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The Geometrical and Quantization Foundations of the Oyibo Grand Unification Theorem

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Abstract Two fundamental arguments against the Oyibo grand unification theorem (GUT) as a possible mathematical basis for a grand unification theory are (1) the obscure nature of the unspecified "physical" or "geometrical" meaning of the group of transformation in Oyibo's definition of conformal invariance and (2) the expectation that the yet-to-be resolved controversy on the possibility of quantizing the Einstein general theory of relativity (GTR) into a valid quantum theory of gravitation will be inherent in the Oyibo GUT. To resolve these arguments, it is demonstrated here that the torus can be used as an invariant geometrical object for the Oyibo GUT. It is also demonstrated that the Oyibo GUT can be created in N+1 dimensional background required for a theory of quantum gravity and that it can also be quantized as required for it to be adopted in the formulation of the quantum aspect of the Oyibo theory. The conclusion reached as in previous studies, is that the Oyibo GUT is a sound mathematical basis for a grand unified theory and therefore needs more attention of the scientific community.

Keywords Grand Unified Theory, Quantum Gravity, Grand Unification Theorem, Conformal Invariance, Torus

1. Introduction

The unified force field theory (UFFT) is expected initially to unify the four known forces such as gravitational, electromagnetic, strong and weak forces. Such accomplishment which is now more generally referred to as the grand unified theory is expected to explain all phenomena in the universe and beyond, both on the microscopic and macroscopic scales, thereby culminating in the theory of everything (TOE). In section II of [1], we have provided a pedagogical review of the statement and philosophy of Oyibo grand unified theory, starting from the Greek first proposal that all phenomena of nature can be explained through four 'elements': fire, earth, air and water, to Einstein seminal quest for a UFFT when he attempted to incorporate electromagnetism into his general theory of relativity (GTR) to modern quest for the UFFT [2-4]. In Oyibo's opinion, the grand unified theory which will account for both the four known forces and yet-to-be-known forces such as the proposed fifth force [5, 6], should possibly explain God Almighty who he considered as the ultimate force [7, 8]. Thus he branded his theorem which he considered as 'a physically sound or credible set of mathematical equations from which to determine or formulate the UFFT' [7] as God Almighty grand unified theorem and the theory from it as God Almighty grand unified theory (GAGUT). This apparently sets both works in the thick trench of science-religion moot point [9-12]. It is pertinent to quickly point out that his postulation is in consonance with that of both Hawking [9] and Davis [10] with the former asserting that "Know the final equations and you will know the mind of God" [9].

Now we have recovered a number of important results from the Oyibo's theorem [1, 13-16] and therefore prefer to refer to it as Oyibo grand unified theorem (GUT): here and as in general, a theorem is a mathematical statement that is proved using rigorous mathematical reasoning while a theory is a set of ideas explaining physical behaviours of a given category of physical systems and is therefore capable of producing experimental predictions for them [13, 16]. Since we have reasons to believe that Oyibo GUT has sound mathematical basis to be a potential candidate for the grand unified theory, our goal has been to advance and popularize it.

The Oyibo's approach to his GUT emanated from his methodology for solving the Navier-Stokes equation in fluid mechanics using invariance of an arbitrary function under a group of conformal transformations [17, 18]. As a common knowledge, invariance of equations can be achieved by appropriate group transformation of such equations from one space to another [19]. Oyibo used this approach to prove the Einstein's GTR [20] and then linked it up with the Maxwell electromagnetic field [7, 8]. Thus he

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claimed his approach can be used to unify gravitation, electromagnetism and the other known forces and as well as possibly future new forces to be discovered, which is a long standing problem in man's quest for UFFT. This esoteric approach is different from the Einstein and co-workers general coordinates transformations, Riemannian and non-Riemannan geometry methodologies which had limited successes [21]. The main fundamental problems with this approach in which modelling is reduced to algebraic operations (hence it can be considered as a new formation of algebrodynamic – algebraic nature of physical geometry and dynamics) rather than differential equations for most problems, are (1) the obscure nature of the unspecified "physical" or "geometrical" meaning of the group of transformation in Oyibo's definition of conformal invariance and (2) the expectation that the yet-to-be resolved controversy on the possibility of quantizing the Einstein GTR into a valid quantum theory of gravitation will be inherent in the Oyibo GUT.

