

# Riemann Hypothesis, Birch.S-Dyer, and Navier Stokes Conjectures Solved. Platonian Theory of Everything

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**Abstract** An extension to my previous articles [1] & [2], articles that I was credited and awarded in December 2019 with the Cockburn memorial medal by the European and Australian art & science organization for proving the 400 B.C Plato's conjecture "Our Universe structure is simply a projection of the mathematical universe structure! The aim of this mathematical physical research paper is to prove the validity and the consistency of my Platonian Theory of Everything by solving Riemann Hypothesis, Birch-Swinnerton-Dyer BSD conjectures, and the Navier Stokes based on the equation of the T.o.E [1] & [2]! Where the physical Navier stokes is just an application to this Platonian T.o.E! A theory that describes the mathematical foundation of general system's structures & distributions. First, we solve the concept of infinity, which is a primordial problem confronted by mathematicians & physicists! Link algebraic number theory to algebraic geometry & topology, and harmonic analysis, by building bridge between analytical L-functions and hyperbolical geometry of low dimension through algebraic variety! The paper consists of describing the packing of the sets  $\mathbb{Z}$ ,  $\mathbb{F}_p$ ,  $\mathbb{Q}$ , and its extensions, based on an irregular sphere packing method, then describes the geometrical & topological representation of these sets, describes the structure & the modular dynamical discrete system that is based on an infinite fractal horocycle flow expanding hyperbolically "Big Bang in Number Theory", solves the splitting & the density volume of the prime numbers, then determines the singularity of the fractal connected hyperspace manifold system to solve R.H & BSD, and based on we prove as an application to the theory: the Navier Stokes conjecture, that shows its footprint in the structure and dynamical structure of the invariant mathematical system!. This mathematical model is considered as the foundation of mathematics & science to disclose the hidden truth behind the mathematical universe by analyzing its physical structure and its dynamical system. An Ultimate Grand Unification Theory of Mathematics & Theory of Everything Model. A theory of winding and intertwining based on the mathematical invariants: Legendre symbol in Algebraic number theory, linking number in knot theory & winding number in analysis that show similarity for an asymmetrical entangled system! A model that will ease the understanding of the structure of many natural & artificial physical systems and its chaotic distributions! A theory that contradicts Gödel's incompleteness theorem due to the periodical fractal properties of the mathematical invariant universe system! At the end of the paper, I will be providing the shortest Equation for the T.o.E!

**Keywords** Riemann Hypothesis Birch Swinnerton Dyer Navier Stokes conjectures Theory of Everything Entanglement Number Theory Algebraic geometric

## 1. Introduction

In the year 1990 while I was studying in the college "Mathematique Supérieur, Nice, France", I was inspired by this paradox idea that came to my mind "Numbers are in dynamic, but numbers are independent from spacetime" [1] & [2]. My second intrigued question is about the concept of infinity that confronted mathematicians and physicist to solve many problems. This paper from biggining to the end

is a formulated step by step to answer and solve the equation of Navier Stokes, which is an application to the Platonian T.o.E, based on Riemann Hypothesis, and Birch.S-Dyer conjectures. This paper is a continuation to my previous papers to unveil the mystery hidden behind the structure of the discrete system defined by integers and their extensions.

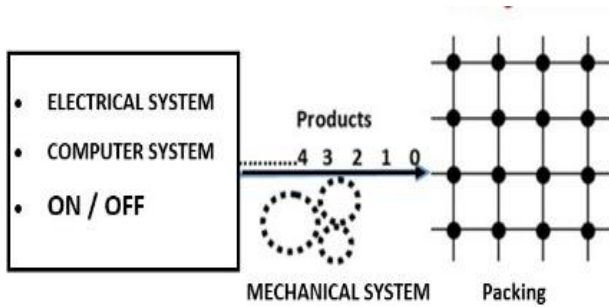
**The manufacturing process system of mathematics:** That describes the representation geometric, topologic & analytic of the numbers by its analogous: points, loops, or volume. A diagram that shows the physical structure & the flow of the invariant mathematical system. A model to ease the understanding of all kinds of natural & artificial systems evolution, through this manufacturing process system, where the discrete system represents the products in this manufacturing system, a giant abstract machine!

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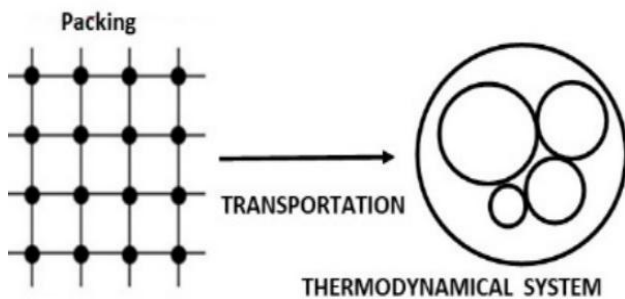
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Received: Oct. 30, 2022; Accepted: Nov. 16, 2022; Published: Nov. 24, 2022

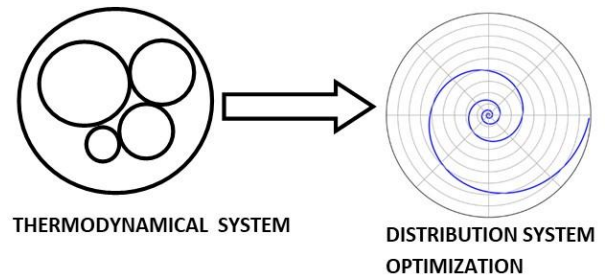
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The concept of manufacturing system discovered by humankind in the industrial field, existed already in the abstract invariant mathematical system. A very interesting fact about the sequence  $1, 2, 3, \dots$  Set of  $\mathbb{Z}$  & Its extensions is that the sequence is simply the product of a machinery process system lined on a belt as a strict line on a 1-dimensional lattice, but in the reality the transformation or the sequence results from an abstract manufacturing numerical system process with multiple generators or symmetry transformations like the Rubik cube that describe a lattice in dynamic in higher dimension! The numerical system is generated by an intelligent super-computer system that codes, decodes and corrects error codes, combined with general systems for a manufacturing process: from mechanical, electrical, thermodynamical, communication system through optimal packing process to distributing, a cyclic periodical process of transformations. A model that will lead us to unify fields of mathematics and interconnect it with physics and other science & art fields by describing its technical process and physical functionality. The invariant mathematical system is in dynamic that we are not aware off, a giant abstract intelligent machine! An abstract conscious living natural system that exists and self-aware of any error in its system, while logic steers and control its cybernetic system "The Universe of Information".



The mathematical universe is an entangled, bipolar asymmetrical system that doesn't tangle, with structure magnetic "Hopf's fibration". Gravity exists even in the numerical system, defined by curvature of the modular lattices while in dynamics! We mentioned in the previous papers that the equation of spacetime is embedded in the mathematical universe, defined by two intertwining modular lattices that swaps roots & fixed points.



Once we introduce the concept of motion in mathematics, we discover those hidden properties. In this article we going to discover the mechanism of the frequencies of the heartbeat of the mathematical universe and its template projection, through a special modular knot analogous to frequencies to define the hyperbolic volume determined by the flow of its respiratory system, an analogous property to solve Navier Stokes equation which is naturally & abstractly live in the realm of the mathematical invariant system.

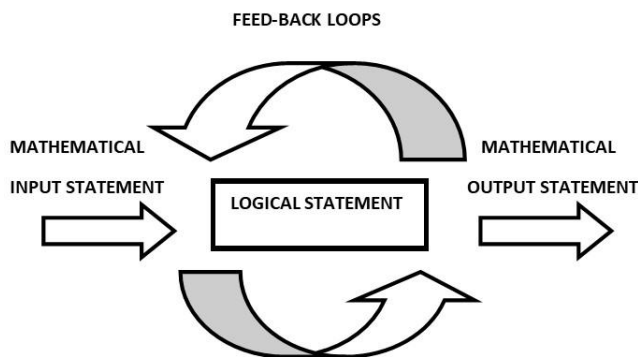
**The Code of our Reality** This article is writing with simple mathematical methods, to ease the understanding for everyone the concept of our reality, since the truth based on simple equations. Our inspiration by the code of creation, gave us no choice but to choose the red pill to discover that hidden matrix: The magnificent wonderland of our reality! Mathematic is elegantly beautiful by nature but by uncovering its hidden secret then there is more to discover, from elegance of its structure to divine truth to it! The beauty here is defined by the flow of its projected artistic geometric picture. A challenging tour for our mind to explore the realm of the mathematical ideal universe; a marvelous place where we will indulge the melody and the rhythmic harmony of the physical abstract Mathematical universe! A colorful tapestry which is seen as an artistic piece of art that shows beauty in its infinite fractal flow of coded patterns with abstract rhythmic musical beats that unfold in time [10]. A true philosophical metaphor that I proved in this comparison theory between the concrete universe and the abstract ideal universe where I attempt to show similitude of properties between the two holographic realms of universes. This theory finally will disclose that hidden bridge between the school of philosophical thoughts to prove the consistency of the 400 BC Plato's conjecture and defeat Aristotle's school of concrete narrowed thought. Our spacetime equation is hidden theoretically in the structure and the flow of the mathematical system! Our universe and its embedding fractal cosmic systems reality are simply a copy of the projection of the mathematical universe reality.

**The Asymmetrical entangled Mathematical Universe:** Mathematic is asymmetrical by nature, defined by 3 kinds of asymmetries: Invariant/variant, internal asymmetry, and external asymmetry.

- 1- invariant/variant mathematics: Invariant mathematics described by a chain of interconnected invariant mathematical systems under symmetry

transformations, generated by cyclic transformations, while variant mathematical systems are changeable that degenerate into the invariant systems. Although invariant theories are of two kinds: Abelian and Non-Abelian then you have solvable & non solvable groups.

- 2- Internal asymmetry seen in its asymmetrical mathematical abstract entities: binary system  $0/1$ ,  $+/-$ ,  $</>$ ,  $\mathbb{F}_p$ ,  $12/21$ .
- 3- External asymmetry described by the asymmetrical system  $\{\text{math}/\text{logic}\}$ , where math is an extension to logic while logic is mathematical that derive from mathematics seen as feedback loops that control the mathematical input statements of the system. Logic acts as the consciousness or generator that control the entropy of the mathematical cybernetic system.



**The mathematical loop network system:** Loops and feedback loops are one of the characteristic primordial to define the action or reaction of an abstract or concrete system related indirectly to time and motion, it encompasses our everyday life activities from communication, transportation, friendship to feeling described by a graph, network or by web. Which give birth to Category Theory In mathematics, where we define loops by functors, maps, functions, transformations, relations, array, string, fibers, knot, edge, strands, step, and word categorized by a mathematical input statement and a mathematical output statement while between the two there is a logical argument as a feedback loop that control the mathematical statements.

**Structure of the mathematical Universe Template:** The foundation of the invariant mathematical system is based on a stack of fibers, an asymmetrical fibered **space** described as a fractal of asymmetrical embedded fibered spaces. By cutting through a hyper-plan the asymmetrical stacked fibers, the cross section then describes a discrete lattice, an asymmetrical granular space which is the dual of the fibered space with discrete structure that generates a lattice field corresponding to an  $n$ -simplex  $A_n$  a set of regular polytopes. There exists an interesting connection between prime numbers, points, loops & volume of polytopes that we going to prove in this article to describe the mathematical modular coded asymmetrical template to solve Riemann Hypothesis & Navier Stokes conjecture.

**The Entangled Mathematical system:** To study the macro mathematical system, we need first to analyze the

structure and the flow of its micro mathematical system based on the set of two asymmetrical loops with cases that split, splice, inert or ramify, properties that are common between its embedding mathematical systems defined by the family of moduli spaces. By introducing a coded binary relation to the system to describe the coded hyper-graph of the mathematical micro-system, and through an asymmetrical hyper-invariant discrete lattice it will generate the macro mathematical system, which is the infinite cyclic flow generated by torus knots with hyperbolic modular flow. The micro asymmetrical mathematical system is analogous to the logical system which is defined by the basic foundation set  $G = \{\text{True} = T, \text{False} = F\}$  with the elements defined by the loop logical statements true or false analogous to the binary system  $\{0,1\}$  described sometimes by  $\mathbb{Z}_2$  or  $(-1)^n$  through an alternate oscillation.

**The Hidden dimension of the truth:** One of the important elements of  $G = \{\text{True} = T, \text{False} = F\}$  is the axis of the system, since our theory based on motion, the two asymmetrical abstract entities considered as loops " $T$  &  $F$ " are orbiting in opposite direction around an axis! Here we represent the continuous system by the dynamical of the discrete system, extension of two asymmetrical points! The vector field then defined by a rotation, translation, or a twist, defined either in a hyperbolic space, analytic, or combined. The axis is with great importance, a reason why qubit system such as "quantum qubit system" is with great performance to detect the path between initial state and output final state, rather than our classic computer system which is based on binary system. Eventually the mystery is hidden in the axis of the system the backbone of the mathematical system! By unveiling the hidden properties of the axis, we going to discover the hidden dimensions, the geometrical shape, and the code of this micro system defined by 2 asymmetrical entities, properties that will lead us to analysis Navier Stokes to define the flow of the fluid & gas systems.

Let's denote by the micro asymmetrical system defined by the set  $G = \{F = xy, T = yx\}$  of two inverse loops with opposite orientation and by  $\{J_1, J_2\}$  the axis of symmetry for each loop. Consider that the loop  $xy$  orbiting around the axis  $J_1$  and the loop  $yx$  orbiting around  $J_2$  with a cyclic transformation. For a connected closed system, the 2 loops create a vortex ring model, orbiting oppositely around the axis with different periods  $\omega_1$  and  $\omega_2$  mapping a torus shape. the system is continuous, closed and connected, then entangled and in dynamic that we just discovered in the abstract universe. The question is why the system does not tangle? What is the nature and topological structure of those loops then? To resolve this problem first we going to analyze experimentally with an interesting transformation that results from how the axis of the system splits to two loops  $\rightarrow \dots$ . take a ribbon, glue its extremities, and cut longitudinally in the middle by trying the 4 cases of the ribbon twists  $0, \pi, 2\pi$  and  $3\pi$ . When  $J_1$  &  $J_2$  are strictly connected with 0 twist then the two loops  $xy$  and  $yx$  are trivial loops with 0 intersection. But when  $J_1$  &  $J_2$  connected by  $\frac{1}{2}$  twist with an angular transformation of  $\pi$  the two loops are connected and generate a mobius strip with argument  $\pi$ , equivalent to a

twisted cylinder topologically with no boundary, then by 1 twist or  $2\pi$  the two loops  $xy$  and  $yx$  generate a special knot defined by Hopf link with argument  $\frac{\pi}{2}$ . By a third twist or  $3\pi$  the loops  $xy$  and  $yx$  describe a trefoil knot with argument  $\frac{\pi}{3}$ ! As a result, we come up to determine the shape of the asymmetrical loops that orbit along and map a torus known by torus knots  $T(p, q)$  fibered knots. Although mobius strip can be contracted to a circle, while a torus knot degenerates to the product of 2 circles. But how many twists we can proceed? Topologically it shows how the set through different axis ramifies and splits into two asymmetrical loops "invariant knots" torus knots that describe the fibers of a torus with short range and long range from trivial knot, Hopf's links to a trefoil knots "the 4 cases of the axis of symmetries described are relevant to a 4d-Torus. A dimension that we can also prove by the determining the Betti numbers based on Pascal's triangle through cross section and the genus formulate for a 5-simplex polytope related to the dihedral groups. To determine the number of the twists we should base on Conway Hexad of invertible loops! There is an interesting hidden concept in the construction generalized by Conway [17], along the hexad system for the inverse loops that we are going to describe! An interesting concept with a solid mathematical interconnection defined in an invariant system that relate mathematical fields from knot theory, number theory, combinatory, algebraic curves, symmetry transformations, simplex geometry, graph theory and analysis!

