International Journal of Traffic and Transportation Engineering 2016, 5(4): 96-102
DOI: 10.5923/j.ijtte.20160504.03

3D Position Estimation Performance Evaluation of a Hybrid Two Reference TOA/TDOA Multilateration System Using Minimum Configuration

Abdulmalik S. Yaro[1,*], Muazu J. Musa[1], Salisu Sani[1], Abdulrazaq Abdulaziz[2]

[1]Department of Electrical and Computer Engineering, Ahmadu Bello University, Zaria, Nigeria
[2]Federal University Gusau, Zaria Road Gusau, Nigeria

Abstract A passive multilateration surveillance system estimate aircraft location using time difference of arrival (TDOA) with a lateration algorithm. It has advantage over multiangulation system as it can estimate aircraft positions in 3 Dimension (3D). In this paper, two reference hybrid TOA/TDOA close form lateration algorithm is develop using the minimum number of antenna stations for 3D position estimation (PE). Using a triangular configuration, the performance in aircraft PE of the developed lateration algorithm is compared with the two fix reference TDOA close form lateration using Monte Carlo simulation for some selected aircraft locations and TOA error standard deviation range of 0 meters to 2 meters. Simulation results comparison shows that the performance of the hybrid TOA/TDOA lateration algorithm in aircraft PE depends on the aircraft location relative to the antenna station configuration. The hybrid system can only estimate aircraft horizontal coordinate with high accuracy outside the antenna station constellation. It fails compare to the two fix reference lateration algorithm to estimate the altitude of the aircraft for at any location within or outside the antenna station constellation.

Keywords Multilateration, 3D position estimation, Hybrid TOA/TDOA, Minimum configuration

1. Introduction

Passive multilateration (MLAT) surveillance system estimate aircraft location in two steps [1]. The first step involves estimating the time difference of arrival (TDOA) of the signal at antenna pairs. The second step involves using the estimated TDOA measurements from the first stage as input to a position estimation (PE) algorithm known as lateration. 2D or 3D aircraft PE depends on the number of antenna deployed [2]. For a 3D aircraft PE, a minimum of 4 antennas are required. Several techniques for estimating TDOA have been reported in literatures [3-7] but the classical approach use in air traffic surveillance is the TOA approach [8]. TDOA estimation using the TOA approach involves a pair wise difference of TOA measurements of the signal estimated at each antenna. TOA of the signal is the time taken for the transmitted signal from the aircraft to be detected at any of the antennas. Several techniques for TOA estimation has been reported in literatures [9-13] but this research is focused on using these TOA measurements for PE process. Thus, the focus of this research is the lateration algorithm.

TDOA measurements when converted to distance results in hyperbolic equations. N number of TOA measurements will result in $N-1$ hyperbolic equations [14]. In the hyperbolic equation, there is a nonlinear relationship between the TDOA measurements and the aircraft location. To solve for the aircraft location, a linear relation between the two parameters (TDOA measurements and the aircraft location) must be obtained. Several articles have reported on how this linear relationship is obtained which resulted in the different types of lateration algorithm. The different types of lateration algorithms can be summarized in to open form and close form [15]. Open form lateration algorithm involves the use of linearization techniques such as Taylor series method to linearize the hyperbolic equation and a random initial aircraft location to be inputted which is then refined using the iteration method [14-19]. This approach suffers from convergence issue if the initial aircraft location inputted is far from the actual aircraft location [19]. Close form lateration algorithm linearized the two parameters through algebraic manipulation of the hyperbolic equations [20-24]. This approach does not require any initial aircraft location thus, do not suffer convergence issue. For this reason, the close form lateration algorithm is adopted in this work. The use of multiple reference antenna to improve the PE accuracy of the lateration algorithm has been discussed in [25].

PE using hybrid TOA/TDOA measurement has been

* Corresponding author:
yaroabdulmalik@yahoo.com (Abdulmalik S. Yaro)
Published online at http://journal.sapub.org/ijtte

discussed in several articles [26-31]. The use of hybrid TOA/TDOA to reduce wireless signal overhead associated with TOA systems and number of antenna for TDOA system is presented in [26]. A comparison between hybrid TOA/TDOA 2D positioning system and 2D TDOA system in terms of PE accuracy is done in [31]. It was founded out that 2D PE accuracy of the hybrid TOA/TDOA system is superior to the 2D TDOA system for far area emitter locations while the opposite is said for emitter locations in the near area.