I will attempt to provide answers to these two problems here in the next two sections. Thereafter, I will conclude.

2. The Geometrical Background of the Oyibo GUT

It is well documented today that the early criticism of the Einstein GTR is that the principle of general covariance adopted by Einstein in its formulation is technically sound but lacks physical content and therefore cannot be the source of any significant physical concept or theory [22-24]. However, there is consensus today that the Einstein GTR is a remarkable framework for gravitation after more than a century of its study [24-26]. Ipso facto, the perceived obscure nature of the Oyibo transformation can be resolved by enhancing its interpretation and understanding as done by Animalu in his review [19]. There, by providing a direct relationship between Oyibo conformal transformations and the usual characterization of conformal invariance in projective space-time geometry, he was able to construct various realizations of the Oyibo conformal group in space-time Minkowskian geometry of special relativity theory, Riemannian (differential) geometry of Einstein's general relativity theory as well as other subtly different Riemannian theories of gravitation, and in classical and quantum field theories, in order to bring them within the purview of the Oyibo's GUT. He was therefore led to the conclusion that projective geometry is key to the Oyibo GUT. Following this line of thought, I will use projective geometry to obtain the geometric object for the Oybo GUT.

Oyibo had admitted that, "One of the most challenging problem in the field unification problem geometrically is one of finding a common geometric object, entity or platform in which all force fields could be combined or unified [7]. He therefore considered his GUT formulation to be geometro-dynamic in that the universe is considered to be a unified field in which matter is just a concentration of the field which fits a toroidal description of the universe. Thus a torus can be considered as the geometric object for the Oyibo GUT formulation of a grand unified theory and this is in line with the Hawking's suggested path to the theory of everything. This can be inferred from Carl Segan assertion in his *Introduction* to *Hawking's popular book* entitled *A brief History of Time* [9] about a universe that has the form of a torus:

"Hawking embarks on a quest to answer Einstein's famous question whether God had any choice in creating the universe. Hawking is attempting, as he explicitly states, to understand the mind of God. And this makes all the more unexpected the conclusion of the effort: a universe with no edge in space, no beginning or end in time [like a torus] and nothing for the Creator to do".

The conclusion of the effort in question, as Hawking stated on p. 116 of [9], is "the possibility (when quantum mechanics is taken into account) that space-time was finite but had no boundary (and no end) …" This description of the universe fits a torus.

There have been other postulations that toroidal structures are detected in astrophysical objects of various types [27] and this has galvanized toroidal models of cosmology [28]. There are also suggestions that the torus is the fundamental pattern needed to investigate the elementary particles [29, 30]. Therefore, there is growing interest in considering the torus as the fundamental pattern for all creation: from galaxies to planets to atoms to photons to nano-torus [31, 32].

Thus one is motivated to demonstrate that the torus can be adopted as the geometric object for the Oyibo GUT formulation and the first step is to show that it is invariant under the Lorentz transformation. To do so, one can start from the Pythagoras right angle triangle shown in Figure 1a which is an invariant geometric object in nature and it is represented by the Pythagoras theorem:

$$a^2 + b^2 = c^2 \tag{1}$$

which is valid for finite values of a and b. Further, the consistency of the Pythagorean triples depends on a and b.

There are already several ways of projecting a torus from the Pythagoras triangle as illustrated in Figure 1b. Now if the triangle with sides A, B, C is projected to a triangle with sides A', B', C' and then to a triangle with sides A", B", C", then we can construct a torus as shown in Figure 2. It is straightforward to see that the projected torus depends on A and B so that it is geometrically invariant.

The equation of the torus in Cartesian coordinates that is symmetric about the z-axis using Figure 2 is

$$\left(\tfrac{1}{2}(A'' + A') - \sqrt{x^2 + y^2}\right)^2 + z^2 = [\tfrac{1}{2}(A'' - A')]^2 \tag{2}$$

The parametric equations are

$$x = (\tfrac{1}{2}(A'' + A') + \tfrac{1}{2}(A'' - A')\cos\theta)\cos\phi \tag{3a}$$

$$y = (\tfrac{1}{2}(A'' + A') + \tfrac{1}{2}(A'' - A')\cos\theta)\sin\phi \quad (3b)$$

$$z = \tfrac{1}{2}(A'' - A')\sin\theta \quad (3c)$$

where θ is the latitude and ϕ is the meridian angle.

