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A Hollow Spherical Gaseous Shell Approaching Horizon

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Abstract The definition of horizon in gravitation is based on the GM factor. This factor defines the Schwarzschild radius and hence the radius of the horizon. Evidently, this factor depends on density and radius alone. There is no direct connection to curvature. In a solid sphere one may change mass by either density or radius. If mass and density are fixed, radius must be fixed. It is shown here that for a hollow shell, mass and density may be fixed, while radii change. By change of shell's thickness and radius, one may transform a shell to a black hole while keeping mass and density fixed. Thus, the transition through horizon and formation of a black hole may be the result of curvature change alone, without an increase of neither mass nor density. It is also shown how a black hole can be created from a free fall collapse of a shell's outer and inner edges, while mass and density are kept fixed. For a non-rotating, neutral dust, of non-interacting particles and with no internal radiation processes, the internal pressure of a hollow sphere is investigated. The minimal value of pressure needed to hold against collapse is considered.

Keywords Shell, Sphere, Schwarzschild radius, Gaseous hollow shell, Pressure

1. Introduction

Is it possible to take a black hole and turn it to a standard object?

The answer is of course that one can do it by either removing parts of its mass (reducing M), or, by increasing its volume, so that its density is reduced.

Black holes are assumed to be of spherical shapes. Either symmetric with J=0, or non-symmetric with J≠0. They may be charged or neutral [1-5]. Hollow spherical shells have also been discussed [7-16]. Hollow cylindrical dust shells, with pressure conditions of Einstein-Bose gas have been discussed recently [21-25]. Here we examine the internal pressure under various gas pressure-volume parameters.

In all cases, one assumes that changes of an object from a standard object (SOB), to a black hole (BH), requires an increase in mass or density or both, enough to make its radius R smaller than its Schwarzschild radius r_s.

In this work, we will show that a hollow shell of finite size and thickness can collapse into a black hole, while keeping a fixed density and fixed mass. This, in contrast to the assumption that a black hole is a result of a given object, collapsing under gravitational forces, due to an enormous increase in density or mass, while reducing its radius R. (here R is its radius in case of a symmetrical sphere, or max(R_1, R_2, R_3) in case of an asymmetrical rotating ellipsoid with radii R_1, R_2 and R_3).

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It will also be shown, that during free fall of the two edges of a shell under central gravitational force, the shell will retain its density and mass while shrinking below the Schwarzschild radius, transforming to a black hole. This black hole cannot be discerned from a spherical black hole by a remote observer. One must conclude that the reason for black holes is not their mass nor their densities. Rather, it is the amount of curvature of spacetime alone, that determines the transition.

It is also shown that during free fall of non-relativistic particles of a dust shell, the shell's thickness will preserve the required conditions for a fixed density. Hence, the transition to a black hole will depend on the radii alone while mass and density are kept fixed.

2. Comparing a Full Sphere to a Hollow Spherical Shell

Consider a full sphere of radius R and mass M and density ρ, and compare it to a hollow spherical shell of same mass M and density ρ, with outer and inner radii, r_{out} and r_{in}, respectively.

For all objects the Schwartzschild radius is defined by $r_s = \frac{2GM}{c^2}$.

The criteria for transition from a BH to SOB is whether the objects radius R is greater or smaller than r_s.

$$R < r_s \leftrightarrow \text{BH}$$

$$R > r_s \leftrightarrow SOB$$

Since the relation between mass M and raius R are determined by the volume and density, this may be reformulated as

$$R^2\rho > \frac{3c^2}{8\pi G} \leftrightarrow \text{BH}$$
$$R^2\rho < \frac{3c^2}{8\pi G} \leftrightarrow SOB$$

This shows that there are two ways to create a BH:

1. Keep R fixed and increase ρ.
2. Keep ρ fixed and increase R.

For a full sphere, one cannot keep **both** the mass M and the density ρ fixed constant, and convert SOB ↔ BH.

