

# An Optimal Solution for a Real Transportation Problem with Lingo Code

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**Abstract** The transportation problem deals with finding the minimum cost of transporting a single product from a certain number of sources to a given number of destinations. The problem can be expressed by the formulation of a linear model, and it can be solved using the simplex algorithm. But due to its particular structure of the linear model, it can be solved with more efficient methods. LINGO is one of the compelling tools which can be used to solve optimization problems. It includes integrated packages that include powerful language for expressing related optimization models, a full-featured environment for both building and editing problems, and a set of fast built-in solvers. In this paper, a lingo code is prepared based on the formulation of transportation problems to solve an actual case study. The optimal solution of the studied case study reduces the total ton-kilometers by 9.1702 % of the real distribution amounts. The real data, the lingo code, the optimal solution, and a comparison between the real and optimal solutions are both included.

**Keywords** Transportation problem, an actual case study, LINGO

## 1. Introduction

Transportation problem is one of the most useful and applied branches of linear programming. It deals with finding the optimal distribution of goods from several sources to several destinations. The optimal distribution means that this distribution costs the lowest transportation cost, and there is no other distribution than the optimal one that costs less. Transportation problems can be solved to minimize transportation cost, distance, time, deterioration of goods, vehicle fuel consumption, etc... The transportation problem can also be solved to maximize the companies' earnings, profits, etc.

The transportation problem was first formalized by the French mathematician Gaspard Monge (1871) [1]. Tolstói, A.N. was one of the first to study the transportation problem mathematically in the 1920s. In 1930, on the collection Transportation Planning Volume number 1, for the National Commissariat of Transportation for the Soviet Union, he published a paper Methods of Finding the Minimal Kilometers in Cargo-transportation in space. Tolstói (1939) illuminated his approach by applications to the transportation of salt, cement, and other cargo between sources and destinations along the railway network of the

Soviet Union. In particular, for that large-scale, instance of the transportation problem was solved to optimality. F.L. Hitchcock (1941) worked on the distribution of products from several sources to numerous localities. Koopman also worked on the optimum utilization of the transportation system and used a model of transportation, in activity analysis of production and allocation. It is known as the Hitchcock Koopman transportation problem [2].

Usually, the solution to the transportation problem passes through three main steps. These steps include finding the initial basic feasible solution, the test of optimality, and moving towards optimality. Enormous researches were prepared to find the initial basic feasible solution as well as the other two steps [3-5]. But different ways give the optimal solution directly. Out of these methods are the Lingo codes and excel solver.

More advanced researches were carried out to solve the fuzzy transportation problem, multi-objective transportation problem, multiple criteria transportation problem, multi-index, multi-level transportation problem, and transshipment [2,6-12].

In this paper, we prepared a lingo code to solve the transportation problem while applying this code to solve a real-world problem with actual data. The prepared lingo code based on the transportation problem technique is used to solve a real transportation problem. The actual problem deals with re-distributing flour, which is produced in middle Egypt milestones to warehouses. The company milestones exist in 16 different cities. The flour was required to be distributed to 5 governorate cities, which are 26 major cities.

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## 2. Transportation Problem

The main parts of the transportation problem are the number of sources “ $m$ ”, the number of destinations “ $n$ ” and the unit costs “ $c_{ij}$ ” of transporting goods from each source “ $i=1,2,\dots,m$ ” to each destination “ $j=1,2,\dots,n$ ”. The amounts of goods that should be transported from source “ $i$ ” to destination “ $j$ ” is  $x_{ij}$ . The total transportation cost “ $Z$ ” can be calculated using the following transportation model:

$$\text{Minimize } Z = \sum_{i=1}^m \sum_{j=1}^n c_{ij} x_{ij}, j = 1, 2, \dots, n; i = 1, 2, \dots, m$$

Subject To:

$$\sum_{j=1}^n x_{ij} = a_i, i = 1, 2, \dots, m$$

$$\sum_{i=1}^m x_{ij} = b_j, j = 1, 2, \dots, n$$

$$x_{ij} \geq 0 \text{ for all } i \text{ and } j.$$

By solving the above model of the transportation problem, we'll get the optimal solution to the problem. This model is used to build a lingo code to solve an actual transportation problem, as in the following sections.