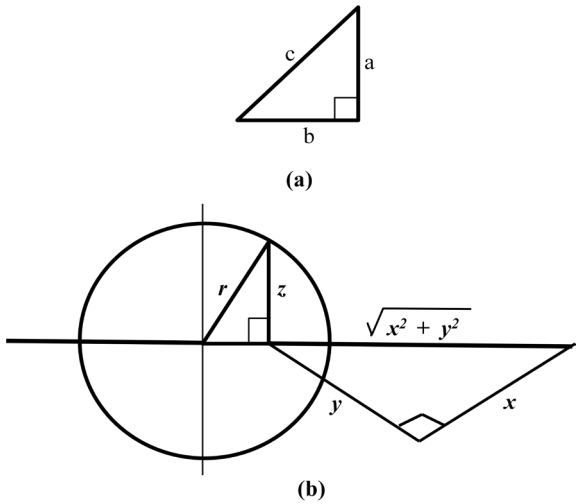

Figure 1. Simple illustrations of (a) a pythogoras triangle and (b) the construction of torus ring from the Pythagoras triangle

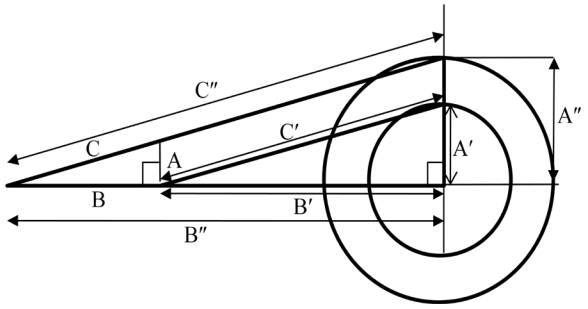

Figure 2. A torus projected from the Pythagoras' theorem

If one follows the approach of [33] in reformulation of the Namba-Goto principle for a torus built from matter and antimatter, then we can adopt the transformation:

$$\tfrac{1}{2}(A'' - A') = s \text{ and } \tfrac{1}{2}(A'' - A') = ct \quad (4)$$

where *s* is the proper distance while t and c are time and speed of light respectively.

Substituting the new transformed parameters in Eq.(4) into Eq.(3) and then substituting the re-expressed parametric equations into Eq.(2), we will obtain after eliminating the θ and ϕ, an equation of torus of the form:

$$s^2 + c^2t^2 - x^2 - y^2 - z^2 = 2s\sqrt{c^2t^2 - z^2}\,. \quad (5)$$

Observe that the LHS of Eq.(5) represents the integrated Oyibo space-time invariant solution from elemental path,

$$(\partial s)^2 = \varepsilon(c\partial t)^2 + \alpha(\partial x)^2 + \beta(\partial y)^2 + \gamma(\partial y)^2 \quad (6)$$

obtained for a curve *r(s)* parameterized by an arc of length *s* from some point on the curve in [1] subject to the constraining constants; $\varepsilon = -1,\ \alpha = 1,\ \beta = 1,\ \gamma = 1$.

We can see that Eq.(5) is the extended integrated Oyibo space-time invariant solution from elemental path for a torus. Interestingly, Eq.(5) is also invariant under the usual Lorentz transformation:

$$ct' = \gamma(ct - (v/c)z, x' = x, y' = y, z' = \gamma(z - vt) \quad (7a)$$

where $\gamma = 1/\sqrt{1 - (v^2/c^2)}$ and it can be verified directly:

$$s^2 + c^2t'^2 - x'^2 - y'^2 - z'^2 = 2s\sqrt{c^2t'^2 - z'^2} \quad (7b)$$

This is a signature that the torus is a geometric object founded on the Lorentz transformation which is the kernel of the Einstein's principle of relativity. The implication is that the Oyibo GUT can also be formulated using the torus as a geometric object since Eq.(5) is an extended integrated Oyibo space-time invariant solution. The above demonstration is very significant as it supports the suggestion in [19] that the Oyibo conformal transformations has a sound geometrical foundation like the usual characterization of conformal invariance in projective space-time geometry [34, 35]. One can then postulate that this common geometrical foundation is responsible for the remarkable correspondence of the Oyibo GUT with the Einstein's unified field equation for conformal (including scale) invariant field theories.