In [20], a performance evaluation of a minimum antenna configuration MLAT system for 3D PE is carried out. This work propose to improve the PE accuracy of the MLAT system using a two reference hybrid TOA/TDOA lateration algorithm. The 3D PE accuracy of the proposed hybrid system will be evaluated and compared with two fix reference TDOA lateration algorithm used in [20] using Monte Carlo simulation for a triangular antenna configuration.

The reminder of the paper is organized as follows. The two reference hybrid TOA/TDOA lateration algorithm is described in section 2 which is then followed by simulation results and discussion in section 3. Finally the conclusion from the research is presented in section 4.

2. Two Reference Hybrid TOA/TDOA Lateration Algorithm

This section gives a detail description of the two reference hybrid TOA/TDOA lateration algorithm. It is assumed that the TOA measurements of the received signal have already been estimated at each antenna.

Let τ_i be the TOA measurement estimated at the i-th antenna station with coordinates (x_i, y_i, z_i). The equation for the TOA measurement of the signal at the i-th antenna station transmitted from an emitter located at $\mathbf{x} = (x, y, z)$ is

$$\tau_i = \frac{\sqrt{(x-x_i)^2 + (y-y_i)^2 + (z-z_i)^2}}{c} \tag{1}$$

where $c = 3\times10^5\, km$

From eq. 1, there is a nonlinear relationship between the aircraft location ($\mathbf{x}$) and the TOA measurement (τ). Taking the square of both sides in eq. 1 will results to

$$(\tau_i \times c)^2 = (x-x_i)^2 + (y-y_i)^2 + (z-z_i)^2 \tag{2}$$

Further simplification of eq. 2 will results in

$$\begin{aligned}(\tau_i \times c)^2 &= \left(x^2+y^2+z^2\right) - \left(2xx_i + 2yy_i + 2zz_i\right) \\ &+ \left(x_i^2 + y_i^2 + z_i^2\right)\end{aligned} \tag{3}$$

Let

$$\begin{aligned}\tau_i \times c &= R_i \\ R &= \left(x^2+y^2+z^2\right) \\ K_i &= \left(x_i^2+y_i^2+z_i^2\right)\end{aligned} \tag{4}$$

Substituting eq. 4 into eq. 3, the resulting expression is

$$R_i^2 = R - \left(2xx_i + 2yy_i + 2zz_i\right) + K_i \tag{5}$$

For $N = 4$, the TOA equations are

$$R_1^2 = R - \left(2xx_1 + 2yy_1 + 2zz_1\right) + K_1 \tag{6a}$$

$$R_2^2 = R - \left(2xx_2 + 2yy_2 + 2zz_2\right) + K_2 \tag{6b}$$

$$R_3^2 = R - \left(2xx_3 + 2yy_3 + 2zz_3\right) + K_3 \tag{6c}$$

$$R_4^2 = R - \left(2xx_4 + 2yy_4 + 2zz_4\right) + K_4 \tag{6d}$$

In eq. 6, the variable R is a function of the aircraft location which is unknown in passive systems. Subtracting eq. 6b from eq. 6a and 6d from 6c will result into two equation.

$$\begin{aligned}R_1^2 - R_2^2 &= 2x(x_1-x_2) + 2y(y_1-y_2) \\ &+ 2z(z_1-z_2) + K_1 - K_2\end{aligned} \tag{7}$$

$$\begin{aligned}R_3^2 - R_4^2 &= 2x(x_3-x_4) + 2y(y_3-y_4) \\ &+ 2z(z_3-z_4) + K_3 - K_4\end{aligned} \tag{8}$$

Eq. 7 and eq. 8 represent a linear relationship between the TOA measurements and the aircraft location. Using eq. 7 and eq. 8 to solve for the aircraft location will not be sufficient as there are 3 unknowns and 2 equations. Using TDOA approach, additional equation can be obtained.

The TDOA of received signal at two separate antenna stations can be obtained by performing a pairwise difference between the TOA measurements from each antenna station. Mathematically, TDOA measurement between the i-th and the j-th antenna station pair is obtained as

$$\begin{aligned}\tau_{ij} = \tau_i - \tau_j &= \frac{\sqrt{(x-x_i)^2 + (y-y_i)^2 + (z-z_i)^2}}{c} \\ &- \frac{\sqrt{(x-x_j)^2 + (y-y_j)^2 + (z-z_j)^2}}{c}\end{aligned} \tag{9}$$

