

Spin Two-Body Problem of Classical Electrodynamics with Radiation Terms (II) – Existence of Solution of the Spin Equations

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Abstract The primary purpose of the present paper is to continue our studies from previous papers where the spin equations were derived. Here we prove an existence of a periodic solution of the spin equations system using fixed point method. As a consequence, we obtain that the general two-body problem of classical electrodynamics with radiation terms and spin is already solved.

Keywords Two body problem of classical electrodynamics, Spin equations, Periodic solutions, Radiation terms

1. Introduction

The present paper is an immediate consequence of [1]. In [1] we have derived a general system of equations of motion describing two-body problem with radiation terms and spin. The results obtained rely on the previous papers [2] - [19]. We note another approach based on Wheeler-Feynman ideas and realized by D.-A. Deckert group (cf. [20] - [23]).

The general system describing motion of two mass charged particles with radiation terms and spin in the frame of classical electrodynamics derived in [1] is:

$$\begin{aligned} d\lambda_r^{(1)} / ds_1 &= e_1 \left(F_{rs}^{(2)} \lambda_s^{(1)} + F_{rs}^{(1)\text{rad}} \lambda_s^{(1)} \right) / (m_1 c^2) \\ d\sigma_{ij}^{(1)} / ds_1 &= e_1 \left[\left(F_{ik}^{(1)} + F_{ik}^{(1)\text{rad}} \right) \sigma_{kj}^{(1)} - \sigma_{ik}^{(1)} \left(F_{kj}^{(1)} + F_{kj}^{(1)\text{rad}} \right) \right] / (m_1 c^2) \\ d\lambda_r^{(2)} / ds_2 &= e_2 \left(F_{rs}^{(1)} \lambda_s^{(2)} + F_{rs}^{(2)\text{rad}} \lambda_s^{(2)} \right) / (m_2 c^2) \\ d\sigma_{ij}^{(2)} / ds_2 &= e_2 \left[\left(F_{ik}^{(2)} + F_{ik}^{(2)\text{rad}} \right) \sigma_{kj}^{(2)} - \sigma_{ik}^{(2)} \left(F_{kj}^{(2)} + F_{kj}^{(2)\text{rad}} \right) \right] / (m_2 c^2) \end{aligned} \quad (1)$$

where Einstein summation convention is valid. This system is overdetermined. In other words, the number of equations is more than the number of unknown functions. In [6] and [1] we have proved that if the system

$$\begin{aligned} d\vec{\lambda}^{(p)} / ds_p &= (Q_p / c^2) \left[\vec{P}^{(pq)} \left(\left\langle \vec{\xi}^{(pq)}, \vec{\lambda}^{(p)} \right\rangle - c^2 \tau_{pq} / \Delta_p \right) - \left(\left\langle \vec{\lambda}^{(p)}, \vec{P}^{(pq)} \right\rangle + c^2 M_{pq} / \Delta_p \right) \vec{\xi}^{(pq)} \right] + \\ &+ (Q_p / 2c^2) \left[\vec{P}^{p,\text{ret}} \left(\left\langle \vec{\xi}^{(p)\text{ret}}, \vec{\lambda}^{(p)} \right\rangle - c^2 \tau_p^{\text{ret}} / \Delta_p \right) - \left(\left\langle \vec{\lambda}^{(p)}, \vec{P}^{p,\text{ret}} \right\rangle + c^2 M_{p,\text{ret}} / \Delta_p \right) \vec{\xi}^{(p)\text{ret}} \right] - \\ &- (Q_p / 2c^2) \left[\vec{P}^{p,\text{adv}} \left(\left\langle \vec{\xi}^{(p)\text{adv}}, \vec{\lambda}^{(p)} \right\rangle - c^2 \tau_p^{\text{adv}} / \Delta_p \right) - \left(\left\langle \vec{\lambda}^{(p)}, \vec{P}^{p,\text{adv}} \right\rangle + c^2 M_{p,\text{adv}} / \Delta_p \right) \vec{\xi}^{(p)\text{adv}} \right] \end{aligned} \quad (2)$$

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$$\begin{aligned}
d\vec{\sigma}^{(p)}/ds_p = & (Q_p/c^2) \left[\vec{P}^{(pq)} \times (\vec{\sigma}^{(p)} \times \vec{\xi}^{(pq)}) + \tau_{pq} \vec{P}^{(pq)} \times (\vec{\sigma}^{(p)} \times \vec{\lambda}^{(p)}) - \right. \\
& \left. - \vec{\xi}^{(pq)} \times (\vec{\sigma}^{(p)} \times \vec{P}^{(pq)}) - (iP_4^{(pq)}/c) \vec{\xi}^{(pq)} \times (\vec{\sigma}^{(p)} \times \vec{\lambda}^{(p)}) \right] + \\
& + (Q_p/2c^2) \left[\vec{P}^{(p)\text{ret}} \times (\vec{\sigma}^{(p)} \times \vec{\xi}^{(p)\text{ret}}) + \tau_p^{\text{ret}} \vec{P}^{(p)\text{ret}} \times (\vec{\sigma}^{(p)} \times \vec{\lambda}^{(p)}) - \right. \\
& \left. - \vec{\xi}^{(p)\text{ret}} \times (\vec{\sigma}^{(p)} \times \vec{P}^{(p)\text{ret}}) - (iP_4^{(p)\text{ret}}/c) \vec{\xi}^{(p)\text{ret}} \times (\vec{\sigma}^{(p)} \times \vec{\lambda}^{(p)}) \right] - \\
& - (Q_p/2c^2) \left[\vec{P}^{(p)\text{adv}} \times (\vec{\sigma}^{(p)} \times \vec{\xi}^{(p)\text{adv}}) + \tau_p^{\text{adv}} \vec{P}^{(p)\text{adv}} \times (\vec{\sigma}^{(p)} \times \vec{\lambda}^{(p)}) - \right. \\
& \left. - \vec{\xi}^{(p)\text{adv}} \times (\vec{\sigma}^{(p)} \times \vec{P}^{(p)\text{adv}}) - (iP_4^{(p)\text{adv}}/c) \vec{\xi}^{(p)\text{adv}} \times (\vec{\sigma}^{(p)} \times \vec{\lambda}^{(p)}) \right]
\end{aligned} \quad (3)$$

(containing 12 equations for 12 unknown functions) has a solution then the rest ones from (1) possess a solution too (pq) = (12), (21).

In [7] we have proved an existence-uniqueness of a periodic solution of equations of motion for two-body problem with corrected radiation terms, namely system (2). It remains to prove an existence of periodic solution of (3). On the right side of the equation (3) are the speeds and trajectories of the moving particles. Their existence is proven in [7]. We will consider them as known functions in (3).

The main goal of the present paper is to prove an existence of periodic spin functions $\sigma_\alpha^{(p)}$ ($p=1,2; \alpha=1,2,3$) satisfying (3). To do this we transform (3) using the known relations from vector calculus and obtain:

$$\begin{aligned}
\vec{P}^{(pq)} \times (\vec{\sigma}^{(p)} \times \vec{\xi}^{(pq)}) &= \langle \vec{P}^{(pq)}, \vec{\xi}^{(pq)} \rangle \vec{\sigma}^{(p)} - \langle \vec{P}^{(pq)}, \vec{\sigma}^{(p)} \rangle \vec{\xi}^{(pq)} - \vec{\xi}^{(pq)} \times (\vec{\sigma}^{(p)} \times \vec{P}^{(pq)}) \\
&= -\langle \vec{\xi}^{(pq)}, \vec{P}^{(pq)} \rangle \vec{\sigma}^{(p)} + \langle \vec{\xi}^{(pq)}, \vec{\sigma}^{(p)} \rangle \vec{P}^{(pq)} \left[\tau_{pq} \vec{P}^{(pq)} - (iP_4^{(pq)}/c) \vec{\xi}^{(pq)} \right] \times (\vec{\sigma}^{(p)} \times \vec{\lambda}^{(p)}) \\
&= \langle \tau_{pq} \vec{P}^{(pq)} - (iP_4^{(pq)}/c) \vec{\xi}^{(pq)}, \vec{\lambda}^{(p)} \rangle \vec{\sigma}^{(p)} - \langle \tau_{pq} \vec{P}^{(pq)} - (iP_4^{(pq)}/c) \vec{\xi}^{(pq)}, \vec{\sigma}^{(p)} \rangle \vec{\lambda}^{(p)} \\
&= \langle \tau_{pq} \vec{P}^{(pq)}, \vec{\lambda}^{(p)} \rangle \vec{\sigma}^{(p)} - (iP_4^{(pq)}/c) \langle \vec{\xi}^{(pq)}, \vec{\lambda}^{(p)} \rangle \vec{\sigma}^{(p)} - \tau_{pq} \langle \vec{P}^{(pq)}, \vec{\sigma}^{(p)} \rangle \vec{\lambda}^{(p)} + (iP_4^{(pq)}/c) \langle \vec{\xi}^{(pq)}, \vec{\sigma}^{(p)} \rangle \vec{\lambda}^{(p)}.
\end{aligned}$$

Therefore

$$\begin{aligned}
&\vec{P}^{(pq)} \times (\vec{\sigma}^{(p)} \times \vec{\xi}^{(pq)}) - \vec{\xi}^{(pq)} \times (\vec{\sigma}^{(p)} \times \vec{P}^{(pq)}) + \left(\tau_{pq} \vec{P}^{(pq)} - (iP_4^{(pq)}/c) \vec{\xi}^{(pq)} \right) \times (\vec{\sigma}^{(p)} \times \vec{\lambda}^{(p)}) \\
&= \langle \vec{P}^{(pq)}, \vec{\xi}^{(pq)} \rangle \vec{\sigma}^{(p)} - \langle \vec{P}^{(pq)}, \vec{\sigma}^{(p)} \rangle \vec{\xi}^{(pq)} - \langle \vec{\xi}^{(pq)}, \vec{P}^{(pq)} \rangle \vec{\sigma}^{(p)} + \langle \vec{\xi}^{(pq)}, \vec{\sigma}^{(p)} \rangle \vec{P}^{(pq)} + \\
&+ \langle \tau_{pq} \vec{P}^{(pq)}, \vec{\lambda}^{(p)} \rangle \vec{\sigma}^{(p)} - (iP_4^{(pq)}/c) \langle \vec{\xi}^{(pq)}, \vec{\lambda}^{(p)} \rangle \vec{\sigma}^{(p)} - \tau_{pq} \langle \vec{P}^{(pq)}, \vec{\sigma}^{(p)} \rangle \vec{\lambda}^{(p)} + (iP_4^{(pq)}/c) \langle \vec{\xi}^{(pq)}, \vec{\sigma}^{(p)} \rangle \vec{\lambda}^{(p)} \\
&= \langle \vec{\xi}^{(pq)}, \vec{\sigma}^{(p)} \rangle \vec{P}^{(pq)} - \langle \vec{P}^{(pq)}, \vec{\sigma}^{(p)} \rangle \vec{\xi}^{(pq)} + \left(\tau_{pq} \langle \vec{P}^{(pq)}, \vec{\lambda}^{(p)} \rangle - (iP_4^{(pq)}/c) \langle \vec{\xi}^{(pq)}, \vec{\lambda}^{(p)} \rangle \right) \vec{\sigma}^{(p)} + \\
&+ \left((iP_4^{(pq)}/c) \langle \vec{\xi}^{(pq)}, \vec{\sigma}^{(p)} \rangle - \tau_{pq} \langle \vec{P}^{(pq)}, \vec{\sigma}^{(p)} \rangle \right) \vec{\lambda}^{(p)}.
\end{aligned}$$