3. The Quantization Background of the Oyibo GUT

The second problem which is basically emanating from the long standing controversy on the possibility of quantizing the Einstein GTR [36-38] has two schools of thought: the researchers who think that the Einstein GTR cannot be quantized into a theory of quantum gravity [39, 40] and the opposing school of thought that believe it can be quantized [4, 23, 41]. I take a more subtle position on both debates. My reasons are as follows: (1) the relevance of the proof of Einstein GTR by Oyibo using his new mathematical approach is a common practice in mathematics and physics whereby we first obtain previous results from a new approach to verify the level of its applicability. Therefore, the successful demonstration as in the case of the Oyibo's new approach in the proof of the Einstein GTR [20], gives a boost to its extended applications. (2) Based on this first reason, it follows that even if it is assumed that the Einstein GTR cannot be quantized into quantum theory of gravitation, there is no scientific justification that the Oyibo approach cannot be used to achieve quantum theory of gravitation on its own merit. Extending this line of thinking, then one expect that the problem of moving to higher dimensions to achieve quantization [42] is not likely to be inherent in the Oyibo GUT. For the suggestion to move into higher dimensions was due to the need to add more degrees of freedom to the GTR [43]. Put more tacitly, the essential difference between gravity in 3D and gravity in more than

3D is that the former has no local degrees of freedom, whereas the latter do have such local degrees of freedom which become vital in introducing quantum effects [42, 44].

It is pertinent to point out that there are a large number of degrees of freedom inherent in the Oyibo GUT. Therefore, in order to be in conformity with the other workers' 5-dimensional background adopted in earlier attempts to unify the gravitational force field and electromagnetic force field [7, 45] and quantum gravitation in general [42], it will be straightforward to demonstrate here how to create and quantized a 5-dimensional background for Oyibo system whose boundary conditions and material specific requires such dimensional description.

3.1. Demonstration of the Quantization of the Oyibo GUT in N+1 Dimension

The Oyibo generic conservation equation which is an arbitrary function of space and time coordinates (x, y, z, t), velocities $(\dot{x},\dot{y},\dot{z})$, density (ρ), fluid or gas viscosity (μ), temperature (T), pressure (P), etc is given by (See pedagogical study in [1]):

$$G_{mn}(x,y,z,t,\dot{x},\dot{y},\dot{z},\rho,\mu,T,P,....)=0. \tag{8}$$

Therefore it can also be expressed as grand unified field theory equation for general hyper-dimensional space N which is the given number of independent dimensions as [7]

$$(G_{jk})_{x_k}=0 \tag{9}$$

where k = 0, 1, 2, 3, …, N-1.

The general solution which becomes the solutions of the generic equations are generic functions of the absolute invariant subgroups of transformation η_n given by

$$\begin{aligned}\eta_n = g_{n0}x_0^{n+1}+g_{n1}x_1^{n+1}+g_{n2}x_2^{n+1}+\\ g_{n3}x_3^{n+1}+...+\ g_{nN-1}x_{N-1}^{n+1}\end{aligned} \tag{10}$$

Again, it is emphasized here that the generic nature of the general solutions means their applications depend on the boundary conditions and other specifics of the systems. For example, to apply it to a four space-time dimensional (that is, N = 4) system, Eq. (9) which can be re-expressed as

$$\begin{aligned}&(G_{j0})_{x_0}+(G_{j1})_{x_1}+(G_{j2})_{x_2}+(G_{j3})_{x_3}+...\\ &+\ (G_{jN-1})_{x_{N-1}}=0\end{aligned} \tag{11}$$

reduces to

$$(G_{j0})_{x_0}+(G_{j1})_{x_1}+(G_{j2})_{x_2}+(G_{j3})_{x_3} \tag{12}$$

where $x_0=t$, $x_1=x$, $x_2=y$, $x_3=z$. (13)

It is then straightforward to re-express the solutions in Eq.(10) as

$$\eta_n = g_{n0}t^{n+1}+g_{n1}x^{n+1}+g_{n2}y^{n+1}+g_{n3}z^{n+1}. \tag{14}$$

where η_n is a function of the local-time and space coordinates (t, x, y, z), and of the "metric" parameters (g_{n0}, g_{n1}, g_{n2}, g_{n3}) as well as *n*.

