

A New Approach to Duality of Electron

Sultan Muhammad*, Miss Omama

Qurtuba University Phase 3 Hayatabad Peshawar, Pakistan

Abstract I decipher quantum duality of electron in Young's Double-Slit experiment. Hypothesis intends to decode interaction of knocked-electrons with observer, and perturbative disappearance of interference pattern. Hypothesis is based on Bohr's Atomic Model, and the theoretical concepts of Quantization of electron. The hypothesis proposes a universal field, similar to Higg's field, that conserves the potential energy of electron through interaction with knocked-electrons, utilizing phenomena of pair-production. The hypothesis provides comprehensive theoretical and mathematical solutions to possibly elaborate, in a broader context, why electrons exhibit duality and the role of observer in Young's Double-Slit experiment through introduction of universal field (SM Field). The interactions between photon and knocked-electrons have been discussed. Through using Schrodinger wave equation (SWE), a mathematical model has been derived, that is used to explain role of the observer, and duality of electron by using SM field as a supplement.

Keywords Electron, Wave-Particle Duality, Quantum

1. Introduction

The postulation and understanding of wave-particle dual nature was a controversial topic to begin with. It is closely entangled with the origin of the Quantum theory. Many questions are associated with it. Why are electrons dual in nature? Why does the electron deter from presumed particle nature, and exhibit wave nature? What is the role of the observer in the change in behaviour of electron? Why does it behave differently when in external environment, i.e outside the nucleus? The solutions to these quantum mechanical problems were developed with assistance of currently existing theories, such as thermionic emission, De Broglie relation, Schrodinger wave equation.

The double slit experiment is the heart of Quantum mechanics (Feynman) [1] following the experiment:

If the beam of electrons passes through two slits, we result with diffraction pattern, instead of envisioned single line. [5] [6]. The dual nature of electron is observed. But an intriguing event occurs when a detector is positioned in order to observe the electron going through slit 'A' or 'B'. The wave pattern disappears [7] [8]. What ramificates the electron? What role does the detector play? How to elucidate this phenomena?

We know that electron gun produces knocked electron in the experiment. [10] Most quotidian method is thermionic emission. [11,12,13] The electrons are ejected from the surface of the metal with the virtue of temperature. But one

thing to be discerned, is that enough energy is to be supplied to overcome the wave function, in order to liberate the electrons, or, to over come the electrostatic force. That minimum energy can be stated as "Ionization Potential Energy". [14,15,16]

The electron has a distinctive property to have "n" numbers of energies when attainable [2,3]. This implies that it can occupy energy state, and expel it through the process of absorption and emission, respectively. It is understood that, from Bohr's model, when electron is confined to the nucleus, the electron jumps to higher energy-level or principle Quantum number when it absorbs the energy, and jumps back when emission occurs. [4]

(see appendix "A")

Likewise, unless the electron emits the energy, it persists in that upper energy-level. If no higher energy-level exists, then the electron is liberated from the atom, and henceforth, is known as 'knocked electron'.

For Hydrogen, the ionization-energy can be written as (See appendix "B")

2. Problem

As it is discussed that the electron is dual in nature, and it is also verified in the context of double slit experiment; The question arises, what is the explanation? Why does the electron, being matter, behave as a wave? Where does this dual nature stem from? Perhaps, it is just like De Broglie suggested in his hypothesis, that the motion is the agent to the wave nature of electron. But then, this would raise a serious objection.

If the wave nature of electron is dependent on the relative motion of electron, then we can establish a statement. "When

* Corresponding author:

Chemistryloversm@gmail.com (Sultan Muhammad)

Received: Dec. 15, 2020; Accepted: Dec. 29, 2020; Published: Dec. 31, 2020

Published online at <http://journal.sapub.org/ijtmp>

a detector is positioned to measure the position of electron, the wave function is collapsed". Therefore, from the above statement, we could postulate that "Either the electron has become stationary, or the wave length has approached infinity (as the relation suggests)".

$$\lambda = \frac{h}{m\vec{v}} \quad (a)$$

Anomaly:

Theoretically, if the wave function collapses, then we can assume the electrons are stationary, but, in the experiment the electrons are observed (detected) on the screen. figure (1) & (2) shows the results of double-slit experiment:

Diffraction pattern of electrons using two slits

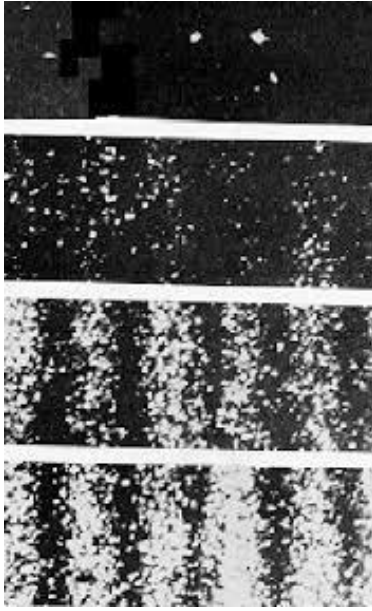


Figure 1

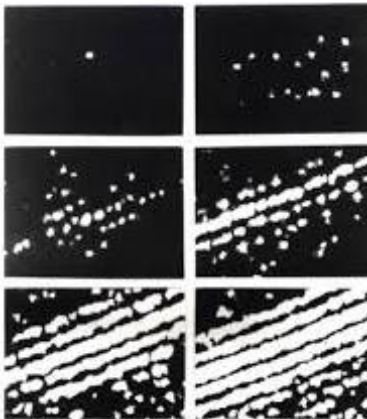


Figure 2

When a detector takes the measurement, the interference pattern disappears, but despite this, the electrons after the measurement are not stationary. They obey a straight line path, and are detected on the screen with no interference pattern. They have velocity, yet, the wave nature disappears.

Question:

What is the role of detector in change of behaviour, and how is velocity is related to wave nature of electron? What are the hidden parameters causing this change in behaviour?

3. A Field Model to Explain the Potential Energy of Knocked Electron

A theoretical field is proposed to explain the nature of electron.

We can characterize the electrons on the basics of Quantization of the properties of electron, such as:

- 1) Mass
- 2) Charge
- 3) Energy

So, to interact with the "Knocked- electron", we need a "field" or "condensate" with any of above mentioned quantized property.

A field of neutral Boson [18] is proposed with quantized energy, with capability to trigger Pair-production when the knocked electron enters the field. Thus, the potential energy of the electron is retained, and the knocked electrons are no longer considered with "ZERO POTENTIAL". The potential energy outside the Principle Quantum Number is due to the field.

Interaction of field with Electron

When the electrons are knocked out of the Principle- Shell, the electrons are subjected to uniform non- changing field, just like Higgs field. The field is neutral, but has a minimum function of energy, and interacts with the Ionization Potential-energy of the electron, subsequently triggering pair-production [19] at any instance. The produced pair of electron and positron interacts with the knocked electron. The interaction is repulsive in nature, as most knocked electron and produced electron have with each other. This pair-production phenomena occurs at the "Amplitude of the wave of knocked electron".

As soon as the knocked electron (not bounded to the nucleus) descends from amplitude peak, the produced pair annihilate each other, and the energy is returned to the field and electron:

$$A = \frac{\lambda}{\sin(wt + \theta)} \quad (1.1)$$

Where $\theta = 0$, "w" is frequency, and it depends on Potential energy of the electron $w = \frac{U}{h}$ and "t" is the time period that is taken to form single wave length. It depends on the kinetic energy of the electron: $t = \frac{h}{KE}$. λ is the wave length from DeBroglie hypothesis [20].

By putting the values in equation (1.1):

$$A = \frac{\lambda}{\sin(\frac{U}{KE})} \quad (1.2)$$

Where the “U” is potential energy derived from Bohr’s model, which depends on the nature of atom from which the electron is knocked out, and it can be calculated as:

$$U = \frac{(Z_{eff})^2 e^4 m_e}{4\epsilon_0^2 n^2 h^2}$$

Where “KE” is the kinetic energy of the knocked electron, as we use the electron gun to accelerate the electron. Thus, ϵ_0 is vacuum permittivity [23,24], “h” is plank’s constant [25], “e” is elementary charge [26,27,28], “ m_e ” is the mass of electron [29,30,31], “ Z_{eff} ” is effective nuclear charge [32,33,34]

KE becomes:

$$E = \frac{n^2 eV}{4} \quad (\text{“V” is the voltage applied})$$

Putting the values of “U” & “KE” in equation (1.2):

$$A = \frac{\lambda}{\sin\left(\frac{(Z_{eff})^2 e^3 m_e}{\epsilon_0^2 n^4 h^2 V}\right)} \quad (1.3)$$

The equation (1.3) is the final form to calculate amplitude of the wave of knocked electron.