Then the system (3) becomes

$$\begin{aligned}
d\vec{\sigma}^{(p)}/ds_p = & (Q_p/c^2) \left[\langle \vec{\xi}^{(pq)}, \vec{\sigma}^{(p)} \rangle \vec{P}^{(pq)} - \langle \vec{P}^{(pq)}, \vec{\sigma}^{(p)} \rangle \vec{\xi}^{(pq)} + \right. \\
& + \left(\tau_{pq} \langle \vec{P}^{(pq)}, \vec{\lambda}^{(p)} \rangle - (iP_4^{(pq)}/c) \langle \vec{\xi}^{(pq)}, \vec{\lambda}^{(p)} \rangle \right) \vec{\sigma}^{(p)} + \left((iP_4^{(pq)}/c) \langle \vec{\xi}^{(pq)}, \vec{\sigma}^{(p)} \rangle - \tau_{pq} \langle \vec{P}^{(pq)}, \vec{\sigma}^{(p)} \rangle \right) \vec{\lambda}^{(p)} \right] + \\
& + (Q_p/2c^2) \left[\langle \vec{\xi}^{(p)\text{ret}}, \vec{\sigma}^{(p)} \rangle \vec{P}^{p,\text{ret}} - \langle \vec{P}^{p,\text{ret}}, \vec{\sigma}^{(p)} \rangle \vec{\xi}^{(p)\text{ret}} + \left(\tau_p^{\text{ret}} \langle \vec{P}^{p,\text{ret}}, \vec{\lambda}^{(p)} \rangle - (iP_4^{p,\text{ret}}/c) \langle \vec{\xi}^{(p)\text{ret}}, \vec{\lambda}^{(p)} \rangle \right) \vec{\sigma}^{(p)} + \right. \\
& + \left((iP_4^{p,\text{ret}}/c) \langle \vec{\xi}^{(p)\text{ret}}, \vec{\sigma}^{(p)} \rangle - \tau_p^{\text{ret}} \langle \vec{P}^{p,\text{ret}}, \vec{\sigma}^{(p)} \rangle \right) \vec{\lambda}^{(p)} \right] - (Q_p/2c^2) \left[\langle \vec{\xi}^{(p)\text{adv}}, \vec{\sigma}^{(p)} \rangle \vec{P}^{p,\text{adv}} - \langle \vec{P}^{p,\text{adv}}, \vec{\sigma}^{(p)} \rangle \vec{\xi}^{(p)\text{adv}} + \right. \\
& + \left(\tau_p^{\text{adv}} \langle \vec{P}^{p,\text{adv}}, \vec{\lambda}^{(p)} \rangle - (iP_4^{p,\text{adv}}/c) \langle \vec{\xi}^{(p)\text{adv}}, \vec{\lambda}^{(p)} \rangle \right) \vec{\sigma}^{(p)} + \left((iP_4^{p,\text{adv}}/c) \langle \vec{\xi}^{(p)\text{adv}}, \vec{\sigma}^{(p)} \rangle - \tau_p^{\text{adv}} \langle \vec{P}^{p,\text{adv}}, \vec{\sigma}^{(p)} \rangle \right) \vec{\lambda}^{(p)} \right].
\end{aligned}$$

In coordinate form (3) is ($\alpha=1,2,3$):

$$\begin{aligned}
 d\sigma_\alpha^{(p)} / ds_p = & (Q_p / c^2) \left[\left\langle \vec{\xi}^{(pq)}, \vec{\sigma}^{(p)} \right\rangle P_\alpha^{(pq)} - \left\langle \vec{P}^{(pq)}, \vec{\sigma}^{(p)} \right\rangle \xi_\alpha^{(pq)} + \right. \\
 & + \left(\tau_{pq} \left\langle \vec{P}^{(pq)}, \vec{\lambda}^{(p)} \right\rangle - \left(i P_4^{(pq)} / c \right) \left\langle \vec{\xi}^{(pq)}, \vec{\lambda}^{(p)} \right\rangle \right) \sigma_\alpha^{(p)} + \\
 & + \left. \left(\left(i P_4^{(pq)} / c \right) \left\langle \vec{\xi}^{(pq)}, \vec{\sigma}^{(p)} \right\rangle - \tau_{pq} \left\langle \vec{P}^{(pq)}, \vec{\sigma}^{(p)} \right\rangle \right) \lambda_\alpha^{(p)} \right] + \\
 & + (Q_p / 2c^2) \left[\left\langle \vec{\xi}^{(p)\text{ret}}, \vec{\sigma}^{(p)} \right\rangle P_\alpha^{p,\text{ret}} - \left\langle \vec{\xi}^{(p)\text{adv}}, \vec{\sigma}^{(p)} \right\rangle P_\alpha^{p,\text{adv}} - \right. \\
 & - \left\langle \vec{P}^{p,\text{ret}}, \vec{\sigma}^{(p)} \right\rangle \xi_\alpha^{(p)\text{ret}} + \left\langle \vec{P}^{p,\text{adv}}, \vec{\sigma}^{(p)} \right\rangle \xi_\alpha^{(p)\text{adv}} + \\
 & + \left(\tau_p^{\text{ret}} \left\langle \vec{P}^{p,\text{ret}}, \vec{\lambda}^{(p)} \right\rangle - \left(i P_4^{p,\text{ret}} / c \right) \left\langle \vec{\xi}^{(p)\text{ret}}, \vec{\lambda}^{(p)} \right\rangle \right) \sigma_\alpha^{(p)} - \\
 & - \left(\tau_p^{\text{adv}} \left\langle \vec{P}^{p,\text{adv}}, \vec{\lambda}^{(p)} \right\rangle - \left(i P_4^{p,\text{adv}} / c \right) \left\langle \vec{\xi}^{(p)\text{adv}}, \vec{\lambda}^{(p)} \right\rangle \right) \sigma_\alpha^{(p)} + \\
 & + \left(\left(i P_4^{p,\text{ret}} / c \right) \left\langle \vec{\xi}^{(p)\text{ret}}, \vec{\sigma}^{(p)} \right\rangle - \tau_p^{\text{ret}} \left\langle \vec{P}^{p,\text{ret}}, \vec{\sigma}^{(p)} \right\rangle \right) \lambda_\alpha^{(p)} - \\
 & - \left. \left(\left(i P_4^{p,\text{adv}} / c \right) \left\langle \vec{\xi}^{(p)\text{adv}}, \vec{\sigma}^{(p)} \right\rangle - \tau_p^{\text{adv}} \left\langle \vec{P}^{p,\text{adv}}, \vec{\sigma}^{(p)} \right\rangle \right) \lambda_\alpha^{(p)} \right].
 \end{aligned} \tag{4}$$

Remark. It is easy to verify that if $(\sigma_1^{(p)}, \sigma_2^{(p)}, \sigma_3^{(p)})$ is a solution of the system (4) then $(-\sigma_1^{(p)}, -\sigma_2^{(p)}, -\sigma_3^{(p)})$ is a solution too.

2. Formulation of the Periodic Problem for Spin Equations System

Recall denotations for quantities relating to the particles.

The space-time coordinates of the moving particles are

$$(x_1^{(p)}(t), x_2^{(p)}(t), x_3^{(p)}(t), x_4^{(p)} = ict) \equiv (\vec{x}^{(p)}, ict), (p = 1, 2);$$

c is the speed of the light; L_p ($p = 1, 2$) – world lines; m_p ($p = 1, 2$) – proper masses; e_p ($p = 1, 2$) – charges;

$Q_p = e_p^2 / m_p$ ($p = 1, 2$). We have $|e_1| = |e_2|$. But the interesting case is $e_1 e_2 < 0$. Then $Q_p = -e_p^2 / m_p < 0$ ($p = 1, 2$);

$$\xi^{(pq)} = (\vec{\xi}^{(pq)}, \xi_4^{(pq)}) = (x_1^{(p)}(t) - x_1^{(q)}(t - \tau_{pq}), x_2^{(p)}(t) - x_2^{(q)}(t - \tau_{pq}), x_3^{(p)}(t) - x_3^{(q)}(t - \tau_{pq}), ic\tau_{pq})$$

$(pq) = (12), (21)$ are components of the null vector lying on the light cone; $(u_1^{(p)}(t), u_2^{(p)}(t), u_3^{(p)}(t)) = \vec{u}^{(p)}(t)$, ($p = 1, 2$) are velocities of the moving particles;

$$\langle \vec{u}^{(p)}(t), \vec{u}^{(p)}(t) \rangle = \sum_{\gamma=1}^3 u_\gamma^{(p)}(t) u_\gamma^{(p)}(t),$$

$$\Delta_p = \sqrt{c^2 - \langle \vec{u}^{(p)}(t), \vec{u}^{(p)}(t) \rangle}, (\lambda_1^{(p)}, \lambda_2^{(p)}, \lambda_3^{(p)}, \lambda_4^{(p)}) = (\vec{\lambda}^{(p)}, ic / \Delta_p) = (\vec{u}^{(p)} / \Delta_p, ic / \Delta_p)$$

are components of the unit tangent vectors to world lines;

$$\lambda^{(q)} = \left(\frac{u_1^{(q)}(t - \tau_{pq})}{\Delta_{pq}}, \frac{u_2^{(q)}(t - \tau_{pq})}{\Delta_{pq}}, \frac{u_3^{(q)}(t - \tau_{pq})}{\Delta_{pq}}, \frac{ic}{\Delta_{pq}} \right) = (\vec{u}^{(pq)}(t - \tau_{pq}) / \Delta_{pq}, ic / \Delta_{pq})$$

where

$$\Delta_{pq} = \sqrt{c^2 - \langle \vec{u}^{(q)}(t - \tau_{pq}), \vec{u}^{(q)}(t - \tau_{pq}) \rangle}; ds^{(p)} = \Delta_p dt, \langle \lambda^{(p)}, \lambda^{(q)} \rangle_4 = \left(\langle \vec{u}^{(p)}, \vec{u}^{(q)} \rangle - c^2 \right) / \Delta_p \Delta_{pq};$$

$$\langle \lambda^{(p)}, \xi^{(pq)} \rangle_4 = \left(\langle \vec{\xi}^{(pq)}, \vec{u}^{(p)} \rangle - c^2 \tau_{pq} \right) / \Delta_p; d\lambda_\alpha^{(p)} / ds_p = \dot{u}_\alpha^{(p)} / (\Delta_p^2) + u_\alpha^{(p)} \langle \vec{u}^{(p)}, \dot{\vec{u}}^{(p)} \rangle / (\Delta_p^4);$$

$$d\lambda_4^{(p)} / ds_p = ic \langle \vec{u}^{(p)}, \dot{\vec{u}}^{(p)} \rangle / (\Delta_p^4); d / ds_p = (1 / \Delta_p) d / dt; , \frac{d}{ds_q} = \frac{1}{\Delta_{pq}} \frac{d}{dt_{pq}} = \frac{1}{\Delta_{pq}} \frac{dt}{dt_{pq}} \frac{d}{dt}, \frac{dt}{dt_{pq}} \equiv D_{pq}$$

where the derivative is calculated from the equation

$$t - t_{pq} = (1/c) \sqrt{\sum_{\gamma=1}^3 \left[x_\gamma^{(p)}(t) - x_\gamma^{(p)}(t_{pq}) \right]^2}, \text{ i.e. } \frac{dt}{dt_{pq}} = \frac{c \sqrt{\langle \vec{\xi}^{(pq)}, \vec{\xi}^{(pq)} \rangle} - \langle \vec{\xi}^{(pq)}, u^{(q)} \rangle}{c \sqrt{\langle \vec{\xi}^{(pq)}, \vec{\xi}^{(pq)} \rangle} - \langle \vec{\xi}^{(pq)}, u^{(p)} \rangle} \equiv D_{pq};$$

$$d\lambda_\alpha^{(q)} / ds_q = D_{pq} \dot{u}_\alpha^{(q)} / (\Delta_{pq}^2) + D_{pq} \langle \vec{u}^{(q)}, \dot{\vec{u}}^{(q)} \rangle u_\alpha^{(q)} / \Delta_{pq}^4; d\lambda_4^{(q)} / ds_q = ic D_{pq} \langle \vec{u}^{(q)}, \dot{\vec{u}}^{(q)} \rangle / \Delta_{pq}^4;$$

$$\langle \xi^{(pq)}, d\lambda^{(q)} / ds_q \rangle_4 = D_{pq} \left[\langle \xi^{(pq)}, \dot{u}^{(q)} \rangle / (\Delta_{pq}^2) + \left(\langle \xi^{(pq)}, u^{(q)} \rangle - c^2 \tau_{pq} \right) \langle u^{(q)}, \dot{u}^{(q)} \rangle / \Delta_{pq}^4 \right];$$

$$P_\alpha^{(pq)} = - \left(1 + \langle \xi^{(pq)}, d\lambda^{(q)} / ds_q \rangle_4 \right) \lambda_\alpha^{(q)} / \left(\langle \lambda^{(q)}, \xi^{(pq)} \rangle_4^3 \right) +$$

$$+ \left(1 / \left(\langle \lambda^{(q)}, \xi^{(pq)} \rangle_4^2 \right) \right) d\lambda_\alpha^{(q)} / ds_q = M_{pq} u_\alpha^{(q)} + N_{pq} \dot{u}_\alpha^{(q)}$$

where

$$M_{pq} = \frac{\Delta_{pq}^2 + D_{pq} \langle \vec{\xi}^{(pq)}, \dot{\vec{u}}^{(q)} \rangle}{\left(c^2 \tau_{pq} - \langle \vec{\xi}^{(pq)}, \vec{u}^{(q)} \rangle \right)^3}; N_{pq} = \frac{D_{pq}}{\left(c^2 \tau_{pq} - \langle \vec{\xi}^{(pq)}, \vec{u}^{(q)} \rangle \right)^2} \text{ and}$$

$$P_4^{(pq)} = - \frac{1 + \langle \xi^{(pq)}, d\lambda^{(q)} / ds_q \rangle_4}{\langle \lambda^{(q)}, \xi^{(pq)} \rangle_4^3} \lambda_4^{(q)} + \frac{1}{\langle \lambda^{(q)}, \xi^{(pq)} \rangle_4^2} \frac{d\lambda_4^{(q)}}{ds_q} = ic \frac{\Delta_{pq}^2 + D_{pq} \langle \vec{\xi}^{(pq)}, \dot{\vec{u}}^{(q)} \rangle}{\left(c^2 \tau_{pq} - \langle \vec{\xi}^{(pq)}, \vec{u}^{(q)} \rangle \right)^3} = ic M_{pq}.$$

Consequently

$$P_\alpha^{(pq)} \langle \lambda^{(p)}, \xi^{(pq)} \rangle_4 = M_{pq} \langle \lambda^{(p)}, \xi^{(pq)} \rangle_4 u_\alpha^{(q)} + N_{pq} \langle \lambda^{(p)}, \xi^{(pq)} \rangle_4 \dot{u}_\alpha^{(q)}; P_4^{(pq)} = ic M_{pq}.$$

We recall the basic assumption $\sqrt{\langle \vec{u}^{(p)}, \vec{u}^{(p)} \rangle} \leq \bar{c} < c$.