A path that led Conway to describe the path to the quaternions and octonions through the group automorphism generated by the orthogonal groups  $O(n)$  and  $SO(n)$  & the exceptional Lie groups  $F_4; G_2; \dots E_8$  that relate the 4 division algebras [c] along the hexad. Giving a binary relation  $xy$ , neutral element  $e$  & an inverse  $*$ , with  $xy = z$  then  $xx^* = yy^* = zz^* = e$ . The elements  $x, y, z, x^*, y^*, z^*$  map a hexad. The asymmetrical system is characterized by duality  $\{(x, x^*), (y, y^*), (z, z^*)\}$  and triality  $\{(x, y, z), (x^*, y^*, z^*)\}$  Elements orbiting around an axis system defined by the set  $\{J_1, J_2\}$  that are the analogue of the inverse loops. By introducing the isotopies, monotopies and their companions that preserve the relation: if  $\sigma$  is a monotopy there exist  $\alpha$  &  $\beta$  so that  $xy = z \rightarrow x^\alpha y^\beta = z^\sigma$  and by defining the left and right ideals of the loop companions  $a$  &  $b$ : with  $L_a: x \rightarrow ax$  and  $R_b: x \rightarrow xb$  Then  $\{J_1, J_2\}$  coincide with  $\{L_a, R_b\}$ , for more detail see Conway [17] The left and right ideals for the group automorphism that generates invertible loops along the axis characterized by twists. As a conclusion due to the triality and to the asymmetrical properties of the system, the number of the twists is 3. The system is entangled but not tangled due to the effect of Paul Dirac belt trick generalized by the quaternions along the transformation of  $SO(3)$ , more detail see Kauffman [2] But we will discover the rest of the flow by analyzing the hyperbolic dynamical of the discrete system in algebraic number theory! Since this dynamic is limited to elliptic modular only. In this paper we going to define integers by points representation and loops representation to describe its algebraic, geometric &

topological structure while in dynamic.

**Equation for the theory of everything \*\*\*\*\***  
**Discovered 2012 after a decade T=10, here we reconfirm again a fortiori the validity & the consistency of the Equation & T.o.E:** Giving a set  $M = \{1, 2, \dots, 99\}$  embedded in a hyper-sphere  $S_{666}$ . Then the dynamical system of  $M$  and sphere  $S_{666}$  define the equation for the theory of everything (See: [1] & [2])

**The equation [1] & [2] was defined and based on the following sequences:** The set  $M$  composed of 99 elements, has 25 prime numbers, 73 composites and number 1. A mysterious triangular sequences that call for the distribution of the primes, and integers inside the sphere  $S_{666}$ ! Sequences that generate an analytical function that we going to discover:  $\sum_1^{36} i = 666$ ;  $\sum_1^{25} i = 37 + 288$ ;

$$\sum_1^{73} i = (4 \times 666) + 37; \quad \sum_1^{99} i = (7 \times 666) + 288$$

$$\sum_0^{99} \sqrt{k} \approx \int_1^{100} \sqrt{t} dt = \left[ \frac{2}{3} t\sqrt{t} \right]_1^{100} = 666,666 - \frac{2}{3} = 666.$$

**Generalization of the equation & The Concept of Infinity:** In the previous papers [1] & [2], we did analyze the structure & the flow of the 2 sets:  $M_1 = \{1, 2, \dots, 9\}$  &  $M_2 = \{1, 2, 3, \dots, 99\}$ . Let's denote by  $M_0 = ]0, 1]$ , and by  $M_3 = \{1, 2, \dots, 999\}$ ,  $M_4 = \{1, 2, \dots, 9999\}$ ,  $M_n = \{1, 2, \dots, 99999 \dots\}$ , "9999.." with  $n$  digits, and by  $\theta_k = k \in M_n$ . It follows that the sequences  $\sum_{k=1}^{999\dots} \theta_k = \sum_{k=1}^{999\dots} k \equiv 0, \pm 288[666]$ .

**Interpretation:** To find out the period of the system, we need to evaluate the sequence:  $\sum_1^{36} k = 666$ , since  $\theta_{k+1} = k + 1 = \theta_k + 1$ , it shows 666 represent  $\frac{1}{10}$  of  $2\pi$ , then period  $T = 10$  and  $\omega = \frac{2\pi}{10} = \frac{\pi}{5}$ . The sets of triangular sequences  $T_{99}, T_{999}, \dots$  Expand by oscillating  $S_{666}$  with the angle  $\beta = 0 \text{ or } \pm \frac{288}{180}\pi = \frac{1}{5}\pi, \frac{4}{5}\pi$ , points that are considered vertices of a fractal tetrahedral mapped to  $\infty$ . We have then 2 glued asymmetrical tetrahedral in  $\mathbb{Q}[\zeta_5]$  the abelian cyclotomic group. Tetrahedral that we consider with coordinates the points  $0, 1, z$  &  $\frac{z-1}{z}, \frac{1}{1-z}, z$ , related to  $SL(2, \mathbb{Z})$  and the symmetry group  $A_4$ . This dynamical is based on a  $A_4$  tetrahedral rotation, and  $S_4$  reflexion group for the twist transformation, a transformation that will allow us to go from  $S_4 \rightarrow S_3$ , to describe the transformation of an hyperelliptic curve to a elliptic curve. Orbital defined by a spiral of Fibonacci curve, that orbits along a fractal dodecahedral space dual of an icosahedral, that results from a quintic algebraic curve singularity. A homology sphere, an interconnection related to hyperbolic modular flow & volume of polytopes at singularity [1] & [2]. Let's denote now by  $r_k = \sqrt{k}$  with  $k \in M_n = \{1, 2, \dots, 99999 \dots\}$ , then since  $f(x) = \sqrt{x}$ , monotone increasing, we have the following: with  $\frac{2}{3} = 0.666$   $r_k^2 = k = \sum_0^{99} r_k = \sum_0^{99} \sqrt{k} \approx \int_1^{100} \sqrt{t} dt = \left[ \frac{2}{3} x\sqrt{x} \right]_1^{100} = 666,666 - \frac{2}{3} = 666$

$$\begin{aligned}\sum_{100}^{999} r_k &= \sum_{100}^{999} \sqrt{k} \approx \int_{100}^{1000} \sqrt{t} dt \\ &= \left[ \frac{2}{3} x\sqrt{x} \right]_{100}^{1000} = \frac{2}{3} 10^4 \sqrt{10} - \frac{2}{3} 10^3 \\ \sum_{100}^{9999} r_k &= \sum_{100}^{9999} \sqrt{k} \approx \int_{100}^{10000} \sqrt{t} dt \\ &= \left[ \frac{2}{3} x\sqrt{x} \right]_{100}^{10000} = \frac{2}{3} 10^6 - \frac{2}{3} 10^3 \\ &= 666.10^3\end{aligned}$$

**Interpretation:** The sphere expands by  $10^3$ , by  $\int_{10^{2k}}^{10^{2k+2}} \sqrt{t} dt = \left[ \frac{2}{3} x\sqrt{x} \right]_{10^{2k}}^{10^{2k+2}} = 666.10^{3k}$  with radius:  $R_k = \sqrt{x} = 10^1, \dots, 10^k$  to pack the sets  $M_2 = \{1, 2, \dots, 99\} - M_0, M_4 - M_2, M_6 - M_4, \dots$

$\sum_1^{n-1} r_k = \sum_1^{n-1} \sqrt{k} \approx \int_1^n \sqrt{x} dx = \frac{2}{3} n\sqrt{n} - \frac{2}{3}$  just volume of a sphere of radius  $\sqrt{n}$  minus volume of a sphere unity of radius 1. The interesting thing about this sequence is that for each radius of  $10^k$  the integral  $\int_{10^{2k-2}}^{10^{2k}} \sqrt{x} dx \equiv 0[666]$  for even powers  $\dots -2, 2, 4, \dots$ . It also generates a fractal expansion that extend in  $M_0 = ]0, 1]$  for the values in the interval  $[10^{-2k-2}, 10^{-2k}]$ , a concept that is seen also in the Riemann trivial zeros, which is due to the modular asymmetrical flow that we will describe later. Which also prove the packing of  $\frac{1}{\sqrt{x}}$  with a hyperbolic flow related to a translation transformation, and due to the optimal density of this irregular spheres packing "A fractal dynamical system of packing", it follows then the convergence of the sequence. The Sphere expand then by  $10^{\pm 3}$  with a rate of  $\sqrt{10^{2k}} \rightarrow 10^{\pm k} \sqrt{10^{2k}}$ . A periodical expansion of the radius defined by the period  $T_k = 10^k$ , similar to the hyperbolic function  $e^{\pm k}$ , since:  $T_k = 10^k = e^{k \ln 10} \approx e^{2k}$  since  $\ln 10 = 2.30$ . Note that  $\int_0^{100} \sqrt{t} dt = 666.666$  &  $\int_0^1 \sqrt{t} dt = \frac{2}{3} \approx 0.666$  volume of unit sphere by multiplying with  $\int_0^{2\pi} d\theta$ . This rational sequence has an expansion to p-adic extension:  $\int_1^{10^{2n}} \sqrt{x} dx = \frac{2}{3} (10^{3n} - 1) = \frac{2}{3} \cdot 9 \cdot \sum_1^{3n-1} 10^i = 6 + 60 + 600 \dots$

**Conclusion:** Let's denote by the quadratic equation  $x^2 = k$  with  $k \in \mathbb{Z}$ , then this packing is related to the an irregular spheres packing of the set  $\mathbb{Z}$  inside the sphere  $S_{666}$ . Our goal in this paper is to define the spectrum  $p_i$  of the prime numbers and determine the packing of the set  $\mathbb{F}_i$  of the prime numbers, also the packing of the set of rational numbers  $\mathbb{Q}$  and its abelian extensions based on the quadratic equation, to solve R.H & B.S.D conjectures.

**System Model:** Therefore, our system is a closed compact manifold, a fractal periodical space endowed with a metric space, defined by combining these 2 dynamical flows of a homogenous space & time with proportional parameters, that give a rise to a vector field with rotation, twist, and translation, both dynamical are periodic with common period  $T_k = 10^k$ . Let's denote by:  $z_k = r_k \cdot e^{\pm i\theta_k}$ , the flow described as a hyperbolic Fibonacci modular flow of an infinite cyclic covering of linked torus knots with a

horocycle flow! To analyze such a flow, we need to determine the properties of the following parameters  $\theta_k = k, e^{\pm i\theta_k}$ ,  $r_k = k^{\pm \frac{1}{2}}$  and  $z_k = r_k \cdot e^{\pm i\theta_k}$  with an expansion by  $e^{\pm k}$  generated by the following matrix transformations:  $\phi^t = \begin{pmatrix} e^{+t} & 0 \\ 0 & e^{-t} \end{pmatrix}$  & by  $\phi^t = \begin{pmatrix} e^{+it} & 0 \\ 0 & e^{-it} \end{pmatrix}$  for the vector field defined by the matrix,  $L = \begin{pmatrix} 1 & k \\ 0 & 1 \end{pmatrix}$  and  $M = \begin{pmatrix} 1 & i \\ 0 & 1 \end{pmatrix}$ . Before we start describing the flow of the system we need to make some quite connection between the structure & the path of the prime numbers and our modeling system!

**Prime Numbers Distribution:** Prime numbers are the building block of the integers, any composite number can be written as a product of prime numbers,  $\prod p_i^{\beta_i}$ ,  $\beta_i$  integer. Some primes ramify, some inert, and some split in the ring of integers. Any integer including primes is a trans-palindrome, or is a palindrome self-symmetric, like number 11 in the set  $\mathbb{Z}$ , while in  $\mathbb{Z}$  extensions it splits as conjugate elements. Following Dirichlet theorem for the arithmetic progression of the primes, any prime number  $p_i \equiv \pm 1[6]$  or  $\equiv \pm 1[4]$  except the 2 primes 2 & 3. Then the product  $\prod p_i^{\beta_i} \equiv \pm 1[6]$  or  $\equiv \pm 1[4]$ , we can locate the primes then following the square & hexagram diagram! From these 2 diagrams we can conclude the existence of two sort of prime numbers: Eisenstein primes and Gaussian primes related to the modular hexagonal & square lattices defined by  $p = a^2 + b^2 - ab$  in the ring  $\mathbb{Z}[\omega]$  and  $p' = a^2 + b^2$  in the ring  $\mathbb{Z}[i]$ , although there is other form of prime numbers, for detail see Daniel Chong for quadratic reciprocity [12], where  $p = a^2 - db^2$  when discriminant  $D = 4d$ , where  $p$  is quadratic residue mod[d], related to the quadratic equation:  $x^2 = D_k = k$  with  $k \in \mathbb{Z}$  an extension of  $\mathbb{Z}$  defined by the field  $\mathbb{Q}[\sqrt{D}]$ . Since the integers are in dynamic, then we need to find the exact path of the spectrum  $\mathbb{F}_{p_i}$  prime's orbital, by finding the shape geometric of the curve dynamical, for this reason we need to analyze the structure, the flow, and the singularity of the curve according to the discriminant  $x_k^2 = D_k$  to determine the packing & the distribution of the integers and prime numbers related to our discovered periodical manifold Space defined by  $S_{666}$ . In the class field theory, we know that there exists a coherent relationship between, law of quadratic reciprocity, split & density of primes along the Galois extension of  $\mathbb{Q}$ . Every finite abelian extension of  $\mathbb{Q}$ , lie in some cyclotomic abelian field  $\mathbb{Q}[\zeta_n]$ , and for any finite abelian extension of  $\mathbb{Q}\sqrt{-d}$  there is special values of automorphic functions over it, and there is a unique lattice  $\mathbb{Z} + \tau\mathbb{Z}$  of an imaginary quadratic field.