At this point, the pair-production is triggered, and a repulsion occurs. The pair -produced electron exerts a repulsive force, which can be calculated using Coulombic force [21]:

$$F_c = \frac{Kq_e q_o}{r^2} \quad (1.4)$$

Where “K” is Coulombic constant [22] $K = \frac{1}{4\pi\epsilon_0}$, “ q_e ” is the charge of electron and “ q_o ” is the charge of the pair produced electron.

We know that:

$$E = F_c \cdot A \quad (1.5)$$

Where “A” is the amplitude of the wave of electron:

$$U = \frac{Kq_e q_o}{r^2} A \quad (1.6)$$

$$U = \frac{Kq_e q_o A}{r^2} \quad (1.6a)$$

But from Bohr’s Energy, we know that:

$$U = \frac{(Z_{eff})^2 e^4 m_e}{4\epsilon_0^2 n^2 h^2} \quad (1.7)$$

Comparing equation (1.6a) with (1.7):

$$\frac{Kq_e q_o A}{r^2} = \frac{(Z_{eff})^2 e^4 m_e}{4\epsilon_0^2 n^2 h^2} \quad (1.8)$$

Now, we can isolate “r”, as we are interested in the

field, repulsion of knocked electron and pair-produced electron. The equation yields the expected distance of the pair-production from the amplitude of the electron wave:

$$\frac{1}{r^2} = \frac{(Z_{eff})^2 e^4 m_e}{4\epsilon_0^2 n^2 h^2 A q_e q_o K} \quad (1.8a)$$

As “ q_e ” is charge of electron, and it equals to elementary charge “e”, and “ q_o ” is the charge of pair-produced electron, which also equals to elementary charge “e”. Correspondingly, equation (1.8a) becomes:

$$r^2 = \frac{4\epsilon_0^2 n^2 h^2 A e^2 K}{(Z_{eff})^2 e^4 m_e} \quad (1.9)$$

$$r^2 = \frac{4\epsilon_0^2 n^2 h^2 A K}{(Z_{eff})^2 e^2 m_e} \quad (2.0)$$

When the electron approaches amplitude, pair-production is triggered. The knocked electron and the potential energy acts as an agent.

The knocked electron, at the amplitude, experiences a repulsive force:

$$F_c = \frac{Kq_e q_o}{r^2} \quad (1.4)$$

As “ q_e ” and “ q_o ” are elementary charges:

$$F_c = \frac{K e^2}{r^2} \quad (1.4a)$$

Substituting the value of “ r^2 ” from equation (2.0) in equation (1.4a):

$$F_c = \frac{(Z_{eff})^2 e^4 m_e}{4\epsilon_0^2 n^2 h^2 A} \quad (2.1)$$

That force is responsible for potential energy of electron, even apart from nucleus.

4. Hypothesis

To answer the question, we initially try to describe the motion of electron, which is a general wave motion, but, a wave motion can be described by two parameters in our case:

- 1) Wave length
- 2) Amplitude / Frequency

The wave-motion may be evaluated as the amalgamation of two fully contrasting motions. Straight-line motion or “translatory” motion of electron along x-axis, and a vibratory motion along y-axis. The vibratory motion is due to the SM field, and its interaction with knocked electron. The two motions of the particle are additive.

Let us suppose two particles “A” & “B”.

The particle A has some kinetic energy. It progresses along x-axis in a straight line. Consequently, this motion can be termed as translatory motion.

Now, consider Particle “B”.

The particle B has some energy (may or may not be kinetic), and it vibrates along y-axis only. Accordingly, this motion can be termed as vibratory motion.

