# Derivation of Planck's Constant from Two Strings Coupling

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**Abstract**   The existence of strings has not yet been proven, but it will be shown in the following, that if a classical fermion is considered as made up of two coupled strings, then the coupling between these two strings creates tension in the strings, and this tension is proportional to the coupling force via the Planck constant.

**Keywords**   String coupling, Planck constant, Schroedinger equation

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## 1. Introduction

Classical quantum theory is the basis for our concept of modern physics elementary particles theory. Ever since its introduction in the early years of the 20th century.

The birth of quantum mechanics is commonly attributed to the discovery of the Planck relation. In order to explain black-body radiation, Planck postulated that the radiation energy is transmitted in packages ("energy quanta"). Einstein later has found that light is absorbed by an electron in small "packets", which, like Planck's "energy quanta", is proportional to the light frequency $\nu$. This relation is now called the Planck relation or Planck–Einstein relation: $E = h\nu$, where the constant "h" is "Planck's constant". Its value is $6.662607015 \times 10^{-34}$ JSec and it is usually appears as $\frac{h}{2\pi} = \hbar = 1.054571817 \times 10^{-34}$ and is pronounced h bar.

It has become one of the most important universal constants in physics. Yet, the exact physical meaning of Planck's constant is unknown; it has not been derived based on first principles.

Several approaches have been described recently (e.g. Lipovka [1,2], Brucholz [3] and Chang [4]), trying to derive h from basic principles.

Lipovka [1,2] shows that the Planck constant is actually the adiabatic invariant of the electromagnetic field, characterized by scalar curvature of space of the Riemann – Cartan geometry. The main result of his work was to obtain the ratio between Riemannian scalar curvature of the Universe R, the Cosmological   constant Λ and Planck's constant h.

Brucholz [3] claims that since a photon must have a geometric boundary (which is why it behaves like a particle), the integration of its energy density (based on Maxwell equations) over a bounded volume must have $E = h\nu$.

Chang [4], by using the Maxwell theory, have, in a similar manner to Bruchholz, assumed a finite size photon. Thus, a relationship between the total electromagnetic energy of a single photon, its frequency, its width (Q factor) and the dielectric qualities of the vacuum. This provides a similar relation   $E = h\nu$.

In this work, a totally different approach to quantum mechanics was used. Referring to wave functions as a combination of real fields and observing of the differential equations as representing geometrical qualities of coupled classical strings. By assuming the coupled string like real wave functions, undergo some exchange interaction, leads us to the understanding that Planck constant h is the result of these exchange interactions between two coupled strings.

Though this work uses the classical strings, it may be just as well extended to the concept of strings as the basic structure units of elementary particles [5-9].

## 2. A Real Presentation of Schrödinger Equation

The basic equation of quantum mechanics is the one particle time-dependent Schrödinger equation:

$$-i\hbar \frac{\partial}{\partial t} \psi(x,t) = \mathcal{H}\psi(x,t) \qquad (1)$$

where $\hbar$ is the reduced Planck constant which is h/$2\pi$, $\psi(x,t)$ is the complex wave function of the quantum system, x is the position in a one-dimensional coordinate system, and t the time. $\mathcal{H}$ is the Hermitian Hamiltonian operator (which characterizes the total energy of the system under consideration).

By decomposing the complex wave function into real and imaginary components

$$\psi(x,t) = \Psi = \varphi_1 + i\varphi_2 \qquad (2)$$

the Schrödinger equation may be written:

$$-i\hbar\frac{\partial}{\partial t}\Psi = \mathcal{H}\Psi = (\mathcal{H}_r + i\,\mathcal{H}_i)(\varphi_1 + i\varphi_2) \quad (3)$$

$$+\hbar\frac{\partial}{\partial t}\varphi_2 = \mathcal{H}_r\varphi_1 - \mathcal{H}_i\varphi_2 \qquad (4)$$

$$-\hbar\frac{\partial}{\partial t}\varphi_1 = \mathcal{H}_i\varphi_1 + \mathcal{H}_r\varphi_2 \qquad (5)$$

In other words, the traditional Schrödinger equation is in fact two coupled equations of real wave functions, with real operators on a real 3-dimensional space [1].

The same procedure is also applicable to the relativistic Dirac equation, but this issue will not be discussed in here.

