)\}^e$ is nodal forces at t, $\{R_q(t)\}^e$ is nodal discharge matrix at t, and usually is zero [8]. $[K_e]=\int_{V^e}[B]^T[D][B]dv$, $[K_c]=\int_{V^e}[B]^T[M][\bar{N}]dv$ and $[K_s]=\int_{V^e}[B_s]^T[\bar{k}][B_s]dv$ are the element stiffness matrix, element coupled matrix and element seepage matrix respectively, $[N]=[N_1 I\ N_2 I\cdots N_i I\cdots N_n I]$ and $[\bar{N}]=[\bar{N}_1\ \bar{N}_2\cdots\bar{N}_j\cdots\bar{N}_m]$ are the shape function matrixes for displacement and pore pressure respectively, **I** is a 3×3 unit matrix, N_i and $\bar{N}_j$ are the shape functions for displacement and pore pressure respectively, n and m are the total nodal numbers of displacement and pore pressure in a element respectively, $\{M\}=\{1\ 1\ 1\ 0\ 0\ 0\}^T$, $\theta(1>\theta\geq 0.5)$ is an integral constant, and usually 0.5 or 2/3,

$$[B_s]=[B_1\ B_2\cdots B_j\cdots B_m],$$

$$[B_j]=\begin{bmatrix}\frac{\partial\bar{N}_j}{\partial x} & \frac{\partial\bar{N}_j}{\partial y} & \frac{\partial\bar{N}_j}{\partial z}\end{bmatrix}^T,$$

$$[\bar{k}]=\frac{1}{\gamma_w}\begin{bmatrix} k_x & 0 & 0 \\ 0 & k_y & 0 \\ 0 & 0 & k_z\end{bmatrix}.$$

The assumed data for the verification are: R_w=5cm, $R_s=2R_w$, $R_e=20R_w$, $k_w=2\times 10^{-2}$cm/s, $k_s=6\times 10^{-8}$cm/s, $k_h=k_v=k_s$; $m_v=5\times 10^{-4}kPa^{-1}$, elastic modulus E=1246.15kPa and Poisson's ratio μ=0.35.

The one-eighth of soil is considered because of its symmetry. The soil is vertically divided into some layers, and the elements in a layer are shown in Fig.4. The length of the drain, H=10m, is considered, and three cases: (1) vacuum loading only, (2) surcharge loading only and (3) combined vacuum and surcharge loading, are analyzed. Pentahedron element with 15 displacement nodes and 6 pore pressure nodes for the drain, and hexahedron element with 20 displacement nodes and 8 pore pressure nodes for the smeared zone and undisturbed zone, are used for the analysis.

The degree of consolidation for soil is calculated according to Eq.(25). It is notable that S_t in Eq.(25) is the average vertical displacement at the top of soil,which is got by FEM.

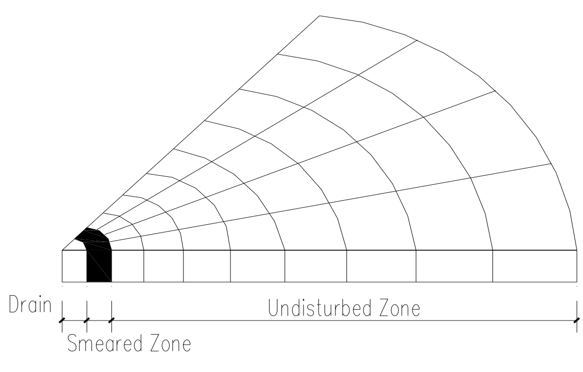


Figure 4. Schematic picture of a layer element mesh of soil

5.1. For Instantaneous Loading

For the exerted loadings $q(t) = q(0) = q_u = 80kPa$ and $p(t) = p(0) = p_u = -80kPa$, the comparisons between the calculated degree of consolidation by the FEM and got one from Eq.(26) are shown in Fig.5. It is clearly evident that both are in good agreement.

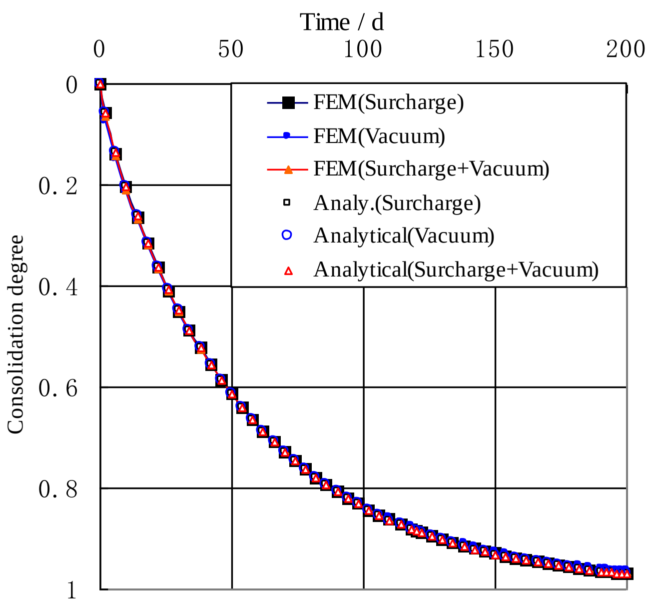
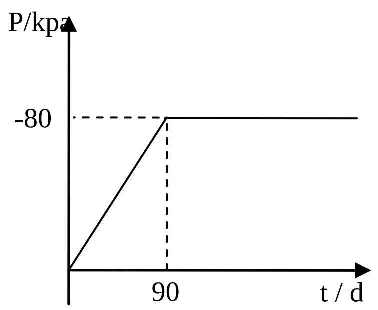
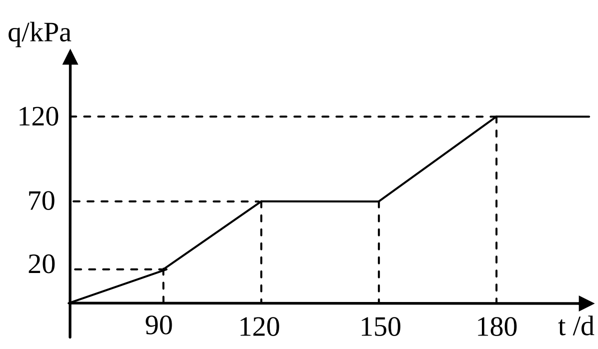


