

Magnetostatics and Gravity

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Abstract According to general relativity, the gravity is a property of the curvature of space-time. A curved 3-dimensional space is also considered to be equivalent to a gravitational field. On the basis of this assumption, a connection between the curvature of 3-dimensional space and stationary electric currents is found. Starting from this connection, a relation between the metric of 3-dimensional space and magnetic vector potential is established. Several applications are given.

Keywords Weyl vector, Weyl connection, Magnetic field, Stationary electric currents, Long straight wire carrying current, Helical motion

1. Introduction

According to GR, the gravity is a curvature of space-time. We consider that the same is true for the 3-dimensional space, so that, without gravity, the 3-dimensional space possesses Euclidean geometry, and the curvature tensor is identically zero. Also, according to GR, a measure of the curvature of space-time is given by Riemann-Christoffel curvature tensor. Again, we consider that the same is true for the 3-dimensional space. So, let us consider a 3-dimensional space, equipped with a Riemannian metric

$$ds^2 = g_{ij} dx^i dx^j \quad (i, j = 1, 2, 3) \quad (1)$$

that is conformal to the Euclidean metric

$$ds^2 = f^2 \delta_{ij} dx^i dx^j \quad (2)$$

where δ_{ij} is the Kronecker symbol

$$\delta_{ij} = \begin{cases} 1, & \text{if } i = j \\ 0, & \text{if } i \neq j \end{cases}$$

and $f = f(x^i)$ is a function of the Cartesian coordinates

$$x^1 = x, \quad x^2 = y, \quad x^3 = z$$

Therefore, as in [1] the metric tensor of such a metric is given by

$$g_{ij} = f^2 \delta_{ij} \quad (3)$$

2. The Field Equations

Starting from the metric of space (1), we can deduce the well-known RC curvature tensor, as in [2]

$$R_{jkl}^i = \partial_l \left\{ \begin{matrix} i \\ j \ k \end{matrix} \right\} - \partial_k \left\{ \begin{matrix} i \\ j \ l \end{matrix} \right\} + \left\{ \begin{matrix} i \\ s \ k \end{matrix} \right\} \left\{ \begin{matrix} s \\ j \ l \end{matrix} \right\} - \left\{ \begin{matrix} i \\ s \ l \end{matrix} \right\} \left\{ \begin{matrix} s \\ j \ k \end{matrix} \right\} \quad (4)$$

where the symbol ∂_k means partial derivative with respect to x^k , and the quantities in brackets are Christoffel symbols of the second kind, or what we call the Levi-Civita connection

$$\left\{ \begin{matrix} i \\ j \ k \end{matrix} \right\} = \frac{1}{2} g^{il} (\partial_k g_{jl} + \partial_j g_{kl} - \partial_l g_{jk}) \quad (5)$$

Substituting now (3) into (5), the Levi-Civita connection becomes the Weyl conformal connection

$$\left\{ \begin{matrix} i \\ j \ k \end{matrix} \right\} = \delta_j^i \psi_k + \delta_k^i \psi_j - \delta_{jk} \psi^i \quad (6)$$

where we introduced the notations

$$\psi_k = \partial_k \ln f \quad (7)$$

$$\psi^i = \delta^{ik} \psi_k \quad (7')$$

for the components of Weyl vector. From the RC curvature tensor (4), we can get the well-known contractions. First is the symmetric Ricci tensor

$$R_{jl} = R_{jil}^i = R_{lj} \quad (8)$$

and the other is the scalar curvature

$$R = g^{jl} R_{jl} = R_j^j \quad (9)$$

Therefore, we can build the well-known Einstein field equations, as in [3]

$$G_{jl} = -\frac{8\pi G}{c^4} T_{jl} \quad (10)$$

where we consider that T_{jl} is the energy density tensor of the gravitational field, and G_{jl} is the well-known Einstein tensor

$$G_{jl} = R_{jl} - \frac{1}{2} R g_{jl} \quad (10')$$

From the Bianchi identities,

$$\nabla_m R_{jkl}^i + \nabla_k R_{jlm}^i + \nabla_l R_{jmk}^i = 0 \quad (11)$$

where the symbol ∇_l represents the covariant derivative with respect to x^l , we deduce that the covariant divergence of the energy density tensor is identically zero

