

Modeling and Simulation of Metro Transit Station Walkway as a State-dependent Queuing System based on the Phase-Type Distribution

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Abstract Modeling and Discrete-Event Simulation (DES) is usually used in the planning, design and control of traffic and transportation systems. The existing design code for analysis of metro transit station walkways ignores the randomness of passenger arrival and service time of walkways. Moreover the existing analytical approach for analysis is highly complicated and leads to challenging problems. The study reported in this paper details a new DES approach for the analysis of metro transit station walkway considering both the randomness in passenger arrival process and service time of the walkway. The metro transit station walkway is considered as a state-dependent, finite capacity queuing system with the passenger arrival and walkway service process based on the Phase-type (PH) distribution. The three necessary performance measures are obtained by developing a DES model of walkway in *SimEvents* software. The DES results are then compared to the analytical model for the verification purpose. Moreover, the numerical experiments are conducted to assess the differences in the result of the new DES model and the existing models for different walkway configuration. This research provides a new and simplified approach for the analysis of walkways at metro transit stations.

Keywords Queuing System, Discrete-Event Simulation, Metro Transit Station, Walkway

1. Introduction

A metro transit station is an operational system consisting of a framework of infrastructures, facilities, personnel which collectively provide services to passengers. Metro transit stations are points of connection between arrivals and departures. It must ensure an easy transition from a system of continuous arrivals (which is the passenger flow) to a system of discrete departures (which are the scheduled trains).

Queuing occur, usually in case when there are insufficient resources at metro transit stations. The service facilities (ticket service facilities, elevators, stairways, walkways, escalators, platform and etc.) at metro transit stations are often of finite capacity, which means that there is limited space available or limited number of facilities for passengers to get services from the service facilities. The performance assessment of these service facilities may tell us something about the mean (average) time that the

facilities will be in use, or the mean time that a passenger have to wait. Information of such important performance measures can help the planners and designers to make intelligent decisions regarding the demand and capacity as well as when and how to upgrade the service facilities.

The Transit Capacity and Quality of Service Manual (TCQSM) [1] for the planning and design of metro transit station service facilities considers the passenger arrival process (or arrival rate to service facility) and service time (time spent in getting services) of service facilities as deterministic under a given Level of Service (LOS). It means that the service facilities are considered as a D/D/1/C queuing system. The design manual neglects the variation in passenger arrival rate (demand variation) and fluctuation in the service time of the service facilities. Therefore; a new technique is of urgent importance that takes into account randomness in passenger arrival process and service time of service facilities. To narrow the scope of the study, our paper covers only the walkway facility at metro transit stations.

Studies have been conducted in the past using both analytical queuing models and simulation techniques to assess the performance and optimize the service facilities at metro transit stations. Jiang et al. [2] for the first time

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modelled the metro transit station walkway facility as a M/G/1 queuing system with the passenger arrival process based on the exponential distribution and a service time the walkway based on the general random distribution. Ying et al. [3] developed a queuing simulation model for the ticket windows of the metro transit station and optimized the number of ticket windows by considering different parameters. Jiang and Lin [4] developed a Discrete-Event Simulation (DES) model for performance evaluation and optimization of the Ticket Vending Machines (TVM) at metro transit stations using the log-normal distribution for fitting the arrival interval and gamma distribution for fitting the TVM service time. Xu et al. [5] analyzed the Metro transit station Capacity (SSC) as M/G(n)/C/C state dependent queuing network. A new concept according to the gathering and scattering process was defined. Cruz et al. [6] and Khalid et al. [7] established an M/G/C/C state-dependent DES model for pedestrian flow in corridors and network of corridors.

Jiang et al. [8] established a G/G(n)/C/C state-dependent simulation model for the optimization of the metro transit corridor width under the different LOS of the corridor. They have used a gamma distribution to fit the arrival process of passengers and log-normal distribution to fit the service process of the corridor.

From the review of advanced stochastic processes study, we found that the Phase-Type (PH) distribution has substituted the exponential distribution in several fields including; the queuing systems, manufacturing processes and communication systems. The reason to use PH distribution for fitting the arrival interval and service time in queuing system is its own apt analyticity, universality, and computability [9]. Theoretically, it can be fit to any positive random number infinitely that resulted in the emergence of ample PH-based queuing models including PH/PH/1/C by Alfa and Zhao [10] while Jiang et al. [11] for the first time applied the PH distribution to fit the passengers' flow arrival interval distribution at metro transit station that has revealed a good data fitting effect. It has opened the way for using PH distribution in the field of traffic and transportation. Reijsbergen et al. [12] proposed a methodology of constructing stochastic performance model for public transportation network using PH distribution. Hu et al. [13] presented an analytical PH/PH(n)/C/C state-dependent queuing model for the analysis and design of metro transit stations walkway. The state-dependent analytical model of metro transit station walkway is highly complicated and the closed form solution for performance assessment requires some advanced study of stochastic processes and numerical analysis for problem solving. None of the researchers so far considered DES approach based on PH distribution for the analysis of metro transit station walkways. Therefore; in this study, we model and analyze the metro transit station walkway using the DES approach.