Figure 5. Comparison between the calculated consolidation degree by FEM and got one by analytical solution



(a) Vacuum preloading



(b) Surcharge preloading

Figure 6. Assumed vacuum and surcharge preloading

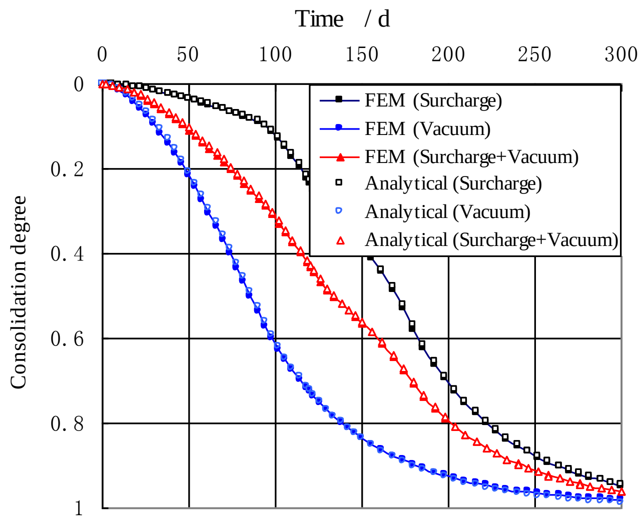


Figure 7. Comparison between the calculated consolidation degree by FEM and got one by analytical solution

5.2. For Multi-Loading

The assumed vacuum and surcharge loadings are shown in Fig.6. The comparisons between the calculated degree of consolidation by FEM and got one from Eq.(26) is shown in Fig.7. It is obvious that both are also good consistent.

6. Conclusions

(1) The solution for the degree of consolidation of soils by vertical drains under both arbitrary surcharge and vacuum preloading, including instantaneous preloading, is proposed taking into account the well resistance and the smear effect, radial and vertical direction flow, and an analytical solution for the degree of consolidation under linear time-dependent loading is also presented.

(2) The degree of consolidation for soil is dependent on the totality of surcharge and vacuum loading rather than their respective proportion. It is right that to convert the vacuum loading into the equivalent amount of surcharge when calculating the degree of consolidation.

(3) Carrillo's method is not directly used to calculate the degree of consolidation for soils by vertical drains under the condition of equal vertical strain.

REFERENCES

[1] Barron R.A., Consolidation of Fine Grained soils by drained wells[J], Trans., ASCE, 1948, 113: 718-733.

[2] Yoshikumi, H, Nakanodo, H,, Consolidation of Soils by Vertical Drain Wells with Finite Permeability[J], Soils and Foundations, 1974, 34(2): 35-46.

[3] Hansbo, S., Consolidation of Fine-grained soils by Refabricated Drains[J], Prec. 10th, (3), 1981, 677-682, Stockbolm.

[4] Xie Kanghe, Zeng Guoxi, Consolidation Theories for Drain Wells under Equal Strain Condition[J], Chinese Joural of Geotechnical Engineering, 1989, 11(2): 3-17. (in Chinese)

[5] Dong Zhiliang, Analytical Theory of Consolidation of Sand Drain's Ground under Combined Vacuum and Surcharge Preloading[J], Port &Waterway Engineering, 1992, (9): 1-7. (in Chinese)

[6] YIN Jing, LIU Shuguang, DONG Zhiliang, Three-dimensional Consolidation Analysis of Sand Drain's Ground under Combined Vacuum and Surcharge Preloading Considering Loading Process[J], Joural of Tongji University (Natural Science), 2009, 37(9):1174-1225. (in Chinese)

[7] Xiao-Wu Tang, Katsutada Onitsuka, Consolidation by Vertical Drains under time-dependent loading[J], Int. J. Numer. And Mech. Geomech.., 2000, 24:739-751.

[8] Gong Xiaonan, Geotechnical Engineering of Computer Analysis [M], Beijing, China Architecture & Building Press, 2000. (in Chinese)