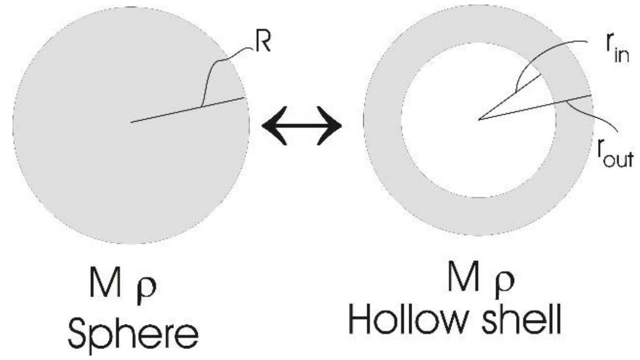

Figure 1. Changing outer and inner radii of a hollow shell in comparison to a full spherical object of radius R, while keeping both object's masses and densities equal and fixed

But, looking at a hollow shell, defined by its mass M and radii r_{in} and r_{out}, and comparing it to a full sphere of radius R, both having the same mass M and same density ρ, one can keep the mass M (~$R^2\rho$) and density ρ fixed, and yet allow the hollow shell to undergo the transformation SOB ↔ BH.

This transformation can be achieved by changing both r_{in} and r_{out} according to:

$$r_{out}^3 - r_{in}^3 = R^3 \quad (1)$$

Where R is the radius of the original object.

This condition merely represents the fact that the volume of the sphere and hollow shell is equal and thus if densities are kept fixed, so is the mass M for both.

3. Transforming a Standard Spherical Shell into a Black Hole into

Consider a non-rotating spherically symmetric stellar object of mass M, of radius R, of density ρ, and with a Schwarzschild radius $r_s = \frac{2GM}{c^2}$.

We know that whenever $r_s < R$, it is considered a black hole, and when $r_s > R$ it is considered a standard stellar object.

Suppose we take this object and transform its mass into **a very thin** spherical shell of radius $R_{sh} = kR$, and of width ΔR_{sh}. We also denote its new density by ρ_{sh}.

By comparing the two objects, before and after the transformation:

$$\frac{4\pi R^3}{3}\rho = 4\pi R_{sh}^2 \, \Delta R_{sh} \, \rho_{sh} \quad (2)$$

In a sphere the density cannot be changed without change of radius, as there is only one radius. But, it is possible to do so in a spherical hollow shell. It has inner and outer radii and so one can change both simultaneously in such a manner that keeps the density constant.

By keeping the same density for both objects, one obtains:

$$\frac{R^3}{3} = R_{sh}^2 \, \Delta R_{sh} \quad (3)$$

$$\frac{R}{3\Delta R_{sh}} = k^2 \quad (4)$$

We see how one may transform a sphere into a thin shell and vice versa, while keeping the mass and the density fixed. As a result, both objects will keep the Schwarzschild radius unchanged and the same for both.

In the more general case, where the shell is not thin, assume its inner and outer radii R_0 and R respectively.

Comparing the shell's mass to the sphere's mass M

$$M = \frac{4\pi R^3}{3}\rho = \frac{4\pi R^3}{3}\left(1 - \left(\frac{R_0}{R}\right)^3\right)\rho \quad (5)$$

This can only hold true for lim $(R - R_0) = 0$, namely an infinitesimally thin shell of radius R.

For the most general case, where a sphere of radius R is transformed into a shell of internal and external radii r_{in} and r respectively, one obtains (keeping the densities of both objects fixed):

$$R = r\left(1 - \left(\frac{r_{in}}{r}\right)^3\right)^{\frac{1}{3}} \quad (6)$$

Which can be solved for r_{in}

$$r_{in} = (r^3 - R^3)^{1/3} \; (R < r) \quad (7)$$

This means that one can find a shell which outer radius r is outside the Schwarzschild radius with an internal radius which is either inside or outside this radius. Setting the inner radius of the shell to be $r_{\text{in}} = r_{\text{s}}$ gives condition for the shell, with relation to the Schwarzschild radius:

$$r^3 \geq r_s^3 + R^3 \; (R < r) \quad (8)$$

If the radius of the object is $R = r_{\text{s}}$ (the object has just become a black hole, the outer radius of the shell will be $r = 1.2599 r_s$. The larger the radius R, the thinner the shell.

Notice, that so far there has been no assumption made about the nature of the matter involved. It is so far true for dust like gas as well as for charged, high density, high pressure object. Though, no energy may escape in form of radiation processes.

For a thin enough shell, one can expand it indefinitely and keep the characteristics of the original object:

1. *The mass M of the shell is kept, hence the Schwarzschild radius $r_s = \frac{2GM}{c^2}$ remains the same.*
2. *From a far enough distance, both the original sphere and the expanded shell will have the same gravitational attraction.*
3. *The density is kept the same for both objects, thus one cannot attribute the Black hole to an indefinite*

increase in density.