DES is the computer evaluation of Discrete-Event Dynamic system models in which chronological sequence of events represents the operation of system [14]. The

SimEvents software offers an integrated setting for modeling dynamic systems that contain discrete-event, and discrete and continuous-time components. By *SimEvents*, the development of a DES model to simulate the flow of passenger through a metro transit station walkway and performance evaluation is possible. In this paper, we develop the metro transit station walkway DES model using PH distribution to evaluate the performance measures. The performance measures obtained by the DES model are then compared with analytical model to assess differences in results.

2. Mathematical Background of PH Distribution

Before, going into the details of PH/PH(n)/C/C DES model based on PH distribution, we first give a brief introduction to PH distribution. PH distribution has been used in queuing modeling in the past because of its versatility and powerful applicability to the solution of various stochastic models that are based on Matrix analytic methods [13]. Phase-Type distribution was introduced first by Neuts [15]. The PH distribution is a probability distribution that is constructed by convolution of exponential distributions. It results from the system of one or more interrelated Poisson processes occurring in phases. The distribution is represented by a random variable that describes the time until absorption of Markov process with 1 absorbing state. Let a Continuous-Time Markov Chain (CTMC) as shown in Figure 1 with state space $S = \{0, 1, 2, 3, 4\}$ where 1, 2, 3, 4 states are the transient states represented by white circles and 0 is the absorbing state represented by a black circle. The λ represents rate of transition from one state to another state.

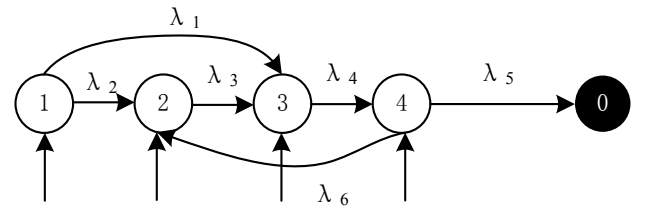


Figure 1. CTMC representations of phase-type distributions

It has initial state probability vector $(a_0, \alpha) = (a_0, a_1, a_2, \dots, a_4)$ with $\sum_{i=1}^4 a_i = 1$ and infinitesimal generator matrix Q given by Eq. (1);

$$Q = \begin{bmatrix} 0 & 0 \\ T^0 & T \end{bmatrix} \quad (1)$$

Where T is a $m \times m$ Matrix and rows in T Matrix must sum to 0. Where T^0 is given by Eq. (2);

$$T^0 = -T1 \quad (2)$$

1 represents a column vector of all 1's.

3. Description of Walkway as a Queuing System

3.1. Passengers' arrival Process

The passengers' arrival process to the metro transit station walkway is specified by the first and second moment of the inter-arrival interval. More specifically, the arrival rate λ and c_a^2 are the traffic descriptors that describe the passenger traffic flow. During the planning and design phase of metro transit stations λ and c_a^2 can be deduced from peak hour volume (q), peak-hour factor (ε), mean headway (h) between trains and squared-coefficient of variation of headway c_h^2 . The peak hour factor can be calculated by using Eq. (3).

$$\varepsilon = \frac{q}{4q_{15}} \quad (3)$$

Where;

q : Peak-15 minute flow rate.

The arrival rate can be obtained by using peak-15 minute flow rate given in [1] by Eq. (4)

$$\lambda = \frac{q}{3600\varepsilon} \quad (4)$$

The value of c_a^2 can be calculated by different ways [11].

If only peak-hour factor is available, then c_a^2 can be calculated using Eq. (5)

$$c_a^2 = \frac{e^{6.819\varepsilon}(\varepsilon-1)^2}{(4\varepsilon-1)} \quad (5)$$

If headway (h) and c_h^2 between the trains are available, then c_a^2 can be calculated by using Eq. (6)

$$c_a^2 = e^{0.503c_h^2} \left[\left(\frac{qh}{3600\varepsilon} \right) - 1 \right] \quad (6)$$

When the number of passengers increases on the metro transit station walkway; the mean and standard deviation of their speed decrease because the number of passengers on the walkway are limited to walkway capacity ' C '. The speed decreases approximately to 0 when the number of passengers is $C = 5LW$. Here 5ped/m^2 is the density of passengers on the walkway [16].