For a time-independent classical Hamiltonian of a free particle, with mass m:

$$\mathcal{H} = \frac{p^2}{2m}$$

$$\mathcal{H}_r = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} \; \mathcal{H}_i = 0$$

When separated into real and imaginary components, these are equivalent to:

$$= +\hbar\frac{\partial}{\partial t}\varphi_2 \; \mathcal{H}_r\varphi_1 = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\varphi_1 \qquad (6)$$

$$-\hbar\frac{\partial}{\partial t}\varphi_1 \; \mathcal{H}_r\varphi_2 = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\varphi_2 = \qquad (7)$$

Which gives two coupled equations of the two real wave functions:

$$\frac{\partial\varphi_1}{\partial t} = +\frac{\hbar}{2m}\frac{\partial^2\varphi_2}{\partial x^2} \qquad (8)$$

$$\frac{\partial\varphi_2}{\partial t} = -\frac{\hbar}{2m}\frac{\partial^2\varphi_1}{\partial x^2} \qquad (9)$$

It does not worth any extra efforts to deal with the non-relativistic Schrödinger equation, because it will not render any new results as this is only a matter of representation change from complex to real. The results will be the same.

However, it is interesting to see, that the real representation and solutions displays two real entities (wave functions) instead of a single, complex entity. And this can reveal an interesting new approach.

It will be assumed herewith, that the quantum description and characteristics of a single particle are the result of a coupling interaction between two components (fields) which composes the single "particle".

Based on this assumption, it will be described in the following, how can this real interpretation suggest an explanation to the non-relativistic Schrödinger equation through an interacting coupled two-strings classical model.

## 3. Tension in a Classical String

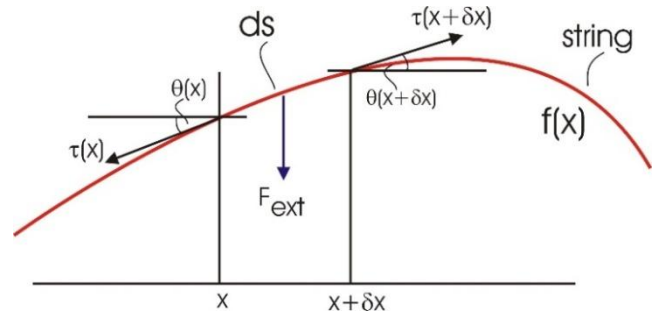Let us start with a description of the forces in a classical one-dimensional, time independent, string.



**Figure 1**

Let the spatial distribution of a 1-dimensional string of mass density **ρ** be described by the function f(x). Internal tension forces on the string are at two opposite directions. We will assume that the magnitude of the tension $\tau(x)$ is the same along the string.

Additionally, there is an external force Fext, y acting vertically on the infinitesimal element **ds**. This external force is due to some external interaction.

The total horizontal component $F_{tot,x}$ of the force on the elemental **ds** is given by

$$F_{tot,x} = \tau(x+\delta x)\cos\theta(x+\delta x) - \tau(x)\cos\theta(x) \quad (10)$$

While the total vertical component $F_{tot,y}$ of the force on the elemental **ds** is given by

$$F_{tot,y} = \tau(x+\delta x)\sin\theta(x+\delta x) - \tau(x)\sin\theta(x) + F_{ext,y} \qquad (11)$$

For infinitesimal small element **ds**, one may replace $\sin\theta \approx \tan\theta\ (=\frac{\partial f(x)}{\partial x})$.

Hence

$$F_{tot,y} \approx \tau(x+\delta x)\frac{\partial f(x+\delta x)}{\partial x} - \tau(x)\frac{\partial f(x)}{\partial x} + F_{ext,y} \quad (12)$$

$$F_{tot,x} \approx \tau(x+\delta x) - \tau(x) = \frac{\partial T(x)}{\partial x}\delta x \qquad (13)$$

Thus

$$F_{tot,y} \approx \tau(x)\left[\frac{\partial f(x+\delta x)}{\partial x} - \frac{\partial f(x)}{\partial x}\right] + F_{ext,y} \qquad (14)$$

And so

$$F_{tot,y} \approx \tau(x)\frac{\partial^2 f(x)}{\partial x^2}\delta x + F_{ext,y} \qquad (15)$$