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$$\nabla_l T^{jl} = \partial_l T^{jl} + T^{kl} \left\{ \begin{matrix} j \\ k \ l \end{matrix} \right\} + T^{jk} \left\{ \begin{matrix} l \\ k \ l \end{matrix} \right\} = 0 \quad (12)$$

This means that we have ensured the conservation law of gravitational field energy. Now, inserting the Christoffel symbols (6) into (4), we get

$$R_{jkl}^i = \delta_j^i (\psi_{kl} - \psi_{lk}) + \delta_k^i \psi_{jl} - \delta_l^i \psi_{jk} + \delta_{jl} \psi_k^i - \delta_{jk} \psi_l^i \quad (13)$$

where we introduced the identities

$$\psi_{kl} = \partial_l \psi_k + \psi_k \psi_l - \frac{1}{2} \delta_{kl} \psi^2 \quad (14)$$

$$\psi^2 = \delta^{kl} \psi_k \psi_l = \psi^l \psi_l \quad (15)$$

$$\psi_l^i = \delta^{ij} \psi_{jl} \quad (16)$$

But, according to (7), we have

$$\psi_{kl} - \psi_{lk} = \partial_l \partial_k \ln f - \partial_k \partial_l \ln f = 0$$

Therefore, we find the following expressions for the Ricci tensor and scalar curvature

$$R_{jl} = (n-2) \psi_{jl} + \delta_{jl} \Omega \quad (17)$$

$$R = 2(n-1) f^{-2} \Omega \quad (18)$$

where we introduced the identities

$$\Omega = \delta^{kl} \psi_{kl} = \psi_i^i = \text{div } \vec{\psi} - \frac{1}{2} \psi^2 \quad (19)$$

$$\text{div } \vec{\psi} = \partial_i \psi_i = \partial^i \psi_i = \partial_i \psi^i \quad (20)$$

$$n = \delta_{jl} \delta^{jl} = \delta_i^i = 3 \quad (21)$$

$$\vec{\psi} = \nabla \ln f \quad (22)$$

Further, we try to find the physical significance of this Weyl vector field. Substituting the above identities into gravitational field equations (10), the Einstein tensor becomes

$$G_{jl} = \partial_j \psi_l + \psi_j \psi_l - \delta_{jl} \text{div } \vec{\psi} \quad (23)$$

Applying the partial derivative with respect to x^j , we get

$$\partial^j G_{jl} = \partial^j \partial_j \psi_l + \psi_j \partial^j \psi_l + \psi_l \partial^j \psi_j - \partial_l (\text{div } \vec{\psi}) \quad (24)$$

This Euclidean divergence of Einstein's tensor is no longer identically zero. If we introduce the notation

$$\partial^j \partial_j \psi_l = \delta^{ij} \partial_{ij}^2 \psi_l = \Delta \psi_l \quad (25)$$

after a rearrangement of terms, the Euclidean divergence of Einstein's tensor can be written as follows

$$\partial^j G_{jl} = \Delta \psi_l - \partial_l (\text{div } \vec{\psi}) + \psi_l \text{div } \vec{\psi} + \psi_j \partial^j \psi_l \quad (26)$$

We can now observe that the expression $\Delta \psi_l - \partial_l (\text{div } \vec{\psi})$ is the left-hand side of the differential equation which relates the magnetic vector potential to the electric current density, in stationary regime

$$\Delta \vec{A} - \nabla (\text{div } \vec{A}) = -\mu_0 \vec{J} \quad (27)$$

written in terms of components. Therefore, if we identify

$$\psi_k = \alpha A_k \quad (28)$$

where α is an arbitrary constant, the Weyl vector $\vec{\psi}$ becomes proportional to the magnetic potential vector \vec{A} . With this identification, the expression (26) can be written again as follows

$$\partial^j G_{jl} = -\alpha \mu_0 J_l + \alpha^2 (A_l \text{div } \vec{A} + A_j \partial^j A_l) \quad (29)$$

where J_l are the components of the electric current density vector \vec{J} .

3. The Metric Tensor

According to (22) and (28), we have the relation

$$\nabla \ln f = \alpha \vec{A} \quad (30)$$

Therefore, we can deduce

$$\ln f = \alpha \int \vec{A} d\vec{r} \quad (31)$$

or

$$f = e^{\alpha \int \vec{A} d\vec{r}} \quad (32)$$

Replacing this identity into (3), we obtain the metric tensor

$$g_{ij} = e^{2\alpha \int \vec{A} d\vec{r}} \delta_{ij} \quad (33)$$

4. Applications

In this section we calculate the magnetic vector potential and the well-known components of RC curvature tensor for different types of magnetic field.