## 3. Lingo

LINGO is a well-known and powerful Operational Research software tool available today designed to make building and solving problems easier and more efficient. It is designed to build and solve Linear, Nonlinear, Quadratic, Quadratically Constrained, Semi-Definite, Second Order Cone, Integer and Stochastic optimization models faster, easier, and more efficient.

LINGO includes integrated packages that include a powerful language for the aim of expressing optimization models, a full-featured environment for editing and building problems, and a set of fast built-in solvers.

Lingo lets us formulate our linear, nonlinear and integer

problems quickly in a highly readable format. LINGO modeling language allows us to express models in a straight-forward intuitive manner using summations and subscripted variables, much like we would with pencil and paper. The key benefits of LINGO are easy model expression, convenient data options, create turn-key applications and extensive documentation and help [13].

Models become more comfortable to build, easier to understand, and, therefore, much easier to maintain. LINGO can also exploit multiple CPU cores for faster model generation. Lingo codes were prepared to solve transportation problem examples as in [14-16].

## 4. Case Study

M.E.M.C. is a big company that produces and distributes flour in 5 governorates in Egypt. Their mills exist in 16 cities while the company distributes flour to 26 major cities. The company produces several types of flour.

The collected data of this problem include the distances in kilometers between all cities where mills exist, to all cities where they want to send flour. Also, all mills availabilities and all major cities requirements in tons through the studied month. Table (1) represents the collected data.

A lingo code is prepared to solve the transportation problem case study that is studied in this research. Figure (1) illustrates the prepared lingo code for the case study. The prepared lingo code based on the transportation problem formulation is applied to solve the problem and redistribute flour from the company mills to the major cities. It is found that the total ton-kilometers was 973,060 for the actual distribution for the studied month. But it is reduced to become 883,828 ton-kilometers for the optimal distribution for the same month. The total saving was found to be 89,232 ton-kilometers. That is 9.1702% in savings.

To compare the results of the LINGO code, i.e. the optimal solution with the actual distribution of the problem, we add the actual distribution of the studied transportation problem case study in Table (2). Total ton-kilometers of the actual distribution is 973,060.

**Table 1.** Transportation problem collected data sources capacity and destination demands and distances between them

Distance between sources and destinations		TO Cities Destinations																										Capacity
		St1	St2	St3	St4	St5	St6	St7	St8	St9	St10	St11	St12	St13	St14	St15	St16	St17	St18	St19	St20	St21	St22	St23	St24	St25	St26	
Sources	W1	219	419	28	218	0	151	135	18	120	138	629	84	87	39	514	53	207	193	64	10	182	252	154	247	25	106	382
	W2	123	373	124	122	96	248	39	114	24	235	533	180	183	135	418	149	111	97	160	86	86	126	58	151	71	10	1536
	W3	272	522	45	271	53	98	188	35	173	85	682	31	34	14	517	0	260	246	15	63	235	275	257	300	88	159	1011
	W4	169	419	78	168	50	202	85	680	70	189	579	134	137	89	464	103	157	143	114	40	132	172	104	197	25	56	3580
	W5	372	622	144	371	152	53	288	134	273	40	782	86	97	113	667	99	360	346	114	162	335	375	307	40	177	259	1048
	W6	279	45	452	278	424	571	289	442	304	563	205	508	511	463	90	477	267	253	488	414	242	282	270	307	399	318	281
	W7	345	595	117	344	125	26	261	107	246	13	755	59	70	86	640	72	33	319	87	135	308	348	280	373	150	232	1490
	W8	37	287	210	36	182	334	47	200	62	321	447	266	269	221	321	235	25	11	246	172	0	40	28	65	157	76	2012
	W9	294	544	67	293	75	76	210	57	195	63	704	9	20	36	589	22	282	218	37	85	257	297	229	322	100	181	615
	W10	77	303	194	71	166	318	31	184	46	305	463	250	253	205	337	219	65	51	230	156	40	80	12	105	141	60	3140
	W11	327	577	100	321	108	109	243	90	228	96	737	24	53	69	622	55	315	301	70	118	290	330	262	355	133	214	2297
	W12	148	398	99	147	71	223	64	89	49	210	558	155	158	110	443	124	136	122	135	61	111	151	83	176	46	35	1877
	W13	194	444	53	193	25	176	110	43	95	163	604	109	112	74	489	88	182	168	99	15	157	197	129	22	0	81	156
	W14	237	487	10	231	18	133	153	0	138	120	647	61	69	21	532	35	225	211	46	28	200	240	172	215	43	124	104
	W15	99	349	148	98	120	272	15	138	0	259	509	204	207	159	394	173	87	73	184	11	62	102	34	127	95	14	154
	W16	61	311	234	12	206	358	71	224	86	345	471	290	293	245	345	259	49	32	270	196	24	64	52	41	181	100	2910
Demand		643	32	797	1123	1809	1744	1505	89	745	1295	171	210	979	897	313	27	1108	1294	774	1307	156	413	1449	460	2234	1019	

```

model:
!solving a real 16 sources and 26 destination transportation problem ;
sets:
Sources / S1 S2 S3 S4 S5 S6 S7 S8 S9 S10 S11 S12 S13 S14 S15 S16
/:Capacity;
Destinations / D1 D2 D3 D4 D5 D6 D7 D8 D9 D10 D11 D12 D13 D14 D15 D16
D17 D18 D19 D20 D21 D22 D23 D24 D25 D26 / : Demand;
links ( Sources ,Destinations ): Distance , Volume ;
endsets

! the objective ;
min = @sum(links (I,J): Distance ( I,J )* Volume (I,J));
! THE Demand constraints ;
@for(Destinations (J): @SUM(Sources(I) : Volume(I,J))= Demand(J));
! The Capacity constraints ;
@for (Sources (I): @sum ( Destinations (J) :
Volume(I,J))<= Capacity (I))

! the data ;
data:
Capacity = @ole('C:\Transportation model\Transportation model
data') ;
Demand = @ole('C:\Transportation model\Transportation model
data') ;
Distance = @ole('C:\Transportation model\Transportation model
data') ;
@ole('C:\Transportation model\Transportation model data')= Volume ;
enddata
end