We can say both particles A & B have some motion, but none of them have wave motion (Sine-wave). But, what if we

have another particle “C” that is allowed to have both motions simultaneously. Or, we can say, particle “C” is in articulate motion, that is additive of Translatory and Vibratory motion. Such motions result in the “wave motion”. Figure (3) demonstrates the analogy:

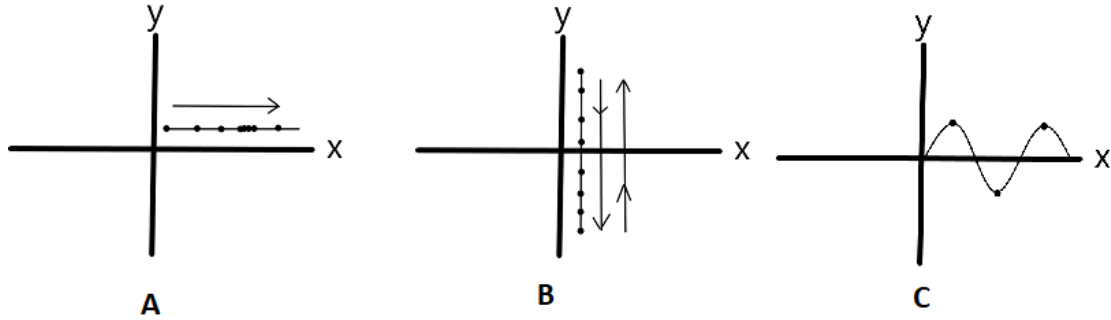


Figure 3

We can describe the wave motion as a function. Thus, wave function is the combination of two functions. Before we can solve the functions, we may describe the energy of the free particle in wave motion.

The translatory motion is caused by kinetic energy, in our case of knocked electrons this energy is gained by the electron from electrical acceleration provided by the electron gun.

The vibratory motion triggered by the potential energy gained by electron, in order to overcome the work function from the shell of the atom. The energy is termed as ‘Ionization energy’ in case of non-metals, and ‘work function’ in case of metals.

The electron, being energy carrier, will carry the ‘ionization potential-energy’ or ‘work function’ when it is ejected from the shell. This argument is valid if we eject an electron from the shell of atom, and the electron is then allowed to re-enter the shell. Thus, same amount of energy can be obtained. We can say that the energy is carried by electron. The carried energy then interacts with the SM field, which results in vibratory motion by triggering pair production, so that we can have pair-produced electron, and knocked-electron repulsion through Coulombic force, thus retaining potential energy.

We know that:

- 1) The ionization potential energy is converted to vibratory motion of electron.
- 2) The translatory motion is the conversion of electrical potential to kinetic energy.
- 3) Wave motion is the additive function of translatory and vibratory motions.
- 4) The total energy of the knocked electron is the sum of kinetic energy and work function/ potential energy of the electron, that is equal to Hamiltonian function. Thus, Schrodinger wave equation can be used to determine the total energy of the electron.

If we use above assumptions, we might be able to explain the role of observer in double slit experiment:

5. Equations and Derivations

To solve the stated problems, we use Schrodinger wave equation, The developed mathematics is similar to the prevalent problem of physics “Particle in box 1D”.

$$\frac{-h^2}{8\pi^2m} \nabla^2 \psi + U\psi = E\psi \quad (3)$$

As

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

As the electron maneuvers in straight line, we consider that electron is moving along x-axis only. Thus, the **Laplace** operator reduces to:

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + 0 + 0 = \frac{\partial^2}{\partial x^2}$$

We can rewrite the equation (3):

$$\frac{-h^2}{8\pi^2m} \frac{\partial^2 \psi}{\partial x^2} + U\psi = E\psi \quad (3.1)$$

It can be observed that the equation (3.1) has the same form as the Schrodinger wave-equation for ‘particle in a box 1D’.

Thus, we can use same wave function:

$$\psi = A \sin kx$$

Where:

$$k = \frac{n\pi}{a} \quad (3.2)$$

Where:

$$a \rightarrow \lambda$$

Unlike particle in one dimension we make some modifications:

- 1) Electron has some potential energy. The non-zero potential energy is due to the fact that “Electron acts as energy carrier”, and there is a field external to the atom. Thus, the potential energy is nothing but the ionization potential-energy, or, the work-function of the metal used as source in the electron gun.
- 2) The parameter “a” in equation (3.2) is to be replaced with wavelength of the moving electron, as the electron is fired from electron gun, we consider the wave length as a discrete box, where it interacts with the external field, where the electron exists for a period of time and then moves to second one, i.e “from one wave length to another”. To find wavelength, we use De-Broglie relation [9]:

$$\lambda = \frac{h}{p} \quad (3.3)$$

But we know that:

$$\vec{p} = m_e \vec{v} \quad (3.3a)$$

We can rewrite De-Broglie equation:

$$\lambda = \frac{h}{m_e \vec{v}} \quad (3.4)$$

Putting the value of ‘ λ ’ into equation (3.2):

$$k = \frac{n\pi m_e \vec{v}}{h} \quad (3.5)$$

Now, we have modified value of kappa. The wave function becomes,

$$\psi = A \sin \left[\frac{n\pi m_e \vec{v}}{h} x \right] \quad (3.6)$$

Now, we solve the equation to find the solution:

$$\frac{-h^2}{8\pi^2 m} \frac{\partial^2 \psi}{\partial x^2} + U\psi = E\psi \quad (3.1)$$

To find the solution, we take partial-derivative with respect to “x”:

$$\psi = A \sin \left[\frac{n\pi m_e \vec{v}}{h} x \right] \quad (3.6)$$

$$\frac{\partial \psi}{\partial x} = \frac{\partial A \sin \left[\frac{n\pi m_e \vec{v}}{h} x \right]}{\partial x} \quad (3.7)$$

$$\frac{\partial \psi}{\partial x} = \left[\frac{n\pi m_e \vec{v}}{h} \right] A \cos \left[\frac{n\pi m_e \vec{v}}{h} x \right] \quad (3.8)$$

We take partial derivative again, with respect to “x”:

$$\frac{\partial^2 \psi}{\partial x^2} = \frac{\partial \left[\frac{n\pi m_e \vec{v}}{h} \right] A \cos \left[\frac{n\pi m_e \vec{v}}{h} x \right]}{\partial x} \quad (3.9)$$

$$\frac{\partial^2 \psi}{\partial x^2} = - \left[\frac{n\pi m_e \vec{v}}{h} \right]^2 A \sin \left[\frac{n\pi m_e \vec{v}}{h} x \right] \quad (4.0)$$

From equation (3.6):

$$\psi = A \sin \left[\frac{n\pi m_e \vec{v}}{h} x \right]$$

Substituting the value from equation (3.6) into equation (4.0):

$$\frac{\partial^2 \psi}{\partial x^2} = - \left[\frac{n\pi m_e \vec{v}}{h} \right]^2 \psi \quad (4.1)$$

Substituting equation (4.1) in Schrodinger wave equation (3.1):

$$- \left[\frac{n^2 \pi^2 m_e^2 \vec{v}^2}{h^2} \right] \psi \frac{-h^2}{8\pi^2 m} + U\psi = E\psi \quad (4.2)$$

$$\psi \left(\left[\frac{n^2 \pi^2 m_e^2 \vec{v}^2}{h^2} \right] \frac{h^2}{8\pi^2 m} + U \right) = E\psi \quad (4.2 a)$$

$$E = \frac{n^2 m_e \vec{v}^2}{8} + U \quad (4.3)$$

As we use Electric potential-energy to accelerate electrons, the kinetic energy can be related to Electric potential energy.

$$m_e \vec{v}^2 = 2eV \quad (4.4)$$

Where “e” is elementary charge and “V” is Voltage.

Substituting the equation (4.4) in equation (4.3):

$$E = \frac{n^2 eV}{4} + U \quad (4.5)$$

Recall 1st Postulate from the hypothesis: “The potential energy of the knocked-electron is equal to Ionization Potential energy”, thus:

$$U = \frac{(Z_{eff})^2 e^4 m_e}{4\epsilon_0^2 n^2 h^2} \quad (4.6)$$

Substituting value of “U” from equation (4.6) in equation (4.5):

$$E = \frac{n^2 eV}{4} + \frac{(Z_{eff})^2 e^4 m_e}{4\epsilon_0^2 n^2 h^2} \quad (4.7)$$

The equation (4.7) is the total energy of the knocked-electron, that is accelerated by electron gun. The knocked electron is considered in SM field, therefore, the electron retains its Potential energy through pair-production in the field.

6. Solution to Double Slit Experiment

Thought experiment:

The kinetic part of the equation is the force that accelerates the electron. The acceleration is linear, a straight line. Let us suppose that we have a tiny piece of matter, and we provide some force, so that the tiny piece starts to move. We always get a straight line motion.

The electron always had some potential energy while being in orbits of the atom. That potential-energy function was being neutralized by working against electrostatic force between electron and proton (Hydrogen).