According to traffic flow theory; the flow rate (μ) is given as follows;

$$\mu = kV \quad (7)$$

In case of the metro transit station walkways, flow rate is the number of passengers passing through the walkway section per second. The reciprocal of flow rate $1/\mu$ is referred as the time interval of the passengers leaving the walkway. This time interval $T_n = 1/\mu$ is the service time of the walkway. The service time is expressed as $T_n = L/V_n$. Therefore; the state-dependent service time of the walkway is actually due to state-dependence of passengers speed on that walkway. The state-dependent passengers speed V_n and squared coefficient of variation $c_{s,n}^2$ of the walkway is given by Eq. (8) and Eq. (9) respectively;

$$V_n = V_1 \exp \left[- \left(\frac{n-1}{\omega} \right)^\gamma \right], \text{ where } n=1,2,3,\dots,C \quad (8)$$

$$c_{s,n}^2 = \left[\left(\frac{\delta_1}{v_1} \right) \exp \left(\left(\frac{n-1}{\omega} \right)^\gamma - \left(\frac{n-1}{\omega'} \right)^{\gamma'} \right) \right]^2 \quad (9)$$

Where;

$$\gamma = \frac{\ln \left(\frac{v_a}{v_1} \right) - \ln \left(\frac{a-1}{b-1} \right)}{\ln \left(\frac{v_b}{v_1} \right) - \ln \left(\frac{a-1}{b-1} \right)}, \quad \omega = (a-1) \left[\ln \left(\frac{v_1}{v_a} \right) \right]^{1/\gamma}$$

$$\gamma' = \frac{\ln \left(\frac{\delta_a}{\delta_1} \right) - \ln \left(\frac{a-1}{b-1} \right)}{\ln \left(\frac{\delta_b}{\delta_1} \right) - \ln \left(\frac{a-1}{b-1} \right)}, \quad \omega' = (a-1) \left[\ln \left(\frac{\delta_1}{\delta_a} \right) \right]^{1/\gamma'}$$

$$a = 2LW, b = 4LW$$

The values of v_1 and δ_1 are the mean and standard deviation of speed when there is only one passenger on the walkway. The values of v_a and δ_a are the mean value and standard deviation of speeds for $a = 2LW$ passengers on the walkway respectively. The values v_b and δ_b are the mean value and standard deviation of speed for $b = 4LW$ passengers on the walkway respectively. The values $(v_1, \delta_1), (v_a, \delta_a), (v_b, \delta_b)$ are the representative points for calculation of state-dependent service rate given by Hu et al. [13].

The passengers after entrance to walkways occupy the spaces. Each space on the walkway act like a server and passenger flow is represented by arrows in Figure 2. Since, the capacity of the walkway is C , thus there are C servers that provide the services to the passengers on the walkway. Hu et al. [13] have achieved good fitting for arrival interval to metro transit stations as well as for state-dependent service time of walkway using the PH distribution with any coefficient of variation. Therefore; in this research, we use the PH distribution to fit the arrival interval and state-dependent service time of the walkway and the passengers-walkway system can be described as

PH/PH(n)/C/C state-dependent queuing system. In order to simplify the multi-server passengers-walkway queuing system, the C servers are equivalently transformed to single server with an average service time interval equal to $T_n = \frac{1}{n\mu_n}$. It means that PH/PH (n)/C/C queuing system is also equivalently converted to PH/PH (n)/1/C queuing system as shown in Figure 2. The average state-dependent service rate of walkway can now be written as Eq. (10) and squared coefficient of variation $c_{s,n}^2$ can be obtained from Eq. (9).

$$\mu_n = \frac{nV_1 \exp\left[-\left(\frac{n-1}{\omega}\right)\right]}{L}, \text{ where } n = 1, 2, 3, \dots, C \quad (10)$$

4. Fitting PH Distribution to Arrival and Service Process

As discussed in the previous section, λ and c_a^2 are two parameters that describe the arrival of passengers to the walkway whereas the other two parameters that describe

walkway state-dependent service process specified by μ_n and $c_{s,n}^2$. The PH distribution for arrival and the services processes is represented by an initial probability vector α and a Matrix T and β_n, S_n respectively.

Arrival process: $A \sim \text{PH}(\alpha, T)$

Service process: $B_{(n)} \sim \text{PH}(\beta_n, S_n) \quad n = 1, 2, \dots, C$

The parameters $(\alpha, T), (\beta_n, S_n)$ can be calculated by using λ, c_a^2, μ_n and $c_{s,n}^2$ under certain conditions that are described [18] and [13] given in Table 1.

5. Discrete-Event Simulation Model Architecture

The DES approach based on the PH distribution involve the generation of random variates from the PH distributions. Neuts [15] proposed a ‘Play and Count’ approach for the generation of PH random variates. The DES approach eliminates the need to solve large matrices and equations as in analytical approach in order to estimate the performance measures.

