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Buckling by Constant Stiffness and Curvature-Deflection Resonance

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Abstract The extensive use of plates and frames in construction regularly brings the engineer to confront buckling failure and its analysis and estimates of strength ;this makes it necessary to continue to explore easier ways of finding and verifying buckling solutions. Buckling by the constant initial elastic stiffness matrix is desirable but difficult to find in the literature ;the use of stability functions to modify the stiffness matrices is more readily found. By replacing the curvature in the Euler equation for buckling by its relative value, a successive approximation of steady state relative curvature -deflection ratio is, in-fact, found to be the buckling coefficient. In this way, a new practical buckling criterion is invoked as the resonant relative curvature-deflection ratio . Bars and rigid-frames are examined in this study.

Keywords Column Buckling , Bending Moment , Relative Moment(Curvature,Acceleration) , Constant Stiffness Matrix, Deflection, Computer Solution , Buckling Criterion , Resonance

1. Introduction

The use of a constant initial tangent stiffness in the analysis of buckling of bars and rigid frames is most desirable but is ,as yet, very difficult to find in the literature A constant stiffness method, as it is the case of non-linear finite-element analysis demonstrated by Zienkiewich[1] and further studied by Duncan and Johnarry[2] always promises to be easier than those that rely on tangent-stiffness-changing iterations with load stages. Iterations based on stability functions alteration of the stiffness matrix according to the extent of the axial buckling forces is amply demonstrated by Ping-Chun Wang(1965),[3], Wood(1974),[4] relied on stiffness alteration in his stiffness distribution buckling studies. Georgios Mageirou and Charles Gantes(2006),[5] and Messaoud Bourezane(2012),[6] ,among others, relied on Stability functions-based matrices in their buckling research on beams and frames . For example, in the slope-deflection equation, the familiar rotational beam stiffness, $4EI/L=(k_{11})$ will change to (k_{11}) + function(c,E,I,L) ;c=compression. There are several other approximate methods for critical loads which by-pass the use of stability functions; the treatment by Horne,1975,[7] ,which relied on elastic story drifts ,stands out for its simplicity as it requires the elastic solution and the employment of a formula on the solution displacements/rotations ;the formula was not completely proven. In recent studies Johnarry[8,9] has been examining the employment of acceleration-deflection resonance in new alternative solutions for plates and re-definition of buckling criterion .(curvature ≡ acceleration)

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2. Analysis

2.1. Resonance as a Buckling Criterion

In the presence of fiction ,the Euler equation will be,

$$Y_{,xx} + (P/EI)Y = Noise/EI = S^* \;;(\ (Noise/EI) \rightarrow 0\ ) \quad (1)$$

For a pin ended bar , try

$$Y = \Sigma A_m \operatorname{Sin} m\pi x/L$$

Continuing ,

$$A_m = [-4LK/(m\pi)] / (P/EI - m^2 . \pi^2 ./L^2) \quad (2)$$

$A_m \rightarrow \infty$ as $P \rightarrow EI(m^2)\pi^2 / L^2$;(m=1 for mode-1)

Buckling is sudden appearance of uncontrollably large displacement , by this view. Now, in this case

$$(Y_{,xx})_{shape} = Y_{shape} = \operatorname{Sin} m\pi x/L$$

Can it also be generalized that buckling is a condition of resonance between curvature and displacement ?

Can relative-acceleration be substituted for acceleration ? Clearly in this pin-ended case there is no standing support fixing moments and the relative acceleration is everywhere the same as the acceleration. A relative curvature envelop has an immediate common property with its relevant displacement envelop; both have zero end-values if the end values of displacements are prescribed zero. In the event the shapes of the two envelops are similar there is then resonance ,buckling, vibration.

2.2. Relative Acceleration – Displacement Analysis

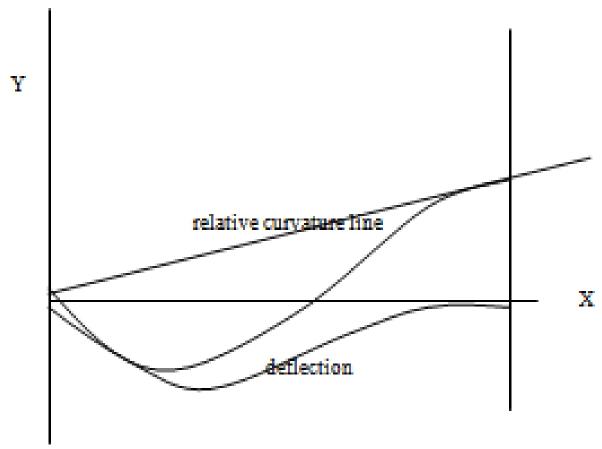

Figure 1. Pinned – Clamped Column

As an example, in Fig.1 ,consider the pinned-fixed column. Chose the Y-function to be able to satisfy all geometric boundary conditions; let,

$Y = \text{Sin } K X + \text{Cos } KX + A\ X + B$;($-K^2(\text{Cos } KX) = 0$;so, $\text{Cos}KX=0$ and $B=0$) (3)

$Y_{,xx}. = - K^2 \text{ Sin } K X$ (4)

Let the relative curvature be ,

$Y_{,xx})_{relative\text{-}zero\text{-}ends} = - K^2 \text{ Sin } K X + R X + S$ (5)

S=0 from starting conditions

Also, in relative curvature analysis, terminal ends of a line have zero relative curvatures; so,

At x=L , $(Y_{,xx})_{rel} .=0$;so, $R=(K^2 \text{ Sin } KL) /L$

Compare shapes of 'Y' and '$Y_{,xx\text{-}rel\text{-}zeroends}$.'

$N^* (\text{Sin } K X + AX) = K^2 (-\text{Sin } KX + (X/L)\text{Sin } KL)$ (6)

At X=L, Y=0; so $A= - (\text{Sin}KL)/L$;from Eq.3 (BCA)

$Y_{,xx\text{-}rel} / Y = K^2 (-\text{Sin } KX + (X/L)\text{Sin } KL) / [N^*(\text{Sin } KX – (X/L)\text{Sin } KL)]$ (7a)

$= -K^2 / N^* = 1$ (7b)

AT X=L; $Y_{,x} = 0$; so, $K \text{ Cos}KL + A = 0$, and with Eq-7,

$KL = \tan KL$; $KL=4.49$; (8a)

and,

$K^2=N^* = 20.16/ L^2 . = (Y_{,xx})_{rel}/ Y$ (8b)

So far in this section, no buckling force-P, has been mentioned , but from Euler's buckling relation for a pin-ended bar,

$EI\ Y_{,xx}+ P\ Y = 0$ (9a)

$EI\ Y_{,xx} .= EI\ (Y_{,xx\text{-}rel} +Y_{,xx\text{-}0}) = -P\ Y$ (9b)

$-P = EI(Y_{,xx\text{-}rel}) /Y + EI\ (Y_{,xx\text{-}0}). / Y$ (9c)

$(Y_{,xx\text{-}rel} . / Y) \rightarrow$ resonant , $Y_{,xx\text{-}rel} /Y >> Y_{,xx\text{-}0}/Y$; $Y \rightarrow \infty$, at buckling

So,

$E\ I\ Y_{,xx\text{-}rel} = -P\ Y$

$P = E\ I\ (\ Y_{,xx\text{-}rel}\ . /\ Y\ )$ (9d)

So, buckling(sudden large displacement) is indicated if relative acceleration and displacement become resonant.