Let's consider the following polynomials:  $f(x) = x^4 - 1$  with roots  $\pm i, \pm 1$  &  $g(x) = x^6 - 1$  with roots  $\pm \omega, \pm 1$ , and  $w(x) = x^5 - 1 = (x - 1)(x^4 - x^3 - x^2 - x + 1)$ , with  $GCD(f, g)(x) = h(x) = x^2 - 1$ , and since  $GCD(w, f, h)(x) = x - 1$  then all this polynomial degenerate at the point  $x = 1$ , polynomials that are related to the cyclic polynomial defined in the abelian cyclotomic field  $\mathbb{Q}[\zeta_n]$  extension of  $\mathbb{Q}$ . **PSL(2,  $\mathbb{R}$ )/PSL(2,  $\mathbb{Z}$ ), the**

**Trefoil Knot, Hopf's link, and the space rings of  $\mathbb{Z}[i]$  &  $\mathbb{Z}[\omega]$**  An interesting interconnection is that  $f_1(x) = x^2 + 1$  and  $g_1(x) = x^2 - x + 1$  interchange roots and fixed points, for the units values  $\pm i$  &  $\pm \omega$  but  $h(x) = x^2 - 1$  interchanges roots and fixed points with  $L(x) = x^2 - x - 1$  for the values  $\pm 1$  &  $\varphi$  (*Golden ratio*). Per stereography projection for  $f_1$  we have:  $\frac{-1}{x} = x$  and for  $g_1$  we have  $\frac{x-1}{x} = x$  with matrix:  $N = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$  resp  $M = \begin{pmatrix} 1 & -1 \\ 1 & 0 \end{pmatrix}$  with  $\det = 1$  elements of  $SL(2, \mathbb{Z})$  for more details see [20], while  $M^2 = N^3$ . Since  $PSL(2, \mathbb{Z}) \cong \mathbb{Z}_2 \times \mathbb{Z}_3$  related to the elements  $M$  &  $N$ . We notice the 2 units of the Rings  $\mathbb{Z}[i]$  &  $\mathbb{Z}[\omega]$  connected by the relation:  $i^2 + \omega^3 = 0$  &  $\omega^3 - 1 = 0$  then  $i, \omega$ , and  $-1$  elements of the torus knot  $T(2,3)$  a trefoil knot, also since  $i^2 + 1 = 0$  then  $i$  &  $1$  are elements of the Hopf's link, defined by the 2 linked circles in  $\mathbb{C}$ . The map:  $PSL(2, \mathbb{R}) \rightarrow PSL(2, \mathbb{Z})$  in the Poincare half disk shows that the flow is generated by a translation & a rotation transformation in the space of  $\mathbb{Z}[i]$  &  $\mathbb{Z}[\omega]$  that describe a discrete subgroup of an infinite co-volume space [15], defined by Seifert surfaces through a hyperbolic flow generated by the compliment of the hyperbolic 8 knot  $K_{4_1}$ . Now for an algebraic curve:  $\Delta = \{z_1 z_2 \in \mathbb{C}, z_1^2 = z_2^3\}$ , the trefoil  $K = \Delta \cap S^3$  then  $PSL(2, \mathbb{R})/PSL(2, \mathbb{Z}) \cong S^3 - K$  and  $\pi_1(S^3/K) :< M^2 = N^3 \cong B_3$  Braid group, for more details see Birman [14].

**Trefoil Knot  $K_{3_1}$  8 knot  $K_{4_1}$ , and Hopf's link:**

Let  $P(z) = z^2 + 1$  &  $Q(z) = 1 - z^2$ , for  $z = \sqrt{t} - \frac{1}{\sqrt{t}}$  then we have then the Jones polynomials  $K_{3_1}$  and  $K_{4_1}$   $P(z) = t - 1 + \frac{1}{t}$  and  $Q(z) = -(t - 3 + \frac{1}{t})$ . While Hopf's link with polynomial:  $R(z) = -t - \frac{1}{t} = -\frac{(1+t^2)}{t}$ . Extension Polynomials, [9] that are related to the cyclotomic polynomial  $\phi_4$  of  $\mathbb{Q}[\zeta_4^k]$ ,  $\phi_6$  of  $\mathbb{Q}[\zeta_6^k]$ , and  $\phi_{10}$  of  $\mathbb{Q}[\zeta_{10}^k]$ . The hyperbolic knot  $K_{4_1}$  also constructed by gluing 2 ideal tetrahedral with the points:  $0, 1, \frac{1}{1-z}, \frac{z-1}{z}$ , and  $z$ , related to  $PSL(2, \mathbb{Z})$ . We can conclude then that Hopf's link and trefoil knot are connected and swaps root with fixed point for the values:  $\pm i$  &  $\pm \omega$  and the 8 knot  $K_{4_1}$  swaps roots with  $1 - t^2$  for values:  $\pm 1$  &  $\varphi$  (*Golden ratio*) transformations that involve polynomial characteristic and Seifert surface defined by  $\text{Det}[M - xI]$  and  $\text{Det}[M - xM^T]$ , while polynomial characteristic involves the discriminant of the quadratic equation.

**Flow of the system:** This dynamical system generated by 2 types of flows, one is related to infinite cyclic covering of a linked torus knots, where the Seifert surface related to the complement of the trefoil knot and Hopf link a fibered knots, that give rise to a elliptic modular curves, which expand exponentially to create a hyperbolic Fibonacci modular flow, this combination of flows is related to the correspondence of the trefoil knot, Hopf's link, trivial knots and the 8 knot. It shows that the convergence of the sequences defined by  $\sum r_k = \sum \sqrt{k}$  and  $\sum s_k = \sum \frac{1}{\sqrt{k}}$  is trivial for even powers, a

similar case of the R.H trivial zeros. This is seen in its distribution as irregular sphere packing with optimal density in the sphere  $S_{666}$ . Our aim is to define an analytical function or Serie  $\mathfrak{Z}_k \sim \xi(s)$  with common properties to describe the condition of convergence and the zeros of the R, for this reason we need to analyze the flow and the convergence of the following sequences:

- $\sum \theta_k = \sum k$
- $\sum r_k = \sum k^{\pm \frac{1}{2}}$
- $\sum e^{\pm i\theta_k}$
- $\sum \frac{1}{\sqrt{k}} e^{\pm i\theta_k}$

**Evaluation of the twist & rotation transformations of the system:** We know:  $\sum \theta_k = \sum k \equiv 0, \pm 288[666]$ , related to  $0, \frac{2}{5}\pi, \frac{3}{5}\pi$  for the period  $T=10$ . Let's denote by the number of twists  $\theta_k = k \in \mathbb{Z}$  for the torus knots  $T(K+1, 1)$  &  $T(K+1, K)$  defined by the rational tangles:

$$\begin{cases} T(1, k+1) = 1 + \theta_k = k + 1 \\ T(k+1, k) = \frac{\theta_k}{\theta_{k+1}} = \frac{k}{k+1} \end{cases}$$

Due to our closed system, the flow describes 2 asymmetrical dynamics, with opposite directions, with stable & instable curves, for this reason, we are going also to analyze the asymmetrical twists.

The flow of the system is defined in  $PSL(2, \mathbb{R})/PSL(2, \mathbb{Z}) \cong S^3 - K$ , for an horocycle with stable flow determined by the right multiplication of the nilpotent group  $\{\varphi_k = \begin{pmatrix} 1 & \sqrt{k} - \frac{1}{\sqrt{k}} \\ 0 & 1 \end{pmatrix}, k \in \mathbb{Z}^*\}$  by the matrix of the hyperbolic flow defined by the diagonal group  $\{\psi^k = \begin{pmatrix} e^s & 0 \\ 0 & e^{-s} \end{pmatrix}, k \in \mathbb{Z}^*\}$ .

$$\begin{aligned} \varphi_k \times \psi^s &= \begin{pmatrix} 1 & \sqrt{k} - \frac{1}{\sqrt{k}} \\ 0 & 1 \end{pmatrix} \begin{pmatrix} e^s & 0 \\ 0 & e^{-s} \end{pmatrix} = \\ &= \begin{pmatrix} e^s & (\sqrt{k} - \frac{1}{\sqrt{k}})e^{-s} \\ 0 & e^{-s} \end{pmatrix} \\ \psi^s \times \varphi_k &= \begin{pmatrix} e^s & 0 \\ 0 & e^{-s} \end{pmatrix} \begin{pmatrix} 1 & \sqrt{k} - \frac{1}{\sqrt{k}} \\ 0 & 1 \end{pmatrix} = \\ &= \begin{pmatrix} e^s & \sqrt{k} - \frac{1}{\sqrt{k}} \\ 0 & e^{-s} \end{pmatrix} \end{aligned}$$

We have then:  $\psi^s \varphi_k \neq \varphi_k \psi^s = \psi^{-2sk} \varphi_k$ . Which shows that the flow is asymmetrical, non-abelian with stable and instable flow, a flow which was analyzed by Birman & Ghys to solve the Lorenz modular flow for detail see [15] & [14]

- **Twist  $T(1, k+1)$** , the matrix correspondent  $U = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ , with  $\det U = 1$ , then its polynomial characteristic  $P_2(t) = \det(U - tI) = (1 - t)^2$  that degenerates for  $t = 1$  & its Seifert surface:  $S_1(t) = \det(U - tU^T) = t^2 - t + 1 = t[t - 1 + t^{-1}]$  compliment of a trefoil knot  $S \setminus K_{3_1}$ .
- **Twist  $T(1 + k, 1)$**  the matrix correspondent  $U_1 = \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}$  with  $\det U_1 = -1$ , we recognize this



Fibonacci fraction defined by:  $\frac{1}{1+z}$  where  $\varphi = \frac{1}{1+\frac{1}{1+\frac{1}{1+\dots}}}$

the rational tangle. Due to Cassini identity:  $\det M = f_{n-1}f_{n-2} - f_n f_n = (-1)^{n+1}$ . Modular when  $\det M = 1$ .

Then its polynomial characteristic  $P_3(t) = t^2 - t - 1$  degenerates at the Golden ratio  $\varphi$ .

And its Seifert surface:  $S_3(t) = -(1-t)^2$  complement of the trivial knot.

Since our twist defined by  $\frac{1}{k+1}$  with  $\det U_1 = -1$  element of  $GL(2, \mathbb{Z})$ , then due to continuity of the system, the twist going to ramify to combine with the modular flow. Hence  $\det(U_1^2) = 1$  defined by the fraction:  $f^2(z) = f(\frac{1}{1+z}) = \frac{1+z}{2+z}$  with the matrix correspondent:  $U_2 = \begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix}$ , the polynomial characteristic is:  $P_4(t) = t(t-3+t^{-1})$  related to a hyperbolic 8 knot  $K_{41}$ , while its Seifert surface  $S_4(t) = (1-t)^2$  complement of the trivial knot.

- **Twist  $T(k+1, k)$**  we have the matrix correspondent  $V = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$ , with  $\det V = 1$ , its polynomial characteristic  $P_2(t) = (1-t)^2$  degenerates for  $t=1$ , and its Seifert surface  $S_2(t) = t(t-1+t^{-1})$  complement of a trefoil knot  $S \setminus K_{31}$ . Notice that the matrix  $U$  &  $V$  are products of the previous matrix  $M$  &  $N$  elements of  $SL(2, \mathbb{Z})$ , we have  $MN = U$  and  $M^{-1}N = V$ .
- **Twist  $T(k, K+1) = \frac{k+1}{k}$**  with the correspondent matrix  $U_3 = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}$  with  $\det = -1$  element of the  $GL(2, \mathbb{Z})$ , The polynomial characteristic then:  $P_5(t) = t^2 - t - 1$  that converge toward the golden ratio  $\varphi$ , and its Seifert surface:  $S_5(t) = -(1-t)^2$ , then also for this flow to combine with our modular flow, its twist going to ramify.  $U_3^2 = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$ , with  $\det = 1$ , Then the polynomial characteristic  $P_6(t) = t(t-3+t^{-1})$  related to  $K_{41}$  knot, and its Seifert Surface  $S_5(t) = (1-t)^2$ .

**Interpretation:** The flow defined by 2 asymmetrical reverse flows determined by 2 Seifert surfaces complement of a trefoil knot & complement of the 8 knot that swaps & interchange path through roots & fixed points system, while the trefoil knot related to the cyclic modular torus knot flow and the 8-knot related to the modular hyperbolic flow that converge toward the golden ration. This is not a Fibonacci iteration flow, only  $f = \frac{1}{1+z}$  &  $f^2 = \frac{1+z}{z+2}$  that are involved in our flow, there is a lot of modular knots with  $\det = 1$  of their matrix that are not embedded or involved for this flow.

**Conclusion:**  $(\varphi, K_{41}) \leftrightarrow (K_{31}, \omega)$ . The asymmetrical flow degenerates alternatively for  $t=1$  related alexander polynomial, one side of the flow degenerates, while the other side converges toward  $\omega$  or  $\varphi$ . Since the hyperbolic flow is not limited only in  $\mathbb{R}$ , but also expand in  $\mathbb{C}$ . Then for the twist  $T(1, i+1) = i+1$  with the matrix correspondent:  $M = \begin{pmatrix} 1 & i \\ 0 & 1 \end{pmatrix}$  with  $\det M = 1$ , have a polynomial

characteristic  $P(x) = (1-t)^2$ , and a Seifert surface  $S(t) = t(t-3+t^{-1})$  defined by the compliment of  $K_{41}$ .

**Existence of other flows in our system:** An interesting sequence that I will be decoding to determine the rest of the Seifert surface of our system.  $2^2 + 3^2 + 5^2 + 7^2 + 11^2 + 13^2 + 17^2 = 666$  Equation of a hypersphere with radius  $\sqrt{666}$  with origin the point  $O(2,3,5,7,11,13,17)$  defined by the formal equation:  $(x_1-2)^2 + (x_2-3)^2 + (x_3-5)^2 + (x_4-7)^2 + (x_5-11)^2 + (x_6-13)^2 + (x_7-17)^2 = 666$ , while  $666 = 2 \times 3^2 \times 37$ . We are going then to calculate the Seifert Surfaces related to its origin point, the following discriminants  $D_k = 2, 3, 5, 7, 11, 13, 17$  &  $37$  as the complement of the Alexander knot polynomial defined by:

$\det[M - tM^t]$ . Let's denote by  $M = \begin{pmatrix} 1 & \sqrt{D_k} \\ 0 & 1 \end{pmatrix}$  then  $\det[M - tM^t] = t^2 + (D_k - 2)t + 1 = \Delta_k$ , and due to the orientation of the flow, replacing  $t$  by  $-z$ .

Now we will recognize some of its Seifert surfaces:

$k=2$  then  $S_f = \mathcal{E}(\Delta_2 = z^2 + 1)$  complement of Hopf's Link.

$k=3$  then  $S_f = \mathcal{E}(\Delta_3 = z^2 - z + 1)$  complement of a trefoil Knot  $K_{31}$ .