Finally we write down (4) in the form

$$d\sigma_\alpha^{(p)} / ds_p = (Q_p / c^2) \left[\langle \vec{\xi}^{(pq)}, \vec{\sigma}^{(p)} \rangle P_\alpha^{(pq)} - \langle \vec{P}^{(pq)}, \vec{\sigma}^{(p)} \rangle \xi_\alpha^{(pq)} + \right.$$

$$+ \left(\tau_{pq} \langle \vec{P}^{(pq)}, \vec{\lambda}^{(p)} \rangle - (i P_4^{(pq)} / c) \langle \vec{\xi}^{(pq)}, \vec{\lambda}^{(p)} \rangle \right) \sigma_\alpha^{(p)} + \left((i P_4^{(pq)} / c) \langle \vec{\xi}^{(pq)}, \vec{\sigma}^{(p)} \rangle - \tau_{pq} \langle \vec{P}^{(pq)}, \vec{\sigma}^{(p)} \rangle \right) \lambda_\alpha^{(p)} \Big] +$$

$$+ (Q_p / 2c^2) \left[\langle \vec{\xi}^{(p)\text{ret}}, \vec{\sigma}^{(p)} \rangle P_\alpha^{p,\text{ret}} - \langle \vec{P}^{p,\text{ret}}, \vec{\sigma}^{(p)} \rangle \xi_\alpha^{(p)\text{ret}} + \left(\tau_p^{p,\text{ret}} \langle \vec{P}^{p,\text{ret}}, \vec{\lambda}^{(p)} \rangle - (i P_4^{p,\text{ret}} / c) \langle \vec{\xi}^{(p)\text{ret}}, \vec{\lambda}^{(p)} \rangle \right) \sigma_\alpha^{(p)} + \right.$$

$$+ \left((i P_4^{p,\text{ret}} / c) \langle \vec{\xi}^{(p)\text{ret}}, \vec{\sigma}^{(p)} \rangle - \tau_p^{p,\text{ret}} \langle \vec{P}^{p,\text{ret}}, \vec{\sigma}^{(p)} \rangle \right) \lambda_\alpha^{(p)} \Big] - (Q_p / 2c^2) \left[\langle \vec{\xi}^{(p)\text{adv}}, \vec{\sigma}^{(p)} \rangle P_\alpha^{p,\text{adv}} \right.$$

$$- \langle \vec{P}^{p,\text{adv}}, \vec{\sigma}^{(p)} \rangle \xi_\alpha^{(p)\text{adv}} + \left(\tau_p^{p,\text{adv}} \langle \vec{P}^{p,\text{adv}}, \vec{\lambda}^{(p)} \rangle - (i P_4^{p,\text{adv}} / c) \langle \vec{\xi}^{(p)\text{adv}}, \vec{\lambda}^{(p)} \rangle \right) \sigma_\alpha^{(p)} +$$

$$+ \left((i P_4^{p,\text{adv}} / c) \langle \vec{\xi}^{(p)\text{adv}}, \vec{\sigma}^{(p)} \rangle - \tau_p^{p,\text{adv}} \langle \vec{P}^{p,\text{adv}}, \vec{\sigma}^{(p)} \rangle \right) \lambda_\alpha^{(p)} \Big].$$

(5)

3. Operator Presentation of the Periodic Problem

In this section we formulate an operator presentation of the periodic problem for (5).

First we transform (5) in order to obtain a suitable form of the spin equations system:

$$\begin{aligned}
 d\sigma_{\alpha}^{(p)} / ds_p = & (Q_p / c^2) \left[P_{\alpha}^{(pq)} \sum_{\gamma=1}^3 \xi_{\gamma}^{(pq)} \sigma_{\gamma}^{(p)} - \xi_{\alpha}^{(pq)} \sum_{\gamma=1}^3 P_{\gamma}^{(pq)} \sigma_{\gamma}^{(p)} + \right. \\
 & + \left(\tau_{pq} \sum_{\gamma=1}^3 P_{\gamma}^{(pq)} \lambda_{\gamma}^{(p)} - (iP_4^{(pq)} / c) \sum_{\gamma=1}^3 \xi_{\gamma}^{(pq)} \lambda_{\gamma}^{(p)} \right) \sigma_{\alpha}^{(p)} + \left((iP_4^{(pq)} / c) \sum_{\gamma=1}^3 \xi_{\gamma}^{(pq)} \sigma_{\gamma}^{(p)} - \tau_{pq} \sum_{\gamma=1}^3 P_{\gamma}^{(pq)} \sigma_{\gamma}^{(p)} \right) \lambda_{\alpha}^{(p)} \Big] + \\
 & + (Q_p / 2c^2) \left[P_{\alpha}^{p,\text{ret}} \sum_{\gamma=1}^3 \xi_{\gamma}^{(p)\text{ret}} \sigma_{\gamma}^{(p)} - P_{\alpha}^{p,\text{adv}} \sum_{\gamma=1}^3 \xi_{\gamma}^{(p)\text{adv}} \sigma_{\gamma}^{(p)} - \xi_{\alpha}^{(p)\text{ret}} \sum_{\gamma=1}^3 P_{\gamma}^{p,\text{ret}} \sigma_{\gamma}^{(p)} + \xi_{\alpha}^{(p)\text{adv}} \sum_{\gamma=1}^3 P_{\gamma}^{p,\text{adv}} \sigma_{\gamma}^{(p)} + \right. \\
 & + \left(\tau_p^{\text{ret}} \sum_{\gamma=1}^3 P_{\gamma}^{p,\text{ret}} \lambda_{\gamma}^{(p)} - (iP_4^{p,\text{ret}} / c) \sum_{\gamma=1}^3 \xi_{\gamma}^{(p)\text{ret}} \lambda_{\gamma}^{(p)} \right) \sigma_{\alpha}^{(p)} - \left(\tau_p^{\text{adv}} \sum_{\gamma=1}^3 P_{\gamma}^{p,\text{adv}} \lambda_{\gamma}^{(p)} - (iP_4^{p,\text{adv}} / c) \sum_{\gamma=1}^3 \xi_{\gamma}^{(p)\text{adv}} \lambda_{\gamma}^{(p)} \right) \sigma_{\alpha}^{(p)} + \\
 & + \left((iP_4^{p,\text{ret}} / c) \sum_{\gamma=1}^3 \xi_{\gamma}^{(p)\text{ret}} \sigma_{\gamma}^{(p)} - \tau_p^{\text{ret}} \sum_{\gamma=1}^3 P_{\gamma}^{p,\text{ret}} \sigma_{\gamma}^{(p)} \right) \lambda_{\alpha}^{(p)} - \left((iP_4^{p,\text{adv}} / c) \sum_{\gamma=1}^3 \xi_{\gamma}^{(p)\text{adv}} \sigma_{\gamma}^{(p)} - \tau_p^{\text{adv}} \sum_{\gamma=1}^3 P_{\gamma}^{p,\text{adv}} \sigma_{\gamma}^{(p)} \right) \lambda_{\alpha}^{(p)} \Big]
 \end{aligned}$$

or

$$d\sigma_{\alpha}^{(p)}(t) / \Delta_p dt = F_{\alpha}^{(p)}(t, \sigma_1^{(p)}, \sigma_2^{(p)}, \sigma_3^{(p)}) = F_{\alpha,L}^{(p)}(t, \sigma_1^{(p)}, \sigma_2^{(p)}, \sigma_3^{(p)}) + F_{\alpha,\text{rad}}^{(p)}(t, \sigma_1^{(p)}, \sigma_2^{(p)}, \sigma_3^{(p)}) \quad (7)$$

where

$$\begin{aligned}
 F_{\alpha,L}^{(p)}(t, \sigma_1^{(p)}, \sigma_2^{(p)}, \sigma_3^{(p)}) = & Q_p / (c^2) \times \left[\sum_{\gamma=1}^3 \sigma_{\gamma}^{(p)} \left(P_{\alpha}^{(pq)} \xi_{\gamma}^{(pq)} - \xi_{\alpha}^{(pq)} P_{\gamma}^{(pq)} + \right. \right. \\
 & + \left. \left. (iP_4^{(pq)} \lambda_{\alpha}^{(p)} / c) \xi_{\gamma}^{(pq)} - \tau_{pq} \lambda_{\alpha}^{(p)} P_{\gamma}^{(pq)} \right) + \sigma_{\alpha}^{(p)} \sum_{\gamma=1}^3 \left(\tau_{pq} P_{\gamma}^{(pq)} - (iP_4^{(pq)} / c) \xi_{\gamma}^{(pq)} \right) \lambda_{\gamma}^{(p)} \right], \\
 F_{\alpha,L}^{(p)}(t, \sigma_1^{(p)}, \sigma_2^{(p)}, \sigma_3^{(p)}) = & Q_p / (c^2) \times \left[\sum_{\gamma=1}^3 \sigma_{\gamma}^{(p)} \left(P_{\alpha}^{(pq)} \xi_{\gamma}^{(pq)} - \xi_{\alpha}^{(pq)} P_{\gamma}^{(pq)} + \right. \right. \\
 & + \left. \left. (iP_4^{(pq)} \lambda_{\alpha}^{(p)} / c) \xi_{\gamma}^{(pq)} - \tau_{pq} \lambda_{\alpha}^{(p)} P_{\gamma}^{(pq)} \right) + \sigma_{\alpha}^{(p)} \sum_{\gamma=1}^3 \left(\tau_{pq} P_{\gamma}^{(pq)} - (iP_4^{(pq)} / c) \xi_{\gamma}^{(pq)} \right) \lambda_{\gamma}^{(p)} \right], \\
 F_{\alpha,\text{rad}}^{(p)}(t, \sigma_1^{(p)}, \sigma_2^{(p)}, \sigma_3^{(p)}) = & Q_p / (2c^2) \times \left[\sigma_{\gamma}^{(p)} \left[\sum_{\gamma=1}^3 \left(P_{\alpha}^{p,\text{ret}} \xi_{\gamma}^{(p)\text{ret}} - P_{\alpha}^{p,\text{adv}} \xi_{\gamma}^{(p)\text{adv}} - \right. \right. \right. \\
 & - \left. \left. \xi_{\alpha}^{(p)\text{ret}} P_{\gamma}^{p,\text{ret}} + \xi_{\alpha}^{(p)\text{adv}} P_{\gamma}^{p,\text{adv}} \right) + \sigma_{\alpha}^{(p)} \sum_{\gamma=1}^3 \lambda_{\gamma}^{(p)} \left(\tau_p^{\text{ret}} P_{\gamma}^{p,\text{ret}} - \tau_p^{\text{adv}} P_{\gamma}^{p,\text{adv}} - \right. \right. \\
 & - \left. \left. (iP_4^{p,\text{ret}} / c) \xi_{\gamma}^{(p)\text{ret}} + (iP_4^{p,\text{adv}} / c) \xi_{\gamma}^{(p)\text{adv}} \right] + \lambda_{\alpha}^{(p)} \sum_{\gamma=1}^3 \sigma_{\gamma}^{(p)} \left((iP_4^{p,\text{ret}} / c) \xi_{\gamma}^{(p)\text{ret}} \right. \right. \\
 & - \left. \left. (iP_4^{p,\text{adv}} / c) \xi_{\gamma}^{(p)\text{adv}} - \tau_p^{\text{ret}} P_{\gamma}^{p,\text{ret}} + \tau_p^{\text{adv}} P_{\gamma}^{p,\text{adv}} \right) \right].
 \end{aligned}$$

In what follows we call $F_{\alpha,L}^{(p)}(t, \vec{\sigma}^{(p)})$ Lorentz term of the spin equations, while $F_{\alpha,\text{rad}}^{(p)}(t, \vec{\sigma}^{(p)})$ – radiation term.