Consistent displacements can be introduced and adjusted until the relative curvature - displacement in Eq.9d is constant at each station and when that happens the force ,P, assumes a critical value- P_{cr} .

This is the exact solution for the problem. In a computer stiffness matrix solution no displacement function is called for ; the slope deflection equation matrix formulation provides initial consistent nodal displacements.

The point-wise constant relative acceleration-displacemen t ratio can also be presented to vibration relation, clearly.

2.2.1. Energy Method Versus New

Acceleration-displacement ratio method

In the energy method the buckling load is found from

$$P_{cr} = EI \int (Y_{,xx})^2\ dx \Big/ \int (Y_{,x})^2.dx \qquad (10a)$$

In the present method it is found from,

$$P_{cr} = \int M_{rel} \Big/ \int Y\ ;\ (M_{rel})_{shape} = Y_{shape} \qquad (10b)$$

If $(M_{rel})_{curve\ shape} \neq Y_{curve\ shape}$, then rationalize,

$(M_{rel}) = H\ Y$; find scalar- H ,so that,

$\int (M_{rel})\ Y / \int Y.Y = H$

Examine a pin-ended column by the restricted function , $Y= X(1-X)$

$M_{fixity} =0$; physical support condition ,so, $M_{relative} = M$

$M_{rel} = EI\ (-2) = H\ .\ Y$

$\int EI(-2)\ Y\ dx = \int H\ Y.\ Y\ dx$; $H=10\ EI$

$Pcr = 10\ EI$; exact $= (\pi^2)EI$

The energy integral Eq.-10a will give a less approximate result of $P_{cr}=12EI$.Eq.-10b is easier than Eq.-10a in many instances.

Fig.2 gives an indication of how the fixed-fixed column is to be handled(suitable equation Y=Cos Kx +C) .The pin-pin Column is illustrated in Fig.3 divided into 38-elements and later solved.

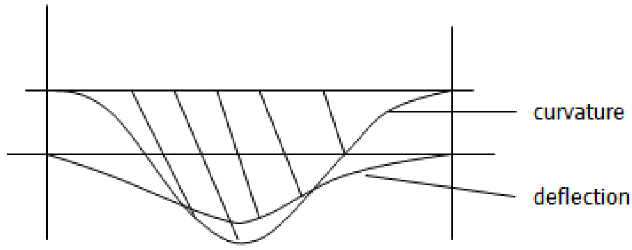

Figure 2. fixed – fixed column

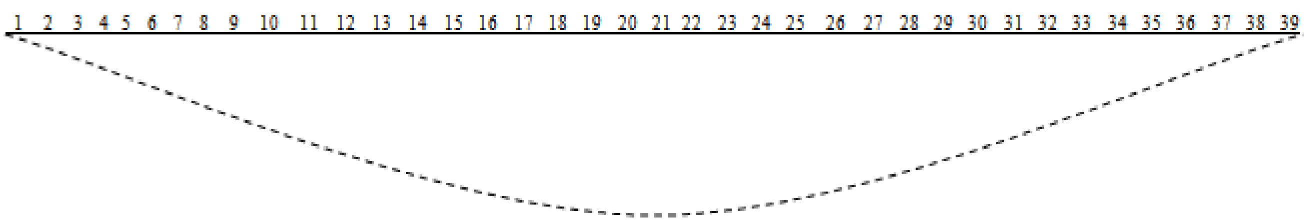

Figure 3. Pin-Pin Column, 38-elements, 39-nodes

2.3. Computer Stiffness Solution

The equation Y = J (Y″ + RX + S) is solved by successive approximation.

$$EI\,(Y_{,xx-rel})_x = EI\,Y_{,xx} - M_{fixity-x} = M_{rel} \quad (11)$$

The right hand side(rhs) of eq-11 is available from stiffness analysis , point-wise.

Start by applying a transverse load (say uniform load of any magnitude)

$$M_{fixity-x} .= M_{support-1} . + R\,X \quad (12)$$

$$R = (M_{support-2} - M_{support-1}) / L_{1-2} \quad (13)$$

Interpret the bending moments so found as the next set of applied loads and re-solve.

(other acceleration expressions can be tried)

A solution is found when ,point-wise,

$$P_{cr}, = (M_{rel}) / Y = \text{Constant} \quad (14)$$

is obtained; the displacement and the relative acceleration curves are then similar or of constant ratio point-wise.

The basic beam stiffness can be stated as:

$$F_{local} . = K_{local} . (\text{Vector})_{local} \quad (15a)$$

! N ! ! -a* 0 0 0 0 0 ! u1

! V1 ! ! 0 -12r/L^2. -6r/L 0 12r/L^2 . -6r/L ! v1

! M1 ! = ! 0 6r/L 4r 0 -6r/L 2r ! θ1 (15b)

! V2 ! ! 0 12r/L^2. 6r/L 0 -12r/L^2. 6r/L ! v2

! M2 ! ! 0 6r/L 2r 0 -6r/L 4r ! θ2 ….

So, find for (N=axial force,M=moment,V=shear-force;member from point-1 to2); a*=EA/L

$$K_{global} . = T^T . K_{local} . T . \quad (15c)$$

and conduct routine stiffness analysis . The stiffness matrix once found will remain unaltered in the iterations and produces very fast analysis. The results for the pinned-fixed column, the fixed-fixed column and a portal frame in axial compression against its mirror are presented

2.3.1. Pinned-Fixed Column and Others in Computer Solutions

The fixed-pinned column is ,often, a severe test in buckling studies and so the member is divided into 38 equal elements and 39- nodes. The record “moyope” means $(m_{rel})/(y)/P_E$.;(P_E .= reference Euler load)

The quotient is deficient in the vicinities of zero deflections and rapidly changing bending moments. Table 1 shows that the predominant ratio of 2.08 at many nodes is virtually exact. Where the ratio is much different from this the station weight measured by (y m/mmax) will be very small. Tables-1,2,3 show results for iterations =1,3,20. Table 4 compares iterations and the quality of results leading to the conclusion that three iterations are sufficient in a given situation.

As shown in Table-5 for the pin-pin column(20-elements) the result of 1.002 is ,practically, exact and it must be recalled that the stiffness solution did not assume a sinusoidal Y-variation that, in a manual solution, would have led to the same correct result. The result confirms that a column buckles into a sine-curve .