$k=5$  then  $S_f = \mathcal{E}(\Delta_5 = z^2 - 3z + 1)$  complement of an 8 knot  $K_{41}$ .

$k=7$  then  $S_f = \mathcal{E}(\Delta_7 = z^2 - 5z + 1)$ .

$k=11$  then  $S_f = \mathcal{E}(\Delta_{11} = z^2 - 9z + 1)$ .

$k=13$  then  $S_f = \mathcal{E}(\Delta_{13} = z^2 - 11z + 1)$ .

$k=17$  then  $S_f = \mathcal{E}(\Delta_{17} = z^2 - 13z + 1)$ .

This quadratic equation defined by  $z^2 - (D_k - 2)z + 1$  have a discriminant  $\Delta = D_k(D_k - 4)$ , follows from Dirichlet theorem, although the factorization of  $666 = 2 \times 3^2 \times 37$  shows that for  $k=37$  the discriminant of its complement Seifert surface is  $D = 37.11.3$  connected to other discriminants. Now for  $z=0$ , all polynomial degenerate to  $\Delta = 1$ , and the Seifert surface is then the complement of the circles, and from the Poincare conjecture, the complement of a circle is the sphere  $S^2$ . The polynomial with  $k=7, 11, 13$  &  $17$  shown that are interconnected to other polynomials when solved in  $\mathbb{F}_2, \mathbb{F}_3$ , and  $\mathbb{F}_5$ . Next we going to see that all the primes split in the complement of the hyperbolic space related to the hyperbolic eight knot  $K_{41}$ .

**The Splitting of all the prime numbers  $p_i$ :** For  $f \in \mathbb{Z}_p[x]$ , let's denote by the quadratic equation:  $f(x) = x^2 - p$ , with  $\gcd(p_i, p) = 1$  then for  $p_i \in \text{Spl}(f) \Leftrightarrow$

$$\begin{cases} p_i \equiv \square[p] & \text{if } \begin{cases} p \equiv 1[4] \Rightarrow \left(\frac{p_i}{p}\right) = 1 \\ p \equiv 3[4] \Rightarrow \left(\frac{p_i}{p}\right) = -1 \end{cases} \end{cases}$$

Following law of reciprocity, [7] the split is based on the Legendre symbol  $\left(\frac{p_i}{p}\right)$ . Since  $\mathbb{Q} \subset \mathbb{Q}[\zeta_p]$  then  $p_i \in \text{Spl}(\mathbb{Q}(f)) \Rightarrow p_i \in \text{Spl}(\mathbb{Q}_p) \Rightarrow p_i \equiv 1[p]$ . Now for  $p=5 \equiv 1[4]$ , with  $\mathbb{F}_5^* = \{1, 2, 3, 4\}$ , we notice 1 & 4 elements of  $\mathbb{F}_5$  are square residues mod [5], 1 or  $4 \equiv \square[5]$  but not 2 & 3. By Fermat little theorem,  $p_i^{\varphi(p)} \equiv 1[p]$  then  $p_i^4 \equiv 1[5]$ , although some primes splits in  $\mathbb{Q}[\sqrt{5}]$  like 11,

31, 41...with the property  $p_i \equiv 1[5]$  and some split in  $\mathbb{Q}[\sqrt[4]{5}]$  like 19, 29.... With the property  $p_j^2 \equiv 1[5]$ , Then 5 does not split completely, but all primes  $p_i$  split completely in  $\mathbb{Q}[\sqrt[8]{5}]$ , then split in  $\mathbb{Q}[\zeta_{5^4}]$ . Since  $p_i^4 \equiv 1[5]$ , or  $p_i^4 - 1 \equiv 0[5]$ , then  $1 + p_i + p_i^2 + p_i^3 \equiv 0[5]$  with  $p_i$  not splitting in  $\mathbb{Q}(\sqrt{5})$ .

**Conclusion:** By Little Fermat theorem, all the primes  $p_i$  split in  $\mathbb{Q}[\sqrt[8]{5}]$ , then split in  $\mathbb{Q}[\zeta_{5^4}]$ , with  $1 + p_i + p_i^2 + p_i^3 \equiv 0[5]$  a formulate that will lead us to construct the algebraic variety for  $B = \{1, p_i, p_i^2, p_i^3 \dots\}$ . By Hensel's lemma, the group of units  $\mathbb{Z}_{p_i}^*$  has  $(p_i - 1)$ th roots of unity. This splitting descends to the quotient rings  $\mathbb{Z}/p_i^n$  and its group of units, giving an isomorphism  $GL(\mathbb{Z}/p_i^n) \cong (\mathbb{Z}/p_i^{n-1}) \oplus \mathbb{Z}/p_{i-1}$ . Well in our case the units of  $\mathbb{Z}(\sqrt{2}, \sqrt[8]{5})$  are the independent elements defined by:  $\pm(2 + \sqrt{5})(\sqrt{2} + \sqrt[4]{5})(\sqrt[4]{2} + \sqrt[8]{5})(\sqrt[4]{2} - \sqrt[8]{5}) = \pm 1$ , where the points  $Q(4, \pm 5)$  elements of the hyperelliptic curve with rank  $\frac{d}{2} = \frac{8}{2} = 4$ . Since the period of our system is  $T = 10$ , then for  $p = 10 = 2.5$ , and since all  $p_i \equiv 1[2]$ , it follows that all the primes  $p_i$  split in  $\mathbb{Q}[\sqrt[4]{2}, \sqrt[8]{5}]$  which is the quadratic extension of  $\mathbb{Q}$  defined by  $\mathbb{Q}[\sqrt[4]{2}, \sqrt[8]{5}]$ , hence  $p_i$  split in the cyclotomic abelian field  $\mathbb{Q}[i, \zeta_{5^4}]$ , and all those primes are unramified due to the discriminant  $\Delta_{10} = 5^3$ . For generalization with  $p_i^4 \equiv 1[5]$ , then  $p_i^{4m} \equiv 1[5^m]$  and all  $p_i \equiv 1[2]$  then for all  $m \in \mathbb{Z}$ , the quadratic equation  $x^2 = 10^m$  imply that  $p_i \in \mathbb{Q}[\sqrt[4]{2}, \sqrt[8]{5}]$ , then all  $p_i$  split in  $\mathbb{Q}[\sqrt[4]{2}, \sqrt[8]{5}]$  which is the extension of  $\mathbb{Q}$ , hence  $p_i$  split in the cyclotomic abelian field defined by the abelian cyclotomic field  $\mathbb{Q}[i, \zeta_{5^4}]$ .

**Finite cyclic torus covering:** We know that Alexander polynomial describes the homology of the infinite cyclic covering as  $\mathbb{Z}[t^{\pm 1}]$ . Let's denote by  $B = \{1, p_i, p_i^2, p_i^3 \dots\}$  where  $p_i$  are prime numbers, and by  $\Delta_B = \{t \in \mathbb{Q}, \text{ and } k \in \mathbb{Z}, / \sum p_i^k = 0\}$ , its algebraic variety associated to it, then  $\sum_1^n p_i^k = \frac{p_i^{n+1}-1}{p_i-1} = 0$  in our case the period  $T_k = 10^k$ , then for  $T_1 = 10$ ,  $p_i^4 - 1 \equiv 0[10]$ , which proves that all the primes split in  $\mathbb{Q}(\sqrt[4]{2}, \sqrt[8]{5})$ . Then Let's denote then by its extension in the cyclotomic field  $A = \{1, \zeta_n^{-1}, \zeta_n^{-2}, \dots, \zeta_n^{n-1}\}$ , with  $\zeta_n = e^{\frac{2\pi i}{n}}$  the abelian extension of  $\mathbb{Q}$ , and by  $\Delta_A = \{z_k \in \mathbb{C}, \theta_k \in \mathbb{Z}, \sum z_k^{\theta_k} = 0\}$  its algebraic variety, a cyclic covering of linked torus knots defined by the twists  $T(k+1, k) = \frac{\theta_k}{\theta_{k+1}} = \frac{k}{k+1}$ , since  $\zeta_{\left(\frac{k}{k+1}\right)^{k+1}} = \zeta^k$ , although the cyclic polynomial  $\sum_0^{n-1} \zeta^k = \phi_d = \prod_d (x^d - 1)^{\mu\left(\frac{n}{d}\right)} = 0$ , then  $\Delta = \{\zeta \in \mathbb{C}, k \in \mathbb{Z}, \sum \zeta^k = 0\}$ , and the Seifert surface is the  $S^3 \setminus \Delta$ . In our case the period of our dynamical system is  $n = 10$ , infinite cyclic linked torus knot is reduced due to the period of the system. Period defined by the Euler totient associated function:  $\varphi(10) = \varphi(5) = 4$ , which is isomorphic to Mordell-Weil group  $\mathbb{Z}/4\mathbb{Z}$  of order 4, the rank of the abelian variety related to the zero of the associated L-function

related to Birch & Swinnerton-Dyer conjecture. The cyclotomic polynomial is then:  $\phi_{10}(x) = \phi_{2 \times 5}(x) = \phi_5(-x) = x^4 - x^3 + x^2 - x + 1$ . We recognize this polynomial by the Jones polynomial of the hyperbolic 8 knot  $K_{4_1}$  and the Seifert surface  $S = S^3 - K_{4_1}$  the complement of the hyperbolic 8 knot  $K_{4_1}$ . A polynomial with discriminant  $\Delta_{10} = \Delta_5 = (-1)^{\frac{\varphi(n)}{2}} \frac{n^{\varphi(n)}}{\prod_d p^{\frac{\varphi(n)}{p-1}}} = 5^3$  that

shows 5 the only ramified prime number in the cyclotomic field  $\mathbb{Q}[i, \zeta_{5^4}]$ . We notice for  $n_k = 10^k$ ,  $\varphi(n_k) = 2^{k-1} \times 5^{k-1} \times 4$  and  $\frac{n_k}{\varphi(n_k)} = \frac{5}{2} = \frac{n_m}{\varphi(n_m)}$  for  $m \neq k$ . Now let's denote by  $n = \prod p_i$  product off all unramified primes. Then  $\frac{n}{\varphi(n)} = \frac{\prod p_i}{\prod (p_i-1)} = \prod \frac{1}{1-p_i^{-1}} = \prod (1-p_i^{-1})^{-1} = \zeta_{s \rightarrow 1} = \frac{n}{\varphi(n)}$ . Riemann function when  $s \rightarrow 1$  related to prime numbers.

**Algebraic & Geometric variety structure:** It shows the system is related to the quintic algebraic equation:  $x^2 + y^3 = z^5$ . Let  $f: \mathbb{C}^3 \rightarrow \mathbb{C}$  the complex polynomial  $f(x, y, z) = g(x, y) + z^5 = x^2 + y^3 + z^5$  then  $f^{-1}(0)$  is the complex variety. By extending the line of singular algebraic variety to  $V(f) = \{x^2 + y^3 + z^5 = 0 \text{ \& } |x|^2 + |y|^2 + |z|^2 = 1\}$  then we have per projection a  $\mathbb{Z}_5$  regular cover, where  $V(f)$  is the branched cover of  $S^3$  with branch locus  $K_g$  over the variety  $S^3$ . It generates the equation of a dodecahedral. Let's denote by  $x^2 + y^3 = -z^5 = c$  that when  $c = 0$  then  $t = 0$  variety with cusp 0 at the origin, and when  $c \neq 0$  then the equation has 5 roots and the fixed points form the cone of the trefoil knot, we have then  $K_f = V(f) \cap S^5$  sits inside the dodecahedral for a 5 fold by symmetry. A particular case that corresponds to the homology 3- sphere "Brieskorn"  $\Sigma(2, 3, 5)$  known by the Poincare dodecahedral space with finite fundamental group the binary icosahedral group defined by  $\{x^3 = y^5 = (xy)^2\}$  of order 120 double cover of the regular icosahedral group (alternating group) determined by the quotient space or simply by the double cover that can be constructed from Dehn surgery on knot from a right handed trefoil knot. In our case as a generalization, the construction is based then on the Brieskorn algebraic variety that are associated with polynomial  $f(z) = \sum z_k^{\theta_k}$  with  $z_k \in \mathbb{C}, \theta_k \in \mathbb{Z}$ , with the variety  $V(f) = \{z \in \mathbb{C}^{n+1}, f(z) = 0 \sum |z_i|^2 = 1\}$  where the knot associated as the intersection of the variety algebraic and the sphere of dim  $S^{2n+1}$  defined by  $K^{2n-1} = V(f) \cap S^{2n+1} - \partial K^{2n}$  where  $K$  has an intersection form  $Q_k$  on  $H_n(K, \mathbb{Z})$  and if  $Q_k$  is unimodular then the boundary  $K^{2n-1}$  is a homology, exotic sphere. In our case for  $n = 4$  the knot associated to a manifold of dim 7 defined by  $K_f = V(f) \cap S^{2n+1} = V((x^2 + y^3 + z^5) + t^2 + w^2) \cap S^9$ . We have then:  $\Sigma(2, 3, 5) \subset \Sigma k^7 \subset S^9 \subset \mathbb{R}^{10}$ . And since the homology 3- sphere determined by the quotient  $SO(3)/I$  and the Milnor 7 sphere acts transitively on the group of the quaternions  $SO(3)$ , we have  $SO(3)/I \times \Sigma k^7 / SO(3) \cong \Sigma(2, 3, 5) \times D^4$ . Shows that  $SO(3)$  act between the icosahedral and the 7 dim Milnor sphere through Paul Dirac belt trick, a construction to prove which is based on



using J-homomorphism and Hopf's fibration for a higher dim to describe the fundamental group, for more detail see Kauffman [8]. This reason the asymmetrical flow in our system defined by the asymmetrical cyclic of linked torus knots with hyperbolic flow that doesn't tangle. It shows then the dynamical of the orthogonal system defined by the trans-palindromes in  $\mathbb{Z}$  and the conjugates elements in  $\mathbb{C}$  are linked and orbit in opposite directions without tangle, an asymmetrical flow controlled by Paul Dirac belt trick, including quaternions and their conjugates. The Alexander polynomial torus knot  $K(p, q)$  defined by:  $\Delta(k, t) = \frac{(t^{pq} - 1)(t - 1)}{(t^p - 1)(t^q - 1)}$  for  $T(2, 5)$  we have  $\Delta(k, t) = t^4 - t^3 + t^2 - t + 1$  the 8 knot hyperbolic knot, a construction of the homology sphere  $\Sigma(2, 3, 5)$  that is based on  $T(2, 3)$  and  $T(2, 5)$ .