We recall (cf. [7]) that velocity functions $u_{\alpha}^{(p)}(t), (p=1,2; \alpha=1,2,3)$ belong to the space

$$M_0 = \left\{ u(\cdot) \in M : \int_{kT_0}^{(k+1)T_0} u(t) dt = 0 \ (k = 0, 1, 2, 3, \dots) \right\},$$

where

$$M = \left\{ u \in C_{T_0}^\infty[0, \infty) : |u^{(n)}(t)| \leq U_0 \omega^n e^{\mu t}, t \in [0, T_0] \right\} \ (n = 0, 1, 2, \dots), \mu, \omega, U_0, T_0 = \text{const} > 0, U_0 e^{\mu T_0} \leq \bar{c} < c,$$

$C_{T_0}^\infty[0, \infty)$ is the space of all infinite differentiable T_0 -periodic functions. Following A. Sommerfeld [9], [10] we denote by $\beta = \bar{c} / c < 1$. It follows that trajectories and velocities are T_0 -periodic functions.

Our goal is to prove an existence of T_0 -periodic solution of (6). For that purpose we define an operator on the space $C_{T_0}[0, T_0]$ consisting of all continuous functions such that $\sigma(0) = \sigma(T_0)$ by the formulas

$$\begin{aligned} H &= \left(H_1^{(1)}, H_2^{(1)}, H_3^{(1)}, H_1^{(2)}, H_2^{(2)}, H_3^{(2)} \right) : \left(C_{T_0}[0, T_0] \right)^6 \rightarrow \left(C_{T_0}[0, T_0] \right)^6, \\ H_\alpha^{(p)}(\sigma_1^{(p)}, \sigma_2^{(p)}, \sigma_3^{(p)})(t) &:= \sigma_\alpha^{(p)}(T_0) + \int_0^t F_\alpha^{(p)}(\sigma_1^{(p)}, \sigma_2^{(p)}, \sigma_3^{(p)})(s) ds, \\ t &\in [0, T_0]; \ (p = 1, 2; \ \alpha = 1, 2, 3). \end{aligned} \quad (8)$$

Lemma. Every T_0 -periodic solution of (6) is a fixed point of H and vice versa.

The proof can be found in [24] (M. A. Krasnoselskii).

Consequently, we have to prove an existence of a fixed point of the operator H .

4. Preliminary Lipschitz Estimates of the Lorentz Term of the Spin Equations

First we notice that $U_0 e^{\mu T_0} \leq \bar{c} < c$;

$$\begin{aligned} |\xi_\gamma^{(pq)}| &\leq \bar{c} \tau_{pq} \leq c \tau_{pq}; \ |D_{pq}| \leq (1 + \beta) / (1 - \beta); \ \tau_{pq} \geq r_0 / (2c) \Rightarrow 1 / \tau_{pq} \leq 2c / r_0; \ \omega = 2\pi / T_0; \\ |u_\alpha^{(p)}| / \Delta_p &\leq c / \sqrt{c^2 - \bar{c}^2} = 1 / \sqrt{1 - \beta^2}; \\ c^2 \tau_{pq} - \langle \vec{\xi}^{(pq)}, \vec{u}^{(q)} \rangle &\geq c^2 \tau_{pq} - \sqrt{\langle \vec{\xi}^{(pq)}, \vec{\xi}^{(pq)} \rangle} \sqrt{\langle \vec{u}^{(q)}, \vec{u}^{(q)} \rangle} \geq c^2 \tau_{pq} - \bar{c} c \tau_{pq} = c^2 \tau_{pq} (1 - \beta); \\ |P_\alpha^{(pq)}| &\leq |M_{pq}| |u_\alpha^{(q)}| + |N_{pq}| |\dot{u}_\alpha^{(q)}| \leq \left| \left(\Delta_{pq}^2 + D_{pq} \langle \vec{\xi}^{(pq)}, \dot{\vec{u}}^{(q)} \rangle \right) / \left(c^2 \tau_{pq} - \langle \vec{\xi}^{(pq)}, \vec{u}^{(q)} \rangle \right) \right|^3 |u_\alpha^{(q)}| + \\ &+ \left| D_{pq} / \left(c^2 \tau_{pq} - \langle \vec{\xi}^{(pq)}, \vec{u}^{(q)} \rangle \right) \right|^2 |\dot{u}_\alpha^{(q)}| \leq |u_\alpha^{(q)}| (t - \tau_{pq}) \times \\ &\times \left| \left(c^2 + (1 + \beta) / (1 - \beta) \sum_{\gamma=1}^3 |\xi_\gamma^{(pq)}| |\dot{u}_\gamma^{(q)}| \right) / \left(c^2 \tau_{pq} (1 - \beta) \right) \right|^3 + \\ &+ (1 + \beta) |\dot{u}_\alpha^{(q)}| (t - \tau_{pq}) / (1 - \beta) \left(c^2 \tau_{pq} (1 - \beta) \right)^2 \leq |u_\alpha^{(q)}| (t - \tau_{pq}) / \left(c^4 \tau_{pq}^3 (1 - \beta)^3 \right) + \\ &+ (1 + \beta) / \left(c^5 \tau_{pq}^2 (1 - \beta)^4 \right) |u_\alpha^{(q)}| (t - \tau_{pq}) \left| \sum_{\theta=1}^3 |\dot{u}_\theta^{(q)}| (t - \tau_{pq}) \right| + (1 + \beta) |\dot{u}_\alpha^{(q)}| (t - \tau_{pq}) / \left(c^4 \tau_{pq}^2 (1 - \beta)^3 \right) \leq \\ &\leq (1 - \beta) / \left(c^3 \tau_{pq}^3 (1 - \beta)^4 \right) + \left(3(1 + \beta) \omega + (1 - \beta)(1 + \beta) \omega \right) U_0 e^{\mu_0} / \left(c^4 \tau_{pq}^2 (1 - \beta)^4 \right) \leq \\ &\leq \left[1 / \left(c^3 (1 - \beta)^4 \right) \right] \left[\left((1 - \beta) / \tau_{pq}^3 \right) + \left(4(1 + \beta) \omega / \tau_{pq}^2 \right) \right] \end{aligned}$$

and

$$\left| iP_4^{(pq)} / c \right| \leq \left| \frac{i}{c} \dot{c} \left| \left(\Delta_{pq}^2 + D_{pq} \left\langle \vec{\xi}^{(pq)}, \dot{\vec{u}}^{(q)} \right\rangle \right) / \left(c^2 \tau_{pq} - \left\langle \vec{\xi}^{(pq)}, \vec{u}^{(q)} \right\rangle \right)^3 \right| \right| \leq$$

$$\leq \left[1 / (c(1-\beta))^4 \right] \left[\left((1-\beta) / \tau_{pq}^3 \right) + \left(3(1+\beta) \omega / \tau_{pq}^2 \right) \right].$$

Then

$$\left| H_{\alpha,L}^{(p)}(\sigma_1^{(p)}, \sigma_2^{(p)}, \sigma_3^{(p)})(t) - H_{\alpha,L}^{(p)}(\bar{\sigma}_1^{(p)}, \bar{\sigma}_2^{(p)}, \bar{\sigma}_3^{(p)})(t) \right| \leq$$

$$\leq \int_0^t \left| F_{\alpha,L}^{(p)}(\sigma_1^{(p)}, \sigma_2^{(p)}, \sigma_3^{(p)})(s) - F_{\alpha,L}^{(p)}(\bar{\sigma}_1^{(p)}, \bar{\sigma}_2^{(p)}, \bar{\sigma}_3^{(p)})(s) \right| ds \leq$$

$$\leq \left(|Q_p| / c^2 \right) \left[\sum_{\gamma=1}^3 \int_0^t \left| P_{\alpha}^{(pq)} \xi_{\gamma}^{(pq)} - \xi_{\alpha}^{(pq)} P_{\gamma}^{(pq)} + \right. \right.$$

$$\left. + \left(iP_4^{(pq)} \lambda_{\alpha}^{(p)} / c \right) \xi_{\gamma}^{(pq)} - \tau_{pq} \lambda_{\alpha}^{(p)} P_{\gamma}^{(pq)} \right| \left| \sigma_{\gamma}^{(p)}(s) - \bar{\sigma}_{\gamma}^{(p)}(s) \right| ds +$$

$$\left. + \sum_{\gamma=1}^3 \int_0^t \left| \tau_{pq} P_{\gamma}^{(pq)} - \left(iP_4^{(pq)} / c \right) \xi_{\gamma}^{(pq)} \right| \left| \lambda_{\gamma}^{(p)} \right| \left| \sigma_{\alpha}^{(p)}(s) - \bar{\sigma}_{\alpha}^{(p)}(s) \right| ds \right] \leq$$

$$\leq 2|Q_p| \left(c\sqrt{1-\beta^2} + 1 \right) / \left(c^5 (1-\beta)^4 \sqrt{1-\beta^2} \right) \times$$

$$\times \sum_{\gamma=1}^3 \int_0^t \left[\left(1-\beta + 4(1+\beta) \omega \tau_{pq} \right) / \tau_{pq}^2 \right] \left| \sigma_{\gamma}^{(p)}(s) - \bar{\sigma}_{\gamma}^{(p)}(s) \right| ds + 2|Q_p| / \left(c^5 (1-\beta)^4 \sqrt{1-\beta^2} \right) \times$$

$$\times \sum_{\gamma=1}^3 \int_0^t \left[\left(1-\beta + 4(1+\beta) \omega \tau_{pq} \right) / \tau_{pq}^2 \right] \left| \sigma_{\alpha}^{(p)}(s) - \bar{\sigma}_{\alpha}^{(p)}(s) \right| ds \leq 8|Q_p| \left(c\sqrt{1-\beta^2} + 1 \right) / \left(c^4 (1-\beta)^4 \sqrt{1-\beta^2} \right) \times$$

$$\times \left[\left(c(1-\beta) + 2(1+\beta) \omega r_0 \right) / r_0^2 \right] \sum_{\gamma=1}^3 \int_0^t \left| \sigma_{\gamma}^{(p)}(s) - \bar{\sigma}_{\gamma}^{(p)}(s) \right| ds +$$

$$+ 8|Q_p| \left(c(1-\beta) + 2(1+\beta) \omega r_0 \right) / \left(c^4 (1-\beta)^4 r_0^2 \sqrt{1-\beta^2} \right) \times \sum_{\gamma=1}^3 \int_0^t \left| \sigma_{\alpha}^{(p)}(s) - \bar{\sigma}_{\alpha}^{(p)}(s) \right| ds \leq$$

$$\leq e^{\mu t} \rho \left((\sigma_1^{(1)}, \sigma_2^{(1)}, \sigma_3^{(1)}, \sigma_1^{(2)}, \sigma_2^{(2)}, \sigma_3^{(2)}), (\bar{\sigma}_1^{(1)}, \bar{\sigma}_2^{(1)}, \bar{\sigma}_3^{(1)}, \bar{\sigma}_1^{(2)}, \bar{\sigma}_2^{(2)}, \bar{\sigma}_3^{(2)}) \right) \times$$

$$\times 8|Q_p| \left(c\sqrt{1-\beta^2} + 2 \right) \left(c(1-\beta) + 2(1+\beta) \omega r_0 \right) / \mu \left(c^4 (1-\beta)^4 r_0^2 \sqrt{1-\beta^2} \right).$$