With 4 antenna stations and $i=1$ & $i=2$ as reference pair for obtaining the TDOAs, 4 number TDOA hyperbolic equations are obtained as

$$\begin{aligned}\tau_{13} \times c &= \sqrt{(x-x_1)^2 + (y-y_1)^2 + (z-z_1)^2} \\ &- \sqrt{(x-x_3)^2 + (y-y_3)^2 + (z-z_3)^2}\end{aligned} \tag{10a}$$

$$\begin{aligned}\tau_{14} \times c &= \sqrt{(x-x_1)^2 + (y-y_1)^2 + (z-z_1)^2} \\ &- \sqrt{(x-x_4)^2 + (y-y_4)^2 + (z-z_4)^2}\end{aligned} \tag{10b}$$

$$\tau_{23} \times c = \sqrt{(x-x_2)^2 + (y-y_2)^2 + (z-z_2)^2} - \sqrt{(x-x_3)^2 + (y-y_3)^2 + (z-z_3)^2} \quad (10c)$$

$$\tau_{14} \times c = \sqrt{(x-x_1)^2 + (y-y_1)^2 + (z-z_1)^2} - \sqrt{(x-x_4)^2 + (y-y_4)^2 + (z-z_4)^2} \quad (10d)$$

Further simplification eq.10 will results in 2 plane equations in the form [20]

$$a_{134} = x \times b_{134} + y \times c_{134} + z \times d_{134} \quad (11)$$

$$a_{234} = x \times b_{234} + y \times c_{234} + z \times d_{234} \quad (12)$$

where the coefficients of eq. 4 and eq. 5 for $1 \le i \le 2$ and $3 \le j \le 4$ are

$$a_{i34} = 0.5\left(R_{i4} - R_{i3} + \frac{K_{i4}}{d_{i4}} - \frac{K_{i3}}{d_{i3}} \right)$$

$$b_{i34} = \left(\frac{x_{3i}}{R_{i3}} - \frac{x_{4i}}{R_{i4}} \right)$$

$$c_{i34} = \left(\frac{y_{3i}}{R_{i3}} - \frac{y_{4i}}{R_{i4}} \right)$$

$$d_{i34} = \left(\frac{z_{3i}}{R_{i3}} - \frac{z_{4i}}{R_{i4}} \right)$$

$$x_{ji} = x_j - x_i,\ y_{ji} = y_j - y_i,\ z_{ji} = z_j - z_i$$

$$K_{ij} = (x_i^2 + y_i^2 + z_i^2) - (x_j^2 + y_j^2 + z_j^2) \quad (13)$$

Detail description of the simplification can be found in [20]. Eq. 11 and eq. 12 are the additional equations obtained using the TDOA approach. Combining these equations with eq. 7 and eq. 8 will result in the hybrid TOA/TDOA lateration algorithm as shown below in matrix form of eq. 14.

$$\mathbf{Ax} = \mathbf{b} \quad (14)$$

where

$$\mathbf{A} = \begin{bmatrix} 2(x_1 - x_2) & 2(y_1 - y_2) & 2(z_1 - z_2) \\ 2(x_3 - x_4) & 2(y_3 - y_4) & 2(z_3 - z_4) \\ \left(\frac{x_{31}}{R_{13}} - \frac{x_{41}}{R_{14}} \right) & \left(\frac{y_{31}}{R_{13}} - \frac{y_{41}}{R_{14}} \right) & \left(\frac{z_{31}}{R_{13}} - \frac{z_{41}}{R_{14}} \right) \\ \left(\frac{x_{32}}{R_{23}} - \frac{x_{42}}{R_{24}} \right) & \left(\frac{y_{32}}{R_{23}} - \frac{y_{42}}{R_{24}} \right) & \left(\frac{z_{32}}{R_{23}} - \frac{z_{42}}{R_{24}} \right) \end{bmatrix} \quad (15a)$$

$$\mathbf{b} = \begin{bmatrix} \left(R_1^2 - R_2^2 - K_1 + K_2 \right) \\ \left(R_3^2 - R_4^2 - K_3 + K_4 \right) \\ 0.5\left(R_{14} - R_{13} + \frac{K_{14}}{R_{14}} - \frac{K_{13}}{R_{13}} \right) \\ 0.5\left(R_{24} - R_{23} + \frac{K_{24}}{R_{24}} - \frac{K_{23}}{R_{23}} \right) \end{bmatrix} \quad (15b)$$

$$\mathbf{x} = [x, y, z]^T \quad (15c)$$

The location of the emitter can be obtaining by solving the inverse matrix eq. 14 given the TOA measurements and coordinate of the antenna stations. The matrix eq. 14 is an over determined total least square (TLS) problem which can be solved using Singular Value decomposition (SVD).