The result of the fixed-fixed column (38-elements) is given in Table-6 and the result P/Pe=4.006 compares with the exact value of 4.0

Table 1. fix-pin-col; **iteration=1**; bars = 38, nodes=39

….node	msd	m_{rel}=m	y;	**moyope**;	weight= (m.y/mmax)
…. 1	12697.184	0.000	-0.000	0.000	0.000
…. 3	11541.115	-179.326	-0.086	1.061	0.000
…. 5	9596.420	-1472.886	-0.330	2.261	0.009
…. 7	7105.286	-3312.884	-0.707	2.375	0.046
…. 9	4287.557	-5479.477	-1.181	2.350	0.126
…. 11	1337.197	-7778.702	-1.715	2.298	0.261
…. 13	-1573.286	-10038.050	-2.267	2.243	0.445
…. 15	-4293.961	-12107.589	-2.798	2.192	0.662
…. 17	-6702.02	-13864.517	-3.269	2.148	0.886
…. 19	-8695.847	-15297.203	-3.649	2.111	1.084
…. 21	-10195.279	-16055.500	-3.908	2.081	1.226
…. 23	-11148.308	-16357.393	-4.028	2.057	1.287
…. 24	-11405.808	-16289.325	-4.030	2.047	1.283
…. 27	-11296.713	-15203.526	-3.801	2.026	1.129
…. 29	-10498.644	-13754.321	-3.452	2.018	0.928
…. 31	-9161.334	-11765.877	-2.959	2.014	0.680
…. 33	-7345.712	-9299.119	-2.340	2.014	0.425
…. 35	-5137.435	-6439.707	-1.619	2.015	0.204
…. 37	-2645.315	-3296.451	-0.828	2.018	0.053
…. 39	0.039	-1658.439	-0.000	2.019	0.013

Note: Pc/P_E-weighted= 2.085751; weight-i=miyi/ (mmax.ymax);
Analysis: K Y = Fi = msd=gravity- force; msd=slope defl- moment; moyope = m/y/Pe ;
Pe = Euler critical load

Table 2. fix-pin –col; **iteration=3**, bars = 38

….node	msd	m-rel=m	y;	moyope;	weight
…. 1	18513.324	0.000	-0.000	0.000	0.000
…. 3	17027.420	-61.852	-0.125	0. 250	0.000
…. 5	14578.231	-1561.636	-0.485	1.632	0.006
…. 7	11307.885	-3882.579	-1.045	1.882	0.034
…. 9	7407.607	-6833.453	-1.761	1.965	0.101
…. 11	3099.289	-10192.367	-2.579	2.002	0.220
…. 13	-1371.603	-13713.854	-3.440	2.019	0.395
…. 15	-5749.822	-17142.670	-4.282	2.028	0.614
…. 17	-9795.646	-20239.090	-5.045	2.032	0.855
…. 19	-13285.272	-22779.313	-5.673	2.034	1.082
…. 21	-16029.378	-24574.014	-6.118	2.035	1.258
…. 23	-17877.473	-25472.705	-6.342	2.035	1.352
…. 24	-18434.568	-25555.098	-6.363	2.035	1.361
…. 25	-18735.422	-25381.250	-6.320	2.034	2.343
…. 27	-18551.770	-24248.193	-6.040	2.034	1.226
…. 29	-17343.844	-22090.863	-5.505	2.033	1.018
…. 31	-15180.911	-18978.527	-4.731	2.032	0.752
…. 33	-12179.821	-15028.033	-3.748	2.031	0.471
…. 35	-8510.411	-10409.219	-2.597	2.031	0.226
…. 37	-4375.045	-5324.449	-1.328	2.030	0.059
…. 39	-0.053	-2677.58	-0.000	2.031	0.015

Pc/Pe-weighted = 2.030782; weight – i=miyi/ (mmax. Ymax);
Analysis: K Y = Fi=msd=gravity- force .

Table 3. fix-pin –col; **iteration=20**, bars = 38

….node	msd	mrel=m	y;	moyope	wt
…. 1	-18363.709	0.000	- 0.000	0.000	0.000
…. 3	-16898.070	53.110	0.124	0.217	0.000
…. 5	-14490.811	1518.638	0.481	1.599	0.006
…. 7	-11276.017	3791.699	1.038	1.851	0.033
…. 9	-7434.182	6691.802	1.749	1.938	0.098
…. 11	-3177.589	10006.662	2.563	1.978	0.214
…. 13	-1256.193	13498.712	3.421	1.999	0.386
…. 15	5617.753	16918.541	4.262	2.011	0.603
…. 17	9669.299	20028.354	5.025	2.019	0.841
…. 19	13184.827	22602.150	5.655	2.025	1.068
…. 21	15966.829	24442.420	6.103	2.029	1.247
…. 23	17860.639	25394.496	6.332	2.032	1.344
…. 24	18441.959	25504.951	6.355	2.033	1.355
…. 25	18765.209	25357.334	6.315	2.034	1.338
…. 27	18618.063	24268.455	6.039	2.036	1.225
…. 29	17436.277	22144.939	5.507	2.037	1.019
….. 31	15281.722	19048.650	4.735	2.038	0.754
…. 33	12273.036	15098.232	3.752	2.039	0.473
…. 35	8582.428	10465.893	2.600	2.039	0.227
…. 37	4413.864	5355.596	1.331	2.039	0.060
…. 39	0.070	2693.588	0.000	2.039	0.015

Pc/Pe-weighted= 2.026943, weight – I= miyi/ (mmax. ymax);
Analysis: K Y = Fi =msd=gravity- force;

Table 4. fixed-pined column iterations and P/P_E .comparison

Iterations	Summed weighted P/P_E.	Best nodal P/Pe
1.	2.085	2.057=node-23
3.	2.031	2.035=node-24
20	2.027	2.033=node-24

From Table-4 and from experience three iterations are often sufficient.