## 4. Interacting Strings

Consider next two strings $\varphi_1$ and $\varphi_2$. Let $\varphi_1(x,t)$ represent the amplitude of string 1 at time t and at position x. Let $\tau_s$ be some tension force in the string. As shown above, the net force exerted by this tension, on a small string element **ds** (see figure 2) is connected to the amplitude change along the x axis and is described by:

$$F_s = \tau_s\frac{\partial^2\varphi_1(x,t)}{\partial x^2} \qquad (16)$$

Assume next, a second string is near the first one and is interacting with it by means of some coupling force, which couples the two strings together. Suppose now the second string, described by $\varphi_2(x,t)$, undergoes some small temporal perturbation
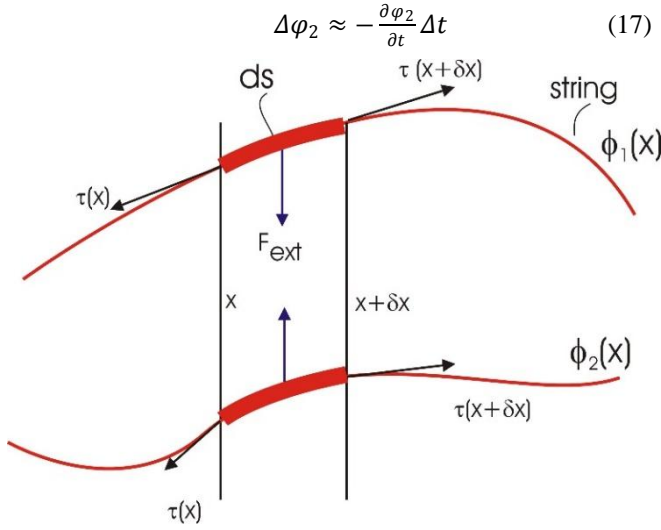
$$\Delta\varphi_2 \approx -\frac{\partial\varphi_2}{\partial t}\Delta t \quad (17)$$



**Figure 2**

This perturbation induces a change in the coupling force $F_{21}$, exerted by string 2 on string 1. This force is proportional to $\Delta\varphi_2$ and attracts or repels string 1, in the opposite direction of $\Delta\varphi_2$.

We denote this proportionality coupling constant by $k_s$.

We will also assume, without loss of generality, that the coupling between the two strings is proportional to the mass of **ds**. This is a reasonable assumption as we may think that the more mass, the stronger the coupling.

All in all here are the assumptions made:

Assumption 1 (Hook's Law): The coupling force is proportional to displacement $\Delta\varphi_2$ of string 2. We will denote this proportionality coupling constant by $k_s$.

Assumption 2 (mass law): The coupling between the two strings is proportional to the mass of the elemental **ds**.

Therefore, the disturbance in the force is described by:

$$\Delta F_{ext} = -\rho\, ds\, (k_s\, \Delta\varphi_2) = -\rho\, ds\, k_s\, \Delta t\, \frac{\partial\varphi_2}{\partial t} \quad (18)$$

$$\Delta F_{ext} = -\rho\, ds\, k_s \Delta\varphi_2 \quad (19)$$

And from the projection of **ds** on x:

$$\Delta F_{ext,y} = -k_s \frac{\partial\varphi_2}{\partial t}\Delta t\, \rho\, ds\, cos\theta = -k_s \frac{\partial\varphi_2}{\partial t}\Delta t\, \rho\, \delta x \quad (20)$$

$k_s$ is the proportionality factor depending on the strength of the coupling interaction. Therefore, by Eq. 15:

$$\Delta F_{tot,y} \approx \left(\frac{\partial T(x)}{\partial t}\frac{\partial^2 f_1(x)}{\partial x^2} - k_s\rho\frac{\partial\varphi_2(x)}{\partial t}\right)\Delta t\, \delta x \quad (21)$$

At equilibrium $\Delta F_{tot,y} = 0$, and so:

$$\frac{\partial\varphi_2(x)}{\partial t} = \frac{1}{\rho\, k_s}\frac{\partial\tau}{\partial t}\frac{\partial^2\varphi_1}{\partial x^2} \quad (22)$$

By symmetry reason, the action of disturbance string 1 on tension in string 2 will be described by (force in the opposite direction)

$$\frac{\partial\varphi_1(x)}{\partial t} = -\frac{1}{\rho\, k_s}\frac{\partial\tau}{\partial t}\frac{\partial^2\varphi_2}{\partial x^2} \quad (23)$$

Equations 23 and 24 represent the coupling of two real strings.