If for the original sphere, the Schwarzschild radius $r_s = \frac{2GM}{c^2}$ was outside R, for the shell, one can pick its thickness to be small enough, so that its radius R_{sh} is much larger than $r_s = \frac{2GM}{c^2}$, thus making the shell into a standard (non-black hole) object.

4. Free Fall

Consider now particles at the outer and inner edges of the shell. Assume two particles, at very large distances r_1 and r_2 from a common attracting center point undergo free fall. They fall at accelerations a_1 and a_2 respectively. These free fall accelerations can be easily shown to have the relation

$$\frac{a_2}{a_1} = \frac{{r_1}^2}{{r_2}^2} \tag{9}$$

At constant accelerations, $\dot{r} = at$ and therefore

$$\frac{\dot{r}_2}{\dot{r}_1} = \frac{a_2}{a_1} \tag{10}$$

Which gives

$$\frac{{r_1}^2}{{r_2}^2} = \frac{\dot{r}_2}{\dot{r}_1} \tag{11}$$

And so

$$\frac{dr_1^3}{dt} = \frac{dr_2^3}{dt} \tag{12}$$

Identify r_1 with r and r_2 with r_{in} to obtain

$$r^3 = r_{in}^3 + constant \tag{13}$$

and hence the relation $r_{\text{in}} = (r^3 - R^3)^{1/3}$ (for R<r), is valid for a shell undergoing free fall towards a common attracting center of a central gravitational field.

Therefore, the model of a free-falling shell answers the requirement of having the density fixed.

At least for a collision free gas of non-interacting, free falling particles, the shell model will (under a weak-field assumption) contract towards a common center and can eventually become a black hole. We notice that at both its external edges, the shell's gas is in contact with free space and therefore the edges are at zero temperature. The assumption on collision less gas particles is valid.

This model is different from the model of gravitational collapse due to increased density or increased mass,

The shell model represents gravitational collapse under constant density, constant mass and constant Schwarzschild radius r_s.

The model is limited to free falling particles under dilute, non-rotating, neutral gas conditions and so modifications will be required to include internal interactions due to pressure, charge and nuclear processes.

In the general relativistic case, a falling particle path is determined by

$$\frac{d^2r}{d\tau^2} = -\Gamma^r_{\alpha\beta}\frac{dx^\alpha}{d\tau}\frac{dx^\beta}{d\tau t} \tag{14}$$

$$\Gamma^r_{00} = -\frac{1}{2g_{rr}}\frac{\partial g_{00}}{\partial r} \tag{15}$$

$$\Gamma^r_{rr} = \frac{1}{2g_{rr}}\frac{\partial g_{rr}}{\partial r} \tag{16}$$

The particle's acceleration is thus,

$$\frac{d^2r}{dt^2} = \frac{1}{2g_{rr}}\left[c^2\frac{\partial g_{00}}{\partial r} - \frac{\partial g_{rr}}{\partial r}\left(\frac{dr}{dt}\right)^2\right] \tag{17}$$

According to the Schwarzschild metric,

$$g_{00}(r) = 1 - \frac{r_s}{r} \tag{18}$$

$$g_{rr}(r) = -\frac{r}{(r - r_s)} \tag{19}$$

Thus

$$\frac{\partial g_{00}(r)}{\partial r} = \frac{r_s}{r^2} \tag{20}$$

$$\frac{\partial g_{rr}(r)}{\partial r} = \frac{r_s}{(r - r_s)^2} \tag{21}$$

and so

$$\frac{d^2r}{dt^2} = -\frac{1}{2}\left(\frac{r - r_s}{r}\right)\left[c^2\frac{r_s}{r^2} - \frac{r_s}{(r - r_s)^2}\left(\frac{dr}{dt}\right)^2\right] \tag{22}$$

As before, identify r_1 with r_{out} and r_2 with r_{in} to obtain:

$$\frac{a_2}{a_1} = \frac{\left(\frac{r_{in} - r_s}{r_{in}}\right)\left[c^2\frac{r_s}{{r_{in}}^2} - \frac{r_s}{(r_{in} - r_s)^2}\left(\frac{dr}{dt}\right)^2\right]}{\left(\frac{r_{out} - r_s}{r_{out}}\right)\left[c^2\frac{r_s}{{r_{out}}^2} - \frac{r_s}{(r_{out} - r_s)^2}\left(\frac{dr}{dt}\right)^2\right]} \tag{23}$$

Where r_{out} and r_{in} are the outer and inner radii of the shell, respectively.