Table 5. pin-pin-col;itertn =3, bars = 20

….node	msd	m-rel	y;	moyope;	weight
…. 1	0.033	0.000	0.000	-0.000	0.000
…. 2	-3178.815	-3178.815	-1.607	1.002	5.107
…. 3	-6279.420	-6279.420	-3.173	1.002	19.927
…. 4	-9225.088	-9225.088	-4.662	1.002	43.009
…. 5	-11943.527	-11943.527	-6.036	1.002	72.093
…. 6	-14366.936	-14366.936	-7.261	1.002	104.323
…. 7	-16436.387	-16436.387	-8.308	1.002	136.549
…. 8	-18100.791	-18100.791	-9.150	1.002	165.614
…. 9	-19318.322	-19318.322	-9.766	1.002	188.662
…. 10	-20059.930	-20059.930	-0.142	1.002	203.447
…. 11	-20308.916	-20308.916	-0.268	1.002	208.536
…. 12	-20057.795	-20057.795	-0.142	1.002	203.418
…. 13	-19312.686	-19312.686	-9.765	1.002	188.595
…. 14	-18092.527	-18092.527	-9.149	1.002	165.522
…. 15	-16427.439	-16427.439	-8.307	1.002	136.458
…. 16	-14357.994	-14357.994	-7.260	1.002	104.244
…. 17	-11936.252	-11936.252	-6.035	1.002	72.037
…. 18	-9219.818	-9219.818	-4.661	1.002	42.977
…. 19	-6276.302	-6276.302	-3.173	1.002	19.914
…. 20	-3177.169	-3177.169	-1.606	1.002	5.103
…. 21	-0.019	-3177.169	-0.000	1.002	5.103

ratio,Pc/Pe-weighted = 1.002017, analysis: K Y = msd = gravity- force;

Table 6. fixd-fix- col; itertn 3; bars = 38

….node	msd	m-rel	y;	moyope;	wtmy;
…. 2	9379.820	-146.149	0.016	0.000	0.000
…. 4	8351.551	-1121.576	144.813	3.923	0.014
…. 6	6411.210	-3061.964	387.588	4.002	0.105
…. 8	3778.354	-5694.869	718.157	4.017	0.361
….10	745.159	-8728.111	1100.475	4.018	0.849
…. 11	-819.092	-10292.386	1298.155	4.016	1.181
…. 12	-2357.932	-11831.250	1493.012	4.014	1.561
…. 13	-3829.455	-13302.797	1679.745	4.012	1.975
…. 14	-5194.740	-14668.105	1853.279	4.009	2.403
…. 15	-6416.244	-15889.633	2008.910	4.007	2.821
…. 16	-7463.585	-16936.998	2142.424	4.005	3.207
…. 17	-8308.951	-17782.387	2250.210	4.003	3.537
…. 18	-8928.733	-18402.193	2329.356	4.002	3.789
…. 19	-9307.604	-18781.088	2377.724	4.001	3.947
…. 20	-9435.602	-18909.109	2394.008	4.001	4.001
…. 21	-9308.465	-18781.996	2377.765	4.002	3.947
…. 22	-8930.229	-18403.785	2329.443	4.002	3.789
…. 23	-8310.577	-17784.156	2250.335	4.003	3.537
…. 24	-7466.403	-16940.008	2142.581	4.005	3.208
…. 25	-6419.980	-15893.607	2009.090	4.008	2.822
…. 26	-5197.589	-14671.239	1853.471	4.010	2.404
…. 27	-3831.219	-13304.894	1679.939	4.012	1.976
…. 28	-2359.182	-11832.881	1493.201	4.014	1.562
…. 29	-820.713	-10294.435	1298.333	4.017	1.181
…. 30	743.527	-8730.219	1100.638	4.018	0.849
…. 32	3777.859	-5695.935	718.274	4.017	0.362
…. 34	6411.984	-3061.856	387.655	4.001	0.105
…. 36	8352.875	-1121.014	144.840	3.921	0.014
…. 38	9381.512	-482.707	0.016	3.755	0.003

Pc – weighted = -4.006524
;Analysis: K Y =F =Mrel=gravity-force

Table 7. fixed-base portal + mirror, itertn =3; bars= 34;$(I_{beam})/(I_{col})$=0.1

….node	msd	m-rel=m	y;	moyope;	weight
…. 1	49.213	0.000	-0.000	0.000	0.000
…. 2	-2485.434	-2534.649	-0.815	0.252	2.066
…. 3	-4923.839	-4973.056	-1.599	0.252	7.954
…. 4	-7172.256	-7221.474	-2.322	0.252	16.771
…. 5	-9143.320	-9192.539	-2.956	0.252	27.177
…. 6	-10760.814	-10810.034	-3.477	0.252	37.585
…. 7	-11962.952	-12012.173	-3.864	0.252	46.411
…. 8	-12703.617	-12752.839	-4.102	0.252	52.311
…. 9	-12953.934	-13003.156	-4.182	0.252	54.383
…. 10	-12704.128	-12753.352	-4.102	0.252	52.313
…. 11	-11963.758	-12012.982	-3.864	0.252	46.414
…. 12	-10761.386	-10810.612	-3.477	0.252	37.587
…. 13	-9143.571	-9192.799	-2.956	0.252	27.178
…. 14	-7172.499	-7221.721	-2.322	0.252	16.772
…. 15	-4924.174	-4973.403	-1.599	0.252	7.954
…. 16	-2485.545	-2534.775	-0.815	0.252	2.066
…. 17	49.224	-2534.775	0.000	0.252	2.066
…. 18	49.169	-2534.775	0.000	0.252	2.066
…. 19	-2485.367	-2534.601	-0.815	0.252	2.066
…. 20	-4923.784	-4973.019	-1.599	0.252	7.953
…. 21	-7172.152	-7221.388	-2.322	0.252	16.771
…. 22	-9143.009	-9192.245	-2.956	0.252	27.175
…. 23	-10760.813	-10810.052	-3.477	0.252	37.584
…. 24	-11962.858	-12012.098	-3.864	0.252	46.410
…. 25	-12703.341	-12752.581	-4.102	0.252	52.309
…. 26	-12954.058	-13003.299	-4.182	0.252	54.383
…. 27	-12704.064	-12753.307	-4.102	0.252	52.312
…. 28	-11963.339	-12012.582	-3.864	0.252	46.412
…. 29	-10761.072	-10810.316	-3.477	0.252	37.585
…. 30	-9143.340	-9192.585	-2.956	0.252	27.177
…. 31	-7172.343	-7221.590	-2.322	0.252	16.771
…. 32	-4924.021	-4973.269	-1.599	0.252	7.954
…. 33	-2485.487	-2534.735	-0.815	0.252	2.066
…. 34	49.250	-2534.735	0.000	0.252	2.066

Pcr/Pe = 0.2520134, 1b/Ic = 0.1, analysis: K Y=Fi=msd=gravity-force

Table 8. fixed-base portal + mirror; itertn =3, bars = 34; $I_{beam}/(I_{col})$ =1