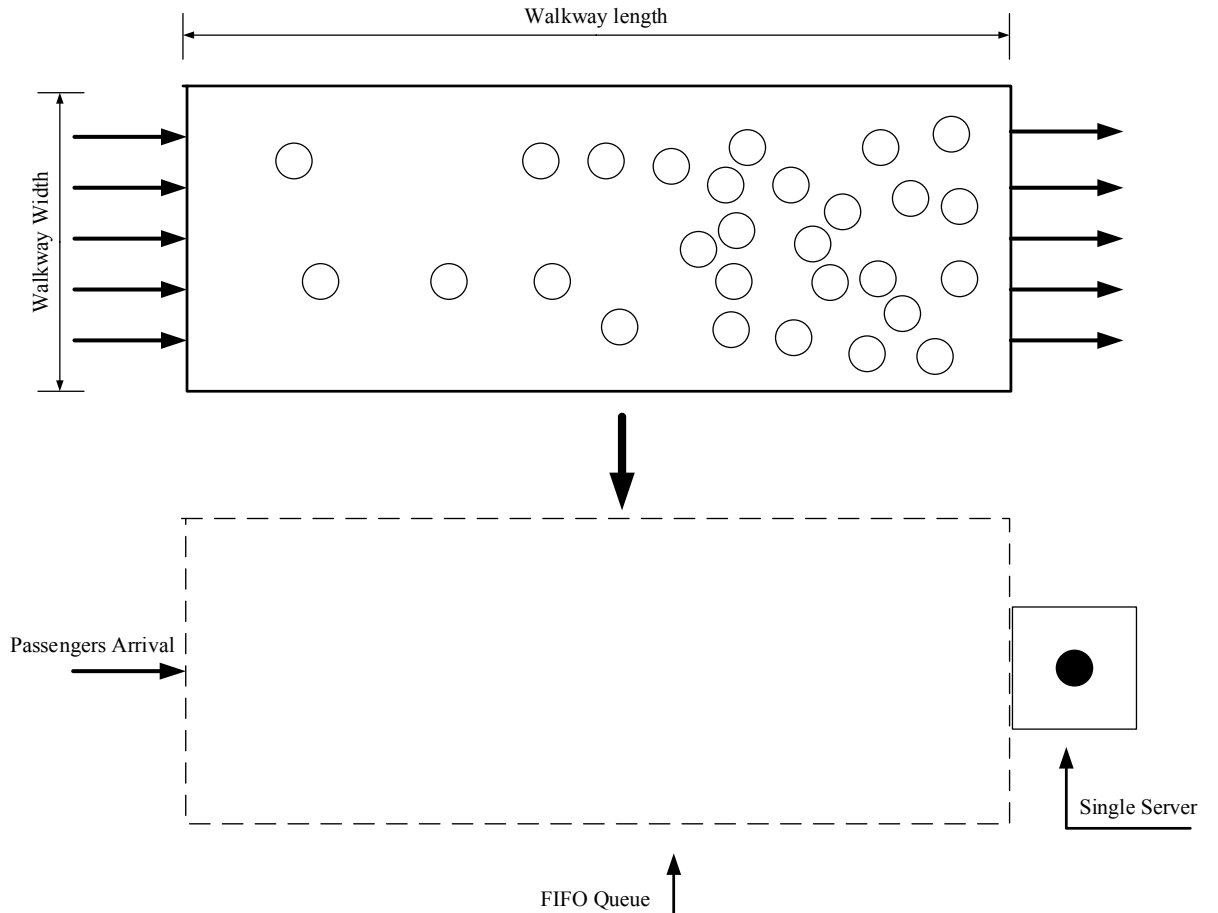


Figure 2. Abstraction of metro transit station walkway as queuing system

Table 1. Conditions for Phase-Type (PH) Distribution

Distribution	c^2	α	T
Hypo-Exponential	<1	$\alpha = (1, 0, \dots, 0)$	$\mathbf{T} = \begin{pmatrix} -\lambda_0 & \lambda_0 & & & \\ & -\lambda_0 & \lambda_1 & & \\ & & \ddots & \ddots & \\ & & & -\lambda_{w-2} & \lambda_{w-2} \\ & & & & -\lambda_{w-1} \end{pmatrix}$
Hyper-Exponential	>1	$\alpha = \frac{1}{2} + \frac{1}{2} \sqrt{\frac{c^2 - 1}{c^2 + 1}}$	$\mathbf{T} = \begin{pmatrix} \frac{-2p}{E[X]} & 0 \\ 0 & \frac{-2(1-p)}{E[X]} \end{pmatrix}$
Exponential	$= 1$	1	$-\lambda$
Fixed-Length	≤ 0.1	Approximate as Erlang-10 Distribution	

Where;

$$\lambda_i = wE[X]^{-1}$$

$$\lambda_{w-1} = \frac{2w}{(2 + w - w^2c^2)E[X]} \left[1 + \sqrt{0.5w(wc^2 - 1)} \right],$$

$$\lambda_{w-2} = \frac{0.5w\lambda_{w-1}}{(\lambda_{w-1}E[X] - w)}$$

for $0 \leq i < w - 2$

5.1. Generation of Phase-Type (PH) Random Variates

The Neuts [15] procedure relies on generating an Erlang-distributed sample with degree f and parameter λ given as:

$$\text{Erl}(f, \lambda) = -\frac{1}{\lambda} \ln \left(\prod_{j=1}^f U_j \right)$$