Case 1: $r_s << r_{in} < r_{out}$

$$\frac{a_{in}}{a_{out}} = \frac{{r_{out}}^2}{{r_{in}}^2}\left(\frac{c^2 - \dot{r}_{in}^2}{c^2 - \dot{r}_{out}^2}\right) \tag{24}$$

and for a non-relativistic speed $c^2 \gg \dot{r}^2$

$$\frac{a_{in}}{a_{out}} = \frac{(r_{out})^2}{(r_{in})^2} \tag{25}$$

For the non-relativistic speed, the ratio $\frac{a_{in}}{a_{out}} \to \frac{{r_{out}}^2}{{r_{in}}^2}$

Implies, as before, $r_{\text{in}} = ({r_{\text{out}}}^3 - R^3)^{1/3}$

Therefore, for a non-relativistic case, at far distance, the collapsing (under free fall) hollow shell retains a constant volume andthe mass and density are kept similar to a full spherical object of radius R and of same density and mass.

Case 2: $r_{in} \to r_s$

From energy considerations for a free-falling particle of mass m and energy $\mathcal{E}$:

$$\left(\frac{dr}{dt}\right)^2 = \left(1 - \frac{r_s}{r}\right)^2 - \frac{m^2c^4}{\mathcal{E}^2}\left(1 - \frac{r_s}{r}\right)^3 \tag{26}$$

Which shows that $\lim_{r_{in}\to r_s}\left(\frac{dr}{dt}\right) = 0$, and also

$$lim_{r_{in}\to r_s}\left[\frac{r_s}{(r_{in} - r_s)^2}\left(\frac{dr}{dt}\right)^2\right] = 0 \tag{27}$$

So, when the shell's inner skin approaches the horizon,

$$lim_{r_{in}\to r_s}\left(\frac{a_{in}}{a_{out}}\right) = \left(\frac{r_{in} - r_s}{r_{out} - r_s}\right)\left(\frac{r_{out}}{r_{in}}\right)^3 = 0 \tag{28}$$

5. Internal Pressure

We will consider a simplified model of gaseous material of non-interacting particles, uncharged, with zero angular momentum and with no thermonuclear sources.

In the non-relativistic case $\frac{dr_{in}^3}{dt} = \frac{dr_{out}^3}{dt}$, this means that the shell's internal volume is constant in time. Assuming the shell is isolated from external radiation, and if no internal nuclear processes take place, the pressure must be constant too. Otherwise, radiation from the inside will flow out, or radiation from the outside will flow into the shell.

But, when approaching the horizon, the inner skin acceleration a_{in}, and its speed $\frac{dr_{in}}{dt}$ reduces to zero. On this ocasion, there is no limitation yet on the outer shell's skin and so it continues in its approach towards center. This means that the shell's volume must be reduced at the cost of increased internal pressure inside the shell.

Once the internal pressure becomes high enough, the outer skin of the shell will not be able to continue its propagation and the shell will become stable, with fixed radii. This may be a function of the approach rate and it may result in fluctuations of the outer skin, and stability will not be reached until energy dissipation processes will act to reduce fluctuations. But this is a subject for evolved thermodynamics and quantum considerations [17,18].

Assume the inner skin has reached a near stop at horizon $r_{in} = r_s$ and the outer skin $r = r_{out}$ is moving still towards center. The internal pressure $\mathcal{P}(t)$ increases while the volume decreases. We will assume an adiabatic change so that the internal temperature remains constant.

For a stellar gas [19,20] the relation between pressure and temperature is given by

$$\mathcal{P}V^{\alpha} = kT \tag{29}$$

Where $\alpha = 1$ for an ideal gas.

At relatively low densities, the pressure of a fully degenerate gas can be derived by treating the system as an ideal Fermi gas, in this way

$$\mathcal{P}V^{5/3} = kT \tag{30}$$

This is well suited for the description of neutrons in a neutron star, and electrons in a white dwarf.

In condensed matter physics, a Bose–Einstein condensate (BEC) is a state of matter which is typically formed when a gas of bosons at low densities is cooled to temperatures very close to absolute zero.

At very high densities, where most of the particles are forced into quantum states with relativistic energies, the pressure is given by

$$\mathcal{P}V^{4/3} = kT \tag{31}$$

To have the most general description, write $\mathcal{P}V^{\alpha} = kT$, with α between 1 and 1.66 according to the gaseous state.