….node	msd	mrel=m	y;	moyope	weight
…. 1	4050.175	0.000	-0.000	0.000	0.000
…. 2	3197.095	-853.044	-0.091	0.759	0.078
…. 3	1979.468	-2070.655	-0.221	0.758	0.458
…. 4	539.338	-3510.767	-0.376	0.757	1.320
…. 5	-957.547	-5007.634	-0.537	0.756	2.689
…. 6	-2341.093	-6391.163	-0.686	0.755	4.388
…. 7	-3455.925	-7505.977	-0.807	0.754	6.059
…. 8	-4178.111	-8228.146	-0.886	0.753	7.287
…. 9	-4427.671	-8477.688	-0.913	0.753	7.738
…. 10	-4177.179	-8227.179	-0.886	0.753	7.285
…. 11	-3454.260	-7504.242	-0.807	0.754	6.057
…. 12	-2338.832	-6388.796	-0.686	0.755	4.385
…. 13	-955.033	-5004.980	-0.537	0.756	2.687
…. 14	-541.753	-3508.177	-0.376	0.757	1.318
…. 15	1981.568	-2068.344	-0.221	0.758	0.457
…. 16	3198.630	-851.264	-0.091	0.758	0.077
…. 17	4050.921	-851.264	0.000	0.758	0.077
…. 18	-4048.568	-851.264	0.000	0.758	0.077
…. 19	3196.140	-853.702	-0.091	0.761	0.078
…. 20	1979.168	-2070.656	-0.221	0.759	0.458
…. 21	539.675	-3510.132	-0.376	0.757	1.318
…. 22	-956.450	-5006.239	-0.537	0.756	2.687
…. 23	-2339.314	-6389.086	-0.686	0.755	4.383
…. 24	-3453.683	-7503.438	-0.807	0.754	6.053
…. 25	-4175.589	-8225.326	-0.885	0.753	7.279
…. 26	-4425.183	-8474.902	-0.912	0.753	7.730
…. 27	-4175.013	-8224.715	-0.885	0.753	7.278
…. 28	-3452.514	-7502.198	-0.807	0.754	6.051
…. 29	-2337.696	-6387.362	-0.686	0.755	4.381
…. 30	-954.511	-5004.161	-0.537	0.756	2.685
…. 31	541.716	-3507.916	-0.375	0.757	1.317
…. 32	1981.006	-2068.608	-0.221	0.759	0.457
…. 33	3197.648	-851.948	-0.091	0.760	0.077
…. 34	4049.579	-851.948	0.000	0.760	0.077

Pcr/Pe = 0.7541421 analysis: K Y=Fi=msd=gravity-force

Table 9. fixed-base portal + mirror; itertn =3, bars = 34;$(I_{beam})/(I_{col})$=100

….node	msd	mrel=m	y;	moyope;	weight
…. 1	3244.744	0.000	-0.000	0.000	0.000
…. 2	2988.031	-256.702	-0.020	1.026	0.005
…. 3	2273.164	-971.563	-0.077	1.023	0.075
…. 4	1214.141	-2030.581	-0.161	1.020	0.328
…. 5	-23.941	-3268.657	-0.261	1.017	0.852
…. 6	-1252.209	-4496.919	-0.359	1.014	1.616
…. 7	-2286.689	-5531.394	-0.443	1.012	2.451
…. 8	-2974.469	-6219.168	-0.499	1.010	3.103
…. 9	-3214.998	-6459.692	-0.518	1.010	3.349
…. 10	-2973.289	-6217.977	-0.499	1.010	3.101
…. 11	-2284.499	-5529.182	-0.443	1.012	2.449
…. 12	-1249.460	-4494.137	-0.359	1.014	1.614
…. 13	-20.931	-3265.602	-0.260	1.017	0.850
…. 14	1216.973	-2027.693	-0.161	1.020	0.327
…. 15	2275.329	-969.331	-0.077	1.023	0.074
…. 16	2989.185	-255.469	-0.020	1.025	0.005
…. 17	3244.692	-255.469	-0.000	1.025	0.005
…. 18	3244.485	-255.469	-0.000	1.025	0.000
…. 19	2987.831	-256.807	-0.020	1.026	0.005
…. 20	2273.053	-971.579	-0.077	1.023	0.075
…. 21	1214.116	-2030.510	-0.161	1.020	0.328
…. 22	-23.880	-3268.501	-0.261	1.017	0.851
…. 23	-1252.085	-4496.701	-0.359	1.014	1.616
…. 24	-2286.520	-5531.129	-0.443	1.012	2.451
…. 25	-2974.296	-6218.899	-0.499	1.010	3.102
…. 26	-3214.854	-6459.453	-0.518	1.010	3.349
…. 27	-2973.171	-6217.764	-0.499	1.010	3.101
…. 28	-2284.431	-5529.019	-0.443	1.012	2.449
…. 29	-1249.402	-4493.984	-0.359	1.014	1.614
…. 30	-20.939	-3265.515	-0.260	1.017	0.850
…. 31	1216.906	-2027.665	-0.161	1.020	0.327
…. 32	2275.229	-969.336	-0.077	1.023	0.074
…. 33	2989.059	-255.501	-0.020	1.026	0.005
…. 34	3244.554	-255.501	-0.000	1.026	0.005

Pcr/Pe= 1.012245; analysis: K Y= Fi= msd = gravity-force

Table 10. pin-base, 1-cell-portl-dirct itrtn= 5; bars = 34; Ib/Ic=0.01

node	msd	mrel=m	y;	moyope	wt
…. 1	-0.648	0.000	-0.000	0.000	0.000
…. 2	-2383.575	-2383.575	-19.935	0.010	47.518
…. 3	-4751.339	-4751.339	-39.863	0.010	189.405
…. 4	-7047.854	-7047.854	-59.777	0.010	421.297
…. 5	-9317.235	-9317.235	-79.668	0.009	742.284
…. 6	-11416.626	-11416.626	-99.530	0.009	1136.297
…. 7	-13402.577	-13402.577	-119.357	0.009	1599.686
…. 8	-15174.580	-15174.580	-139.141	0.009	2111.412
…. 9	-16773.662	-16773.662	-158.879	0.009	2664.980
…. 10	-18250.369	-18250.369	-178.564	0.008	3258.857
…. 11	-19487.324	-19487.324	-198.192	0.008	3862.234
…. 12	-20539.998	-20539.998	-217.760	0.008	4472.782
…. 13	-21348.705	-21348.705	-237.263	0.007	5065.258
…. 14	-21991.242	-21991.242	-256.700	0.007	5645.147
…. 15	-22559.107	-22559.107	-276.068	0.007	6227.845
…. 16	-22814.303	-22814.303	-295.366	0.006	6738.561
…. 17	-23003.789	-23003.789	-314.592	0.006	7236.811
…. 18	3.691	-23003.789	-0.000	0.006	7236.811
…. 19	-2383.833	-2383.833	-19.935	0.010	47.522
…. 20	-4731.493	-4731.493	-39.863	0.010	188.611
…. 21	-7054.062	-7054.062	-59.776	0.010	421.663
…. 22	-9289.369	-9289.369	-79.667	0.009	740.056
…. 23	11415.491	-11415.491	-99.529	0.009	1136.171
…. 24	-13394.956	-13394.956	-119.355	0.009	1598.759
…. 25	-15188.236	-15188.236	-139.140	0.009	2113.291
…. 26	-16729.049	-16729.049	-158.877	0.009	2657.867
…. 27	-18220.387	-18220.387	-178.562	0.008	3253.476
…. 28	-19477.930	-19477.930	-198.191	0.008	3860.344
…. 29	-20527.857	-20527.857	-217.758	0.008	4470.109
…. 30	-21315.250	-21315.250	-237.262	0.007	5057.293
…. 31	-21971.732	-21971.732	-256.699	0.007	5640.116
…. 32	-22555.922	-22555.922	-276.067	0.007	6226.948
…. 33	-22778.387	-22778.387	-295.365	0.006	6727.943
…. 34	-22797.252	-22797.252	-314.592	0.006	7171.835