## 2. Solution to Riemann Hypothesis Conjecture

**Riemann Hypothesis sequence related to the Jacobi Theta function & Gaussian distribution function:** To solve R.H, we are interested in analyzing the case when the radius  $r_k = \frac{1}{\sqrt{k}}$  related to an analytical modular function & distribution function  $\mathfrak{Z}_k$  that we going to determine so that  $\mathfrak{Z}_k \sim \xi(s)$ , but we end up discovering more solutions for  $s$  for its convergence related also to  $r_k = \sqrt{k}$ . We already showed the system is fractal for the series defined by:  $\sum_1^{10^k} \frac{1}{\sqrt{k}}$  and  $\sum_1^{10^k} \sqrt{k}$ , and with  $T_k = 10^k = e^{k \ln 10} \approx e^{2k}$  since  $\ln 10 = 2.30$  which will lead us to evaluate and analyze the Riemann sequence.

**Riemann's condition of convergence:** Let's denote by the hyperbolic expansion of  $r_k$  defined by  $z_k = \frac{1}{\sqrt{k}} e^{-k}$  and by  $\mathfrak{Z}_k = \sum_0^{+\infty} z_k = z_0 + \sum_1^{+\infty} \frac{1}{\sqrt{k}} e^{-k}$ , with  $z_0 = 1$ , then the sequence defined by the integral  $\int_0^{+\infty} \frac{1}{\sqrt{x}} e^{-x} dx = 2 \int_0^{+\infty} e^{-t^2} dt = \sqrt{\pi}$  the Gaussian distribution of  $z_k$ , and by the prime theorem number:  $\pi(k) = \frac{k}{\ln(k)} \# \{p_i \text{ primes for } p_i \leq k\}$ , evaluating  $z_k = \frac{1}{\sqrt{k}} e^{-k} = \frac{1}{k^{\frac{1}{2}}} e^{-\frac{k(\ln k)}{\ln k}} = k^{-\frac{1}{2}} e^{-\pi(k) \ln k} = k^{-\frac{1}{2}} k^{-\pi(k)} = k^{-\frac{1}{2} - \pi(k)} = \frac{1}{K^{s_k}}$ , with  $s_k = \frac{1}{2} + \pi(k)$ , then  $\mathfrak{Z}_k = \sum_0^{+\infty} z_k = z_0 + \sum_0^{+\infty} \frac{1}{K^{s_k}}$ , the sequence converge, and since Riemann sequence defined by  $\xi_s(n) = 1 + \frac{1}{2^s} + \dots + \frac{1}{n^s} = \mathfrak{Z}_k - e^{-1}$  then since  $\mathfrak{Z}_k$  C.V and with  $|\mathfrak{Z}_k - \xi(s)| = |e^{-1}|$ , it follows that for the Riemann function sequence to converge the condition  $s = s_k = \frac{1}{2} + \pi(k)$ . The same for  $\xi_{1-s}(n) = \sum_1^{\infty} \frac{1}{k^{1-s}}$  with  $1 - s_k = \frac{1}{2} - \pi(k)$ . Then a condition of R.H to convergence for this hyperbolic flow, is  $s = s_k = \frac{1}{2} \pm \pi(k)$ . We notice also a missing part of the convergence related to  $r_k = \sqrt{k}$  for  $z_k = \sqrt{k} e^{-k}$  since  $\int_0^{+\infty} \sqrt{x} e^{-x} dx = 2 \int_0^{+\infty} t^2 e^{-t^2} dt = -\int_0^{+\infty} t [e^{-t^2}]' dt =$

$\int_0^{+\infty} e^{-t^2} dt = \frac{\sqrt{\pi}}{2}$  which proves the convergence of  $\mathfrak{Z}_k = \sum_0^{+\infty} z_k = z_0 + \sum_1^{+\infty} \sqrt{k} e^{-k}$ , it follows that the Riemann sequence defined by  $\xi_s(n) = 1 + \frac{1}{2^s} + \dots + \frac{1}{n^s}$  also converges with the condition  $s = -\frac{1}{2} - \pi(k) = \bar{s} - 1$ . Now let's denote by the rotation & twist transformation of  $r_k$  defined by:  $z_k = \frac{1}{\sqrt{k}} e^{-ik}$ , with  $z_0 = 1$ , then  $\mathfrak{Z}_k = \sum_0^{+\infty} z_k = z_0 + \sum_1^{+\infty} \frac{1}{\sqrt{k}} e^{-ik}$  its sum defined by the integral  $\int_0^{+\infty} \frac{1}{\sqrt{x}} e^{-ix} dx = 2 \int_0^{+\infty} e^{-it^2} dt = 2 \int_0^{+\infty} e^{-iz^2} dz = 2\sqrt{\frac{\pi}{8}}(1-i) = \frac{\sqrt{\pi}}{2}(1-i) = \sqrt{\frac{\pi}{2}} e^{-\frac{i\pi}{4}}$ . And by the prime theorem number:  $\pi(k) = \frac{k}{\ln(k)} \# \{p_i \text{ primes for } p_i \leq k\}$ , evaluating then  $z_k = \frac{1}{\sqrt{k}} e^{-ik} = \frac{1}{\sqrt{k}} e^{ik} = \frac{1}{k^{\frac{1}{2}}} e^{-\frac{ik(\ln k)}{\ln k}} = k^{-\frac{1}{2}} e^{-i\pi(k) \ln k} = k^{-\frac{1}{2}} k^{-i\pi(k)} = k^{-\frac{1}{2} - i\pi(k)} = \frac{1}{K^{s_k}}$ , with  $s_k = \frac{1}{2} + i\pi(k)$ , then  $\mathfrak{Z}_k = \sum_0^{+\infty} z_k = z_0 + \sum_1^{+\infty} \frac{1}{\sqrt{k}} e^{-k} = z_0 + \sum_0^{+\infty} \frac{1}{K^{s_k}}$ , the sequence converge, and since Riemann sequence defined by  $\xi_s(n) = 1 + \frac{1}{2^s} + \dots + \frac{1}{n^s} = \mathfrak{Z}_k - e^{-i}$  then since  $\mathfrak{Z}_k$  C.V and with  $|\mathfrak{Z}_k - \xi(s)| = |e^{-i}| = 1$  it follows for the Riemann sequence  $\xi_s(n) = \sum_1^{\infty} \frac{1}{k^s}$  to converge the condition if and only  $s = s_k = \frac{1}{2} + i\pi(k)$  with  $\pi(k) = \frac{k}{\ln(k)}$ , and this also valid for  $\xi_{1-s}(n) = \sum_1^{\infty} \frac{1}{k^{1-s}}$  for  $s = s_k = \frac{1}{2} - i\pi(k)$ .

**Conclusion:**  $\xi_s(n) = \sum_1^{\infty} \frac{1}{k^s}$  shows properties related to the Gaussian distribution or bump function, if  $s = s_k = \frac{1}{2} \pm \pi(k)$  and  $s = s_k = -\frac{1}{2} \pm \pi(k)$  with  $\pi(k) = \frac{k}{\ln(k)}$ . The convergence follows since the system is fractal hyperbolic with period  $T_k = 10^k = e^{k \ln 10} \approx e^{2k}$  since  $\ln 10 = 2.30$ . Also Riemann Hypothesis for the other case is related to the analytical modular Theta function defined by the Fourier series  $f(k) = \sum_0^{\infty} e^{\pm i k^2}$ , and converge if for  $s \in \mathbb{C}$ , if  $s = s_k = \frac{1}{2} \pm \varepsilon \pi(k)$ , or  $s = s_k = -\frac{1}{2} \pm \varepsilon \pi(k)$  with  $\varepsilon = \{\pm i\}$  and  $\pi(k) = \frac{k}{\ln k}$ .

**Gama Function:**  $\Gamma(z) = \int_0^{+\infty} t^{z-1} e^{-t} dt$  by replacing  $z = 1 - s$  and  $s = \frac{1}{2} + \frac{t}{\ln(t)}$ , then Gama is just a Gaussian distribution:  $\Gamma(z) = \int_0^{+\infty} t^{z-1} e^{-t} dt = \int_0^{+\infty} t^{-s} e^{-t} dt = \int_0^{+\infty} \frac{1}{\sqrt{t}} e^{-t} dt = \int_0^{+\infty} 2 e^{-2t^2} dt = \sqrt{\frac{\pi}{2}}$ .

**Evaluation of the twist:** For the sequence  $f(k) = \sum_0^{\infty} e^{\pm k^2}$ , let's denote by  $\begin{cases} T(1, k+1) = 1 + \theta_k = k+1 \\ T(k+1, k) = \frac{\theta_k}{\theta_k+1} = \frac{k}{k+1} \end{cases}$  then by replacing  $k$  by  $m$  such that  $m = \frac{k^2}{2k+1}$  a rational tangle then  $\frac{m}{m+1} = \frac{k^2}{(k+1)^2}$  which also carry the same properties of  $\theta_k$  of the modular horocycle flow defined in the previous twists for the horocycle defined by

$A = \{1, \zeta_n^{-1}, \zeta_n^{-2}, \dots, \zeta_n^{n-1}\}$ , with  $\zeta_n = e^{\frac{2\pi i}{n}}$  the abelian extension of  $\mathbb{Q}$ , and by  $\Delta = \{z_k \in \mathbb{C}, \theta_k \in \mathbb{Z}, \sum z_k^{\theta_k} = 0\}$  the algebraic variety. The number of integers  $k^2$  coprime to  $T_2 = 10^2$ , for example is the Euler Totient function  $\varphi_{k^2} = 4$ , since  $\varphi_k(T_1) = \varphi_{k^2}(T_2)$  where  $T_n = 10^n$ , with points element of a fractal dodecahedral, or 2 glued asymmetrical tetrahedral that go to  $\infty$ , that define a hyperbolic 8 knot  $K_{4,1}$ . Then we see a generalization of the

flow defined by the function:  $f(z) = \sum_0^\infty e^{\frac{\pm i 2\pi k^n z}{T_n}}$ , with properties of an automorphic function, to solve and generalizes the  $L(E, s)$  functions.

**Infinite Zeros of  $\xi_s$ :** It is trivial that all the zeros of  $\xi_s$  lie on  $s = s_k = \frac{1}{2} \pm \varepsilon \beta \pi(k)$  with  $\varepsilon = \{\pm 1, \pm i\}$  and due to condition of its convergence, and due to our periodic system, defined by the cyclotomic polynomials of the algebraic varieties defined by  $\Delta_k = \{z_k \in \mathbb{C}, \theta_k \in \mathbb{Z}, \sum z_k^{\theta_k} = 0\}$ , by  $\Delta_{k^m} = \{z_n \in \mathbb{C}, k^m \in \mathbb{Z}, \sum z_n^{k^m} = 0\}$ , that show similar properties for their cyclotomic polynomial  $\phi_n = \prod_{d=k^m/n} (x^d - 1)^{\mu(\frac{n}{d})}$ . Then when such a polynomial degenerates, we have infinite of solutions, and when is  $\neq 0$ , then finitely generated, and since  $\xi_s$  related to  $\Delta_{k^2} = \{z_n \in \mathbb{C}, \theta_k \in \mathbb{Z}, \sum z_n^{k^2} = 0\}$  which proves that when  $\xi_s = 0$ , the  $\xi_s$  has infinite of solutions and when  $\xi_s \neq 0$ , then  $\xi_s$  generated by a finite set.

**Volume Hyperbolic of the dynamical system:** A problem of volume that was analyzed by Thurston describing hyperbolic structure on  $S^3 \setminus k$  that degenerate by deforming ideal tetrahedral, also a reference to the hyperbolic volume conjecture by Michio Fuji. We notice this dynamical is in the form of 2 Laurent polynomials  $\mathbb{Z}[t, t^{-1}]$  &  $\mathbb{Z}[\zeta, \zeta^{-1}]$  that swaps roots with coefficients, although we know by Seifert theorem  $P(1) = 1$  &  $P\left(\frac{1}{t}\right) = P(t)$  for symmetric polynomial. In this part we going to show that Riemann sequence related to Gaussian distribution & Riemann sequence related to the analytical function are connected to the hyperbolic volume. swap roots with coefficients based on the symmetric Laurent polynomial, where  $P(t) = \sum_1^n z_k t^{\pm k} = \prod_{k=1}^n (t - b_k)$ . Hence for a closed loop  $P(0) = P(1) = 1$ , and with the condition of symmetric polynomial and Seifert theorem, with  $z_k = \frac{1}{\sqrt{k}} e^{-i\beta k}$ , with  $\beta = \frac{2\pi}{T}$ , then  $P(0) = \prod_1^n b_k = 1$  since  $(\prod_1^n \frac{1}{\sqrt{k}} (e^{i\beta k} + e^{-i\beta k}))^2 = \frac{1}{n!} 2^{2n} (\prod_1^n \cos(i\beta k))^2$ . Although we know the normal distribution is when  $\theta = \beta k = \frac{\pi}{4}$ . Then  $\cos \theta = \frac{\sqrt{2}}{2}$ , which proves the equality  $P(0) = \prod_1^n b_k = (\prod_1^n \frac{1}{\sqrt{k}} (e^{i\beta k} + e^{-i\beta k}))^2 = \frac{1}{n!} 2^{2n} (\prod_1^n \cos(i\beta k))^2 = \frac{2^n}{n!} = 1$ , that corresponds to a volume of a polytope when the system degenerate to  $n=0$ . In our case the volume correspond to  $V_3 = \frac{4}{3}$  for  $n = 3$ , and  $V_4 = \frac{2}{3}$  for  $n = 4$ . Volume that we can prove it through distance  $z_k - 1$  &  $1 - z_{\frac{1}{\sqrt{k}}}$  and then by summing:  $d_k = (1 - \sqrt{k}) + \left(\frac{1}{\sqrt{k}} - 1\right) = -\sqrt{k} + \frac{1}{\sqrt{k}} =$  represents the distances between radiuses so we can

calculate the volume of the polytope  $\frac{2^n}{n!}$  in the unity sphere:  $\int_0^1 -\sqrt{t} + \frac{1}{\sqrt{t}} dt = \frac{4}{3}$  that shows  $n=3$ .

**Prime number theorem:** We will terminate these parts by providing the density volume, the best and better approximation prime theorem defined through volume:

$$\begin{aligned} \#\{\text{primes}, p \leq 10^k\} &= \frac{1}{V_0} \frac{V_k}{\text{Ln}(V_k)} \\ &= \frac{3}{2} \times \frac{666 \dots}{\text{Ln}(666 \dots) + \text{Ln}(V_0)} \end{aligned}$$

$$\text{With } \{666 \dots k \text{ times}, V_0 = \frac{2}{3} = 0.666 \dots\}$$

### 3. Solution Birch-Swinnerton-Dyer Conjecture

First let's give some background on the conjecture, the B.S-D conjecture is under many progresses developed by Gross & Zagier, Barry Mazur, Rubin, and Bhargava [16], and many other mathematicians with related works. The conjecture describes the set of rational solutions to the equations defining elliptic curves, that claims that the rank of the abelian group  $E(K)$  of point of  $E$  is the order of the zero of the Hass-Weil L-function  $L(E, s)_{s \rightarrow 1}$ , when  $s \rightarrow 1$ . From the Mordell theorem, the group of rational points on an elliptic curve has a finite basis, Helmut Hass conjectured that  $L(E, s)$  could converge not only when  $\text{Re}(s) > \frac{3}{2}$  through an analytical continuation. Due to the modularity theorem that implies the analytical statement: to each elliptic curve over  $\mathbb{Q}$ , we may attach a correspondent L-series, a Dirichlet series,  $L(E, s) = \sum_1^n \frac{a_n}{n^s}$ , the generating function of the coefficient  $a_n$  is then  $f(E, q) = \sum_1^\infty a_n q^n$ , with  $q = e^{2\pi i z}$ , [13].