2.1. PE Estimation Using SVD TLS

Let **C** be an augment of **A** and **b** mathematically expressed as

$$\mathbf{C} = [\mathbf{A}, \mathbf{b}] \quad (16)$$

Taking the SVD of **C** as [32]

$$\mathbf{C} = \mathbf{U}\Sigma \mathrm{V}^T = \sum_{i=0}^{n+1} u_i \sigma_i v_i^T \quad (17)$$

The solution to matrix eq. 14 is obtain as

$$x = \frac{-1}{v_{n+1,n+1}} \times \left[v_{1,n+1} \cdots v_{n,n+1} \right]^T \quad (18)$$

3. Simulation Results and Discussion

In this section, the PE performance accuracy of the two reference hybrid TOA/TDOA lateration algorithm is compared with the two fix reference TDOA lateration algorithm implemented in [20]. Monte Carlo simulation is used to generate results for comparison using a triangular antenna station configuration. Position root mean square error (RMSE) is used as the performance measure for the comparison. Horizontal coordinate RMSE and the altitude RMSE are mathematically expressed as

$$Horizonal_{RMSE} = \sqrt{\frac{\sum_{i=1}^{N} \left[(\hat{x}_i - x)^2 + (\hat{y}_i - y)^2 \right]}{N}} \quad (19a)$$

$$Altitude_{RMSE} = \sqrt{\frac{\sum_{i=1}^{N} (\hat{z}_i - z)^2}{N}} \quad (19b)$$

where (x, y, z) correspond to the known aircraft location and (x_i, y_i, z_i) is the estimated aircraft location at the *i*-th Monte Carlo simulation iteration. Monte Carlo simulation results are obtained after 500 iterations.

The error in the TOA measurement error is assumed as to have a Gaussian distribution with zero mean and σ standard deviation. The estimated TOA measurement in distance at the *i*-th GRS is

$$\hat{R}_i = R_i + N(0, \sigma) \quad (20)$$

The distribution for the antenna stations are shown in Figure 1 below

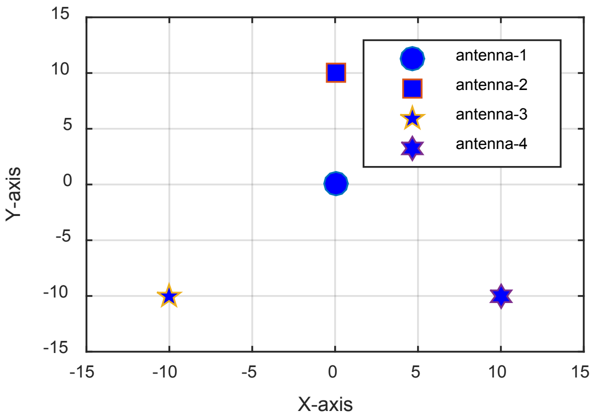


Figure 1. Triangular antenna station configuration with 10 km antenna pair separation

Aircraft locations in surveillance systems are define in terms of range (R), bearing (θ) and altitude (z) which corresponds to the cylindrical coordinate system (R,θ,z). Conversion from cylindrical coordinate system to the Cartesian coordinate system (x,y,z) can be done using the equations below.

$$x = R \times \cos(\theta) \tag{21a}$$

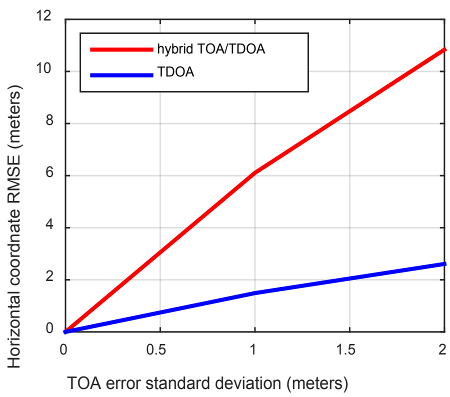
$$y = R \times \sin(\theta) \tag{21b}$$

Comparison between the two lateration algorithms is performed for some selected aircraft locations defined in Table 1 below.