I) THE UNIFORM MAGNETIC FIELD

In this application we calculate the vector potential and the components of RC curvature tensor, for a uniform magnetic field. We suppose that the magnetic induction is a vector parallel to the z axis, having a magnitude B_0

$$\vec{B} = B_0 \vec{k} \quad (34)$$

For the magnetic vector potential, as in [4], we can use the formula

$$\vec{A} = \frac{1}{2} (\vec{B} \times \vec{r}) \quad (35)$$

that yields the following expressions for its components

$$A_x = -\frac{1}{2} B_0 y \quad (36)$$

$$A_y = \frac{1}{2} B_0 x \quad (37)$$

Plugging these identities into (31), we get

$$\ln f = \frac{1}{2} \alpha B_0 (xy - yx) = 0 \quad (38)$$

Thus $f = 1$, and the metric tensor becomes

$$g_{ij} = \delta_{ij} \quad (39)$$

Also, the RC curvature tensor vanishes

$$R_{jkl}^i = 0, (\forall) i, j, k, l = 1, 2, 3 \quad (40)$$

II) THE MAGNETIC FIELD OF A LONG STRAIGHT WIRE CARRYING CURRENT

A straight wire by which the current I flows in the z direction produces, as in [5], the field

$$\vec{B} = \mu_0 \vec{H} = \frac{\mu_0 I}{2\pi\rho} \vec{e}_\varphi \quad (41)$$

where $\rho = \sqrt{x^2 + y^2}$ is the radial distance from the wire, and

$$\vec{e}_\varphi = \cos \varphi \vec{i} - \sin \varphi \vec{j} \quad (42)$$

is the unit vector directed along the circumferential direction. The magnetic vector potential can be chosen as

$$\vec{A} = A_x \vec{i} + A_y \vec{j} \quad (43)$$

where the two components have the expressions

$$A_x = \beta \frac{xz}{x^2 + y^2} \quad (44)$$

$$A_y = \beta \frac{yz}{x^2 + y^2} \quad (45)$$

and β is the constant

$$\beta = \frac{\mu_0 I}{2\pi} \quad (46)$$

Plugging these identities into (31), after integration, we get

$$\ln f = \gamma z \ln(x^2 + y^2) \quad (47)$$

where $\gamma = 2\alpha\beta$. From this we deduce

$$f = e^{\gamma z \ln(x^2 + y^2)} \quad (47')$$

We can now see that as ρ gets larger, so does $\ln(x^2 + y^2)$. So this solution is not acceptable. However let us calculate further. If we substitute (47') into (3), the metric tensor becomes

$$g_{ij} = e^{2\gamma z \ln(x^2 + y^2)} \delta_{ij} \quad (48)$$

From (7) and (47), we find the identities

$$\psi_1 = \psi_x = 2\gamma \frac{xz}{x^2 + y^2} \quad (49)$$

$$\psi_2 = \psi_y = 2\gamma \frac{yz}{x^2 + y^2} \quad (50)$$

$$\psi_3 = \psi_z = \gamma \ln(x^2 + y^2) \quad (51)$$

Substituting these identities into (15), we get

$$\psi^2 = 4\gamma^2 \frac{z^2}{x^2 + y^2} + \gamma^2 \ln^2(x^2 + y^2) \quad (52)$$

Substituting the identities (49) – (52) into (14), we find

$$\psi_{11} = 2\gamma z(\gamma z - 1) \frac{x^2 - y^2}{(x^2 + y^2)^2} - \frac{1}{2} \gamma^2 \ln^2(x^2 + y^2) \quad (53)$$

$$\psi_{22} = 2\gamma z(\gamma z - 1) \frac{y^2 - x^2}{(x^2 + y^2)^2} - \frac{1}{2} \gamma^2 \ln^2(x^2 + y^2) \quad (54)$$

$$\psi_{33} = \frac{1}{2} \gamma^2 \ln^2(x^2 + y^2) - 2\gamma^2 \frac{z^2}{x^2 + y^2} \quad (55)$$

$$\psi_{12} = 4\gamma(\gamma z - 1) \frac{xyz}{(x^2 + y^2)^2} \quad (56)$$

$$\psi_{13} = 2\gamma \frac{x}{x^2 + y^2} [\gamma z \ln(x^2 + y^2) + 1] \quad (57)$$

$$\psi_{23} = 2\gamma \frac{y}{x^2 + y^2} [\gamma z \ln(x^2 + y^2) + 1] \quad (58)$$