**Prove:** As a summary, we showed in this theory, which is based on a periodical fractal system, defined by an abelian algebraic variety described by a chain of finite cyclic covering of torus knots, with modular hyperbolic dynamical system. We defined the period of the system and showed that all the primes  $p_i$  split in both in the abelian field of  $\mathbb{Q}[\sqrt{2}, \sqrt[8]{5}]$  and in its cyclotomic abelian [11] extension  $\mathbb{Q}[i, \zeta_{5^4}]$ . Determined by the abelian variety defined by the complete split of all the prime numbers in the extensions,  $\Delta_{p_i} = \{p_i, 1 + p_i + p_i^2 + p_i^3 \equiv 0[5]\}$ , with  $p_i - 1 \not\equiv 0[5]$ . In our case the period of our dynamical system is  $n = 10$ , infinite cyclic linked torus knot is reduced due to the period of the system. Number of roots defined by the Euler totient associated function:  $\varphi(10) = \varphi(5) = 4$ , which is isomorphic to Mordel-Weil group  $\mathbb{Z}/_{4\mathbb{Z}}$  of order 4, is that the rank of the abelian variety related to the zero of the associated L-function related to Birch & Swinnerton-Dyer conjecture! We going to do more analysis to discover the right rank. By Hensel's lemma, the group of units  $\mathbb{Z}^*_{p_i}$  has  $(p_i - 1)th$  roots of unity. This splitting descends to the quotient rings  $\mathbb{Z}/_{p_i^n}$  and its group of units, giving an isomorphism  $GL(\mathbb{Z}/_{p_i^n}) \cong (\mathbb{Z}/_{p_i^{n-1}}) \oplus \mathbb{Z}/_{p_i-1}$ . Well in our case the units of  $\mathbb{Z}(\sqrt[4]{2}, \sqrt[8]{5})$  are the independent

elements defined by:  $\pm(2 + \sqrt{5})(\sqrt{2} + \sqrt[4]{5})(\sqrt[4]{2} + \sqrt[8]{5})(\sqrt[4]{2} - \sqrt[8]{5}) = \pm 1$ , where the points  $Q(4, \pm 5)$  elements of the hyperelliptic curve with rank  $\frac{d}{2} = \frac{8}{2} = 4$ . it shows that there is a correspondence between an hyperelliptic curve with genus 1 with degree defined by  $2g+2$ , the Galois group is contained in the symmetric group  $S_4$ , a double cover of  $S_2$ . Previously we proved the partial splitting of the primes in the field  $\mathbb{Q}[\sqrt{5}]$ , but a complete splitting of all the primes in the abelian extensions  $\mathbb{Q}[\sqrt[4]{2}, \sqrt[8]{5}]$  and the cyclotomic field  $\mathbb{Q}[i, \zeta_{5^4}]$ . Since for a curve of genus  $g$  and prime  $p_i$  the extension of the curve generated by the points of the 5 torsion, then we have similar situation for the distribution of rational numbers as the prime numbers! the distribution of the rational points depend on the primes ramification! Then the ramification of the curves depends on the prime's ramification in the field  $\mathbb{Q}[\sqrt[4]{2}, \sqrt[8]{5}]$ . Rational numbers lie partially on the elliptic curve, those elliptic curves embedded in the Seifert surfaces. Then after analyzing our system's Seifert surfaces, we can conclude some rational points lie on the complement of the circle "the trivial knot", some lie on the complement of the Hopf's link, some lie on the complement of the trefoil knot, but all the rational points lie completely on the complement of the hyperbolic Eight Knot, describing then the Seifert surfaces defined by the complement of the algebraic varieties described before. Here we connect knots with elliptic & hyperelliptic curves through the quadratic equation. This hyperelliptic curve, is of  $ord(f) = 4 \leq \deg(f) = 5$ , a reduced bilinear equation defined by  $f(x, y) = x^5 \pm z^5$  with degree 5, reduced to the quartic curve defined by:  $y^2 = \sum_1^4 a_k x^{2g+2} = x^4 - x^3 + x^2 - x + 1$ . There are no rational points for the case  $g \geq 2$ , (See [16]). Then we get 2 families of isotropic subspaces of the  $dim = g + 1 = 2$ , defined by the quadratic equations, generated by the symplectic matrices:  $\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \& \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$  seen before, describing the modular twists.

Let's denote by the  $E(\mathbb{Q})$  the Mordell-Weil group of rational on elliptic curve  $E$ ,  $Sel(E)_5$  the 5-Selmer group of  $E$ , and  $W_E[5]$  the 5-part of Tate-Shafarevich group of  $E$ , then we have the following sequence:  $0 \rightarrow E(\mathbb{Q})/_{5E(\mathbb{Q})} \rightarrow Sel(E)_5 \rightarrow W(E)_5 \rightarrow 0$ , see detail for generalization [21]. The rank of  $Sel_5 = r$ , such that  $\#Sel(E)_5 = 5^r$ , then if  $r$  is the rank of the curve,  $rank(E) = r$ , and by the upper bound inequality we have:  $20r - 15 \leq 5^5$ , (See [16]). Then for  $\begin{cases} r = 1 \text{ we have } 20 \leq 20 \\ r = 2 \text{ we have } 40 \leq 40 \end{cases}$  then  $r \in \{1, 2\}$  is an optimum bound. By The parity conjecture then concluded but for partial distribution, with less than 25% with rank 0 and less than 25% with rank 1 of the total rational points. And this is a perfect distribution since we see also as a prove the  $\#\{p_i \equiv 1[5]\} = \#\{p_j^2 \equiv 1[5]\}$ , defined by  $(\#\{11, 31, 41 \dots\}) = (\#\{19, 29, \dots\})$ , while the remaining over 50% defined by  $p_k^4 \equiv 1[5] \Rightarrow \{p_k \cdot p_k^3 \equiv 1[5]\}$  invertible elements through  $S_3$ , by the way  $p_i^3 \equiv 1[5] \Rightarrow \{p_i \equiv 1[5]\}$ , then the elements with  $rank(E) = 0$  and elements with  $rank(E) = 2$  are equal. Now by

observing the bound for the odd and even rank defined by  $12r + 1 \leq 5^r$  for  $r$  even, and  $60r - 55 \leq 5^r$  for  $r$  odd, (See Bhargava details [16]) we have then  $\begin{cases} r = 0, \text{ or } 2 \text{ then } 1 \leq 1, \text{ or } 25 \leq 25 \\ r = 1 \text{ then } 60 \leq 60 \end{cases}$  the rest of rational points by ramification will lie on the quartic equation defined by  $ordf = 4$  determined by the Seifert surface: the complement of the hyperbolic Eight-Knot.

**Conclusion:** All rational lie completely on the quartic elliptic curve with  $ordf = 4$  in the abelian extension  $\mathbb{Q}[\sqrt[4]{2}, \sqrt[8]{5}]$ , and lie partially on the elliptic curves with the  $rank(E) = 0, 1$  or  $2$ . Hence it shows an equidistributional of  $\#\{\text{rational points}\}$  between  $rank(E) = 0$  &  $rank(E) = 1$ , which prove the parity conjecture. While elliptic curve with rank 0 & 2 shows with symmetrical rational points with similar points. Then for this partial distribution, the elliptic curve of  $rank(E) = 0$  & 1 represent less than 50% of rational points, and the remaining over 50% lie on the hyperelliptic curve with  $deg f = 4$  that we going to define. Now back to our Riemann sequence:  $\xi_s = \sum_1^n \frac{1}{k^s} \approx 2 \sum_0^{p-1} e^{\frac{-\beta k^2}{p}}$ , related to Theta function, with  $\beta = 2\pi i$  and  $p = p_2 = 10^2$ , with  $\gcd(k^2, p_2) = 1$ , its Euler totient function  $\varphi_{k^2}(p_2) = \dots = \varphi_{k^m}(p_m) = 4$ , with  $p_m = 10^m = 2^m 5^m$ .  $\Phi_{10}(x) = \Phi_{2 \times 5}(x) = \Phi_5(-x) = x^4 - x^3 + x^2 - x + 1$ . Now returning to our quartic polynomial, which is with discriminant  $\Delta_{10} = \Delta_5 = (-1)^{\frac{\varphi(n)}{2}} \frac{n^{\varphi(n)}}{\prod_{d|n} p^{\frac{\varphi(n)}{p-1}}} =$

$5^3$  that shows 5 the only ramified prime number in the cyclotomic field  $\mathbb{Q}[\zeta_{10}]$ . Since  $\Delta_q = 2^2 \cdot 5$  the discriminant of the quadratic equation, then the associate elliptic curve with discriminant  $\Delta_e = -16(4A^3 + 27B^2) = -2^6 5^3$  or  $= -3^3 \cdot 2^4 \cdot 5^2$  depends on 5 if divides  $A^3$  or  $B^2$ . Note since the system is fractal, modular, and periodic, then by the theorem of elliptic curve over rational set, for points  $P, Q \in C(E)$ , then  $P + Q \in C(E)$ , then all rational points are determined by the fractal system, defined by circles of radius  $T^n = 10^n$ . For example from 2 rational points determined in  $T^n = 10^n$ , we can compute the third rational point in  $T^{n-1} = 10^{n-1}$ , then the algebraic varieties stays the same analogous, with the same singular points, defined by rational points  $\alpha_n = \frac{k^n}{T^n}$ , with  $\varphi_{k^n}(T^n) = \varphi_{k^n}(p_n) = 4$ . Now returning to our quartic polynomial, which is with discriminant  $\Delta_{10} = \Delta_5 = (-1)^{\frac{\varphi(n)}{2}} \frac{n^{\varphi(n)}}{\prod_{d|n} p^{\frac{\varphi(n)}{p-1}}} = 5^3$ . The

primes 2 & 3 are reversible symmetric in  $\mathbb{F}_5, 2, 3 \equiv 1[5]$ , by Fermat little theorem  $2^4 \& 3^4 \equiv 1[5]$

$$\begin{cases} \Delta_e = -2^4(A^3 + 3^3B^2) \equiv -(A^3 + 2B^2)[5] \\ \Delta_e = -2^4(A^3 + 3^3B^2) \equiv 3^2(A^3 + 2B^2)[5^2] \end{cases}$$

Since  $p_i \nmid \Delta_e$ , the same for  $p_i^2 \& p_i^4 \nmid \Delta_e$  the discriminant of the elliptic curve, all the primes except 2 & 5 that are dividing the discriminant of the related elliptic curve, hence we have a good reduction, then  $L(E, s) = \prod_i (1 - a_{p_i} p_i^{-s} + p_i^{1-2s})^{-1}$ , with  $N_{p_i} = p_i + 1 - a_{p_i}$  it follows:  $L(E, s)_{s \rightarrow 1} = \prod_i (1 - a_{p_i} p_i^{-1} + p_i^{-1})^{-1} = \prod_i \frac{p_i}{N_{p_i}}$ . Notice

for  $n$  the product of all ramified primes,  $n = \prod_{i=1}^n p_i^{\alpha_i}$  the Euler totient function  $\varphi(n) = \prod_{i=1}^n p_i^{\alpha_i-1}(p_i - 1)$  hence the fraction related to the prime density:  $\frac{n}{\varphi(n)} = \prod_{i=1}^n (1 - p_i^{-1})^{-1} = \xi_{s \rightarrow 1} = L(E, s)_{s \rightarrow 1} = \prod_{p_i \nmid \Delta_E} (1 - p_i^{-s})^{-1}$  when  $s \rightarrow 1$ , for bad reduction. To evaluate the rank, we need to find conditions on  $a_{p_i}$ , the trace of the Frobenius, related to the density of the primes. We already know the distributions of the primes on the Seifert surfaces defined by our system, determined by the symplectic matrix, this examination will filtrate the values of  $a_{p_i}$  related to the discriminant of the quadratic equations. With  $N_{p_i} = p_i + 1 - a_{p_i}$ , we notice then in the following densities:  $\frac{N_{p_i}}{p_i} - \frac{\varphi(p_i)}{p_i^2}$  with giving  $a_{p_i} = 0, 1, 2 \& 3$  shows some interesting connections to Alexander knot polynomial, and the Seifert surfaces of our system:

For  $a_{p_i} = 0$  we have  $\frac{N_{p_i}}{p_i} - \frac{\varphi(p_i)}{p_i^2} = \frac{p_i+1}{p_i} - \frac{p_i-1}{p_i^2} = \frac{p_i^2+1}{p_i^2}$  related to Hopf's link

For  $a_{p_i} = 1$  we have  $\frac{N_{p_i}}{p_i} - \frac{\varphi(p_i)}{p_i^2} = 1 - \frac{p_i-1}{p_i^2} = \frac{p_i^2-p_i+1}{p_i^2}$  related to trefoil knot

For  $a_{p_i} = 2$  we have  $\frac{N_{p_i}}{p_i} - \frac{\varphi(p_i)}{p_i^2} = \frac{p_i^2-2p_i+1}{p_i^2} = \frac{(p_i-1)^2}{p_i^2}$  related to trivial knots

For  $a_{p_i} = 3$  we have  $\frac{N_{p_i}}{p_i} - \frac{\varphi(p_i)}{p_i^2} = \frac{p_i-2}{p_i} - \frac{p_i-1}{p_i^2} = \frac{p_i^2-3p_i+1}{p_i^2}$  related to the hyperbolic Eight-Knot.

Notice the interesting relation that links the knots of the system:  $\left(\frac{p_i^2-p_i+1}{p_i^2}\right) + \left(\frac{p_i^2-2p_i+1}{p_i^2}\right) - \left(\frac{p_i^2+1}{p_i^2}\right) = \frac{p_i^2-3p_i+1}{p_i^2}$  & by  $\left(\frac{p_i^2-p_i+1}{p_i^2}\right) + \left(\frac{p_i^2-2p_i+1}{p_i^2}\right) + \left(\frac{p_i^2+1}{p_i^2}\right) = \frac{3(p_i^2-p_i+1)}{p_i^2}$  that shows the link of cyclic covering, describe an expanding trefoil knots, and expanding hyperbolically by changing the direction of the flow of the Hopf's link. Hass proved the bound of  $a_{p_i} \leq 2\sqrt{p_i}$ , this bound is not sufficient to determine the rank of  $E$ , then a condition of the rank of an elliptic curve is  $a_{p_i} = 0, 1$  or  $2$ . While  $a_{p_i} = 3$  is related to the rank of the hyperelliptic curve of order 4, defined as a hyperbolic Seifert surface, complement of the hyperbolic 8-knot.