5. Preliminary Lipschitz Estimates of the Radiation Term of the Spin Equations

Here we transform the radiation part of the spin equation using some reasoning from [7]. Indeed, we recall assumption $\tau_p^{\text{ret}} = \tau_p^{\text{adv}} = \tau$ with τ small:

$$\xi_{\alpha}^{(p)\text{adv}} = x_{\alpha}^{(p)}(t + \tau) - x_{\alpha}^{(p)}(t) = \tau u_{\alpha}^{(p)}(t) + (\tau^2 / (2!)) \dot{u}_{\alpha}^{(p)}(t) + \dots \Rightarrow$$

$$\xi_{\alpha}^{(p)a} = \tau u_{\alpha}^{(p)}(t) + O(\tau^2) \Rightarrow \xi_{\alpha}^{(p)a} \approx \tau u_{\alpha}^{(p)}(t);$$

$$\begin{aligned}
u_{\alpha}^{(p)}(t+\tau) &= u_{\alpha}^{(p)}(t) + (\tau/1!) \dot{u}_{\alpha}^{(p)}(t) + (\tau^2/2!) \ddot{u}_{\alpha}^{(p)}(t) + (\tau^3/3!) \dddot{u}_{\alpha}^{(p)}(t) + \dots \Rightarrow u_{\alpha}^{(p)}(t+\tau) = u_{\alpha}^{(p)}(t) + O(\tau); \\
u_{\alpha}^{(p)}(t-\tau) &= u_{\alpha}^{(p)}(t) - (\tau/1!) \dot{u}_{\alpha}^{(p)}(t) + (\tau^2/2!) \ddot{u}_{\alpha}^{(p)}(t) - (\tau^3/3!) \dddot{u}_{\alpha}^{(p)}(t) + \dots \Rightarrow u_{\alpha}^{(p)}(t-\tau) = u_{\alpha}^{(p)}(t) - O(\tau); \\
u_{\alpha}^{(p)}(t) u_{\alpha}^{(p)}(t+\tau) &= \\
&= \left(u_{\alpha}^{(p)}(t)\right)^2 + (\tau/1!) \dot{u}_{\alpha}^{(p)}(t) u_{\alpha}^{(p)}(t) + (\tau^2/2!) \ddot{u}_{\alpha}^{(p)}(t) u_{\alpha}^{(p)}(t) + \dots \Rightarrow u_{\alpha}^{(p)}(t) u_{\alpha}^{(p)}(t+\tau) = \left(u_{\alpha}^{(p)}(t)\right)^2 + O(\tau); \\
u_{\alpha}^{(p)}(t) u_{\alpha}^{(p)}(t-\tau) &= \\
&= \left(u_{\alpha}^{(p)}(t)\right)^2 - (\tau/1!) \dot{u}_{\alpha}^{(p)}(t) u_{\alpha}^{(p)}(t) + (\tau^2/2!) \ddot{u}_{\alpha}^{(p)}(t) u_{\alpha}^{(p)}(t) - \dots \Rightarrow u_{\alpha}^{(p)}(t) u_{\alpha}^{(p)}(t-\tau) = \left(u_{\alpha}^{(p)}(t)\right)^2 - O(\tau); \\
\langle u^{(p)}, u^{(p)\text{adv}} \rangle &= \langle u^{(p)}, u^{(p)}(t+\tau) \rangle = \sum_{\gamma=1}^3 u_{\gamma}^{(p)}(t) u_{\gamma}^{(p)}(t+\tau) \approx \sum_{\gamma=1}^3 u_{\gamma}^{(p)}(t) u_{\gamma}^{(p)}(t) = \langle u^{(p)}, u^{(p)} \rangle; \\
\langle u^{(p)}, u^{(p)\text{ret}} \rangle &\approx \langle u^{(p)}, u^{(p)} \rangle; \quad \langle u^{(p)\text{adv}}, u^{(p)\text{adv}} \rangle \approx \langle u^{(p)}, u^{(p)\text{adv}} \rangle; \quad \langle u^{(p)\text{ret}}, u^{(p)\text{ret}} \rangle \approx \langle u^{(p)}, u^{(p)\text{ret}} \rangle; \\
c^2 \tau_p^{\text{ret}} - \langle \xi^{(p)\text{ret}}, u^{(p)\text{ret}} \rangle &= c^2 \tau - \tau \langle u^{(p)}(t), u^{(p)}(t-\tau) \rangle \approx \tau \left(c^2 - \langle u^{(p)}(t), u^{(p)}(t) \rangle \right) = \tau \Delta_p^2; \\
c^2 \tau_p^{\text{adv}} - \langle \xi^{(p)a}, u^{(p)a} \rangle &= c^2 \tau - \tau \langle u^{(p)}(t), u^{(p)}(t+\tau) \rangle \approx \tau \left(c^2 - \langle u^{(p)}(t), u^{(p)}(t) \rangle \right) = \tau \Delta_p^2.
\end{aligned}$$

Then

$$\begin{aligned}
D_{p,\text{ret}} &= \left(c^2 \tau_p^{\text{ret}} - \langle \xi^{(p)\text{ret}}, u^{(p)\text{ret}} \rangle \right) / \left(c^2 \tau_p^{\text{ret}} - \langle \xi^{(p)\text{ret}}, u^{(p)} \rangle \right) \approx \left(c^2 \tau - \tau \langle u^{(p)}, u^{(p)} \rangle \right) / \left(c^2 \tau - \tau \langle u^{(p)}, u^{(p)} \rangle \right) = 1; \\
M_{p,\text{ret}} &= \left(\Delta_p^2 + D_{p,\text{ret}} \langle \bar{\xi}^{(p)\text{ret}}, \dot{u}^{(p)\text{ret}} \rangle \right) / \left(c^2 \tau_p^{\text{ret}} - \langle \bar{\xi}^{(p)\text{ret}}, \bar{u}^{(p)\text{ret}} \rangle \right)^3 \approx \left(\Delta_p^2 + \tau^2 \langle u^{(p)}(t), \dot{u}^{(p)}(t) \rangle \right) / \tau^3 \Delta_p^6; \\
N_{p,\text{ret}} &= D_{p,\text{ret}} / \left(c^2 \tau_p^{\text{ret}} - \langle \bar{\xi}^{(p)\text{ret}}, \bar{u}^{(p)\text{ret}} \rangle \right)^2 = 1 / \tau^2 \Delta_p^4; \\
P_{\alpha}^{(p)\text{ret}} &= M_{p,\text{ret}} u_{\alpha}^{(p)\text{ret}} + N_{p,\text{ret}} \dot{u}_{\alpha}^{(p)\text{ret}} \approx \left[\left(\Delta_p^2 + \tau^2 \langle u^{(p)}, \dot{u}^{(p)} \rangle \right) u_{\alpha}^{(p)}(t-\tau) / (\tau^3 \Delta_p^6) \right] + \\
&+ \dot{u}_{\alpha}^{(p)}(t-\tau) / \tau^2 \Delta_p^4 \approx \left[\left(\Delta_p^2 + \tau^2 \langle u^{(p)}, \dot{u}^{(p)} \rangle \right) u_{\alpha}^{(p)}(t) / (\tau^3 \Delta_p^6) \right] + \dot{u}_{\alpha}^{(p)}(t-\tau) / \tau^2 \Delta_p^4; \\
P_4^{(p)\text{ret}} &= ic \left(\Delta_p^2 + \tau^2 \langle u^{(p)}, \dot{u}^{(p)}(t-\tau) \rangle \right) / (\tau^3 \Delta_p^6) \approx ic \left(\Delta_p^2 + \tau^2 \langle u^{(p)}(t), \dot{u}^{(p)}(t) \rangle \right) / (\tau^3 \Delta_p^6); \\
D_{p,\text{adv}} &= \left(c^2 \tau_p^{\text{adv}} - \langle \xi^{(p)\text{adv}}, u^{(p)\text{adv}} \rangle \right) / \left(c^2 \tau_p^{\text{adv}} - \langle \xi^{(p)\text{adv}}, u^{(p)} \rangle \right) \approx \left(c^2 \tau - \tau \langle u^{(p)}, u^{(p)} \rangle \right); \\
M_{p,\text{adv}} &= \left(\Delta_p^2 + D_{p,\text{adv}} \langle \bar{\xi}^{(p)\text{adv}}, \dot{u}^{(p)\text{adv}} \rangle \right) / \left(c^2 \tau_p^{\text{adv}} - \langle \bar{\xi}^{(p)\text{adv}}, \bar{u}^{(p)\text{adv}} \rangle \right)^3 \approx \\
&\approx \left(\Delta_p^2 + \tau^2 \langle u^{(p)}(t), \dot{u}^{(p)}(t) \rangle \right) / \tau^3 \Delta_p^6; \quad \left(c^2 \tau - \tau \langle u^{(p)}, u^{(p)} \rangle \right) = 1; \\
N_{p,\text{adv}} &= D_{p,\text{adv}} / \left(c^2 \tau_p^{\text{adv}} - \langle \bar{\xi}^{(p)\text{adv}}, \bar{u}^{(p)\text{adv}} \rangle \right)^2 = 1 / \tau^2 \Delta_p^4; \\
P_{\alpha}^{(p)\text{adv}} &= M_{p,\text{adv}} u_{\alpha}^{(p)\text{adv}} + N_{p,\text{adv}} \dot{u}_{\alpha}^{(p)\text{adv}} \approx \\
&\approx \left[u_{\alpha}^{(p)}(t+\tau) \left(\Delta_p^2 + \tau^2 \langle u^{(p)}, \dot{u}^{(p)} \rangle \right) / (\tau^3 \Delta_p^6) \right] + \left[\dot{u}_{\alpha}^{(p)}(t+\tau) / \tau^2 \Delta_p^4 \right] \approx \\
&\approx \left[u_{\alpha}^{(p)}(t) \left(\Delta_p^2 + \tau^2 \langle u^{(p)}, \dot{u}^{(p)} \rangle \right) / (\tau^3 \Delta_p^6) \right] + \left[\dot{u}_{\alpha}^{(p)}(t+\tau) / \tau^2 \Delta_p^4 \right]; \\
P_4^{(p)\text{adv}} &= ic \left(\Delta_p^2 + \tau^2 \langle u^{(p)}, \dot{u}^{(p)}(t+\tau) \rangle \right) / \tau^3 \Delta_p^6 \approx ic \left(\Delta_p^2 + \tau^2 \langle u^{(p)}(t), \dot{u}^{(p)}(t) \rangle \right) / \tau^3 \Delta_p^6; \\
P_{\alpha}^{(p)\text{ret}} - P_{\alpha}^{(p)\text{adv}} &= \left(\dot{u}_{\alpha}^{(p)}(t-\tau) - \dot{u}_{\alpha}^{(p)}(t+\tau) \right) / \tau^2 \Delta_p^4; \\
P_4^{(p)\text{ret}} - P_4^{(p)\text{adv}} &= \left[ic \left(\Delta_p^2 + \tau^2 \langle u^{(p)}(t), \dot{u}^{(p)}(t) \rangle \right) / (\tau^3 \Delta_p^6) \right] - \left[ic \left(\Delta_p^2 + \tau^2 \langle u^{(p)}(t), \dot{u}^{(p)}(t) \rangle \right) / (\tau^3 \Delta_p^6) \right] = 0.
\end{aligned}$$

Then we transform the radiation part of (5) with accordance of assumptions from [7]:

$$\begin{aligned}
 F_{\alpha, \text{rad}}^{(p)}(t, \sigma_1^{(p)}, \sigma_2^{(p)}, \sigma_3^{(p)}) &= (Q_p / (2c^2)) \left[\sum_{\gamma=1}^3 \left(P_{\alpha}^{p, \text{ret}} \xi_{\gamma}^{(p) \text{ret}} - P_{\alpha}^{p, \text{adv}} \xi_{\gamma}^{(p) \text{adv}} - \right. \right. \\
 &\quad \left. \left. - \xi_{\alpha}^{(p) \text{ret}} P_{\gamma}^{p, \text{ret}} + \xi_{\alpha}^{(p) \text{adv}} P_{\gamma}^{p, \text{adv}} \right) \sigma_{\gamma}^{(p)} + \sigma_{\alpha}^{(p)} \sum_{\gamma=1}^3 \lambda_{\gamma}^{(p)} \left[\tau_p^{\text{ret}} P_{\gamma}^{p, \text{ret}} - \tau_p^{\text{adv}} P_{\gamma}^{p, \text{adv}} - \right. \right. \\
 &\quad \left. \left. - i \left(P_4^{p, \text{ret}} \xi_{\gamma}^{(p) \text{ret}} - P_4^{p, \text{adv}} \xi_{\gamma}^{(p) \text{adv}} \right) / c \right] + \lambda_{\alpha}^{(p)} \sum_{\gamma=1}^3 \left[(i / c) \left(P_4^{p, \text{ret}} \xi_{\gamma}^{(p) \text{ret}} - P_4^{p, \text{adv}} \xi_{\gamma}^{(p) \text{adv}} \right) - \right. \right. \\
 &\quad \left. \left. - \tau_p^{\text{ret}} P_{\gamma}^{p, \text{ret}} + \tau_p^{\text{adv}} P_{\gamma}^{p, \text{adv}} \right] \sigma_{\gamma}^{(p)} \right] = (\tau Q_p / 2c^2) \left[\sum_{\gamma=1}^3 u_{\gamma}^{(p)} \sigma_{\gamma}^{(p)} \left(P_{\alpha}^{p, \text{ret}} - P_{\alpha}^{p, \text{adv}} - P_{\gamma}^{p, \text{ret}} + P_{\gamma}^{p, \text{adv}} \right) + \right. \\
 &\quad \left. + \tau \sigma_{\alpha}^{(p)} \sum_{\gamma=1}^3 \left(P_{\gamma}^{p, \text{ret}} - P_{\gamma}^{p, \text{adv}} - (i u_{\gamma}^{(p)} / c) (P_4^{p, \text{ret}} - P_4^{p, \text{adv}}) \right) \lambda_{\gamma}^{(p)} + \right. \\
 &\quad \left. + \tau \lambda_{\alpha}^{(p)} \sum_{\gamma=1}^3 \left((i u_{\gamma}^{(p)} / c) (P_4^{p, \text{ret}} - P_4^{p, \text{adv}}) - (P_{\gamma}^{p, \text{ret}} - P_{\gamma}^{p, \text{adv}}) \right) \sigma_{\gamma}^{(p)} \right] = \\
 &= (Q_p / 2c^2) \left[\sum_{\gamma=1}^3 (\sigma_{\gamma}^{(p)} u_{\gamma}^{(p)} \tau / \tau^2 \Delta_p^4) \times \left((\dot{u}_{\alpha}^{(p)}(t - \tau) - \dot{u}_{\alpha}^{(p)}(t + \tau)) - (\dot{u}_{\gamma}^{(p)}(t - \tau) - \dot{u}_{\gamma}^{(p)}(t + \tau)) \right) + \right. \\
 &= (Q_p / 2\tau c^2 \Delta_p^4) \left[\sum_{\gamma=1}^3 \left(\dot{u}_{\alpha}^{(p)}(t - \tau) - \dot{u}_{\alpha}^{(p)}(t + \tau) - \dot{u}_{\gamma}^{(p)}(t - \tau) + \dot{u}_{\gamma}^{(p)}(t + \tau) \right) u_{\gamma}^{(p)} \sigma_{\gamma}^{(p)} + \right. \\
 &\quad \left. + (\sigma_{\alpha}^{(p)} / \tau) \sum_{\gamma=1}^3 \lambda_{\gamma}^{(p)} (\dot{u}_{\gamma}^{(p)}(t - \tau) - \dot{u}_{\gamma}^{(p)}(t + \tau)) - (\lambda_{\alpha}^{(p)} / \tau) \sum_{\gamma=1}^3 \sigma_{\gamma}^{(p)} (\dot{u}_{\gamma}^{(p)}(t - \tau) - \dot{u}_{\gamma}^{(p)}(t + \tau)) \right].
 \end{aligned}$$