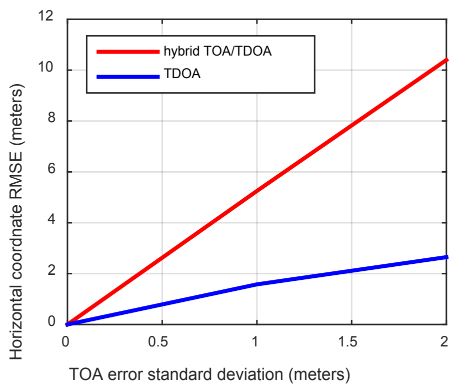
Table 1. Font Specifications for A4 Papers

No.	Location	Range (km)	Bearing (°)	Altitude (km)
1	A	5	30	1
2	B		120	3
3	C	50	240	5
4	D		330	7

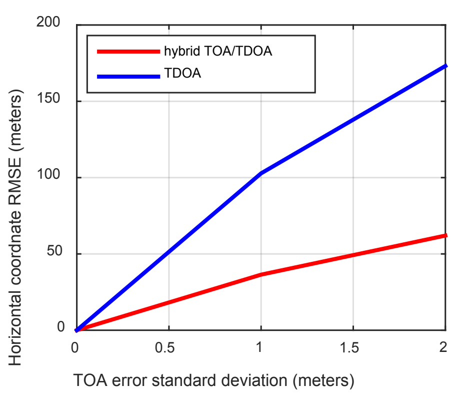
By varying the standard deviation of the error in the TOA from 0 meters to 2 meters, the horizontal coordinate and altitude RMSE obtained using the two reference hybrid TOA/TDOA lateration algorithm is compared with the two fix reference TDOA lateration algorithm in [20] for aircraft locations defined in Table 1. Figure 2 shows the horizontal coordinate RMSE comparison between the two lateration algorithms. Comparison shows that for the selected aircraft locations, there is a linear relation between the horizontal coordinate RMSE and TOA error standard deviation.



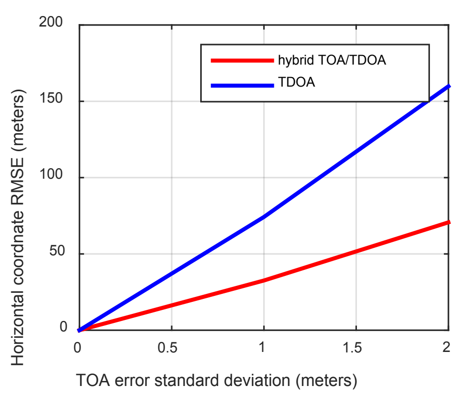
(a) Aircraft location A



(b) Aircraft location B



(c) Aircraft location C



(b) Aircraft location D

Figure 2. Horizontal coordinate RMSE comparison between two reference hybrid TOA/TDOA and two fix reference TDOA lateration algorithm for some selected aircraft locations

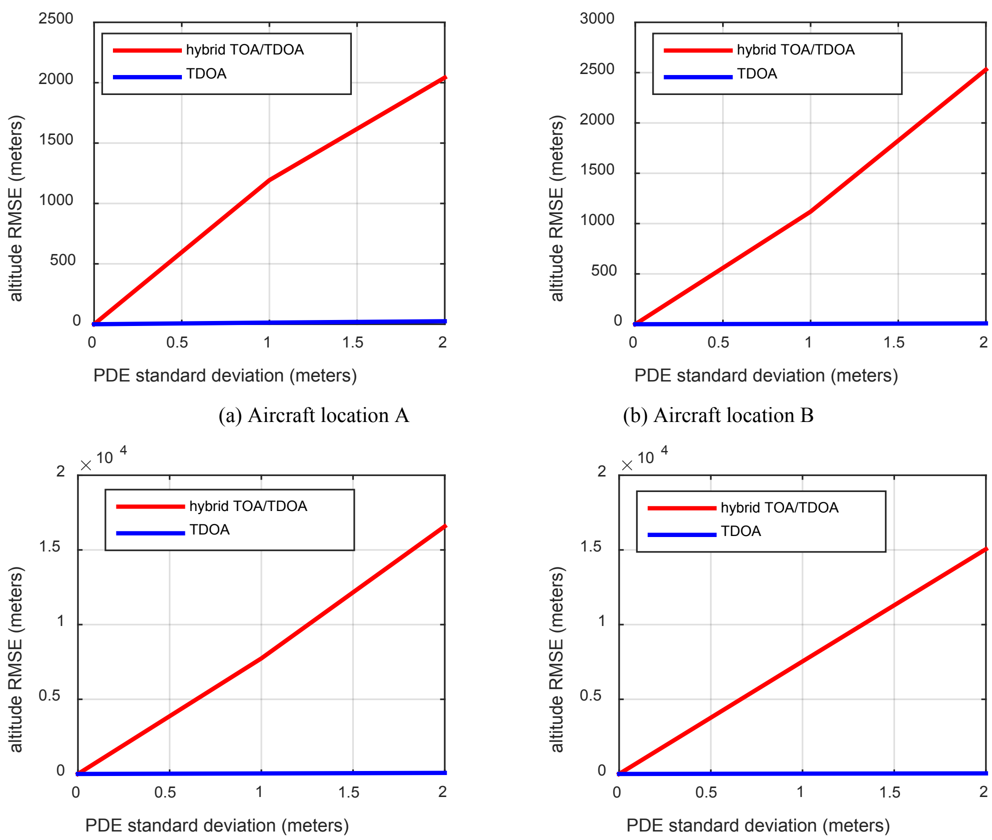
Summary of horizontal coordinate RMSE at TOA error standard deviation of 1 meters can be seen in Table 2 below.