The change in internal pressure is given by

$$\frac{\partial\mathcal{P}(t)}{\partial t} = -\frac{3\alpha}{r(t)}\frac{dr(t)}{dt}\mathcal{P}(t) \tag{32}$$

And with boundary conditions $\mathcal{P}(r=\infty) = 0$ and $\mathcal{P}(r_s) = \mathcal{P}_0$ the solution is

$$\mathcal{P}(r(t)) = \mathcal{P}_0\left(\frac{r_s}{r}\right)^{3\alpha} \tag{33}$$

The force exerted on the outer skin (assumed to have the totality of mass M) is approximated by

$$M\frac{d^2r}{dt^2} = \mathcal{P}_0\left(\frac{r_s}{r}\right)^{3\alpha}4\pi r^2 = +k\frac{1}{r^{3\alpha-2}} \tag{34}$$

with $K = 4\pi\mathcal{P}_0 r_s^{3\alpha}$

The acceleration of the outer skin, towards the horizon under the internal pressure, falls off in proportion to $1/r^{3\alpha-2}$, instead of $1/r^3$ under free fall (Eq. 21).

Gravitation causes acceleration towards center while pressure accelerates outward away from center, so that the total acceleration is given by

$$\left(\frac{d^2r}{dt^2}\right)_{total} = \frac{k}{M}\frac{1}{r^{3\alpha-2}} - \frac{1}{2}\left(\frac{r-r_s}{r}\right)\left[c^2\frac{r_s}{r^2} - \frac{r_s}{(r-r_s)^2}\left(\frac{dr}{dt}\right)^2\right] \tag{35}$$

This allows one to express the steady-state position as a function of pressure and horizon.

As long as $\left(\frac{d^2r}{dt^2}\right)_{total} > 0$, the internal pressure is high enough to prevent collapse.

Equilibrium occurs when $\left(\frac{d^2r}{dt^2}\right)_{total} = 0$. Since we are investigating the situation near horizon, one may neglect the velocity $\left(\frac{dr}{dt}\right) = 0$, and hence

$$\frac{2k}{M} = r^{3\alpha-2}\left(\frac{r-r_s}{r}\right)\left[c^2\frac{r_s}{r^2}\right] \tag{36}$$

By definition $\frac{2k}{M}$ is independent of r, and so by derivation with respect to r one obtains:

$$(3\alpha-4)r^{3\alpha-5} - (3\alpha-5)r^{3\alpha-6}r_s = 0$$

Thus, the solution of Eq. 36 is:

$$r = \frac{(5-3\alpha)}{(4-3\alpha)}r_s \tag{37}$$

For an ideal gas $\alpha = 1$ and so $r = 2r_s$, and as $\alpha \to 4/3$, $r \to \infty$.

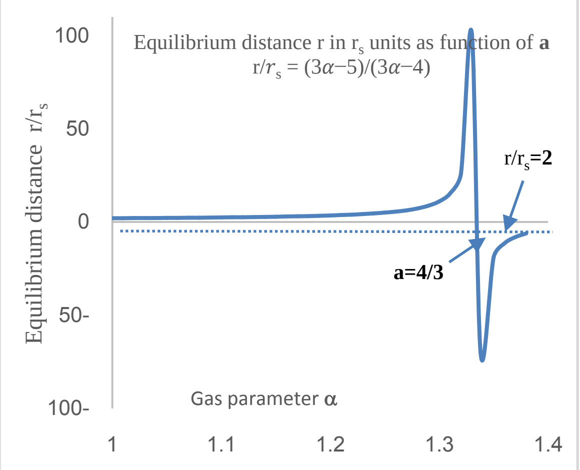

Figure 2. When internal pressure and gravitation collapse are equal, equilibrium is reached at distance r in units of r_s as function of gas characteristic volume dependent constant **α**. This is described by in this figure by $r/r_s = (3\alpha-5)/(3\alpha-4)$

At very high densities, most particles are forced into quantum states with relativistic energies. $\alpha = 4/3$. However, the solution of Eq 36 is dealing with non-relativistic speeds. At relatively low densities, for an ideal Fermi gas $\alpha = 5/3$, and equilibrium will occur at $\mathrm{r} = 0$ which is at complete collapse inside the horizon.

Letting α be a variable representing the various gaseous states (from an ideaal gas, to compressed gas), it may vary in the range $1 \leq \alpha < 4/3$. The following graph depicts the variation of the equilibrium distance r, (in units of r_s).