Consequently in view of

$$\dot{u}_{\alpha}^{(p)}(t - \tau) - \dot{u}_{\alpha}^{(p)}(t + \tau) \approx -2\tau \ddot{u}_{\alpha}^{(p)}(t)$$

we obtain

$$\left| (\dot{u}_{\alpha}^{(p)}(s - \tau) - \dot{u}_{\alpha}^{(p)}(s + \tau)) / \tau \right| \leq 2 \left| \ddot{u}_{\alpha}^{(p)}(s) \right| \leq 2\omega^2 U_0 e^{\mu s} \leq 2\omega^2 U_0 e^{\mu T_0} \leq 2\omega^2 c.$$

Then

$$\begin{aligned}
 &\left| H_{\alpha, \text{rad}}^{(p)}(\sigma_1^{(p)}, \sigma_2^{(p)}, \sigma_3^{(p)})(t) - H_{\alpha, \text{rad}}^{(p)}(\bar{\sigma}_1^{(p)}, \bar{\sigma}_2^{(p)}, \bar{\sigma}_3^{(p)})(t) \right| \leq \\
 &\leq \int_0^t \left| F_{\alpha, \text{rad}}^{(p)}(\sigma_1^{(p)}, \sigma_2^{(p)}, \sigma_3^{(p)})(s) - F_{\alpha, \text{rad}}^{(p)}(\bar{\sigma}_1^{(p)}, \bar{\sigma}_2^{(p)}, \bar{\sigma}_3^{(p)})(s) \right| ds \leq \\
 &\leq \left(Q_p / 2\tau c^2 \sqrt{(1 - \beta^2)^4} \right) \left[\sum_{\gamma=1}^3 \int_0^t \left| u_{\gamma}^{(p)} \right| \left| \sigma_{\gamma}^{(p)}(s) - \bar{\sigma}_{\gamma}^{(p)}(s) \right| \times \right. \\
 &\quad \times \left(\left| \dot{u}_{\alpha}^{(p)}(s - \tau) - \dot{u}_{\alpha}^{(p)}(s + \tau) \right| + \left| \dot{u}_{\gamma}^{(p)}(s - \tau) - \dot{u}_{\gamma}^{(p)}(s + \tau) \right| \right) ds + \\
 &\quad \left. + (2\omega^2 c / \tau) \sum_{\gamma=1}^3 \int_0^t \left| \dot{u}_{\gamma}^{(p)}(s - \tau) - \dot{u}_{\gamma}^{(p)}(s + \tau) \right| \left| \lambda_{\gamma}^{(p)} \right| \left| \sigma_{\alpha}^{(p)}(s) - \bar{\sigma}_{\alpha}^{(p)}(s) \right| ds + \right.
 \end{aligned}$$

$$\begin{aligned}
& + (1/\tau) \sum_{\gamma=1}^3 \int_0^t \left| \dot{u}_\gamma^{(p)}(s-\tau) - \dot{u}_\gamma^{(p)}(s+\tau) \right| \left| \lambda_\alpha^{(p)} \right| \left| \sigma_\gamma^{(p)}(s) - \bar{\sigma}_\gamma^{(p)}(s) \right| ds \Big] \leq \\
& \leq \left(6Q_p \omega^2 (c\sqrt{1-\beta^2} + 1) / c(1-\beta^2)^2 \sqrt{1-\beta^2} \right) \int_0^t e^{\mu s} ds \times \\
& \times \rho \left((\sigma_1^{(1)}, \sigma_2^{(1)}, \sigma_3^{(1)}, \sigma_1^{(2)}, \sigma_2^{(2)}, \sigma_3^{(2)}), (\bar{\sigma}_\gamma^{(1)}, \bar{\sigma}_\gamma^{(1)}, \bar{\sigma}_\gamma^{(1)}, \bar{\sigma}_\gamma^{(2)}, \bar{\sigma}_\gamma^{(2)}, \bar{\sigma}_\gamma^{(2)}) \right) \leq \\
& \leq 6Q_p \omega^2 e^{\mu t} \left(c + \sqrt{1-\beta^2} \right) / \left(\mu c (1-\beta^2)^2 \sqrt{1-\beta^2} \right) \times \\
& \times \rho \left((\sigma_1^{(1)}, \sigma_2^{(1)}, \sigma_3^{(1)}, \sigma_1^{(2)}, \sigma_2^{(2)}, \sigma_3^{(2)}), (\bar{\sigma}_\gamma^{(1)}, \bar{\sigma}_\gamma^{(1)}, \bar{\sigma}_\gamma^{(1)}, \bar{\sigma}_\gamma^{(2)}, \bar{\sigma}_\gamma^{(2)}, \bar{\sigma}_\gamma^{(2)}) \right).
\end{aligned}$$

Therefore

$$\begin{aligned}
& \left| H_\alpha^{(p)}(\sigma_1^{(p)}, \sigma_2^{(p)}, \sigma_3^{(p)})(t) - H_\alpha^{(p)}(\bar{\sigma}_1^{(p)}, \bar{\sigma}_2^{(p)}, \bar{\sigma}_3^{(p)})(t) \right| \leq \left| \sigma_\alpha^{(p)}(T_0) - \bar{\sigma}_\alpha^{(p)}(T_0) \right| + \\
& + \int_0^t \left| F_\alpha^{(p)}(\sigma_1^{(p)}, \sigma_2^{(p)}, \sigma_3^{(p)})(s) - F_\alpha^{(p)}(\bar{\sigma}_1^{(p)}, \bar{\sigma}_2^{(p)}, \bar{\sigma}_3^{(p)})(s) \right| ds \leq \\
& \leq e^{\mu t} \left[e^{\mu T_0} + 4 \left| Q_p \right| (c(1-\beta) + 2(1+\beta)\omega r_0) / (\mu c^3 r_0^2) \times \right. \\
& \times \left(2(1+c\sqrt{1-\beta^2}) / (1-\beta)^4 c\sqrt{1-\beta^2} \right) + \left. \left(6Q_p \omega^2 (1+c\sqrt{1-\beta^2}) / \mu c (1-\beta^2)^2 \sqrt{1-\beta^2} \right) \right] \times \\
& \times \rho \left((\sigma_1^{(1)}, \sigma_2^{(1)}, \sigma_3^{(1)}, \sigma_1^{(2)}, \sigma_2^{(2)}, \sigma_3^{(2)}), (\bar{\sigma}_\gamma^{(1)}, \bar{\sigma}_\gamma^{(1)}, \bar{\sigma}_\gamma^{(1)}, \bar{\sigma}_\gamma^{(2)}, \bar{\sigma}_\gamma^{(2)}, \bar{\sigma}_\gamma^{(2)}) \right).
\end{aligned}$$

It follows

$$\begin{aligned}
& \rho \left(H_\alpha^{(p)}(\sigma_1^{(p)}, \sigma_2^{(p)}, \sigma_3^{(p)}), H_\alpha^{(p)}(\bar{\sigma}_1^{(p)}, \bar{\sigma}_2^{(p)}, \bar{\sigma}_3^{(p)}) \right) \leq \left[e^{\mu T_0} + 4 \left| Q_p \right| (c(1-\beta) + 2(1+\beta)\omega r_0) / (\mu c^3 r_0^2) \times \right. \\
& \times \left(2(2+c\sqrt{1-\beta^2}) / (1-\beta)^4 c\sqrt{1-\beta^2} \right) + \left. \left(6Q_p \omega^2 (1+c\sqrt{1-\beta^2}) / \mu c (1-\beta^2)^2 \sqrt{1-\beta^2} \right) \right] \times \\
& \times \rho \left((\sigma_1^{(1)}, \sigma_2^{(1)}, \sigma_3^{(1)}, \sigma_1^{(2)}, \sigma_2^{(2)}, \sigma_3^{(2)}), (\bar{\sigma}_\gamma^{(1)}, \bar{\sigma}_\gamma^{(1)}, \bar{\sigma}_\gamma^{(1)}, \bar{\sigma}_\gamma^{(2)}, \bar{\sigma}_\gamma^{(2)}, \bar{\sigma}_\gamma^{(2)}) \right).
\end{aligned}$$

Therefore the operator H is continuous one.

6. Existence of a Periodic Solution of the General System

The main result of the present paper is:

Theorem 1. Let the following conditions be fulfilled:

$$U_0 e^{\mu T_0} \leq \bar{c} < c;$$

$$\begin{aligned}
& \left| \sigma_\alpha^{(p)}(T_0) \right| + \left(\left| Q_p \right| / \mu c^3 \right) \left[24(1-\beta + 2(1+\beta)\omega r_0) \times \left(\sqrt{1-\beta^2} + 2 \right) / (1-\beta)^4 r_0^2 \sqrt{1-\beta^2} + \right. \\
& \left. + 6\omega^2 \left(1+c\sqrt{1-\beta^2} \right) / c^2 (1-\beta^2)^{5/2} \right] S_0 \leq S_0.
\end{aligned}$$

Then there exists a periodic solution of (5).

Proof: Introduce the set $\left(C_{T_0}^B[0, T_0] \right)^6$ with a metric

$$\rho \left((\sigma_1^{(1)}, \sigma_2^{(1)}, \sigma_3^{(1)}, \sigma_1^{(2)}, \sigma_2^{(2)}, \sigma_3^{(2)}), (\bar{\sigma}_1^{(1)}, \bar{\sigma}_2^{(1)}, \bar{\sigma}_3^{(1)}, \bar{\sigma}_1^{(2)}, \bar{\sigma}_2^{(2)}, \bar{\sigma}_3^{(2)}) \right).$$

$$= \max \left\{ \rho(\sigma_1^{(1)}, \bar{\sigma}_1^{(1)}), \rho(\sigma_2^{(1)}, \bar{\sigma}_2^{(1)}), \rho(\sigma_3^{(1)}, \bar{\sigma}_3^{(1)}), \rho(\sigma_1^{(2)}, \bar{\sigma}_1^{(2)}), \rho(\sigma_2^{(2)}, \bar{\sigma}_2^{(2)}), \rho(\sigma_3^{(2)}, \bar{\sigma}_3^{(2)}) \right\};$$

$$\rho(\sigma_\alpha^{(p)}, \bar{\sigma}_\alpha^{(p)}) = \sup \left\{ \left| \sigma_\alpha^{(p)}(t) - \bar{\sigma}_\alpha^{(p)}(t) \right| e^{-\mu t} : t \in [0, T_0] \right\}; \quad (p = 1, 2; \alpha = 1, 2, 3)$$

where $C_{T_0}^B[0, T_0] = \left\{ \sigma \in C_{T_0}[0, T_0] : |\sigma(t)| \leq S_0 e^{\mu t} \right\}$, S_0 is a fixed constant.