Table 2. Horizontal coordinate RMSE comparison for different emitter location at TOA error standard deviation of 1 meters

No.	Aircraft location	Horizontal coordinate RMSE (meters)	
		Hybrid TOA/TDOA	TDOA
1	A	5.1	1.4
2	B	5.8	1.5
3	C	38.1	106
4	D	34.5	80.5

The Green fill-in in Table 2 indicate the lateration algorithm with the least PE error among the two lateration algorithms for each aircraft locations. Aircraft location A and B are within the antenna station constellation. It can be seen that at these two locations, the two fix reference TDOA lateration algorithm estimate the aircraft locations with less PE error compared to that two reference hybrid TOA/TDOA lateration algorithm. For aircraft locations outside the antenna station constellation which are indicated by locations C and D, the two reference hybrid TOA/TDOA obtained the least PE error. This means that the performance comparison between the two lateration algorithms depends on the location of the aircraft. For aircraft locations within the antenna station constellation, the two fix reference TDOA lateration algorithm has the high horizontal coordinate PE accuracy compared to the two reference hybrid TOA/TDOA. As for aircraft located outside the antenna station constellation, using the two reference hybrid TOA/TDOA lateration algorithm will estimate the horizontal coordinate with higher accuracy.

Figure 3 shows the altitude RMSE comparison between the two lateration algorithms for aircraft locations defined in Table 1. The same linear relationship between the altitude RMSE and the TOA error standard deviation can be observed. For all locations either within or outside the antenna station constellation, the two fix reference TDOA lateration algorithm has the higher altitude PE accuracy. The two reference hybrid TOA/TDOA lateration algorithm fail in this regards due to high sensivitiy of the algorithm to input noise.



(a) Aircraft location A (b) Aircraft location B

(c) Aircraft location C (b) Aircraft location D

Figure 3. Altitude RMSE comparison between two reference hybrid TOA/TDOA and two fix reference TDOA lateration algorithm for some selected aircraft locations

4. Conclusions

In this work, a two reference hybrid TOA/TDOA lateration algorithm for 3D aircraft PE is developed using the minimum antenna station MLAT configuration. The performance in PE of the developed algorithm is compared with the two fix reference TDOA lateration using Monte Carlo simulation for some selected aircraft locations and TOA error standard deviation range of 0 meters to 2 meters. Simulation results shows that the performance comparison between the two lateration algorithms depends on the location of the aircraft. For aircraft locations within the antenna station constellation, the two fix reference TDOA lateration algorithm estimated the horizontal coordinate with high accuracy compared to the two reference hybrid TOA/TDOA lateration algorithm. Aircraft locations outside the GRS constellation are estimated with high accuracy with the two reference hybrid TOA/TDOA lateration algorithm. As for altitude estimation, the two fix reference TDOA lateration algorithm outperformed the two reference hybrid TOA/TDOA lateration algorithm.

In practice, the two set of lateration algorithms can be used alongside to improve the PE of the MLAT surveillance system. The system can be configured to use the two fix reference TDOA lateration algorithm to monitor and estimate the aircraft horizontal position within the antenna constellation as well as altitude estimation of aircraft within and outside the antenna constellation. For aircraft monitoring outside the antenna constellation, the system can be configured to use the hybrid TOA/TDOA lateration algorithm. Thus, improving the overall 3D PE process of the minimum antenna station MLAT system. Further work will focus on improving the altitude estimation process of the hybrid TOA/TDOA lateration algorithm.

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