Plugging these identities into (13), we obtain

$$R_{212}^1 = \psi_{11} + \psi_{22} = -\gamma^2 \ln^2(x^2 + y^2) \quad (59)$$

$$R_{213}^1 = \psi_{23} = 2\gamma \frac{y}{x^2 + y^2} [\gamma z \ln(x^2 + y^2) + 1] \quad (60)$$

$$R_{232}^1 = \psi_{13} = 2\gamma \frac{x}{x^2 + y^2} [\gamma z \ln(x^2 + y^2) + 1] \quad (61)$$

$$R_{313}^1 = \psi_{11} + \psi_{33} = 2\gamma z(\gamma z - 1) \frac{x^2 - y^2}{(x^2 + y^2)^2} - 2\gamma^2 \frac{z^2}{x^2 + y^2} \quad (62)$$

$$R_{323}^1 = \psi_{12} = 4\gamma(\gamma z - 1) \frac{xyz}{(x^2 + y^2)^2} \quad (63)$$

$$R_{312}^1 = \psi_{23} = 2\gamma \frac{y}{x^2 + y^2} [\gamma z \ln(x^2 + y^2) + 1] \quad (64)$$

$$R_{321}^2 = \psi_{13} = 2\gamma \frac{x}{x^2 + y^2} [\gamma z \ln(x^2 + y^2) + 1] \quad (65)$$

$$R_{313}^2 = \psi_{12} = 4\gamma(\gamma z - 1) \frac{xyz}{(x^2 + y^2)^2} \quad (66)$$

$$R_{323}^2 = \psi_{22} + \psi_{33} = 2\gamma z(\gamma z - 1) \frac{y^2 - x^2}{(x^2 + y^2)^2} - 2\gamma^2 \frac{z^2}{x^2 + y^2} \quad (67)$$

III) THE MAGNETIC FIELD GENERATED BY A CHARGE PARTICLE (ELECTRON) THAT PERFORMS A HELICAL MOTION

For the study of this motion we assume that the electron is in rotation motion around the z axis, having the uniform angular velocity

$$\frac{d\varphi}{dt} = \omega \quad (68)$$

In the same time, the electron is in translation motion along the z direction with the uniform velocity

$$\frac{dz}{dt} = u \quad (69)$$

The parametric equations of this helical curve are

$$\begin{cases} x = \rho \cos \varphi \\ y = \rho \sin \varphi \\ z = p \varphi \end{cases} \quad (70)$$

where $\rho = \sqrt{x^2 + y^2}$ is the radial distance from the z axis, and p is the helical parameter

$$p = \frac{u}{\omega} = \frac{dz}{d\varphi} \quad (71)$$

The equation (69) tells us that the electron produces a current I_z flowing in the z direction. For this component of current, we have to take into account the magnetic vector potential of a straight wire (43)

$$\vec{A} = \beta z \frac{x\vec{i} + y\vec{j}}{x^2 + y^2} + \nabla \Lambda \quad (72)$$

where, since the vector potential is not unique, we have added the gradient of an arbitrary gauge function. But, the equation (68) tells us that we also have a current I_φ that flows in the sense of increasing φ . So, we have to take $z = p \varphi$ in the above expression of potential vector. Therefore, we obtain

$$\vec{A} = \sigma \varphi \frac{x\vec{i} + y\vec{j}}{x^2 + y^2} + \nabla \Lambda \quad (73)$$

or, according to (70),

$$\vec{A} = \sigma \frac{x\vec{i} + y\vec{j}}{x^2 + y^2} \arctan \frac{y}{x} + \nabla \Lambda \quad (74)$$

where $\sigma = \beta p$. The magnetic induction is obtained by taking the curl of this expression. Using the components

$$A_x = \sigma \frac{x}{x^2 + y^2} \arctan \frac{y}{x} + \partial_x \Lambda \quad (75)$$

$$A_y = \sigma \frac{y}{x^2+y^2} \arctan \frac{y}{x} + \partial_y \Lambda \quad (76)$$

$$A_z = \partial_z \Lambda \quad (77)$$

and the fact that $\nabla \times \nabla \Lambda = 0$, we find

$$\vec{B} = \nabla \times \vec{A} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \partial_x & \partial_y & \partial_z \\ A_x & A_y & A_z \end{vmatrix} = \vec{k}(\partial_x A_y - \partial_y A_x)$$