Our first step is to show that the operator H maps $(C_{T_0}[0, T_0])^6$ into itself, that is,

$$H = (H_1^{(1)}, H_2^{(1)}, H_3^{(1)}, H_1^{(2)}, H_2^{(2)}, H_3^{(2)}) : (C_{T_0}[0, T_0])^6 \rightarrow (C_{T_0}[0, T_0])^6.$$

Indeed,

$$\begin{aligned} & \left| H_\alpha^{(p)}(\sigma_1^{(p)}, \sigma_2^{(p)}, \sigma_3^{(p)})(t) \right| \leq \left| \sigma_\alpha^{(p)}(T_0) \right| + \left| H_{\alpha, L}^{(p)}(\sigma_1^{(p)}, \sigma_2^{(p)}, \sigma_3^{(p)})(t) \right| + \left| H_{\alpha, \text{rad}}^{(p)}(\sigma_1^{(p)}, \sigma_2^{(p)}, \sigma_3^{(p)})(t) \right|; \\ & \left| H_{\alpha, L}^{(p)}(\sigma_1^{(p)}, \sigma_2^{(p)}, \sigma_3^{(p)})(t) \right| \leq \int_0^t \left| F_{\alpha, L}^{(p)}(\sigma_1^{(p)}, \sigma_2^{(p)}, \sigma_3^{(p)})(s) \right| ds \leq \\ & \leq \left(|Q_p| / c^2 \right) \left[\int_0^t \left| P_\alpha^{(pq)} \right| \sum_{\gamma=1}^3 \left| \xi_\gamma^{(pq)} \right| \left| \sigma_\gamma^{(p)}(s) \right| ds + \right. \\ & + \int_0^t \left| \xi_\alpha^{(pq)} \right| \sum_{\gamma=1}^3 \left| P_\gamma^{(pq)} \right| \left| \sigma_\gamma^{(p)}(s) \right| ds + \int_0^t \left| \sigma_\alpha^{(p)}(s) \right| \tau_{pq} \sum_{\gamma=1}^3 \left| P_\gamma^{(pq)} \right| \left| \lambda_\gamma^{(p)} \right| ds + \\ & + \int_0^t \left| \sigma_\alpha^{(p)}(s) \right| \left| i P_4^{(pq)} / c \right| \sum_{\gamma=1}^3 \left| \xi_\gamma^{(pq)} \right| \left| \lambda_\gamma^{(p)} \right| ds + \\ & + \int_0^t \left| \lambda_\alpha^{(p)} \right| \left| i P_4^{(pq)} / c \right| \sum_{\gamma=1}^3 \left| \xi_\gamma^{(pq)} \right| \left| \sigma_\gamma^{(p)}(s) \right| ds + \int_0^t \left| \lambda_\alpha^{(p)} \right| \tau_{pq} \sum_{\gamma=1}^3 \left| P_\gamma^{(pq)} \right| \left| \sigma_\gamma^{(p)}(s) \right| ds \Big] \leq \\ & \leq |Q_p| \left[\int_0^t \left((1 - \beta + 4c(1 + \beta)\omega\tau_{pq}) / (c^5\tau_{pq}^2(1 - \beta)^4) \right) \sum_{\gamma=1}^3 \left| \sigma_\gamma^{(p)}(s) \right| ds + \right. \\ & + \int_0^t \left((1 - \beta + 4c(1 + \beta)\omega\tau_{pq}) / (c^5\tau_{pq}^2(1 - \beta)^4) \right) \sum_{\gamma=1}^3 \left| \sigma(s) \right| ds + \\ & + \int_0^t \left| \sigma_\alpha^{(p)}(s) \right| \left((1 - \beta + 4c(1 + \beta)\omega\tau_{pq}) / (c^5\tau_{pq}^2(1 - \beta)^4) \right) \sum_{\gamma=1}^3 \left| \lambda_\gamma^{(p)} \right| ds + \\ & + \int_0^t \left| \sigma_\alpha^{(p)}(s) \right| \left((1 - \beta + 3c(1 + \beta)\omega\tau_{pq}) / (c^6\tau_{pq}^2(1 - \beta)^4) \right) \sum_{\gamma=1}^3 \left| \lambda_\gamma^{(p)} \right| ds + \\ & + \int_0^t \left| \lambda_\alpha^{(p)} \right| \left((1 - \beta + 3c(1 + \beta)\omega\tau_{pq}) / (c^6\tau_{pq}^2(1 - \beta)^4) \right) \sum_{\gamma=1}^3 \left| \sigma_\gamma^{(p)}(s) \right| ds + \\ & + \int_0^t \left| \lambda_\alpha^{(p)} \right| \left((1 - \beta + 4c(1 + \beta)\omega\tau_{pq}) / (c^6\tau_{pq}^2(1 - \beta)^4) \right) \sum_{\gamma=1}^3 \left| \sigma_\gamma^{(p)}(s) \right| ds \Big] \leq \\ & \leq 2|Q_p| \left[\int_0^t \left((1 - \beta + 4c(1 + \beta)\omega\tau_{pq}) / (c^5\tau_{pq}^2(1 - \beta)^4) \right) \sum_{\gamma=1}^3 \left| \sigma_\gamma^{(p)}(s) \right| ds + \right. \end{aligned}$$

$$\begin{aligned}
& + \int_0^t \left| \sigma_{\alpha}^{(p)}(s) \right| \left((1 - \beta + 4c(1 + \beta) \omega \tau_{pq}) / (c^6 \tau_{pq}^2 (1 - \beta)^4) \right) \sum_{\gamma=1}^3 \left| \lambda_{\gamma}^{(p)} \right| ds + \\
& + \int_0^t \left| \lambda_{\alpha}^{(p)} \right| \left((1 - \beta + 4c(1 + \beta) \omega \tau_{pq}) / (c^6 \tau_{pq}^2 (1 - \beta)^4) \right) \sum_{\gamma=1}^3 \left| \sigma_{\gamma}^{(p)}(s) \right| ds \Bigg] \leq \\
& \leq 8 \left| \mathcal{Q}_p \right| \left[\left((1 - \beta + 2c(1 + \beta) \omega r_0) / (c^3 r_0^2 (1 - \beta)^4) \right) \sum_{\gamma=1}^3 \int_0^t \left| \sigma_{\gamma}^{(p)}(s) \right| ds + \right. \\
& + 3 \left((1 - \beta + 2c(1 + \beta) \omega r_0) / (c^3 r_0^2 (1 - \beta)^4 \sqrt{1 - \beta^2}) \right) \int_0^t \left| \sigma_{\alpha}^{(p)}(s) \right| ds + \\
& + \left. \left((1 - \beta + 2c(1 + \beta) \omega r_0) / (c^3 r_0^2 (1 - \beta)^4 \sqrt{1 - \beta^2}) \right) \sum_{\gamma=1}^3 \int_0^t \left| \sigma_{\gamma}^{(p)}(s) \right| ds \right] \leq \\
& \leq 8 \left| \mathcal{Q}_p \right| \left((1 - \beta + 2c(1 + \beta) \omega r_0) / (c^3 r_0^2 (1 - \beta)^4) \right) \left[\sum_{\gamma=1}^3 \int_0^t \left| \sigma_{\gamma}^{(p)}(s) \right| ds + \right. \\
& + \left. \left(3 \int_0^t \left| \sigma_{\alpha}^{(p)}(s) \right| ds + \sum_{\gamma=1}^3 \int_0^t \left| \sigma_{\gamma}^{(p)}(s) \right| ds \right) / \sqrt{1 - \beta^2} \right] \leq 8 \left| \mathcal{Q}_p \right| \left((1 - \beta + 2c(1 + \beta) \omega r_0) / (c^3 r_0^2 (1 - \beta)^4) \right) \times \\
& \times 3 S_0 \left[\sqrt{1 - \beta^2} \int_0^t e^{\mu s} ds + \int_0^t e^{\mu s} ds + \int_0^t e^{\mu s} ds \right] / \sqrt{1 - \beta^2} \leq 8 \left| \mathcal{Q}_p \right| \left((1 - \beta + 2c(1 + \beta) \omega r_0) / (c^3 r_0^2 (1 - \beta)^4) \right) \times \\
& \times 3 S_0 (e^{\mu t} - 1) \left(2 + \sqrt{1 - \beta^2} \right) / \mu \sqrt{1 - \beta^2} \leq 24 \left| \mathcal{Q}_p \right| S_0 e^{\mu t} \left(\sqrt{1 - \beta^2} + 2 \right) \times \\
& \times \left((1 - \beta + 2c(1 + \beta) \omega r_0) / (\mu c^3 r_0^2 (1 - \beta)^4 \sqrt{1 - \beta^2}) \right); \\
& \left| H_{\alpha, \text{rad}}^{(p)}(\sigma_1^{(p)}, \sigma_2^{(p)}, \sigma_3^{(p)})(t) \right| \leq \int_0^t \left| F_{\alpha, \text{rad}}^{(p)}(\sigma_1^{(p)}, \sigma_2^{(p)}, \sigma_3^{(p)})(s) \right| ds \leq \\
& \leq \left(\left| \mathcal{Q}_p \right| / 2 \tau c^2 \right) \left| \int_0^t \left((\dot{u}_{\alpha}^{(p)}(s - \tau) - \dot{u}_{\alpha}^{(p)}(s + \tau)) \sum_{\gamma=1}^3 u_{\gamma}^{(p)} \left((\Delta_p \sigma_{\gamma}^{(p)} + \sigma_{\alpha}^{(p)}) / \Delta_p^5 \right) + \right. \right. \\
& + \left. \left. \left((\Delta_p + 1) u_{\alpha}^{(p)} / \Delta_p^5 \right) \sum_{\gamma=1}^3 (\dot{u}_{\gamma}^{(p)}(t - \tau) - \dot{u}_{\gamma}^{(p)}(t + \tau)) \sigma_{\gamma}^{(p)} \right) ds \right| \leq \\
& \leq (S_0 \left| \mathcal{Q}_p \right| / 2 \tau c^2) \left(\left(c + (1 - \beta^2)^{1/2} \right) / \left(c^4 (1 - \beta^2)^{5/2} \right) \right) \times \\
& \times \left| \int_0^t \left(3(\dot{u}_{\alpha}^{(p)}(s - \tau) - \dot{u}_{\alpha}^{(p)}(s + \tau)) + \sum_{\gamma=1}^3 (\dot{u}_{\gamma}^{(p)}(s - \tau) - \dot{u}_{\gamma}^{(p)}(s + \tau)) \right) e^{\mu s} ds \right| \leq \\
& \leq \left(\left| \mathcal{Q}_p \right| / (2 c^2) \right) \left(\left(c + (1 - \beta^2)^{1/2} \right) / \left(c^4 (1 - \beta^2)^{5/2} \right) \right) \left| \int_0^t (6 \ddot{u}_{\alpha}^{(p)}(s) + 6 \ddot{u}_{\alpha}^{(p)}(s)) e^{\mu s} ds \right| S_0 \leq
\end{aligned}$$

$$\begin{aligned}
&\leq \left(12U_0\omega^2 e^{\mu_0} S_0 |Q_p| / (2c^2)\right) \left(\left(c + (1-\beta^2)^{1/2} \right) / \left(c^4 (1-\beta^2)^{5/2} \right) \right) \int_0^t e^{\mu s} ds \leq \\
&\leq \left(12U_0\omega^2 e^{\mu_0} e^{\mu t} S_0 |Q_p| / (2\mu c^2)\right) \left(\left(c + (1-\beta^2)^{1/2} \right) / \left(c^4 (1-\beta^2)^{5/2} \right) \right) \leq \\
&\leq \left(6\omega^2 e^{\mu t} S_0 |Q_p| / (\mu c^2)\right) \left(\left(c + (1-\beta^2)^{1/2} \right) / \left(c^4 (1-\beta^2)^{5/2} \right) \right).
\end{aligned}$$

Therefore

$$\begin{aligned}
&\left| H_{\alpha}^{(p)}(\sigma_1^{(p)}, \sigma_2^{(p)}, \sigma_3^{(p)})(t) \right| \leq \left| \sigma_{\alpha}^{(p)}(T_0) \right| + \left| H_{\alpha,L}^{(p)}(\sigma_1^{(p)}, \sigma_2^{(p)}, \sigma_3^{(p)})(t) \right| + \left| H_{\alpha,\text{rad}}^{(p)}(\sigma_1^{(p)}, \sigma_2^{(p)}, \sigma_3^{(p)})(t) \right| \leq \\
&\leq \left| \sigma_{\alpha}^{(p)}(T_0) \right| e^{\mu t} + 24 |Q_p| S_0 e^{\mu t} (1-\beta + 2(1+\beta)\omega r_0) \times \left(2 + (1-\beta^2)^{1/2} \right) / \left(\mu c^3 r_0^2 (1-\beta)^4 (1-\beta^2)^{1/2} \right) + \\
&+ 6\omega^2 S_0 e^{\mu t} |Q_p| \left(1 + c(1-\beta^2)^{1/2} \right) / \mu c^5 (1-\beta^2)^{5/2} \leq e^{\mu t} \left| \sigma_{\alpha}^{(p)}(T_0) \right| + 6 |Q_p| S_0 e^{\mu t} [4(1-\beta + 2(1+\beta)\omega r_0) \times \\
&\times \left(2 + (1-\beta^2)^{1/2} \right) / \left(r_0^2 (1-\beta)^4 (1-\beta^2)^{1/2} \right) + \omega^2 \left(1 + c(1-\beta^2)^{1/2} \right) / c^2 (1-\beta^2)^{5/2}] / \mu c^3 \leq S_0 e^{\mu t}.
\end{aligned}$$

Consequently the operator H maps $(C_{T_0}[0, T_0])^6$ into itself.