Looking at the term $\frac{1}{k_s}\frac{\partial T}{\partial t}$, we see that it has units of

angular momentum. We will thus assume:

$$\frac{1}{k_s}\frac{\partial\tau}{\partial t} = -\frac{1}{2}\hbar \quad (24)$$

The above coupled equations now read

$$\frac{\partial\varphi_1}{\partial t} = +\frac{\hbar}{2\rho}\frac{\partial^2\varphi_2}{\partial x^2} \quad (25)$$

$$\frac{\partial\varphi_2}{\partial t} = -\frac{\hbar}{2\rho}\frac{\partial^2\varphi_1}{\partial x^2} \quad (26)$$

These equations are the coupled real presentation similar to Schrödinger equation.

Equation [25] gives a physical meaning to the Planck constant, namely, independent of a particle's mass, the Planck constant $\hbar$ is derived from the internal quality of the real fields. It represents somehow the reaction of the tension of the string fields to perturbations. Up to a proportionality constant,

$$\hbar \approx -\frac{1}{k_s(t)}\frac{\partial\tau_s}{\partial t}$$

The left hand side of this equation is a constant. Therefore, one must have $k_s$ as a time-dependent variable (or else, both $\tau_s$ and $k_s$ are constants).

This leads to the conclusion:

$$\tau_s(t) = -\hbar \int k_s(t)\, dt \quad (27)$$

So, the tension in the strings is proportional to Planck constant $\hbar$, and to the coupling between the two strings.

## 5. Exchange Interaction

From its defining equation $\Delta F_{ext} = -m\,(k_s\,\Delta\varphi_2)$.

The units of $k_s$ are:

$[N] = k_s$ x [m/sec] x [sec] x [kg] = $k_s$ x [m] x [kg]

$[k_s]= [N\,/(kgxm)] = [Kg\; x\; m/sec^2\,/\,(Kgxm)] = [1/sec^2]$

The fact that $[k_s(t)] = 1/sec^2$ is indicative of the interaction type.

It is like the shorter the exchange, the stronger is the interaction.

This is characteristic of an exchange mechanism between the two strings. The higher the rate of exchange (particles/sec), the stronger the interaction.
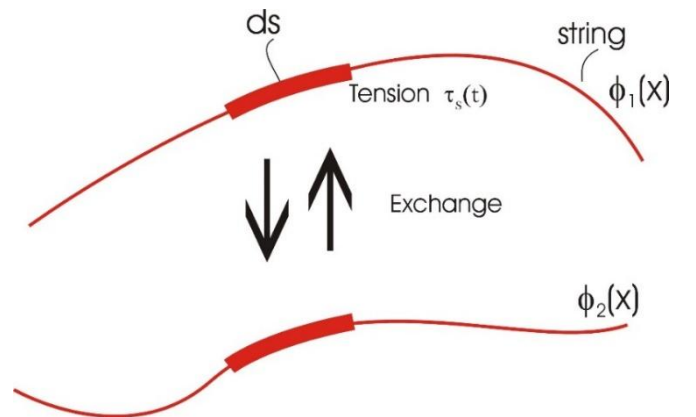


**Figure 3**

Indeed, if the exchange rate is designated by **R**

[particles/sec], then the constant $k_s(t)$ should be proportional with $\mathbf{R}^2$ (two strings interacting with eachother).

Therefore, $k_s(t)$ must have the units of $1/\text{sec}^2$.

So, the tension in the strings is proportional to the Planck constant $\hbar$, and to the coupling between the two strings (Figure 3).

The interaction caused be some sort of exchange mechanism between the two strings, results in tension in the strings, given by $\tau_s(t) = -\hbar \int k_s(t)\, dt$. The proportionality between the exchange force and the tension is the Planck constant $\hbar$.

## 6. Conclusions

Based on the following assumptions:

1. A Classical Fermion is made up of two interacting string-like entities.
2. Tension in the strings is proportional to the coupling between the two strings.
3. The coupling between the two strings is proportional to the amount of time the exchange lasts.

One is lead to conclude, that Planck's constant $\hbar$, is the proportionality constant, between the total exchange (of some sort) between the two strings, and the tension in these strings.

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