In this case, the knocked electrons are not bounded by the nucleus, thus, the potential function is not being neutralized. In case of knocked electrons, the potential function is used to produce oscillations of electrons through interaction with the field.

Now, suppose a tiny oscillating piece of matter, and we provide some force, so that it accelerates and gains momentum. But this time, the particle is oscillating. We know that oscillating particle will produce wave pattern when it moves in a straight line.

From the above supposition, we can assume that the wave nature partially depends on both kinetic and potential energy of the knocked electrons.

The famous double slit experiment is indeed a simple, yet complex procedure, that unlocked many mysteries in quantum mechanics. Yet, a certain case remains unresolved till now. "The role of the detector/Observer". Why does the diffraction pattern disappear when we try to measure the position or path of electron?

From the equation (4.7), it is clear that the total energy of the knocked electron comes in two parts, i.e "kinetic-function" and "Potential-function", and the Potential-function is well quantized. Thus, the quantum nature of the Potential function may interact with other quantum entities around it. Thus, it interacts with the detector's photons, which may be the cause of the potential function to collapse.

No observer:

$$E = \frac{n^2 eV}{4} + \frac{(Z_{eff})^2 e^4 m_e}{4\epsilon_0^2 n^2 h^2} \quad (5)$$

Observer:

$$E = \frac{n^2 eV}{4} \quad (5.1)$$

When the observer is placed to measure the position of the electron, the potential function is collapsed, thus, the motion of the oscillating electron is reduced to simple straight line, making the diffraction pattern disappear.

Richard Feynman used high-intensity light as a detector to measure the position of the electron, and a tiny reflection of the light was seen [1].

We can assume that reflection is due to the interaction of Potential function of knocked-electron, and incoming

Photons of light. The photons are deflected at certain angles, thus, the Potential function is collapsed. This ultimately collapses the wave function of knocked electron. The kinetic function of the knocked electron is not effected or partially effected, yet, the motion of electron continues, and the electrons are detected at the screen with no diffraction pattern.

If the wave function of the knocked electron only depended on the kinetic function (momentum), then we would not detect electrons at the screen when the measurement is made. After the measurement, the electrons are detected, but, only diffraction has disappeared, which supports the hypothesis in suggestion.

7. Interaction of Detector's Photon with Pair-Production

When ever knocked- electron approaches Amplitude of the wave in the SM field, Pair-production occurs due to the potential energy of the knocked electron.

The interaction is demonstrated through Feynman diagram: (Figure: 4)

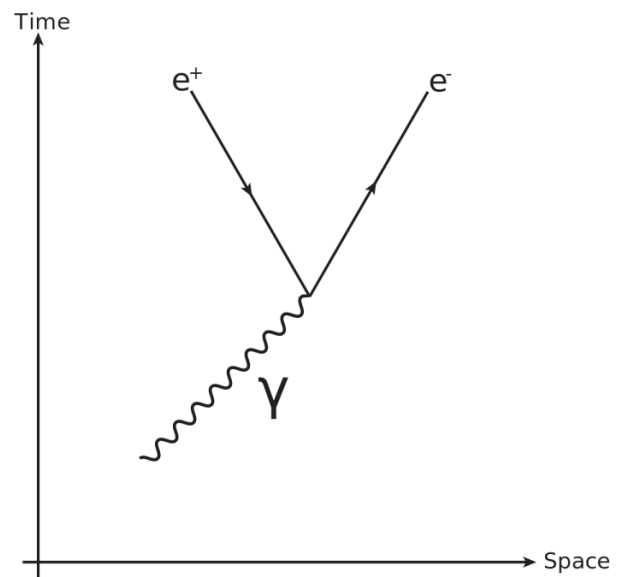


Figure 4

But when the photons from the detector interact with knocked-electron, a second pair production is initiated, which interferes and annihilates the initial pair produced by potential energy of the electron. The annihilation gives rise to two photons. The SM field no longer interacts with the knocked-electron, as the potential function is collapsed. The knocked electron can no longer cause pair production in SM field, thus, only straight line motion is observed in the form of disappearance of interference pattern.