Where, U is uniformly distributed random number $[0, 1]$. Let b_j represent the row vector with 1 at the position j . A pseudo-codes description of the PH random variates used in this work follows:

```

1)  $x_{ph} := 0, f_j = 0$ , for  $j = 1, 2, \dots, n$ , Draw an  $\alpha$ -distributed discrete sample for the initial state.
2) The chain in the state  $j$ ,
   i.  $f_j += 1$ 
   ii. a  $b_j(-\text{diag}(1/t_{jj}, 0)T + I)$ -distributed discrete sample is drawn for the next state,
   iii. in case the next is absorbing state then go to 3 otherwise stay at 2 and repeat
       for  $j=1, 2, \dots, n$ ;
3) do  $x_{ph} += \text{Erl}(f_j, -t_{jj})$ ;
   done
Return  $x_{ph}$ 

```

Figure 3 is a *SimEvents* model of the metro transit station walkway consisting of two phases, i.e., passengers' generation phase; state-dependent service phase. The *SimEvents* software is a "queues and servers" style DES extension of the MATLAB[®] product family.

The main components of the scheme are a *FIFO queue* representing the queueing space, *Single-Server*, *Start timer* and *Read Timer* and *Level-2 MATLAB[®] S-function* blocks that compute and updates the different parameters in the model. In addition to the main blocks, there are two Gateway blocks used in DES model, i.e., *Event to Timed Signal* blocks and *Timed to Event Signal*. They adapt event-driven imperative and the time-driven declarative environments of *SimEvents* respectively. The *Timed to Event Signal* Gateway block output and input signals are identical. But output signal from this block can now be used as an input to the blocks that require event-based input signals. Similarly; *Event to Timed Signal Gateway* block output signals can be used for the time-based dynamics modeling. It gives a single value at given time instant on the simulation clock. Similarly; *Constant blocks* and *Display* blocks are used in the model to input different values and show the results respectively.

5.2. Passengers Generation Phase

The *Level-2 MATLAB[®] S-function block* at the passengers generation phase computes the PH random variates to initiate the simulation process using procedure given by Neuts [15]. The two input parameters for the computation of PH random variates are mean arrival rate λ and squared coefficient of

5.3. State-Dependent Service Phase

To implement this condition, the generated passengers are stored in the *FIFO Queue* block before being delayed by the Single Server block and subsequently being sent to the Entity Sink. There are four *Level-2 MATLAB® S-function blocks* in state-dependent service phase that computes the state-dependent service rates based on PH random variates, average area occupied per passengers, blocking probabilities and judge the number of passengers to prevent them from entry when maximum capacity $C = 5LW$ has been reached. The service rate depends on congestion of the area. The block takes capacity and passengers queue length value as input to compute the state-dependent service rate μ_n and state-dependent squared coefficient of variation of service rate $c_{s,n}^2$ using Eq. (10) and Eq.(9) respectively. The *Digital Clock* and *IC blocks* are used in the computation of throughput values.

6. Computational Experiments

6.1. Verification of PH/PH(n)/C/C DES Model

The average relative error of PH/PH(n)/C/C DES is 2.1% for the average number of passengers on the walkway E [N], 2.9% for the average area occupied per passenger ES and 3.2% for average time spent by the passenger on the walkway E[T]. The comparison shows that PH/PH(n)/C/C DES model has a smaller average relative error and indicates that PH/PH(n)/C/C DES model can be used with good accuracy in performance evaluation of metro transit station walkways. It has been observed that the average number of passengers on the walkway E[N] increase with the increase of passengers' arrival rate λ as shown in the Figure 4.