Now we going to extend our analysis to prove the rank of the elliptic curve, by verifying the Zeta function of an elliptic curve over finite field  $\mathbb{F}_p$  which is a generating function in the field extension  $\mathbb{F}_{p^n}$ . The Zeta function defined by:  $Z(E(\mathbb{F}_{p^n})) = \exp(\sum \#|E(\mathbb{F}_{p^n})| \frac{x^n}{n}) = \frac{1-a_p x + p x^2}{(1-x)(1-px)}$  after evaluation. And for a finite field  $K_n$ , the formula defined by:  $Z(E(K), x) = Z\left(E(K), \frac{1}{px}\right) = \exp(\sum \#|E(K_n)| \frac{x^n}{n}) = \frac{1-a_p x + p x^2}{(1-x)(1-px)}$ . And since:  $e^z = \sum_{n=0}^{\infty} \frac{z^n}{n!}$ , and  $\text{Log} z = \sum_{k=1}^{\infty} (-1)^{k+1} \frac{(z-1)^k}{k}$ , therefore:  $Z((E(K), x) = Z\left((E(K), \frac{1}{px})\right) = e^{L(E(K), x)} = L(E(K), x) + 1$ , and by approximating the rational fraction, with  $0 < A \leq B$ , and condition  $a_p \leq 2\sqrt{p}$  &  $p \leq x$ , notice the fraction:  $\frac{A}{B} \geq 1$  &  $A - B = x N_p$  it

follows then the approximation of the fraction for a max bound determined by:  $\frac{A}{B} = \frac{1-a_p x + p x^2}{(1-x)(1-px)} \approx \frac{1-a_p x + p x^2}{p x^2} \approx \frac{p x^2}{p x^2} + \frac{1}{1} - \frac{a_p x}{x^2} = 1 + \left(1 - \frac{a_p}{x}\right) \approx 1 + \left(1 - \frac{1}{x}\right)^{-a_p}$ , applying  $e^x \approx x + 1$  then  $1 + L(E(K), x) \approx 1 + \left(1 - \frac{1}{x}\right)^{-a_p}$ , it follows  $L(E(K), x) \approx \left(1 - \frac{1}{x}\right)^{-a_p}$ , by using  $\text{Log}(x+1) \approx x$ , then  $L(E, x) = (-1)^{a_p} (\text{Log}(\frac{1}{x}))^{-a_p} \approx (\log x)^{-a_p}$  and since  $L(E, x) = \prod_{p \leq x} \left(\frac{N_p}{p}\right)^{-1}$  then  $\prod_{p \leq x} \frac{N_p}{p} = (\log x)^{a_p}$  with:  $r = a_p$ .

Now the approximation for the inf bound of  $\frac{A}{B} = \frac{1-a_p x + p x^2}{(1-x)(1-px)} \approx \frac{1-a_p x + p x^2}{p x^2} \approx \frac{p x^2}{p x^2} + \frac{1}{1} - \frac{a_p x}{x^2} = 1 + \left(1 - \frac{a_p}{x}\right) \approx 1 + \left(1 - \frac{1}{x}\right)^{a_p}$  and with  $\text{Log} Z(E(K), x) = L(E, x) \approx \text{Log}\left(1 + \left(1 - \frac{1}{x}\right)^{a_p}\right)$ . Which shows that  $L(E, x)$  is analytical at the point  $x = 1$ , since  $\text{Log}\left(1 + \left(1 - \frac{1}{x}\right)^r\right) \approx \sum_{k=1}^{\infty} (-1)^{rk+1} \frac{\left(1 - \frac{1}{x}\right)^{kr}}{k} \Rightarrow L(E, x)_{x \rightarrow 1} = 0$ .

**Conclusion:** The  $\text{rank}(E) = a_p$  is the trace of the Frobenius, with conditions  $a_p = 0, 1$  or  $2$  for elliptic curve and  $\text{rank}=3$  for hyperelliptic curve of order 4 with genus 1. It also proves the parity conjecture that depends on  $(-1)^r$  for  $a_p = r$  odd or even. An interesting conjecture that we going to use in connection to Riemann conjecture to solve Navier stokes conjecture.

## 4. Solution to Navier Stokes Conjecture: Existence & Smoothness of the Navier Stokes Equation

**The analogous Physical parameters that described the mathematical system's flow:** As we notice from our previous analysis of the mathematical discrete system's flow, that fundamentally Navier Stokes shows its footprint in the flow of the mathematical universe. Since we proved the duality between continuous systems & discrete system by analyzing Riemann hypothesis, which results from the flow of the discrete system, by proving the entangled representation of the asymmetrical set of integers and its extensions as points like for an N-Simplex, or as a loop for its analytical structure, or as a knot for its topological structure, or as a polytope volume for its geometric structure, and as an algorithm of string for its fundamental braid group structure. Let's denote by the following analogous parameters of the mathematical universe system:

**$U(x, t)$ : field,  $q$ : density,  $\omega$ : twist "frequency",  $V_{ol}$ : volume,  $E$ : energy,  $P$ : pressure & by  $F$  force.** We proved the dynamical system of integers is defined in a smooth fractal Manifold considered as the spacetime of 4 dimensions, in the Hilbert space, with metric space, endowed with inner product. The system is periodic, fractal, generated by two forces  $f_i$  &  $f_{ext}$ , with  $f_{ext}$  force exterior related to the hyperbolic expansion of the manifold as the packing

sphere, that contains & covers the flow of the internal linked cyclic covering system generated by the hyperbolic force related to the 8-knot, and the other interior force  $f_i$  related to the link of the cyclic covering of the system generated by following 3 forces related to “Trivial knot, Hopf’s Link, and the Trefoil knot”, with a total then of 4 independent and unified forces in the Poincare group defined by symplectic group  $SO(3,1)$  related to the spacetime translation transformation and Lorentz transformation related to the rotation and reflection for its Dihedral group, while we notice in our system that the field  $U(x, t)$  defined in  $\mathbb{Q}[\sqrt[4]{2}, \sqrt[8]{5}]$ , and its cyclotomic extension field  $\mathbb{Q}[i, \zeta_5]$ . Abelian group  $G$ , where its center  $Z(G)=G$  generated by the center groups  $\mathbb{Z}_2, \mathbb{Z}_3, \mathbb{Z}_5$ , and the circle unity  $U(1)$ , which is related to the Gauge field for the Yang Mills theory, normal subgroups that commute resp with  $SU(2)$ ,  $SU(3)$  and  $SU(5)$  or  $SO(10)$ . While the flow of the vector field is asymmetrical generated by rotation, twist, and translation defined by a modular flow in a closed space, which imply the  $\text{div } U = 0$ , defined by 2 asymmetrical curves, one stable related to the velocity  $\nabla^2 U$  and the other flow related to  $U \cdot \nabla U$ , while  $U \cdot \nabla U \ll \nabla^2 U$  for a normal stable system flow. While the turbulence always exists but minimal, the flow is generated by the asymmetrical internal and external waves describing a vortex ring model, that we going to describe by the volume exterior  $V_{ext}$  and its asymmetrical volume interior  $V_i$  that flows in opposite direction, and while  $V_i$  expands, it creates then an interior force from the inside described by the force of pressure, then the energy Kinetic exists and bounded since per definition knots are bounded! And the compliment of the knot is the Seifert surface! Our mathematical system describes all the parameters needed to describe Navier stokes equation. As parameters: we described our numerical flow based on the homogenous spacetime, where space & time have same period  $T_k = 10^k$ , defined by the analytical, or bump function  $U(x, t)$ , **related to the convergence of Riemann series** in a 4-dimensional spacetime analogous to Minkowski spacetime, defined by  $(a, b, c, it)$  for its coordinates, with:  $a^2 + b^2 + c^2 - t^2 = 0$  with

$$\begin{cases} a^2 + b^2 = 1 \rightarrow \text{rotational flow} \\ t^2 - c^2 = 1 \rightarrow \text{hyperbolic flow} \end{cases}, \text{ Then: } \begin{cases} \|U_x\|^2 + \|U_t\|^2 = 1 \rightarrow \text{rotational flow} \\ \|U_y\|^2 - \|U_x\|^2 = 1 \rightarrow \text{hyperbolic flow} \end{cases}$$

The space  $U(U_x, U_y, U_z, U_t)$  defined in  $\mathbb{R}^3 \times [0, +\infty[ \rightarrow \mathbb{R}$  is an embedding, analogous space to Minkowski spacetime, since if we consider the relation between previous knots &  $U_i$  then with,  $\|U_x\|^2 + 1 = K_{31}$ ,  $\|U_y\|^2 + 2 = K_{21}$  “Hopf’s Link”,  $\|U_z\|^2 = K_0$ , “Trivial knot”, and  $\|U_t\|^2 - 1 = K_{41}$ . “8-knot”. We notice the Seifert surfaces related to the vector spacetime  $V(1,1,1,i)$  defined by the symplectic matrix:  $M = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$  &  $N = \begin{pmatrix} 1 & i \\ 0 & 1 \end{pmatrix}$ , are the complement of the trefoil knot  $S^3 \setminus K_{31}$  resp the complement of the hyperbolic 8-knot:  $S^3 \setminus K_{41}$  defined by:  $\det[M - xM^T] = x(x + \frac{1}{x} - 1)$  as  $K_{31}$  &  $\det[N - xN^T] = x(x + \frac{1}{x} - 3)$  as  $K_{41}$ . Then for  $\alpha = \pm 1, \pm i$  with:

$A = \begin{pmatrix} 1 & \alpha U \\ 0 & 1 \end{pmatrix}$  we have:  $\text{Det}[A - xA^t] = x^2 - (2 - (\alpha U)^2)x + 1 \leftrightarrow \text{Det} = x[(\sqrt{x} - \frac{1}{\sqrt{x}})^2 + (\alpha U)^2]$ , Then the Seifert surface covers the entire space if  $\text{Det}[A - xA^t] = 0$ , hence  $U = (\sqrt{x} - \frac{1}{\sqrt{x}})$ , with  $\alpha = \pm 1, \pm i$ . An equation that we are familiar in our paper, and already analyzed related to the convergence of Riemann Hypothesis series & B.S-Dyer rank conjecture. We described the density  $\mathbf{q}$  of the integers, and primes numbers by Euler totient function, also we defined the fractal volume of the packing related to the spheres packing

$$V_{ext} = \int_{10^{2k}}^{10^{2k+2}} \sqrt{x} dx = \left[ \frac{2}{3} x\sqrt{x} \right]_{10^{2k}}^{10^{2k+2}} = 666 \cdot 10^{3k}, \text{ and volume of its asymmetrical inverse flow by } V_{in} = 2 \int_{10^{2k}}^{10^{2k+2}} \frac{1}{\sqrt{x}} dx = [2\sqrt{x}]_{10^{2k}}^{10^{2k+2}} = 18 \cdot 10^k = \frac{2}{3} \times 3^3 \times 10^k$$

sphere of radius 3, that expands by  $10^k$  With  $\partial V_{ol} = V_{ext} - V_{in}$ , **asymmetrical entities defined by volume that flow opposite direction, to create a vortex torus model**, then in the interval  $[0,1]$  the volume is related to the polytope  $\int_0^1 -\sqrt{x} + \frac{1}{\sqrt{x}} dt = \frac{4}{3}$  for  $V_n = V_3 = \frac{2^3}{3!}$ . The frequencies  $\omega$  or the velocity of the system is defined by the modular twists related to the linking in the cyclic covering defined in the algebraic variety. Due to Riemann hypothesis convergences, then based on Fourier transform the series  $\sum U(x, t)$  **converge for both cases: analytic, modular & for Bump function related to the Gaussian distribution** along the period and the expansion with  $T_k = 10^k$ . The analytical modular function defined by the Laplacian function is to describe the behavior of the system along the eigen-values & eigen-function, which return to be a differential equation. Although, we know that the mathematical foundation of the wave, heat equation and the Schrodinger equation in quantum mechanics are based on Theta function. We proved also the singular points related to the algebraic variety in the Riemann Hypothesis section. The physical Navier Stokes equation is known by the equation that describes the chaotic flow of different systems! The foundation of such an equation is mathematical. We can then apply even Navier Stokes equation to describe the dynamical system of the discrete system! This analysis describes then the heartbeat of the mathematical universe flow, by describing the hyperbolic volume of its asymmetrical Seifert surfaces, and the singular points for such a fractal periodic system related to its twists “frequencies”. An abstract theoretical mathematical model that reflects reality, a consistent model to describe the flow of general systems, based on, then we can model the biological heartbeat system defined by sound to describe the air flow for the respiratory system, and blood flow system! The Platonian T.o.E then is just the fundamental mathematical structure of Navier Stokes equation. We proved that Navier Stokes equation which is defined for continuous system have a discrete dual representation based in Number Theory. We also showed the connection between cross polytopes, and the hyperbolic flow of the cyclic covering, properties that are related to the Poincare conjecture! Since we notice a similarity in the construction for the prove of the Poincare conjecture by

Perelman, where he used in hyperbolic geometry Dehn surgery for a linked cyclic covering of the Seifert bundle that extent hyperbolically in a closed space! Both Perelman & Hamilton applied Ricci flow for a smooth differential manifold to prove the compact spaces diffeomorphic to spheres & to get a good understanding of the singularities. Here we showed the singularities are related to the blow up of the Alexander polynomial at  $x = 1$  related to trivial knot "Unknot", while prime number have this structure defined by the Unknot discovered by Barry Mazur, then the complement of a trivial knot defined by  $S^1$  is the Seifert surface  $S^3 \setminus S^1 = \partial S^3 = S^2$  where  $S^1$  is the boundary of the Seifert surface, since by definition a knot is embedded in  $S^3$ . Ricci flow here to describe the exterior forces related to curvature.