8. Conclusions

The hypothesis successfully explains the role of observer

in double-slit experiment and accurately predicts interaction of the detector's photons with the Knocked-electron. It becomes clear that knocked-electron fired from electron gun has total energy, that is the sum of two different energy functions, such as kinetic-function and potential-function due to presence of the SM field (Theoretical), instead of only kinetic, as previously thought which explains the duality of electron.

The hypothesis is strongly supported by the double-slit experiments, and, successfully predicts the behaviour of the knocked-electron when detector is on/off.

The SM field is yet to be proved experimentally, but theoretically, it is the hidden parameter to solve the problem of duality of electron (17).

ACKNOWLEDGEMENTS

I offer concise but sincere acknowledgements to Prof Dr. Fazal-ur-Rehman Sethi (supervisor), Syeda Aiman Jamshaid (Contributor), Zainab, Maham Tariq (Major contributor) and all those who offered their wisdom and time in my research.

Appendix

- A) The equation shows the transition of electron from ground Principle Quantum number to higher Quantum number.

$$E_{\text{photon}} = h\nu = E_2 - E_1$$

$$h\nu = \frac{Z^2 me^4}{8h^2 \epsilon_0^2} \left[\frac{1}{n_1^2} - \frac{1}{n_2^2} \right]$$

- B) It shows the basics wave function of ionization for hydrogen:

$$h\nu = -13.6Z^2 \left[\frac{1}{n_1^2} - \frac{1}{n_2^2} \right] = -13.6\text{eV}$$

REFERENCES

- [1] https://www.feynmanlectures.caltech.edu/III_01.html#Ch1-S4 Section 1-6, paragraph 5
- [2] Anastopoulos, C. (2008). *Particle Or Wave: The Evolution of the Concept of Matter in Modern Physics*. Princeton University Press. pp. 236–237. ISBN 978-0-691-13512-0.
- [3] Thomson, J.J. (1906). "Nobel Lecture: Carriers of Negative Electricity" (PDF). The Nobel Foundation. Archived from the original (PDF) on 10 October 2008. Retrieved 25 August 2008.
- [4] Niels Bohr (1913). "On the Constitution of Atoms and Molecules, Part I" (PDF). *Philosophical Magazine*. 26 (151): 1–24. Bibcode: 1913 PMag... 26.... 1B. doi:10.1080/14786441308634955.
- [5] American Journal of Physics 57, 117 (1989); <https://doi.org/10.1119/1.16104>.
- [6] <https://iopscience.iop.org/article/10.1088/0143-0807/34/3/511/meta>
- [7] Zou, X. Y., Wang, L. J. & Mandel, L. Induced coherence and indistinguishability in optical interference. *Phys. Rev. Lett.* 67, 318–321 (1991).
- [8] <https://doi.org/10.1038/36057>
- [9] <https://doi.org/10.1080/09500830600876565>
- [10] Copeland, Jack; Haeff, Andre A. (September 2015). "The True History of the Traveling Wave Tube". *IEEE Spectrum*. 52 (9): 38–43. doi:10.1109/MSPEC.2015.7226611. S2CID 36963575. Copeland, Jack; Haeff, Andre A. (September 2015). "The True History of the Traveling Wave Tube". *IEEE Spectrum*. 52 (9): 38–43. doi:10.1109/MSPEC.2015.7226611. S2CID 36963575.
- [11] Paxton, William. "THERMIONIC ELECTRONEMISSION PROPERTIES OF NITROGEN-INCORPORATED POLYCRYSTALLINE DIAMOND FILMS" (PDF). Archived (PDF) from the original on 2016-11-23. Retrieved 2016-11-22.
- [12] Thermionic power converter". *Encyclopedia Britannica*. Archived from the original on 2016-11-23. Retrieved 2016-11-22.
- [13] Guthrie, Frederick (October 1873). "On a relation between heat and static electricity". *The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science*. 4th. 46 (306): 257–266. doi:10.1080/14786447308640935. Archived from the original on 2018-01-13.
- [14] Chapter 9: Quantum Mechanics". *faculty.chem.queensu.ca*. January 15, 2018. Retrieved October 31, 2020.
- [15] Lang, Peter F.; Smith, Barry C. (2003). "Ionization Energies of Atoms and Atomic Ions". *Journal of Chemical Education*. 80 (8): 938. Bibcode: 2003 JChEd.. 80.. 938L. doi:10.1021/ed080p938.
- [16] Miessler, Gary L.; Tarr, Donald A. (1999). *Inorganic Chemistry* (2nd ed.). Prentice Hall. p. 41. ISBN 0-13-841891-8.
- [17] Walter Greiner (2001). *Quantum Mechanics: An Introduction*. Springer. ISBN 978-3-540-67458-0.
- [18] Bywater, Jenn (29 October 2015). "Exploring dark matter in the inaugural Blackett Colloquium". *Imperial College London*. Retrieved 29 August 2016.
- [19] <https://doi.org/10.1103/PhysRev.155.1404>
- [20] Feynman, R., QED: *The Strange Theory of Light and Matter*, Penguin 1990 Edition, p. 84.
- [21] Coulomb (1785) "Premier mémoire sur l'électricité et le magnétisme," *Histoire de l'Académie Royale des Sciences*, pp. 569–577 — Coulomb studied the repulsive force between bodies having electrical charges of the same sign: —Coulomb (1785b) "Second mémoire sur l'électricité et le magnétisme," *Histoire de l'Académie Royale des Sciences*, pages 578–611.
- [22] Huray, Paul G., 1941- (2010). *Maxwell's equations*. Hoboken, N.J.: Wiley. pp. 8, 57. ISBN 978-0-470-54991-9. OCLC