Obviously, only values in the range 1 to < 4/3 are valid in this model, since higher values will give $\mathrm{r} < r_s$ and this is below the horizon.

What can be said about $\mathcal{P}_0$?

$$\mathcal{P}_0 = \frac{Mc^2}{8\pi r_s^3}\left(\frac{5-3\alpha}{4-3\alpha}\right)^{3\alpha-4}\frac{1}{5-3\alpha} \tag{38}$$

In the following figure, $\mathcal{P}_0$ is depicted as function of α.

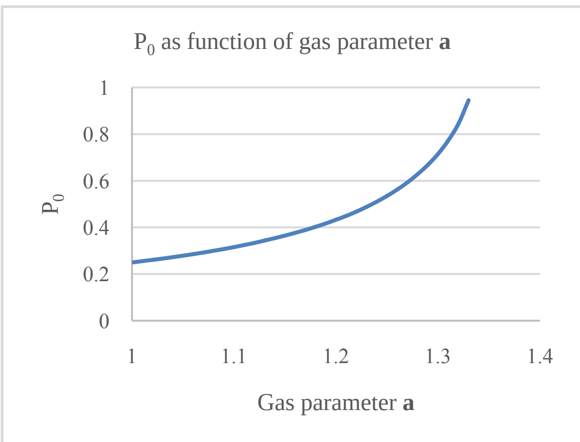

Figure 3. The internal pressure at equilibrium depends on the mass M, the Schwarzschild the gas characteristic volume dependent constant a. This is described by in this figure by $\mathcal{P}_0 = \frac{Mc^2}{8\pi r_s^3}\left(\frac{5-3\alpha}{4-3\alpha}\right)^{3\alpha-4}\frac{1}{5-3\alpha}$

6. Curvature

There is a topological difference between a solid 3D sphere and a hollow 3D shell.

A solid symmetrical sphere is characterized by a single radius r while a solid symmetrical hollow shell is characterized by two radii r_1 and r_2.

If asymmetrical solid sphere is considered, then it is characterized by 3 raddii r_1, r_2 and r_3, whereas an asymmetrical shell requires 6 radii to describe it (r_1, r_2 and r_3 for the inner skin and $\mathrm{r'}_1$, $\mathrm{r'}_2$ and $\mathrm{r'}_3$ for the outer skin.

If we take a solid sphere of mass M, and try to change its density and radius while keeping the mass fixed, we see from

$$\delta M = \delta\rho + 3r^2\delta r = 0 \tag{39}$$

that one cannot change the radius r without a change in density.

However, in a hollow shell, of radii r_1, r_2 and r_3

$$\delta M = \delta\rho(r_1^3 - r_2^3) + 3\rho(r_1^2\delta r_1 + r_3^2\delta r_3 \tag{40}$$

Therefore, one may have $\delta\mathrm{M} = 0$ and $\delta\rho = 0$, while changing two of the radii r_1, r_2 and r_3.

There must be at least 2 degrees of freedom (two radii) to do this, and in a hollow shell we do have. Contrary to a solid spher, where there is only one degree of freedom (a single radius).

By definition, the horizon is given by the Schwarzschild radius $\mathrm{r}_s = 2GM/c^2$. The criterion for the horizon is the mass M, or, density together with radius. If M is fixed, so are the radius and density. One cannot change the Schwarzschild radius by changing density alone nor radius alone. The two must change simultaneously.

However, in the case of a hollow shel of a fixed mass M, one may change the Schwarzschild radius without changing the density, and yet change of radii.

A solid sphere of given fixed mass and density will have a fixed Schwarzschild radius and a fixed radius.

A hollow shell of given fixed mass and density will have a flexible Schwarzschild radius and flexible radii.

In other words, we have a new criterion for turning a spherical shell into a black hole. Rather than its mass M or density, its radii determine its Schwarzschild radius.

Take a shell of given radius and thickness. Increase its thickness while reducing its radius, and it will turn into a black (hollow shell) hole. Reduce its thickness while increasing its radius and it will become of radius larger than r_s and thus stop being a black hole.

We see, that the nature of the object, being a black hole or a standard, is determined by the thickness of the shell and of its radius. One may keep the density and mass fixed, while increasing the shell's radius and reducing its thickness.