```

Figure 1. Lingo code for the studied transportation problem

Table 2. Transportation problem actual distribution between sources and destination in (ton)

VOLUME	St1	St2	St3	St4	St5	St6	St7	St8	St9	St10	St11	St12	St13	St14	St15	St16	St17	St18	St19	St20	St21	St22	St23	St24	St25	St26	Total
W1					382																						382
W2									517																	1019	1536
W3														897		27			87								1011
W4						195														1307					2078		3580
W5										588														460			1048
W6		32													249												281
W7						237				707							546										1490
W8																	562	1294			156						2012
W9											210	405															615
W10	112						1505	74															1449				3140
W11			332			704						574							687								2297
W12			450		1427																						1877
W13																									156		156
W14			15					89																			104
W15									154																		154
W16	531			1123		608					171				64							413					2910
Total	643	32	797	1123	1809	1744	1505	89	745	1295	171	210	979	897	313	27	1108	1294	774	1307	156	413	1449	460	2234	1019	

Table 3. The optimal distribution of M.E.M.C. flour from sources to each destination

VOLUME	St1	St2	St3	St4	St5	St6	St7	St8	St9	St10	St11	St12	St13	St14	St15	St16	St17	St18	St19	St20	St21	St22	St23	St24	St25	St26
W1	0	0	0	0	382	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
W2	0	0	0	0	0	0	0	0	517	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1019
W3	0	0	0	0	0	0	0	0	0	0	0	0	0	237	0	0	0	0	774	0	0	0	0	0	0	0
W4	0	0	247	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1099	0	0	0	0	0	2234
W5	0	0	0	0	0	1048	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
W6	0	32	0	0	0	0	0	0	0	0	171	0	0	0	78	0	0	0	0	0	0	0	0	0	0	0
W7	0	0	0	0	0	195	0	0	0	1295	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
W8	491	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1108	0	0	0	0	413	0	0	0	0
W9	0	0	0	0	0	0	0	0	0	0	0	0	588	0	0	27	0	0	0	0	0	0	0	0	0	0
W10	119	0	0	0	0	0	1337	0	0	0	0	0	0	0	235	0	0	0	0	0	0	0	0	1449	0	0
W11	0	0	446	0	0	501	0	89	0	0	0	210	391	660	0	0	0	0	0	0	0	0	0	0	0	0
W12	0	0	0	0	1427	0	168	0	228	0	0	0	0	0	0	0	0	0	0	0	54	0	0	0	0	0
W13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	156	0	0
W14	0	0	104	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
W15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	154	0	0	0	0	0	0
W16	33	0	0	1123	0	0	0	0	0	0	0	0	0	0	0	0	0	1294	0	0	156	0	0	304	0	0

The optimal distribution of flour is illustrated in Table (3), while Table (4) summarizes the comparison between actual and optimal distributions. Total ton-kilometers of the optimal distribution is 883,828. This reduces the total ton-kilometers by a value of 89,232, which is a 9.1702% in savings.