- 739118459.
- [23] 2018 CODATA Value: vacuum electric permittivity". The NIST Reference on Constants, Units, and Uncertainty. NIST. 20 May 2019. Retrieved 20 May 2019.
- [24] The approximate numerical value is found at: "Electric constant, ϵ_0 ". NIST reference on constants, units, and uncertainty: Fundamental physical constants. NIST. Retrieved 22 January 2012. This formula determining the exact value of ϵ_0 is found in Table 1, p. 637 of PJ Mohr; BN Taylor; DB Newell (April–June 2008). "Table 1: Some exact quantities relevant to the 2006 adjustment in CODATA recommended values of the fundamental physical constants: 2006" (PDF). *Rev Mod Phys.* 80 (2): 633–729. arXiv: 0801.0028. Bibcode: 2008 RvMP... 80.. 633M. doi:10.1103/RevModPhys.80.633.
- [25] McEvoy, J. P.; Zarate, Oscar (2004). *Introducing Quantum Theory*. Totem Books. pp. 110–114. ISBN 978-1-84046-577-8.
- [26] 2018 CODATA Value: elementary charge". The NIST Reference on Constants, Units, and Uncertainty. NIST. 20 May 2019. Retrieved 2019-05-20.
- [27] Robert Millikan: The Oil-Drop Experiment.
- [28] de-Picciotto, R.; Reznikov, M.; Heiblum, M.; Umansky, V.; Bunin, G.; Mahalu, D. (1997). "Direct observation of a fractional charge". *Nature*. 389 (162–164): 162. arXiv: cond-mat/9707289. Bibcode: 1997 Natur. 389.. 162D. doi:10.1038/38241. S2CID 4310360
- [29] Coff, Jerry (10 September 2010). "What Is An Electron". Retrieved 10 September 2010.
- [30] Thomson, J.J. (1897). "Cathode Rays". *Philosophical Magazine*. 44 (269): 293–316. doi:10.1080/14786449708621070.
- [31] Mohr, P.J.; Taylor, B.N.; Newell, D.B. "2018 CODATA recommended values". National Institute of Standards and Technology. Gaithersburg, MD: U.S. Department of Commerce. This database was developed by J. Baker, M. Douma, and S. Kotochigova.
- [32] Clementi, E.; Raimondi, D. L. (1963). "Atomic Screening Constants from SCF Functions". *J. Chem. Phys.* 38 (11): 2686–2689. Bibcode: 1963 JChPh.. 38. 2686C. doi:10.1063/1.1733573.
- [33] Clementi, E.; Raimondi, D. L.; Reinhardt, W. P. (1967). "Atomic Screening Constants from SCF Functions. II. Atoms with 37 to 86 Electrons". *Journal of Chemical Physics*. 47: 1300–1307. Bibcode: 1967JChPh.. 47. 1300C. doi:10.1063/1.1712084.
- [34] https://wiki.ubc.ca/Effective_Nuclear_Charge_-_Definition_and_Trends#:~:text=Effective%20nuclear%20charge%20E2%80%93%20the%20attractive,all%20other%20periodic%20table%20tendencies