It is therefore the curvature

$$R^{\mu}_{\alpha\beta\gamma} = \partial_\beta \Gamma^{\mu}_{\alpha\gamma} - \partial_\gamma \Gamma^{\mu}_{\alpha\beta} + \Gamma^{\mu}_{\sigma\beta}\Gamma^{\sigma}_{\alpha\gamma} - \Gamma^{\mu}_{\sigma\gamma}\Gamma^{\sigma}_{\alpha\beta} \tag{41}$$

Or more precisely, the Kretschmann scalar

$$\mathcal{K} = R_{\alpha\beta\gamma\delta}\, R_{\alpha\beta\gamma\delta} \tag{42}$$

which determines whether the object is a black hole or not.

In case of a hollow shell, there are two curvatures. One for the outer skin and one for the inner skin. However, if the inner skin is inside the Schwarzschild radius, and the outer skin is not, it is not yet a black hole.

One must have both radii to be inside the Schwarzschild radius, in order to have the shell disappear below the horizon.

This means, that for a hollow shell of radii r_1, r_2 with $\mathrm{r}_1 > \mathrm{r}_s$ and $\mathrm{r}_2 \to 0$, the inner curvature becomes infinite. Yet the shell is not a black hole.

It must therefore be the curvature of the outer skin that will determine the criterion for turning a hollow shell into a black hole.

For a Schwarzschild black hole of mass M and radius R[6]:

$$\mathcal{K} = \frac{48G^2M^2}{c^4R^6} = \left(\frac{16\pi}{3}\right)^2\frac{G^2}{c^4}\rho^2 \tag{43}$$

Accordingly, the curvature increases with ρ and decreases with R.

Same equation will be true for a hollow shel, only where R is the external radius.

We see, that the nature of the object, being a black hole or a standard, is determined by the thickness of the shell and of its radius. One may keep the density and mass fixed, while increasing the shell's radius and reducing its thickness.

In contrast to a solid sphere, where $\delta \mathrm{M} = \delta\rho + 3\mathrm{r}^2\delta\mathrm{r}$, in a hollow shell, of radii r_1, r_2 and r_3 $\delta\mathrm{M} = \delta\rho(\mathrm{r}_1^3 - \mathrm{r}_2^3) + 3\rho(\mathrm{r}_1^2\delta\mathrm{r}_1 + \mathrm{r}_3^2\delta\mathrm{r}_3$. Therefore, one may have $\delta\mathrm{M} = 0$ and $\delta\rho = 0$ while changing two of the radii r_1, r_2 and r_3.

The criterion for the horizon is no longer the mass alone (as required by the Schwarzschild radius $2GM/c^2$.

In a hollow shell, one may change the Schwarzschild radius by changes in radii alone while keeping mass M and density ρ fixed.

In other words, we have a new criterion for turning a spherical shell into a black hole. Rather than its mass M or density, it is its curvature.

Take a shell of given radius and thickness. Increase its thickness while reducing its radius, and it will turn into a black spherical shell hole. Reduce its thickness while increasing its radius and it will become of radius larger than r_s and thus stop being a black hole.

It is therefore the curvature

$$R^{\mu}_{\alpha\beta\gamma} = \partial_\beta \Gamma^{\mu}_{\alpha\gamma} - \partial_\gamma \Gamma^{\mu}_{\alpha\beta} + \Gamma^{\mu}_{\sigma\beta}\Gamma^{\sigma}_{\alpha\gamma} - \Gamma^{\mu}_{\sigma\gamma}\Gamma^{\sigma}_{\alpha\beta} \quad (44)$$

Or more precisely, the Kretschmann scalar

$$\mathcal{K} = R_{\alpha\beta\gamma\delta} R_{\alpha\beta\gamma\delta} \quad (45)$$

which determines whether the object is a black hole or not.

For a Schwarzschild black hole of mass M and radius R^6:

$$\mathcal{K} = \frac{48G^2M^2}{c^4R^6} = \left(\frac{16\pi}{3}\right)^2 \frac{G^2}{c^4}\rho^2 \quad (46)$$

Accordingly, the curvature increases with ρ and decreases with R.

7. Conclusions

For a dilute non-rotating uncharged dust like gas, a spherically symmetric object may change its character from being a black hole to standard and vice versa independently of its mass and density.

According to the above argumentation, one may transform the sphere into a shell and accordingly find a shell with the right size so it changes the nature of the object.

By changing the external and internal radii of a shell, its mass and density can be kept fixed, and yet transform from a standard object to become a black hole, and vice a versa, transform from being a black hole shell to become a standard shell. All this without changing the mass and the density.