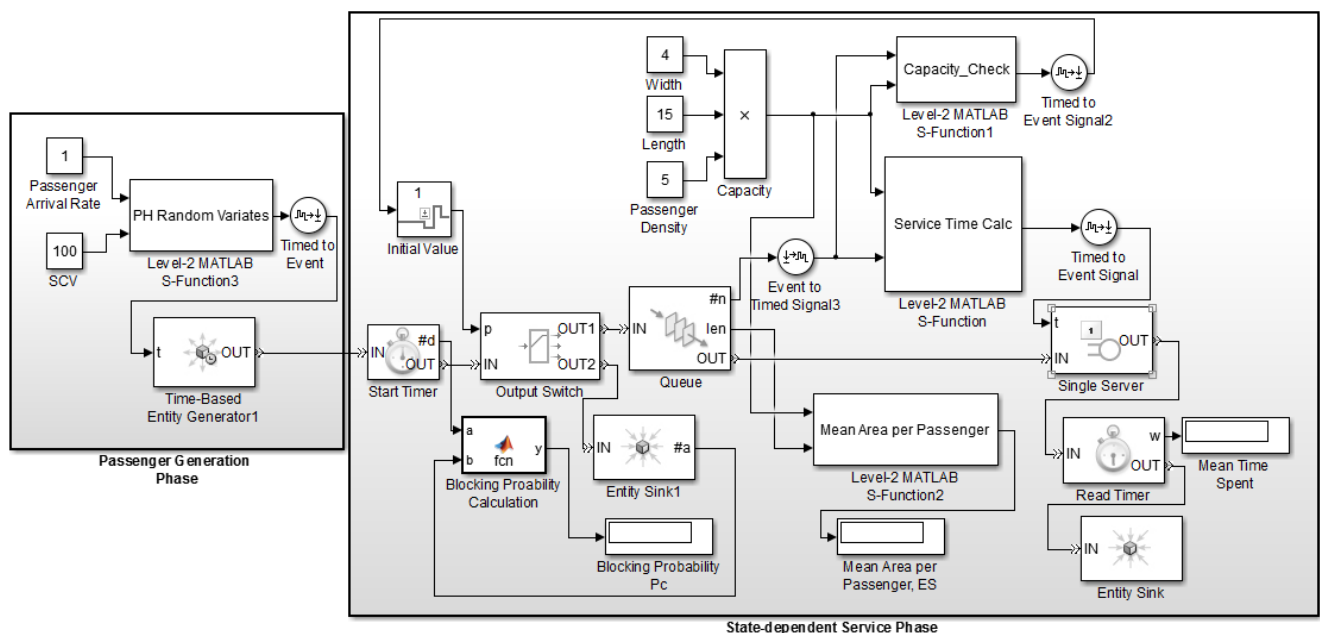


Figure 3. *SimEvents* Implementation of metro transit station walkway

Similarly; the average area occupied per passenger ES decreases with the increase in the arrival rate shown by Figure 5. It is because of the fact that more and more passengers occupy the spaces on walkway thus reducing the distances among themselves. The average time spent by the passengers on the walkway $E[T]$ increases with increase in

arrival rate. It is quite obvious that increase in number of passengers due to high arrival rate results in decreasing walking speed of passengers and thus passengers spend more time on the walkway to travel from one end to another end as in Figure 6.