Therefore, the only non-zero component of \vec{B} is the z component

$$B_z = -\frac{\sigma}{x^2+y^2} \quad (78)$$

From the Stoke theorem, that relates the line integral of vector potential to the flux of magnetic induction

$$\iint \vec{B} d\vec{s} = \oint \vec{A} d\vec{l} \quad (79)$$

we can find the expression of gauge function Λ . Indeed, if we take a circular loop of radius ρ centered along the z axis, the flux through the circular area is, as in [4]

$$\iint \vec{B} d\vec{s} = \pi \rho^2 \vec{k} \vec{B} = -\sigma \pi \quad (80)$$

On the other hand, since the circuit element $d\vec{l}$ is given by

$$d\vec{l} = \rho d\varphi \vec{e}_\varphi = (x\vec{j} - y\vec{i}) d\varphi \quad (81)$$

We find that

$$\vec{A} d\vec{l} = d\vec{l} \nabla \Lambda \quad (82)$$

Therefore, the line integral of the potential vector may be expressed as

$$\oint \vec{A} d\vec{l} = 2\pi \rho \vec{e}_\varphi \nabla \Lambda = 2\pi \rho \nabla_\varphi \Lambda \quad (83)$$

Now, from (80) and (83), we find the gradient of gauge function Λ as follows

$$\nabla \Lambda = \vec{e}_\varphi \nabla_\varphi \Lambda = -\frac{\sigma}{2\rho} \vec{e}_\varphi \quad (84)$$

So, the gradient has a symmetric expression. If we use the spherical polar coordinates

$$\begin{cases} x = \rho \cos \varphi \\ y = \rho \sin \varphi \\ z = \rho \cotan \vartheta \end{cases} \quad (85)$$

where $\rho = r \sin \vartheta$, the magnetic potential vector \vec{A} becomes

$$\vec{A} = A_r \vec{e}_r + A_\vartheta \vec{e}_\vartheta + A_\varphi \vec{e}_\varphi \quad (86)$$

where the components are given by

$$A_r = \sigma \frac{\varphi}{r} \quad (87)$$

$$A_\vartheta = \sigma \frac{\varphi}{r} \cotan \vartheta \quad (88)$$

$$A_\varphi = -\frac{1}{2} \frac{\sigma}{r \sin \vartheta} \quad (89)$$

Finally, according to (84), we can write

$$\vec{i} \partial_x \Lambda + \vec{j} \partial_y \Lambda + \vec{k} \partial_z \Lambda = \frac{\sigma}{2} \frac{y}{x^2+y^2} \vec{i} - \frac{\sigma}{2} \frac{x}{x^2+y^2} \vec{j} \quad (90)$$

This equation gives us the expression of gauge function in Cartesian coordinates

$$\Lambda = \Lambda(x, y) = \frac{\sigma}{4} \left(\arctan \frac{x}{y} - \arctan \frac{y}{x} \right) \quad (91)$$

In spherical polar coordinates this function becomes

$$\Lambda = \Lambda(\varphi) = \frac{\sigma}{4} [\arctan(\cotan \varphi) - \varphi] \quad (92)$$

So, it is a function of φ only. If we substitute (86) into (31), after a simple integration, we obtain

$$\ln f = \tau \varphi \ln \rho - \frac{1}{2} \tau \varphi \quad (93)$$

where $\tau = \alpha \sigma$. This is a very similar situation to the previous application. As ρ gets larger, so does $\ln \rho$. So, instead of trying to calculate again the components of RC curvature tensor, let us try the partial solution

$$\ln f = \alpha \int d\vec{r} \nabla \Lambda \quad (94)$$

After a simple integration, we get

$$\ln f = \frac{\tau}{4} \left(\arctan \frac{x}{y} - \arctan \frac{y}{x} \right) = \alpha \Lambda(x, y) \quad (95)$$