**Table 4.** The optimal versus actual distribution of M.E.M.C.

Actual distribution ton-kilometers	973060
Optimal distribution ton-kilometers	883828
Total savings in ton-kilometers	89232
Total savings percentage (%)	9.1702%

## 5. Conclusions

A LINGO code is prepared and applied to solve an actual transportation problem. Lingo is a potent, fast, and easy tool to solve such transportation problems. The case study is a mills company called M.E.M.C. that exists in middle Egypt. We could achieve the optimal distribution of flour from the company mills which exist in 16 cities to 26 major cities. The total ton-kilometers of the actual distribution was 973,060. Total ton-kilometers of the optimal distribution is 883,828. The generated optimal solution could reduce a total of 89,232 ton-kilometers, which represents 9.1702 % of the actual ton-kilometers. Both lingo code, actual and optimal distribution of flour are included in this paper.

## REFERENCES

- [1] Singh S. Note on transportation problem with new method for resolution of degeneracy. *Universal Journal of Industrial and Business Management*. 2015; 3: 26-36.
- [2] El-Sayed M. Ellaimony, Hilal A. Abdelwali, Jasem M. Al-Rajhi, Mohsen S. Al-Ardhi, Yousef Alhouli. Solution of a class of bi-criteria multistage transportation problem using dynamic programming technique. *International Journal of Traffic and Transportation Engineering*. 2015; 4: 115-22.
- [3] Hillier FS. *Introduction to Operations Research with Access Card for Premium Content*: McGraw-Hill Education; 2014.
- [4] Taha HA. *Operations Research: An Introduction*: Prentice Hall; 2011.
- [5] Lee SM, Olson DL. *Introduction to Management Science*: Thomson; 2005.
- [6] Ellaimony E. *Different Algorithms for Solving Multi-Stage Transportation Problems*. Egypt: Helwan University; 1985.
- [7] Alrajhi J, Hilal A. Abdelwali, Elsayed E.M. Ellaimony and Swilem A.M. Swilem. A Decomposition Algorithm for Solving A Class of Bi-Criteria Multistage Transportation Problem With Case Study. *International Journal of Innovative Research in Science Engineering and Technology*. September, 2013; 2.
- [8] Alkhulaifi JA, Hilal A. Abdelwali, Mohsen AlArdhi, Elsayed E. M. Ellaimony, Khalid. An Algorithm for Solving Bi-criteria Large Scale Transshipment Problems. *The Global Journal of Researches in Engineering*. 2014; Vol 14.
- [9] El-Wahed WFA, Lee SM. Interactive fuzzy goal programming for multi-objective transportation problems. *Omega*. 2006; 34: 158-66.
- [10] Ringuest JL, Rinks DB. Interactive solutions for the linear multiobjective transportation problem. *European Journal of Operational Research*. 1987; 32: 96-106.
- [11] Abdelwali HA, Swilem SM, Shiaty RE, Murad MM. Solving A Transportation Problem Actual Problem Using Excel Solver. *International Journal of Engineering and Technical Research*. December 2019; 9.
- [12] Abdelati MH. *Vehicle Transportation Policy Using an Interactive Fuzzy Multi-objective Technique*. Minia, Egypt: Minia University; October 2019.
- [13] Schrage LE. *Optimization Modeling with LINDO*. 5, revised ed: Duxbury Press; 1997.
- [14] Anand Jayakumar A RP, . Solving a Simple Transportation Problem Using LINGO. *International Journal of Innovative Science, Engineering & Technology*. April 2018; 5.
- [15] AP AJ, Raghunayagan P. Solving a Multistage Transportation Problem Using LINGO. *International Journal of Innovative Science, Engineering & Technology*. April 2018; 5.
- [16] Balaji N, A.N. Balaji. Solving the multi-period fixed cost transportation problem using LINGO solver. *International Journal of Pure and Applied Mathematics*. 2018; 119: 2151-7.