Pc/Pe, weighted= 7.274877E-03; analysis: Fi=msd =gravity-force in F = K Y

Table 11. pin-base, 1-cell-portl-dirct itrtn= 5; bars = 34 ; Ib/Ic = 1

Node	msd	m-rel=m	moyope	y	wt.
…. 1	0.138	0.000	-0.000	0.000	0.000
…. 2	-2618.487	-2618.487	-1.086	0.195	0.008
…. 3	-5212.847	-5212.847	-2.164	0.195	0.033
…. 4	-7759.471	-7759.471	-3.226	0.195	0.073
…. 5	-10236.680	-10236.680	-4.263	0.195	0.128
…. 6	-12618.201	-12618.201	-5.269	0.194	0.195
…. 7	-14884.176	-14884.176	-6.235	0.193	0.272
…. 8	-17014.248	-17014.248	-7.155	0.193	0.357
…. 9	-18988.225	-18988.225	-8.022	0.192	0.447
…. 10	-20787.299	-20787.299	-8.829	0.191	0.538
…. 11	-22396.514	-22396.514	-9.572	0.190	0.629
…. 12	-23806.936	-23806.936	-10.244	0.188	0.715
…. 13	-24997.441	-24997.441	-10.843	0.187	0.795
…. 14	-25965.176	-25965.176	-11.363	0.185	0.865
…. 15	-26699.523	-26699.523	-11.802	0.183	0.924
…. 16	-27188.354	-27188.354	-12.158	0.181	0.969
…. 17	-27439.313	-27439.313	-12.430	0.179	1.000
…. 18	0.042	-27439.313	- 0.000	0.179	1.000
…. 19	-2617.955	-2617.955	-1.086	0.195	0.008
…. 20	-5212.276	-5212.276	-2.164	0.195	0.033
…. 21	-7760.071	-7760.071	-3.226	0.195	0.073
…. 22	-10235.544	-10235.071	-4.263	0.195	0.128
…. 23	-12617.299	-12617.299	-5.269	0.194	0.195
…. 24	-14883.375	-14883.375	-6.235	0.193	0.272
…. 25	-17014.871	-17014.871	-7.155	0.193	0.357
…. 26	-18986.105	-18986.105	-8.021	0.192	0.447
…. 27	-20785.053	-20785.053	-8.829	0.191	0.538
…. 28	-22398.174	-22398.174	-9.571	0.190	0.629
…. 29	-23805.623	-23805.623	-10.244	0.188	0.715
…. 30	-24997.193	-24997.193	-10.843	0.187	0.795
…. 31	-25960.738	-25960.738	-11.363	0.185	0.865
…. 32	-26700.240	-26700.240	-11.802	0.183	0.924
…. 33	-27185.016	-27185.016	-12.158	0.181	0.969
…. 34	-27186.326	-27186.326	-12.430	0.177	0.991

Pc/Pe, weighted= .1863101; analysis: Fi=msd=gravity-force

Table 12. pin-base, 1-cell-portal-direct **itrtn=20**; bars = 34 ; Ib/Ic =100

Node	msd	m-rel=m	y	moyope	wt
…. 1	-0.202	0.000	0.000	0.000	0.000
…. 2	2628.370	2628.370	0.864	0.247	0.009
…. 3	5233.774	5233.774	1.720	0.247	0.037
…. 4	7790.714	7790.714	2.560	0.247	0.082
…. 5	10278.283	10278.283	3.375	0.247	0.142
…. 6	12670.287	12670.287	4.158	0.247	0.216
…. 7	14947.424	14947.424	4.902	0.247	0.300
…. 8	17089.115	17089.115	5.599	0.247	0.392
…. 9	19073.607	19073.607	6.243	0.248	0.487
…. 10	20885.162	20885.162	6.827	0.248	0.583
…. 11	22507.680	22507.680	7.346	0.248	0.677
…. 12	23932.301	23932.301	7.795	0.249	0.763
…. 13	25135.898	25135.898	8.169	0.249	0.840
…. 14	26112.773	26112.773	8.465	0.250	0.904
…. 15	26850.174	26850.174	8.679	0.251	0.954
…. 16	27350.355	27350.355	8.809	0.252	0.986
…. 17	27601.090	27601.090	8.854	0.253	1.000
…. 18	-0.208	27601.090	0.000	0.253	0.000
…. 19	2628.365	2628.365	0.864	0.247	0.009
…. 20	5233.539	5233.539	1.720	0.247	0.037
…. 21	7790.900	7790.900	2.560	0.247	0.082
…. 22	10278.357	10278.357	3.375	0.247	0.142
…. 23	12671.202	12671.202	4.158	0.247	0.216
…. 24	14946.470	14946.470	4.902	0.247	0.300
…. 25	17089.078	17089.078	5.599	0.247	0.392
…. 26	19073.756	19073.756	6.243	0.248	0.487
…. 27	20884.490	20884.490	6.827	0.248	0.583
…. 28	22507.979	22507.979	7.346	0.248	0.677
…. 29	23932.600	23932.600	7.795	0.249	0.763
…. 30	25136.121	25136.121	8.169	0.249	0.840
…. 31	26112.969	26112.969	8.465	0.250	0.904
…. 32	26850.254	26850.254	8.679	0.251	0.954
…. 33	27352.088	27352.088	8.809	0.252	0.986
…. 34	27351.789	27351.789	8.854	0.250	0.991

Pc/Pe, weighted= 0.2494865; analysis: Fi=msd=gravity-force

A fixed-base lone portal frame,Fig.4, is analyzed with its mirror so as to find conservative energy situation, as is the case of the cantilever column. The results are as exact as the isolated columns already examined for various beam-to-column stiffness ratios in Tables 7,8,9 and summarized in Table-13.

In the case of the pin-pin portal frame,Fig.5 there is no need for the mirror part as the displacement curve is conservative directly. Results are given in Tables,10,11,12 for different I-beam /I-col ratios. The exactness of the results is maintained.

Many results of these frames for various I-b/I-c ratios can be easily found as demonstrated . Column strengths and design graphs as found in Wood(1974),[4] and in design Codes,BS-8110,BS-5950[10,11] can be replicated by the present method.; the objective here is to show a very fast buckling solution method without going outside the conventional elastic stiffness analysis that forms the back-bone of bridge and building frames design analyses and studies.