According to (7), this expression leads us to the following identities

$$\psi_1 = \psi_x = \frac{\tau}{2} \frac{y}{x^2+y^2} \quad (96)$$

$$\psi_2 = \psi_y = -\frac{\tau}{2} \frac{x}{x^2+y^2} \quad (97)$$

$$\psi_3 = \psi_z = 0 \quad (98)$$

Substituting these identities into (15), we find

$$\psi^2 = \psi_x^2 + \psi_y^2 = \frac{\tau^2}{4} \frac{1}{x^2+y^2} \quad (99)$$

Substituting the identities (95) – (99) into (14), we obtain

$$\psi_{11} = \frac{\tau^2}{8} \frac{y^2-x^2}{(x^2+y^2)^2} - \tau \frac{xy}{(x^2+y^2)^2} \quad (100)$$

$$\psi_{22} = \frac{\tau^2}{8} \frac{x^2-y^2}{(x^2+y^2)^2} + \tau \frac{xy}{(x^2+y^2)^2} \quad (101)$$

$$\psi_{33} = -\frac{\tau^2}{8} \frac{1}{x^2+y^2} \quad (102)$$

$$\psi_{12} = \frac{\tau}{2} \frac{x^2-y^2}{(x^2+y^2)^2} - \frac{\tau^2}{4} \frac{xy}{(x^2+y^2)^2} \quad (103)$$

$$\psi_{13} = 0 \quad (104)$$

$$\psi_{23} = 0 \quad (105)$$

Therefore, the components of RC curvature tensor become

$$R_{212}^1 = \psi_{11} + \psi_{22} = 0 \quad (106)$$

$$R_{213}^1 = \psi_{23} = 0 \quad (107)$$

$$R_{232}^1 = \psi_{13} = 0 \quad (108)$$

$$R_{313}^1 = \psi_{11} + \psi_{33} = -\frac{\tau^2}{4} \frac{x^2}{(x^2+y^2)^2} - \tau \frac{xy}{(x^2+y^2)^2} \quad (109)$$

$$R_{323}^1 = \psi_{12} = \frac{\tau}{2} \frac{x^2-y^2}{(x^2+y^2)^2} - \frac{\tau^2}{4} \frac{xy}{(x^2+y^2)^2} \quad (110)$$

$$R_{312}^1 = \psi_{23} = 0 \quad (111)$$

$$R_{321}^2 = \psi_{13} = 0 \quad (112)$$

$$R_{313}^2 = \psi_{12} = \frac{\tau}{2} \frac{x^2-y^2}{(x^2+y^2)^2} - \frac{\tau^2}{4} \frac{xy}{(x^2+y^2)^2} \quad (113)$$

$$R_{323}^2 = \psi_{22} + \psi_{33} = -\frac{\tau^2}{4} \frac{y^2}{(x^2+y^2)^2} + \tau \frac{xy}{(x^2+y^2)^2} \quad (114)$$

5. Conclusions

From the relation (33) of the metric tensor

$$g_{ij} = e^{2\alpha \int \vec{A} d\vec{r}} \delta_{ij}$$

we can note that, if the magnetic vector potential is identically zero, then the metric tensor becomes $g_{ij} = \delta_{ij}$, the curvature tensor components R_{jkl}^i become identically zero, and the space becomes Euclidean. Thus, if there is no a magnetic field, then there is neither the curvature of space, nor gravity. Also, from the first application, we conclude that an uniform magnetic field does not produce a curvature of space, so does not produce gravitational field. From the next application, we can see that a variable magnetic field produces a curvature of space. But, from the identities (59) – (67), we can note that as ρ approaches infinity, the curvature tensor, especially the R_{212}^1 component, approaches infinity. This result is not in agreement to the given physical situation. From (41), we observe that when ρ gets to infinity, the intensity of magnetic field gets to zero. If a variable magnetic field produces a curvature of space, then the components of RC curvature tensor should approach zero when the intensity of magnetic field, \vec{H} , approaches zero. But, in our case, these components become infinite in the source and also to infinity. From the third application, we can note how can be removed this contradiction. Instead of the

whole expression (74) of the magnetic vector potential \vec{A} , we introduced in the formula (30) only the gradient of gauge function, $\nabla\Lambda$. In this way, we obtained appropriate solutions to the physical situation, because when ρ approaches infinity, all the components of RC curvature tensor approach zero. This result is in agreement to the given physical situation.

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