**Prove of Navier Stokes statement related to the existence of Smooth solution [18]:** In our model we proved that Riemann hypothesis have two kind of distribution, analytical related to Fourier series, or Theta function, and discrete related to Poisson distribution, or Gaussian distribution for a probability normal distribution of the chaotic system, for this reason you can model any dynamical physical system from discrete to continuous system by a differential function of class  $C^\infty$  or  $C^\omega$  defined in our case by the bump function related to Gaussian distribution of class  $C^\infty$ , or by an analytical modular function of class  $C^\omega$ . Defined by:  $U(x_i, t)_{1 \leq i \leq 3} = \left(\sqrt{x_i} - \frac{1}{\sqrt{x_i}}\right) e^{\beta t}$  defined in  $\mathbb{R}^3 \times [0, +\infty[$ , with  $\beta = \pm 1$  or  $\pm i$  since the system is periodic & fractal! as defined for the condition of the Riemann Hypothesis sequence to converge to cover both the bump Gaussian function and the analytical, or the modular Theta function of the Riemann Hypothesis series. Then  $U_x^0 = U(x, 0) = \left(\sqrt{x} - \frac{1}{\sqrt{x}}\right) = -U\left(\frac{1}{x}, 0\right) = -U_1^0$  with  $f(x) = \sqrt{x}$  &  $\partial f = \frac{1}{2\sqrt{x}}$ . We already know the properties described for this function in our paper,  $f - 2f' \rightarrow \sqrt{x}U_x^0 = f^2 - (f^2)' = f^2 - \partial f^2$  where we proved that  $\sum_{x=1}^n U_x^0$  a fractal sequence related to the volume  $\partial V_{ol} = V_{ext} - V_{in}$ , while  $U_x^0$  bounded since  $(U_x^0)^2 - 1 < (U_x^0)^2 < (U_x^0)^2 + 1 = \left(x + \frac{1}{x} - 1\right)$  equation of a trefoil knot & 8-knots that are embedded in  $S^3$ , while knots are closed curves!  $U_x^0$  is of class  $C^\infty$ ,  $\partial_t U_x^0 = \frac{1}{2\sqrt{x}} \left(1 + \frac{1}{x}\right)$ , and for any  $n$  &  $k$ , we have:  $\partial_t^n U_x^0 = (-1)^{k+1} \times \frac{(2k-3)!!}{2^k} \times \frac{1}{x^{\frac{2k-1}{2}}} \times \left(1 + \frac{2k+1}{2x}\right) \approx C(-1)^{k+1} \frac{1}{x^{\frac{2k-1}{2}}} \times x^k (x - 1)^{\frac{-(2k+1)}{2}} \approx C(-1)^{k+1} (x-1)^{-k} = C(-x+1)^{-k} < C(|x|+1)^{-k}$ . Then  $|\partial_t^n U_x^0| < C(|x|+1)^{-k}$ . The asymmetrical curves defined by the flows shows: The flow of the system is defined in  $\frac{PSL(2, \mathbb{R})}{PSL(2, \mathbb{Z})} \cong S^3 \setminus K$ , and since for  $M = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$  &  $N = \begin{pmatrix} 1 & i \\ 0 & 1 \end{pmatrix}$ ,  $\det[M - xM^T] = x\left(x + \frac{1}{x} - 1\right)$  is  $K_{3_1}$  &  $\det[N - xN^T] = x\left(x + \frac{1}{x} - 3\right)$  is  $K_{4_1}$ . And since  $A = \begin{pmatrix} 1 & \alpha U_x^0 \\ 0 & 1 \end{pmatrix}$  &  $A^T = \begin{pmatrix} 1 & 0 \\ \alpha U_x^0 & 1 \end{pmatrix}$  are

connected by the Seifert matrix determinant, we have  $\det[A - xA^T] = x^2 - \left(2 - (\alpha U_x^0)^2\right)x + 1 \leftrightarrow \det = x \left[\left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)^2 + (\alpha U_x^0)^2\right]$ , then for the Seifert surface to cover the entire space,  $\det = 0$ , hence  $U_x^0 = \alpha \left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)$  with  $\alpha = \pm 1, \pm i$ . Since we are analyzing the hyperbolic horocycle flow based on  $\partial V_{ol} = V_{ext} - V_{in}$ , then we have the asymmetrical stable flows defined when  $t \rightarrow +\infty$  that converge toward the initial state, defined by stable curves determined by the right multiplication of the nilpotent group  $\{\varphi_{U_x^0} = \begin{pmatrix} 1 & \alpha U_x^0 \\ 0 & 1 \end{pmatrix}, x \in \mathbb{Z}^*\}$  by the matrix of the hyperbolic flow, defined by the diagonal group  $\{\psi^t = \begin{pmatrix} e^t & 0 \\ 0 & e^{-t} \end{pmatrix}, t \in \mathbb{Z}\}$ .

$$\varphi_{U_x^0} \times \psi^t = \begin{pmatrix} 1 & \alpha U_x^0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} e^t & 0 \\ 0 & e^{-t} \end{pmatrix} = \begin{pmatrix} e^t & \alpha U_x^0 e^{-t} \\ 0 & e^{-t} \end{pmatrix}_{t \rightarrow +\infty} = \varphi_{U_x^0} = \begin{pmatrix} e^t & \alpha U_x^0 \\ 0 & 1 \end{pmatrix}$$

And for its asymmetrical flow by left multiplication of the matrix of the hyperbolic flow, defined by the diagonal group  $\{\psi^t = \begin{pmatrix} e^t & 0 \\ 0 & e^{-t} \end{pmatrix}, t \in \mathbb{Z}\}$  to its transpose matrix defined by the nilpotent group  $\{\varphi_{U_x^0}^T = \begin{pmatrix} 1 & 0 \\ \alpha U_x^0 & 1 \end{pmatrix}, x \in \mathbb{Z}^*\}$ .

Otherwise, the asymmetrical flow is not stable. Then the asymmetrical flow is stable when  $t \rightarrow +\infty$ :

$$\psi^t \times \varphi_{U_x^0}^T = \begin{pmatrix} e^t & 0 \\ 0 & e^{-t} \end{pmatrix} \begin{pmatrix} 1 & 0 \\ \alpha U_x^0 & 1 \end{pmatrix}_{t \rightarrow +\infty} = \begin{pmatrix} e^t & 0 \\ \alpha U_x^0 e^{-t} & e^{-t} \end{pmatrix}_{t \rightarrow +\infty} = \varphi_{U_x^0}^T = \begin{pmatrix} e^t & 0 \\ \alpha U_x^0 & 1 \end{pmatrix}$$

We notice the asymmetrical hyperbolic flows tend to be asymmetrical equal when  $t \rightarrow +\infty$ ! That tend to  $\rightarrow \alpha U_x^0$ , then the flow is with both directions  $\updownarrow$  hence  $\text{div } U = 0$ . Also I want to clarify for the hyperbolic flow  $e^t$  since we solved the problem of infinity based on the fractal manifold, that expand with a rate of  $T_k = 10^k = e^{k \ln 10} \approx e^{2k}$ , then the parameter  $e^t$  when  $t \rightarrow +\infty$  indicate in that direction the fractal flow of the system! The flow of the system is stable periodic and fractal. Also we proved in our system the connection of the distance  $z = U_x^0 = \left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)$  to the Alexander polynomial knots, described by: Trivial knot, Hopf's link, trefoil knot & the hyperbolic 8-knot by the following equations:  $1 + |U_x^0|^2 = x + \frac{1}{x} - 1$  for  $K_{3_1}$ ;  $1 - |U_x^0|^2 = x + \frac{1}{x} - 3$  for  $K_{4_1}$ ;  $|U_x^0|^2$  for the trivial knot  $x = 1$ , and  $2 + |U_x^0|^2 = x + \frac{1}{x}$  for Hopf's link. Each of these knots related to a force, the system then described by 4 forces. With  $\rho = \frac{1}{V_{ol}}$  for a unit masse, hence the energy Kinetic  $E$  defined by  $\frac{\rho}{2} |U_x^0|^2$  is then bounded since knots are closed curves, embedded in  $S^3$  by definition, and the Seifert surfaces defined by the complement of the knots  $S^3 \setminus K_i$  related to the fields  $\mathbb{Q}[i], \mathbb{Q}[\omega]$ , and  $\mathbb{Q}[\sqrt{5}]$ . And since all prime numbers split in  $\mathbb{Q}[\sqrt[4]{2}, \sqrt[8]{5}]$ , and its cyclotomic extension  $\mathbb{Q}[i, \zeta_{5^4}]$ , then all energy is



distributed in this field of order 4. Now for the analytical, modular flow, described by Theta function in our theory for the Riemann series convergence,  $U(x, t) = (\sqrt{x} - \frac{1}{\sqrt{x}})e^{i\omega t}$  is of class  $C^\omega$ . We have the following analytical modular flow defined by the multiplication of the diagonal group  $\Psi^t = \begin{pmatrix} e^{it} & 0 \\ 0 & e^{-it} \end{pmatrix}, t \in \mathbb{Z}$  but this time by the nilpotent group  $\{\varphi_{U_x^0} = \begin{pmatrix} 1 & U_x^0 \\ 0 & 1 \end{pmatrix}, x \in \mathbb{Z}^*\}$ , With  $x = \omega t = \frac{2\pi}{T_k} t = \frac{\pi}{5} t^k$  for a uniform velocity with period  $T_k = 10^k$  and for  $k = 2$ , we have the convergence of the Riemann series defined by a Theta function as we described in this theory, and based on Fourier transform we determine the temporal and Spatial flow [19]. We also showed that the Euler Totient function is the same for the algebraic varieties of the fractal system  $\varphi_t(T_1) = \dots = \varphi_{t^m}(T_m) = 4$ , with  $T_m = 10^m = 2^m 5^m$ , We have then:  $\varphi_{U_x^0} \times \Psi^t = \begin{pmatrix} 1 & U_x^0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} e^{i\omega t} & 0 \\ 0 & e^{-i\omega t} \end{pmatrix} = \begin{pmatrix} e^{it} & U_x^0 e^{-i\omega t} \\ 0 & e^{-i\omega t} \end{pmatrix}$ , and for the asymmetrical flow:  $\Psi^t \times \varphi_{U_x^0} = \begin{pmatrix} e^{i\omega t} & 0 \\ 0 & e^{-i\omega t} \end{pmatrix} \begin{pmatrix} 1 & 0 \\ U_x^0 & 1 \end{pmatrix} = \begin{pmatrix} e^{i\omega t} & 0 \\ U_x^0 e^{-i\omega t} & e^{-i\omega t} \end{pmatrix}$ . Hence this flow is periodic with order or period 5, known as the critical point.  $\|U_x^0 e^{-i\omega t}\|^5 = \|U_x^0 e^{-i\frac{\pi}{5}t}\|^5 = \|U_x^0\|$ , and  $t$  takes the discrete values in  $[0, 5)$ , for the asymmetrical flow the 5 points are the conjugates, if we want to describe the geometric shape of these points, then these 12 points, including 0 &  $\infty$  lie on the circumsphere of the icosahedron defined by the stereographic projection:  $(a, b, c) \rightarrow \frac{a+ib}{1-c}$ . The 12 vertices have complex coordinates:  $0, (e^{\frac{2\pi i}{5}})^k \left( e^{\frac{2\pi i}{5}} + \left( e^{\frac{2\pi i}{5}} \right)^{-1} \right), \infty, (e^{\frac{2\pi i}{5}})^k \left( (e^{\frac{2\pi i}{5}})^2 + \left( e^{\frac{2\pi i}{5}} \right)^{-2} \right)$  with  $k \in \{0, 1, \dots, 4\}$ . We notice also when we add the 2 flows,  $U_x^0 e^{-i\omega t} + U_x^0 e^{i\omega t} = 2U_x^0 \cos \omega t$  solution of the polynomial:  $x^2 - 2x \cos \beta + 1$  that represent the determinant of the Seifert surface defined by  $\det(M - xM^T)$  for the symplectic matrix  $M = \begin{pmatrix} 1 & \sqrt{\beta} \\ 0 & 1 \end{pmatrix}$  with  $\sqrt{\beta} = 2\sin(\frac{\pi t}{10})$ . And since the system is fractal  $t^k$  take the values in  $[0, 10^k)$ . The energy  $E$  here also bounded as we proved for the hyperbolic bump function that connect  $|U_x^0|^2$  to the bounded knots described above in the fields  $\mathbb{Q}[i]$ ,  $\mathbb{Q}[\omega]$ , and  $\mathbb{Q}[\sqrt[8]{5}, i]$ . The pressure as a force its determination is related to the condition initial of  $\|U_x^0\|$ , and  $\partial V_{ol} = V_{ext} - V_{in}$  We have  $\varrho = \frac{1}{V_{ol}}$  density per unit mass, and since  $F(\text{Force}) \times d(\text{distance}) = \text{Energy}$  and  $P(\text{Pressure}) = \frac{F}{S(\text{Surface})}$  then:  $\nabla P = P - P_0 = \frac{F \times d}{S \times d} = \frac{E}{V_{ol}} = \varrho E = \varrho \frac{1}{2} |U_x^0|^2$  which show that the properties of the pressure are related to that of  $U(x_i, t)_{1 \leq i \leq 3}$  and  $V_{ol}$  then  $P(x, t)$  is of class  $C^\infty$  or  $C^\omega$ . We have then the following construction of Navier stokes eq defined by Newton's 2<sup>nd</sup> law of motion ( $F = ma$ ) or momentum equation, within a

controlled volume, the mass of the flow remains constant with time, and we have:  $\varrho \frac{du}{dt} = F_{int} + F_{ext} = -\nabla p + \mu \nabla^2 u + F_{ext}$  with  $\frac{du}{dt} = \frac{\partial u}{\partial t} + u \cdot \nabla u$ .

**Conclusion:** Navier Stokes equation has its footprint in the mathematical invariant system for a smooth flow, which proves that the mathematical system is conscious! Also aware of non-smooth flows.

## 5. Platonian Theory of Everything Summary

**Finally, The  $\frac{1}{2}$ -inch Equation for the Theory of Everything  $S = 666$**

An Equation which provides & describes:

- The sphere packing of the set  $\mathbb{Z}$  & its extensions.
- The 4 unified forces: related to the 4 unified invariant knots described by trivial knot, Hopf's link, trefoil knot & the hyperbolic 8-knot.
- The probabilistic Quantum Mechanic: Described by Riemann Hypothesis, defined by Theta function, through Fourier transform [19].
- The yang Mills symmetries of transformations related to  $\mathbb{Q}[\sqrt[4]{2}, \sqrt[8]{5}]$ , & its cyclotomic extension  $\mathbb{Q}[i, \zeta_{5^4}]$ .
- The Standard Model / Based on prime Numbers splitting defined in the set  $[1, 100]$ , since it is fractal.
- The Chaotic distribution of general systems.

The equation encompasses many fields and will answer many unsolved questions from: Multiverse ideas, existence of Aliens, wormholes, existence of life outside the earth, how the virology system works, Brain/Mind and consciousness, financial systems, and so on....

## ACKNOWLEDGEMENTS

I want to thank Professor, and artist Robert Pope, "Director, Science-Art Research Centre of Australia, a great mind, who unveiled Plato's hidden knowledge through arts & science", for his encouragement to my research, and for opening me the door to view arts from different perspective, where Mathematics reflects its artistic structure in the reality! "The language of the Truth" Also, I don't want to forget to thank the science & art community, especially the math community for their great achievements and tremendous progress seen along this journey of mathematical history, with a chain of connected mathematical discoveries that led us to discover puzzle after puzzle, the grand picture of the mathematical universe system. Also, I want to dedicate this paper for the memory of my deceased father Mohamed, the year 2012, because of his lost and other reasons, what pushed me to look for the truth and the foundation of the creation.

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