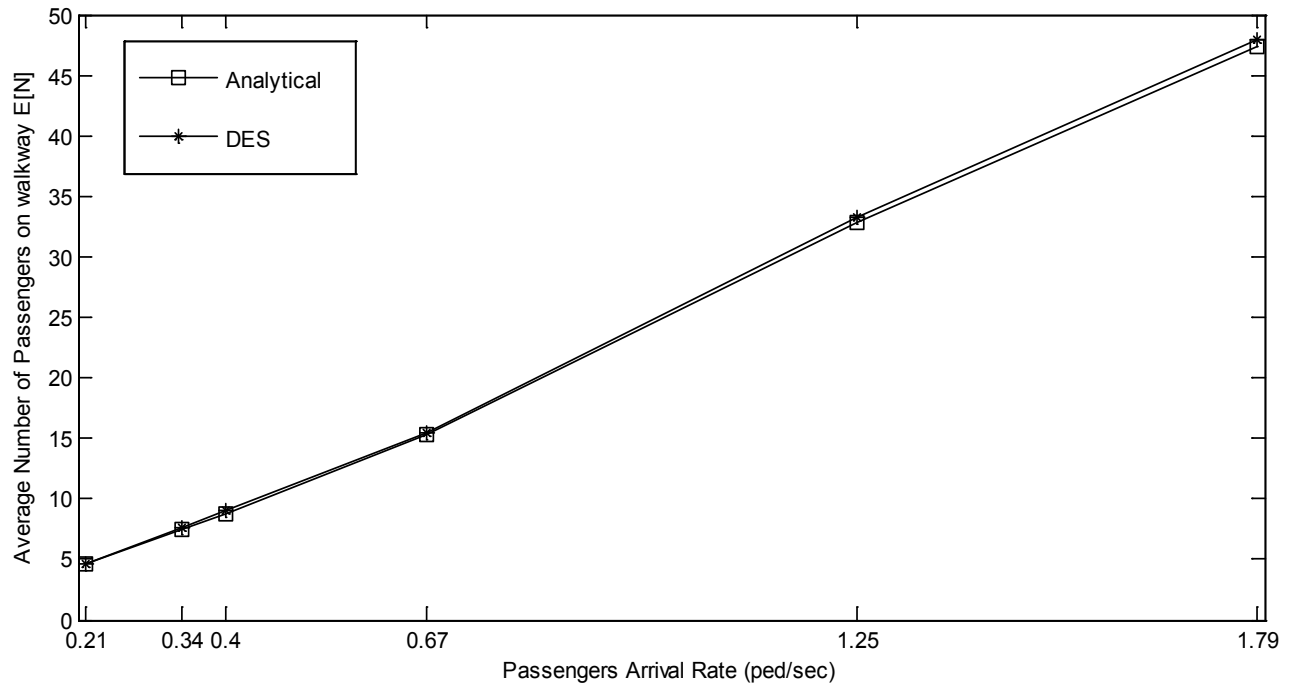


Figure 4. Average Number of passengers on walkway

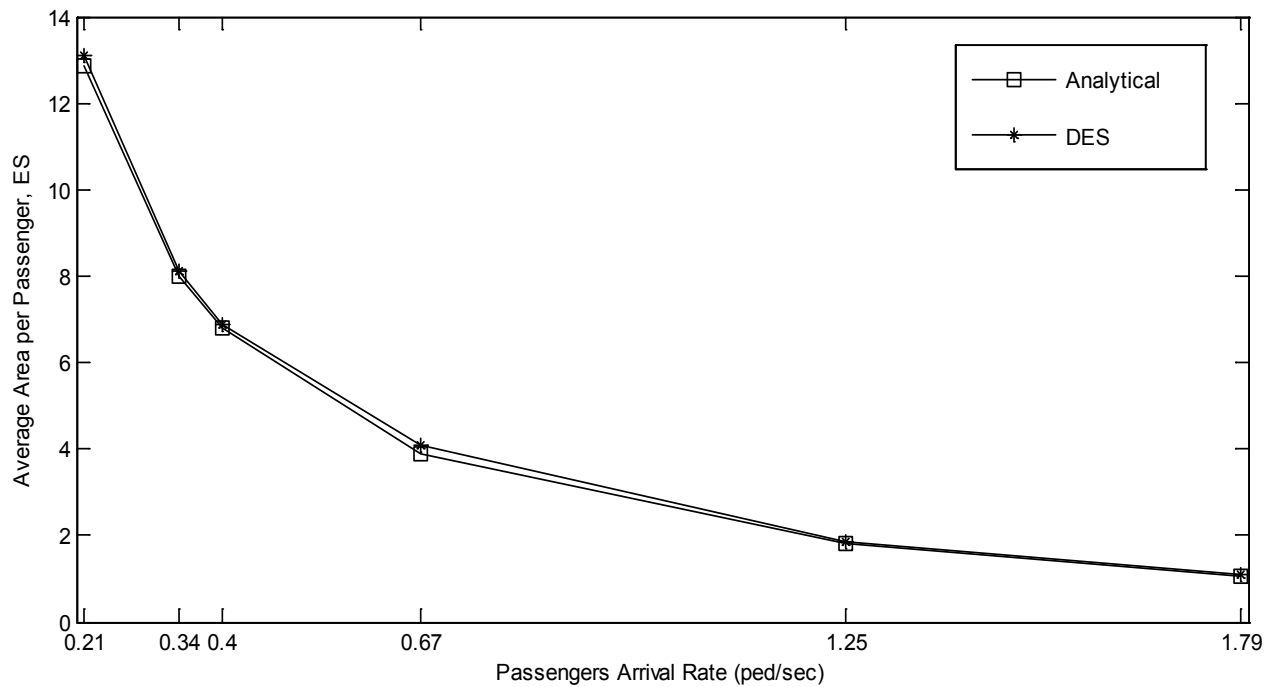


Figure 5. Average area occupied per passenger

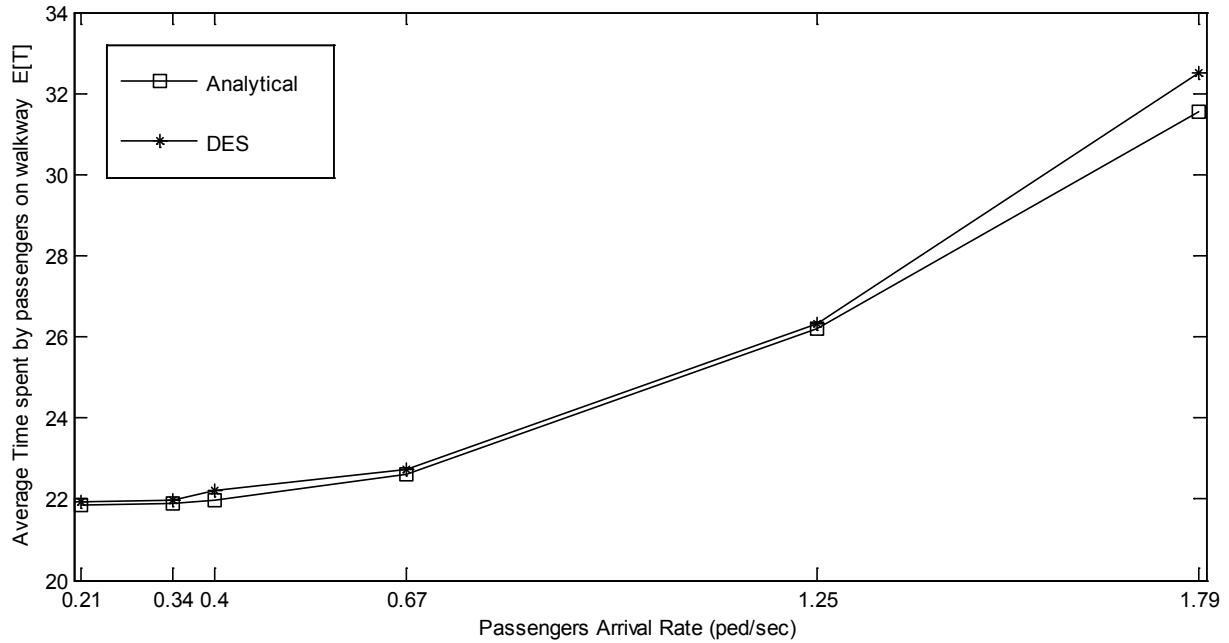


Figure 6. Average time spent on the walkway

Table 2. Comparison of PH/PH(n)/C/C DES model with the existing models

Passenger arrival rate λ	Squared coefficient of variation c_a^2	EN		P_c		ES (m ² /ped)		Throughput θ (ped/s)	
		15x4m ²	30x2m ²	15x4m ²	30x2m ²	15x4m ²	30x2m ²	15x4m ²	30x2m ²
1.50	1/30	17.43	37.75	0.00	0.00	3.44	1.58	1.50	1.50
	1	19.22	40.45	0.00	0.00	3.12	1.48	1.50	1.50
	50	21.53	44.32	0.00	0.00	2.79	1.35	1.50	1.50
	100	25.25	54.64	0.00	0.02	2.37	1.09	1.50	1.47
	500	27.67	109.11	0.00	0.13	2.19	0.54	1.50	1.31
2.00	1/30	23.44	50.03	0.00	0.00	2.63	1.20	2.00	0.00
	1	26.74	58.32	0.00	0.00	2.31	1.03	2.00	0.00
	50	30.05	110.83	0.00	0.03	2.05	0.54	2.00	1.94
	100	35.65	138.25	0.00	0.13	1.71	0.43	2.00	1.74
	500	39.89	154.11	0.00	0.23	1.53	0.39	2.00	1.53

6.2. Comparison with the Existing Models

Table 2 provides the comparison of performance measure of the walkway estimated by using and PH/PH(n)/C/C DES model and existing analytical M/G(n)/C/C [6] and D/D/1/C queuing model [1]. Two different sizes of walkways, i.e., 30x2m² and 15x4m² are considered for the experiments. The passengers arrival rates $\lambda = 1.5$ and $\lambda = 2$ ped/sec and $c_a^2 = 50, 100$ and 300 are chosen according to the actual range in metro transit station. The three representative points for walking speed on walkway are $(v_1 = 1.50, \delta_1 = 0.50)$, $(v_a = 0.64, \delta_a = 0.21)$ and $(v_b = 0.25, \delta_b = 0.08)$ respectively Hu et al. [13]. For comparison purpose, the performance measures of walkway such as, average number of passengers