Under adiabatic collapse, the internal pressure of the shell falss off as $1/r^3$, while the outer skin approaches horizon with acceleration decreasing as $1/r^5$.

The transformation is the result of change in curvature alone and does not depend on the object's mass or density.

REFERENCES

[1] L.D. Landau and E.M. Lifschitz, "The Classical Theory of Fields", 4th ed. Pergamon Press, (1975).

[2] S. Weinberg, "Gravitation and Cosmology, Principles and Applications of the General Theory of Relativity", John Wiley and Sons, (1972).

[3] Misner, K. Thorne and A. Wheeler, "Gravitation", Freeman and Co. (1975).

[4] Ray D'Inverno, ""Introducing Einstein's Relativity", Oxford University Press, (1992).

[5] A. Zee, "Einstein Gravity in a Nutshell", Princeton University Press, (2013).

[6] Richard C. Henry "Kretschmann Scalar for a Kerr-Newman Black Hole". The Astrophysical Journal. The Am. Astronomical Society. 535 (1), 350–353 (2000).

[7] G.L. Alberghi, R. Casadio†, G.P. Vacca, and G. Venturi, "Gravitational Collapse of a Radiating Shell", General Relativity and Quantum Cosmology, Phys. Rev. D64:1040, 12, (2001).

[8] V. de la Cruz, J. E. Chase, and W. Israel, "Gravitational Collapse with Asymmetries", Phys. Rev. Lett. 24, 423 (1970).

[9] T. Marshall, "The Shell Collapsar — A Possible Alternative to Black Holes", Entropy,. 18, 10, p. 363. (2016).

[10] F. Echeverria "Gravitational collapse of an infinite, cylindrical dust shell", Phys. Rev. D 47, 2271 (1993).

[11] P. Szekeres, "Quasispherical gravitational collapse", Phys. Rev. D 12, 2941 (1975).

[12] W. A. Hiscock, "Gravitational entropy of nonstationary black holes and spherical shells", Phys. Rev. D 40, 1336 (1989).

[13] Kuchař, K. "Charged shells in general relativity and their gravitational collapse". Czech J Phys 18, 435–463 (1968).

[14] Chase, J.E. "Gravitational instability and collapse of charged fluid shells". Nuovo Cimento B (1965-1970) 67, 136–152 (1970).

[15] W. Israel, "Singular hypersurfaces and thin shells in general relativity". Nuovo Cimento B (1965-1970) 44, 1–14 (1966).

[16] D. Vogt, P. S. Letelier, "Newtonian and General Relativistic models of spherical shells", *Monthly Notices of the Royal Astronomical Society*, Volume 402, Issue 2, pp. 1313–1316 (2010).

[17] J. Bekenstein, "Black Holes and Entropy". Phys. Rev. D, 7, 2333 (1973).

[18] S. Hawking, "Black Holes and Thermodynamics". Phys Rev. D, 13, 191, (1973).

[19] R. Kippenhahn, A. Weigert, A. Weiss "Stellar Structure and Evolution", Astronomy and Astrophysics Library, Springer Verlag., ISBN 3-540-50211-4 (1990).

[20] O. Pols, "Stellar Structure and Evolution", Astronomical Institute Utrecht, Utrecht University (2011).

[21] Kuei Sun, Karmela Padavi´c, Frances Yang, Smitha Vishveshwara and Courtney Lannert. "Static and dynamic

properties of shell-shaped condensates". arXiv:1712.04428v2 [cond-mat.quant-gas] (2018).

[22] Karmela Padavic´, Kuei Sun, Courtney Lannert and Smitha Vishveshwara. "Physics of hollow Bose-Einstein condensates" arXiv:1612.05809v3 [cond-mat.quant-gas] (2018).

[23] S. Stringari, "Collective excitations of a trapped Bose-condensed gas". arXiv:cond-mat/9603126 (1996).

[24] Ken-ichi Nakao, Yasunari Kurita, Yoshiyuki Morisawa, Tomohiro Harada. "Relativistic Gravitational Collapse of a Cylindrical Shell of Dust". arXiv:gr-qc/0610039 (2006).

[25] Ken-ichi Nakao, Tomohiro Harada , Yasunari Kurita and Yoshiyuki Morisawa, "Relativistic Gravitational Collapse of a Cylindrical Shell of Dust II Settling Down Boundary Condition". arXiv:0905.3968 [gr-qc] (2009).

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