Now if one has to apply the Oyibo GUT to a temperature dependent crystal which is a natural periodic system, we now assume that the projected torus in Fig.(2) is formulated by cubic crystals. Therefore, one can adopt the proposal in [46] that the process of implementing cyclic boundary conditions would require going from the usual 3-dimensional space of a crystal to 5-dimensional space and this will require slightly but very important modification of the transformation in Eq.(4) as

$$\tfrac{1}{2}(A''-A')=aT \ \text{ and } \ \tfrac{1}{2}(A''-A')=ct \tag{15}$$

yielding new expressions for the parametric equations:

$$\begin{aligned}x&=(aT+ct\sin\theta)\cos\varphi,\\ y&=(aT+ct\sin\theta)\sin\varphi,\\ z&=ct\cos\theta\end{aligned} \tag{16}$$

where *T* is temperature so that a is a constant having the dimension of [Length/Temperature], that is, thermal expansion coefficient.

Taking into account these new parametric equations in Eq.(16) into account in Eq,(2), we will obtain after eliminating ϕ, quantization of the torus into 5-dimensional background lattice:

$$\begin{aligned}(ct)^2+(aT)^2-x^2-y^2-z^2&\\ =\ 2(ct)(aT)\sin\theta&\\ \equiv\begin{cases}\pm 2(ct)(aT), \text{ if } \theta=(\mathrm{n}+\frac{1}{2})\pi,\\ 0, \qquad \text{if } \theta=n\pi\end{cases}&\end{aligned} \tag{17}$$

where n = 0, 1, 2.

However, if one consider the condition that the only rotations compatible with translational symmetry of a crystal lattice in three-dimensional space are those for which $2\cos\theta-1=\text{integer}$ [33], then this will yield the limit, $\theta=2\pi/n$, where $\mathrm{n}=1,2,3,4,6$ corresponding to the five degrees of freedom of Oyibo GUT specified in Eq.(14).

4. Conclusions

It has been demonstrated here that the Oyibo GUT has a sound geometrical gravitation and quantization foundation to be a mathematical framework for the unified force field theory which is the unified theory of everything that can explain the functioning and existence of the universe and therefore the holy grail of physics. Interestingly, the lack of such a common mathematical framework has been the bane of a theory of quantum gravity which is considered by some workers to be the most difficult problem in the search for the

UFFT [47]. I am therefore motivated again to advocate for more studies of the Oyibo GUT. This positive disposition is in line with the trend taken to successfully promote all such esoteric works in the past that the ideas or approaches were initially rejected by a number of established colleagues or journals but became celebrated. We have mentioned some cases in our previous communications. In addition, lets quickly mention two classic cases. The first is Enrico Fermi paper on his attempt of a theory of beta radiation based on weak force in 1933, which was first rejected by the highly rated journal he sent it to on the ground that 'it contained speculations too remote from reality to be of interest to the reader' [48]. This, however, became the foundation for his 1938 Nobel Prize "for his work with artificial radioactivity produced by neutrons, and for nuclear reactions brought about by slow neutrons." Remarkably, Enrico Fermi later initially rejected Murray Gell-Mann's explanation for the strange particles by means of displaced isotopic spin multiplets because he thought his own proposal with Feynman for higher angular momentum was the right explanation. However, Gell-Mann was challenged and encouraged to surmount this serious setback and improve on the study when he discovered by secretly reading a letter being prepared by the secretary for Fermi where the latter acknowledged that the Gell-Mann 'displaced isotopic spin multiplets speculation' could be the explanation [49]. But then, the paper Gell-Mann prepared from the work was also initially rejected by the journal he sent it until he did improved on it again before it was published with a modified title and this won him the 1969 Nobel Prize in Physics.

One lucid lesson from all these initially rejected ideas is that the workers were challenged and actuated to improve on their works. This is the response Oyibo and co-workers should give to the critics of their works.

The approach to such response is to first recover previous results from the Oyibo GUT using the appropriate generic solutions by experts in their various aspects of physics. For example, we have recovered the second order partial differential equations (PDF) relevant to physics from the Oyibo GUT [15] and thereby opening the opportunity of extending the Oyibo theory to wide aspects of the related physics. The wave equation from that study has in turn been used to obtain the Klein-Gordon equation [16] from which one can work back to the relativistic mass-energy equation [50] which can then be used to obtain the Dirac equation that is the gateway to quantum field theory.

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