on the walkway $E[N]$, average area occupied per passenger ES, blocking probability P_c and Throughput θ are estimated. In TCQSM [1], the walkway is analyzed by using fixed-length distribution. It means that the analysis procedure in the TCQSM uses fixed arrival rate and a fixed service time which is similar to D/D/1/C analytical queuing model Hu et al. [3]. The c_a^2 is equal to 1/30 (0.03). Moreover, existing model based on the exponential distribution, i.e., M/G(n)/C/C uses $c^2=1$.

The first thing to note that the $E[N]$ increases with increase in arrival rate which is quite obvious as higher the arrival rate, higher the number of passengers on the walkway. It is also observed that with increase in c_a^2 , $E[N]$ and P_c increase while $E[S]$ decrease. It is expected because of the fact that PH/PH(n)/C/C DES model accounts for randomness of

passengers arrival rate as well as randomness and state-dependence of circulation facilities service time compared to M/G(n)/C/C and D/D/1/C model.

Similarly, the blocking probability P_c increase with increase in c_a^2 . The higher c_a^2 results in increase in $E[N]$ but since the capacity of walkway are limited to C, thus the passenger flow is blocked.

7. Concluding Remarks

In this study, metro transit station walkway DES model based on PH distribution was constructed to evaluate the performance of metro transit station walkways. The developed model analyzed the system quite well and achieved the accurate results. The planners and designers of metro transit station can use this model for performance assessment of metro transit station walkway to make intelligent decisions. Moreover, this DES model is more realistic and simple compared to the analytical approach. It has been observed that the squared coefficient of variation has a significant impact on the performance measure, therefore; enough attention should be paid to it during the analysis of walkways which is neglected by the existing models.

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