It remains to show that the set $H(M)$ is equicontinuous. Indeed, for $t_1, t_2 \in [0, T_0]$, $t_1 < t_2$, we have

$$\begin{aligned}
&\left| H_{\alpha}^{(p)}(\sigma_1^{(p)}, \sigma_2^{(p)}, \sigma_3^{(p)})(t_2) - H_{\alpha}^{(p)}(\sigma_1^{(p)}, \sigma_2^{(p)}, \sigma_3^{(p)})(t_1) \right| \leq \\
&\leq \left| \sigma_{\alpha}^{(p)}(T_0) - \sigma_{\alpha}^{(p)}(T_0) \right| + \left| H_{\alpha,L}^{(p)}(\sigma_1^{(p)}, \sigma_2^{(p)}, \sigma_3^{(p)})(t_2) - H_{\alpha,L}^{(p)}(\sigma_1^{(p)}, \sigma_2^{(p)}, \sigma_3^{(p)})(t_1) \right| \\
&+ \left| H_{\alpha,\text{rad}}^{(p)}(\sigma_1^{(p)}, \sigma_2^{(p)}, \sigma_3^{(p)})(t_2) - H_{\alpha,\text{rad}}^{(p)}(\sigma_1^{(p)}, \sigma_2^{(p)}, \sigma_3^{(p)})(t_1) \right| \leq \\
&\leq \int_{t_1}^{t_2} \left| F_{\alpha,L}^{(p)}(\sigma_1^{(p)}, \sigma_2^{(p)}, \sigma_3^{(p)})(s) \right| ds + \int_{t_1}^{t_2} \left| F_{\alpha,\text{rad}}^{(p)}(\sigma_1^{(p)}, \sigma_2^{(p)}, \sigma_3^{(p)})(s) \right| ds \leq \\
&\leq 8 |Q_p| (1-\beta + 2(1+\beta)\omega r_0) / c^3 r_0^2 (1-\beta)^4 \times \left[3S_0 (2 + (1-\beta^2)^{1/2}) / (1-\beta^2)^{1/2} \int_{t_1}^{t_2} e^{\mu s} ds \right] + \\
&+ 12S_0 U_0 \omega^2 e^{\mu_0} |Q_p| \int_{t_1}^{t_2} e^{\mu s} ds \left(1 + c(1-\beta^2)^{1/2} \right) / \left(2c^6 (1-\beta^2)^{5/2} \right) \leq \\
&\leq 48 |Q_p| S_0 \left[\left((1-\beta + 2(1+\beta)\omega r_0)(1 + c\sqrt{1-\beta^2}) / (r_0^2 (1-\beta)^4 \sqrt{1-\beta^2}) \right) + \right. \\
&\left. + \omega^2 \left(1 + c(1-\beta^2)^{1/2} \right) / c^2 (1-\beta^2)^{5/2} \right] (e^{\mu t_2} - e^{\mu t_1}) / \mu c^3.
\end{aligned}$$

Theorem is thus proved.

7. Numerical Confirmation of the Results Obtained for the Hydrogen Atom

Here we show that all assertions obtained concerning two-body problem completely confirm the experimental results for the hydrogen atom. Indeed, we recall that

$$|Q_1| = |e_1 e_2| / m_1 \approx 2,8 \cdot 10^{-8}; \quad |Q_2| = |e_1 e_2| / m_2 \approx 2,8 \cdot 10^{-8} / 1836, \quad c = 3 \cdot 10^8 \text{ m/sec, the radiation time is}$$

$\tau_0 = r_{\text{electron}} / c = 2,82.10^{-15} / 3.10^8 \approx 0,94.10^{-23} = 9,4.10^{-24}$ sec . Since the radius of first Bohr orbit is $r_0 = 0,53.10^{-10} m$ and its velocity is $v_0 = c / 137 = r_0 \omega_0$ ($1/137$ – Sommerfeld fine structure constant [9], [10]), then

$$c / 137 = r_0 \omega_0 \Rightarrow \omega_0 = c / 137 r_0 = 3.10^8 / 137.0,53.10^{-10} \approx 4.10^{16};$$

$$T_0 = 2\pi / \omega_0 = 2\pi.137 r_0 / c \approx 1,52.10^{-16}; \quad f_0 = 1 / T_0 \approx 1 / 1,52.10^{-16} = 0,66.10^{16} Hz;$$

$$\lambda_0 = c / f_0 = 3.10^8 T_0 = 3.10^8.1,52.10^{-16} \approx 4,56.10^{-8} m = 456.10^{-10} \text{ \AA}.$$

Our estimates require $\mu T_0 = \mu_0$ to be a constant. Here $\omega = \omega_0$ and we have to take $\mu > \omega_0$, for instance $\mu = 6.10^{16}$. Then $\mu T_0 = 6.10^{16}.1,52.10^{-16} = 9,12 = \mu_0$. Therefore $\omega_0 / \mu = 2 / 3$ and $\tau_0 \omega_0 = 9,4.10^{-24}.4.10^{16} = 3,76.10^{-7}$, that is condition $\tau_0 \omega_0 = 9,4.10^{-24}.4.10^{16} < 2$ is satisfied. Here $\beta = 1 / 137 \Rightarrow 1 - \beta \approx 1$.

The following inequalities guarantee an existence of periodic solution of the equations of motion with radiation terms (obtained in [7]):

- 1) $(1, 2.10^{-5} + 6,1.10^{-8} + 2,9.10^{-25} + 7,7.10^{-13})(2/3)^{n-1} \leq 1;$
- 2) $(6.10^{-3} + 3.10^{-5} + 2.10^{-23} + 10^{-9})(2/3)^{n-1} \leq 1;$
- 3) $K \approx 2,96142.10^{-9} (2/3)^n < 1;$
- 4) $\dot{K} = 5,92230.10^{-9} (2/3)^n < 1.$

In the above inequalities n might be chosen arbitrarily large (cf. [7]).

We have to verify the inequality from the main theorem

$$\left| \sigma_{\alpha}^{(p)}(T_0) \right| + \left(6S_0 \left| Q_p \right| / \mu c^3 \right) \left[\omega^2 \left(1 + c\sqrt{1-\beta^2} \right) / c^2 \sqrt{(1-\beta^2)^5} + \right. \\ \left. + 4(1-\beta + 2(1+\beta)\omega r_0)(2 + \sqrt{1-\beta^2}) / r_0^2 (1-\beta)^4 \sqrt{1-\beta^2} \right] \leq S_0$$

or in view of $1 - \beta \approx 1$

$$\left| \sigma_{\alpha}^{(p)}(T_0) \right| + 12 \left| Q_p \right| \left[6 \left((1 + 2\omega_0 r_0) / r_0^2 \right) + \omega_0^2 / c \right] / \mu c^3 \leq 1 \Leftrightarrow \\ \left| \sigma_{\alpha}^{(p)}(T_0) \right| + 12 \left(2,8.10^{-8} / 6.10^{16} (3.10^8)^3 \right) \left[(4.10^{16})^2 / 3.10^8 + 6 \left(1 + 2.4.10^{16}.0,53.10^{-10} \right) / (0,53.10^{-10})^2 \right] \leq 1; \\ \left| \sigma_{\alpha}^{(p)}(T_0) \right| \leq 1 - (4,386.10^{-5} + 1,96) / 10^{23}.$$

This means that the initial values of the spin functions should satisfy the last inequality.

8. Conclusions

The present paper completes our investigations on the two-body problem of classical electrodynamics. Beginning with Sygne model we have corrected Dirac radiation term and extended Corben-Stehle spin equations. We have proved an existence of unique periodic solution of the equations of motion in [7]

$$d\vec{\lambda}^{(p)} / ds_p = (Q_p / c^2) \left[\vec{P}^{(pq)} \left(\left\langle \vec{\xi}^{(pq)}, \vec{\lambda}^{(p)} \right\rangle - c^2 \tau_{pq} / \Delta_p \right) - \left(\left\langle \vec{\lambda}^{(p)}, \vec{P}^{(pq)} \right\rangle + c^2 M_{pq} / \Delta_p \right) \vec{\xi}^{(pq)} \right] + \\ + (Q_p / 2c^2) \left[\vec{P}^{p,\text{ret}} \left(\left\langle \vec{\xi}^{(p)\text{ret}}, \vec{\lambda}^{(p)} \right\rangle - c^2 \tau_p^{\text{ret}} / \Delta_p \right) - \left(\left\langle \vec{\lambda}^{(p)}, \vec{P}^{p,\text{ret}} \right\rangle + c^2 M_{p,\text{ret}} / \Delta_p \right) \vec{\xi}^{(p)\text{ret}} \right] - \\ - (Q_p / 2c^2) \left[\vec{P}^{p,\text{adv}} \left(\left\langle \vec{\xi}^{(p)\text{adv}}, \vec{\lambda}^{(p)} \right\rangle - c^2 \tau_p^{\text{adv}} / \Delta_p \right) - \left(\left\langle \vec{\lambda}^{(p)}, \vec{P}^{p,\text{adv}} \right\rangle + c^2 M_{p,\text{adv}} / \Delta_p \right) \vec{\xi}^{(p)\text{adv}} \right]$$

and here an existence of a periodic solution of spin equations

$$\begin{aligned}
d\vec{\sigma}^{(p)} / ds_p = & (Q_p / 2c^2) \times \left\{ 2 \left[\vec{P}^{(pq)} \times (\vec{\sigma}^{(p)} \times \vec{\xi}^{(pq)}) + \tau_{pq} \vec{P}^{(pq)} \times (\vec{\sigma}^{(p)} \times \vec{\lambda}^{(p)}) - \right. \right. \\
& - \vec{\xi}^{(pq)} \times (\vec{\sigma}^{(p)} \times \vec{P}^{(pq)}) - (iP_4^{(pq)} / c) \vec{\xi}^{(pq)} \times (\vec{\sigma}^{(p)} \times \vec{\lambda}^{(p)}) \left. \right] + \\
& + \left[\vec{P}^{(p)\text{ret}} \times (\vec{\sigma}^{(p)} \times \vec{\xi}^{(p)\text{ret}}) + \tau_p^{\text{ret}} \vec{P}^{(p)\text{ret}} \times (\vec{\sigma}^{(p)} \times \vec{\lambda}^{(p)}) - \right. \\
& - \vec{\xi}^{(p)\text{ret}} \times (\vec{\sigma}^{(p)} \times \vec{P}^{(p)\text{ret}}) - (iP_4^{(p)\text{ret}} / c) \vec{\xi}^{(p)\text{ret}} \times (\vec{\sigma}^{(p)} \times \vec{\lambda}^{(p)}) \left. \right] - \\
& - \left[\vec{P}^{(p)\text{adv}} \times (\vec{\sigma}^{(p)} \times \vec{\xi}^{(p)\text{adv}}) + \tau_p^{\text{adv}} \vec{P}^{(p)\text{adv}} \times (\vec{\sigma}^{(p)} \times \vec{\lambda}^{(p)}) - \right. \\
& \left. \left. - \vec{\xi}^{(p)\text{adv}} \times (\vec{\sigma}^{(p)} \times \vec{P}^{(p)\text{adv}}) - (iP_4^{(p)\text{adv}} / c) \vec{\xi}^{(p)\text{adv}} \times (\vec{\sigma}^{(p)} \times \vec{\lambda}^{(p)}) \right] \right\}.
\end{aligned}$$

In this manner we have proved the stability of hydrogen atom and showed that stationary states introduced by N. Bohr are implied by classical electrodynamics. Introducing spin equations in the frame of relativistic Synge formalism we can investigate 3-body problem (and in general N-body) of classical electrodynamics.

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