Table 13. $P./P_E.$ for portal frame for various I_{beam}/I_{col}

$K_b./K_c.$	$P./P_E.$	portal-base-support
0.1	0.252	fixed-fixed
1.0	0.754	fixed-fixed
100	1.01(≈1.0)	fixed-fixed (maximum,1.0)
0.01	0.0073	pin-pin
1.0	0.186	pin-pin
100	0.249(approx,0.25),	pin-pin

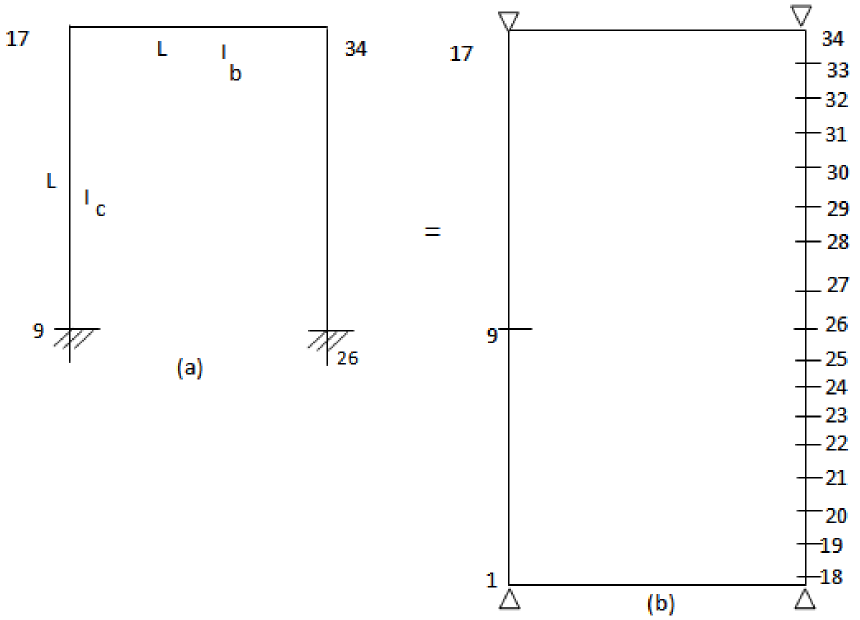

Figure 4. Fixed-base sway frame; (a) alone (b) plus mirror for analysis; $\Theta_9 = \Theta_{26} = O$; nodes 1-9,18-26 = mirror

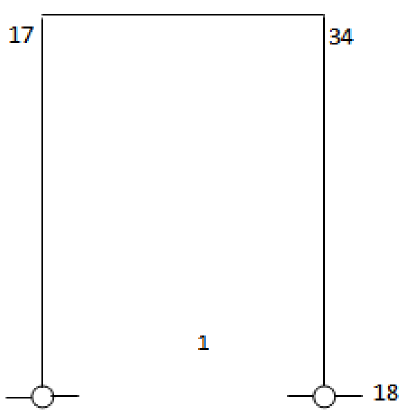

Figure 5. Pin-base single portal-analysis

2.4. Extension to Substitute Frames

The substitute frame method allows a large frame to be reduced to easier- to -handle limited frames in the design process.

In applying the present method to any story the main issue to contend with is the value of the fixity at the base of the story. It is somewhere between fixed and hinge support. The present method is based on elastic analysis and so super-position of two solutions(partial fixity and partial hinge) is studied.

2.4.1. Column-Base Fixity Ratio ,F_R

Beams at joints provide fixity to columns and a fixity ratio is defined as

$F_R = (I/L^2)_{beam} / [(I/L^2)_{beam} + (I/L^2)_{col}]$;

maximum=1.0 whenbeam is infinitely rigid (16)

So, hinge-support ratio = 1 -F_R

The equation is rational and highly effective. It is known that in the buckling environment, moments are proportional to (P.y) or (I/L^2) and moments must be balanced at joints of beams and columns.

2.4.2. Examples of Story Analysis in Multi-story Frames

These frames ,Figures 6,7,8,9 were taken out of the paper by Horne[7]; results summarized in Table 14

..(Example-Figure.6.)..3-storys frame, Figure. 6; Pinned-bases

; try bottom story ;fixity=0; so direct analysis with pinned bases .

Enter program with Icol=76470000 , E=201,000, L-col=4000, L-beam=8000 ;Ibeam=350,830,000

find P_{cr}/P_E =0.2177; $P_{applied}$ =1088Kn.; $P_{cr}/P_{applied}$. = 3.79, ((alternate result quoted by Horne[7]=3.78));

;;try another story if necessary.

..(Example-Figure.7(same as Fig.6 but fixed feet),)) ; ; try story-2

stp1.. $F_R = (350{,}830{,}000/8000^2) / (350{,}830{,}000/8000^2 + 76{,}470{,}000/4000^2) = 0.534$

stp2.. Hinge-factor =1- 0.534 = 0.464

stp3.. $P_{cr\text{-}hinge}/P_E$= 0.2177 –already found above .

st4..With new fixed-fixed boundary conditions ,find (P_{cr}/P_E)=0.881

st5..So, $(P_{cr}/P_E)_{story}$ = 0.881(0.534)+0.2177(0.466) = 0.572 ; P_{cr} = 10,828 Kn.

st6..$(P_{cr}/P_{applied})$ = 10847/628 =17.24

(Example-Fig.7,story-1)..Try bottom story to check if more critical; (F_r = 1; P_{cr} =16,680; $P_{applied}$ = 1088.

$P_{cr}/p_{applied}$ = 15.34; compare exact 14.72 - - Table 14;)

(Example-Figure.8,botm story.).. 8-story frame shown in Figure.8 ;try bottom story

stp1.. $F_R = (236000000/3050^2) / (2360000000/3050^2 + 479160000/5080^2) = 0.577$

stp2.. solve with base fixed-fixed ; Pcr/Pe = 0.573

stp3.. solve with pinned bases ; Pcr/Pe = 0.136

stp4.. $(Pcr/Pe)_{combined}$ = (0.577(0.573) + 0.423(0.136)) = 0.3875

stp5.. Pcr = $0.3875(\pi^2)E(479160000\ (2legs))/(5080^2)$ =

28,547 Kn

stp6.. $P_{applied}$ = 7,120 Kn ; Load-factor $Pcr/P_{applied}$ = 4.0 ((alternate result quoted by Horne[7]=3.95))

(Example-Figure.9,story-2).. 5-story – 2bay frame ; Figure.9 ; try story-two

stp1..first reduce to one bay 5-story frame(i.- add up I-beams ;ii.- add up I-cols and divide by two)

stp2.. find F_R = 0.6591

stp3 solve for fixed bases ; Pcr/Pe = 0.9035

stp4.. solve with pinned bases ;Pcr/Pe = 0.223

stp5.. $(Pcr/Pe)_{combined}$ = 0.671

stp6.. $Pcr/P_{applied}$ = 7.51 for E=201,000N/sq-mm ((alternate result quoted by Horne[7] =7.70))

(Example-Figure.9,story-1)..try story-one ; F_R =1 ; bases actually fixed-fixed ,so pin-pin contribution is none.

..stp1 ; Pcr/Pe = 0.893

..stp2 ; Pcr / $P_{applied}$ = 8.52 > 7.38 for story-2 found above.

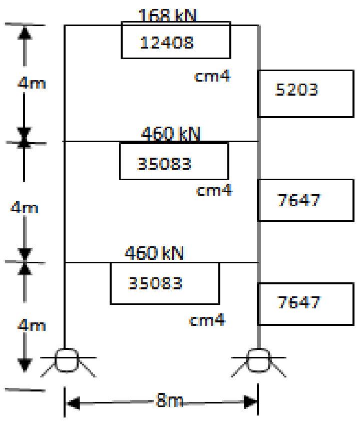

Figure 6. pin-pin frame

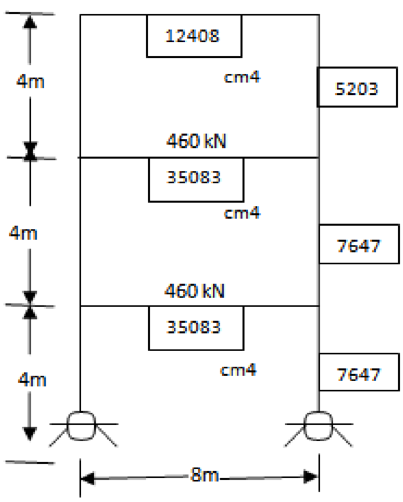

Figure 7. fixed-fixed frame ,same as Fig.6 but fixed

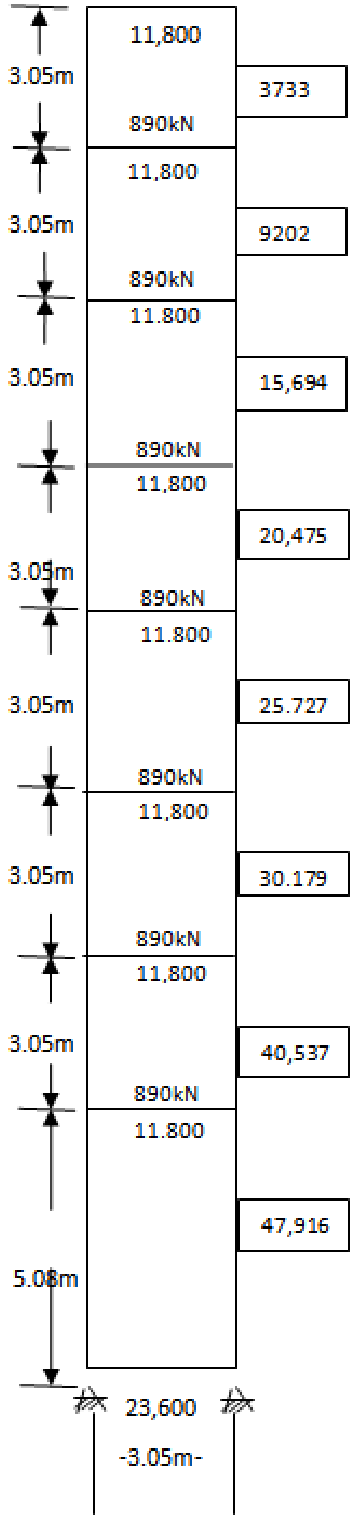

Figure 8. Frame

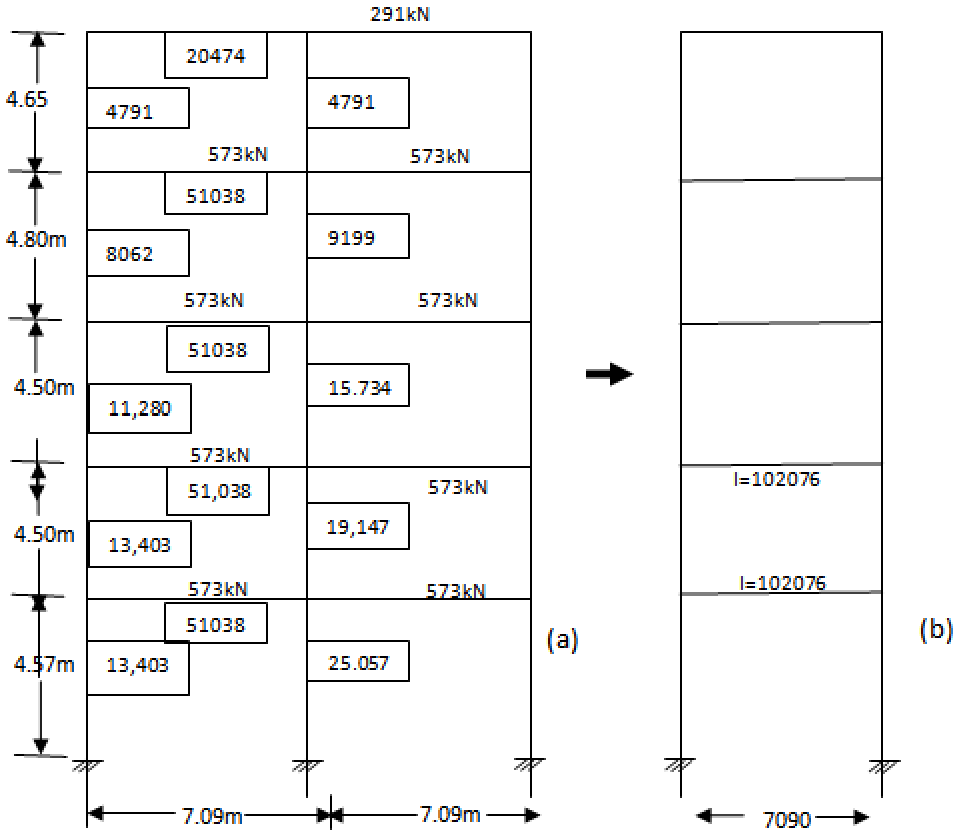

Figure 9. Frame

2.4.3. Quality of Results in Frames

The fixity ratio method of mixed-boundary-conditions studied here led to results within 5-percent of the best of other quoted results in the limited experience of the method for multi-bay multi-story frames.

3. Conclusions

Buckling instability has been successfully defined as the invariant point-wise quotient of the relative-moment and the consistent displacement, easily obtained from an iterative load-displacement analysis using the constant elastic tangent stiffness. Examples in beams and frames were tested in this preliminary study. About three iterative solutions are needed for an analysis. Above the ground story , the upper storys operate in partial fixity(between fully fixed and pinned);partial fixity and partial pinned factors were rationally found to produce a super-posed solution; each solution is purely elastic. The critical load of each story was easily found directly.

ACKNOWLEDGEMENTS

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