

# Addition of Velocities, Forces and Powers using Vector H-number Representation

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**Abstract** This paper analyses the addition of velocities, forces and powers acting on an ideal particle, and represents a continuation and extension of the H-number model presented in an earlier contribution [1]. A hypothetical model, using the fact that mass flow rate is a tangent function, will permit the computation of the Universe's mass.

**Keywords** Vector Hyper-complex numbers, Basic unit multipliers, Velocity, Force, Power, Geometrized system units, Big bang

## 1. Introduction

The calculus with hyper-complex numbers, or H-numbers, and their applications in physics were extensively presented in a paper [1], published in 2015. To be able to understand the present article the reader should first read it, especially the third chapter.

## 2. The V-H Numbers

### 2.1. Extension to H-numbers with Vector Parts

According to the article [1], an ideal particle is characterized by four parameters: time (t), mass (z), momentum (y) and space (x). This particle represents a point in the H space and can be written as an H number:

$$p = t + iz + jy + kx \quad (2.1)$$

The four parameters of the particle are expressed in meter, using the geometrized units System [2]. The symbols i, j and k are fundamental units of H-numbers.

**Table 1.** Units' Multiplication Table

x	1	i	j	k
1	1	i	j	k
i	i	-1	-k	j
j	j	-k	-1	i
k	k	j	i	1

If we consider that space and momentum are vectors, then the particle is represented by a Vector-Hyper-Complex number, or VH-number, which is formally written as:

$$p = t + iz + \bar{j}y + \bar{k}x \quad (2.2)$$

Where,  $t + iz$  represents the scalar part of this number, and  $\bar{j}y + \bar{k}x$  is the vector part. Its geometrical correspondence is a point in an eight-dimensional space.

### 2.2. Scalar and Vector Arguments

According to the paragraph 1.4 of the paper [1], an H-number can be exponentially represented by the following exponential expression:

$$p = \rho e^{i\alpha + j\beta + k\chi} \quad (2.3)$$

$\rho$  is norm of  $p$ ,  $\alpha$  is the imaginary argument,  $\beta$  co-imaginary argument, and  $\chi$  co-real argument. The notion of the unit multiplier is introduced in the paragraph 1.10 of the reference [1].

The initial 3 basic unit multipliers will be now extended for vector-arguments:

- The rotor with imaginary argument is by definition a pure scalar

$$e^{i\alpha} = \cos \alpha + i \sin \alpha$$

- The rotor with co-imaginary argument contains a vector part:

$$e^{j\beta} = \cos \beta + j \sin \beta$$

- The pseudo-rotor with co-real argument contains a vector part, too:

$$e^{k\chi} = \cosh \chi + k \sinh \chi$$

As you see we have introduced vector-arguments in the exponential expression from b and c.

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$$\begin{aligned}\bar{\beta} &= \beta \bar{u} \\ \bar{\chi} &= \chi \bar{u}\end{aligned}$$

where  $\beta$  and  $\chi$  are magnitudes and  $\bar{u}$  is a unit vector.

The imaginary argument  $\alpha$  is a scalar argument and the rotation  $e^{i\alpha} = \cos \alpha + i \sin \alpha$ , acts fully on any VH-number.

The rules for unit multipliers with vector-arguments are postulated as it follows: The rotation with co-imaginary vector-argument and the pseudo-rotation with co-real vector-argument acts on the scalar part of any arbitrary VH-number. However they act only on the vector part which is parallel with their unit vectors. The perpendicular component remains unmodified.

### 3. Transformations for Particles with Constant Rest Mass

The present contribution analyses coordinate transformations, produced by multiplication with arbitrary unit multipliers, from the proper frame to a new frame.

Further we will consider the particles with constant rest mass only, i.e.  $dz_0 = 0$ , where  $dz_0$  is the differential of the rest mass.

#### 3.1. Multiplication by a Pseudo-rotor. Velocities Addition Formula

A particle in its proper frame is expressed by:

$$p_0 = t_0 + iz_0, \quad (3.1)$$

where  $t_0$  and  $z_0$  are the proper time and respectively the proper mass. Now let us consider a differential mapping defined by the following expression:

$$\begin{aligned}dp &= dp_0 e^{k\bar{\chi}} = dt_0 e^{k\bar{\chi}} \\ &= dt_0 \cosh \chi + k\bar{u} dt_0 \sinh \chi = dt + k d\bar{x}\end{aligned} \quad (3.2)$$

The velocity of the now moving particle is:

$$\bar{v} = \frac{d\bar{x}}{dt} = \bar{u} \tanh \chi \quad (3.3)$$

Let us analyze the case of two consecutive pseudo-rotations with arguments  $\bar{\chi}_1$  and  $\bar{\chi}_2$ . After the first pseudo-rotation we get:

$$dp_1 = dt_0 \cosh \chi_1 + k\bar{u}_1 dt_0 \sinh \chi_1 \quad (3.4)$$

If the angle between the vector-arguments is  $\Phi$  then we may write the components of  $dp_1$ , which are parallel respectively perpendicular to  $\bar{u}_2$ , as it follows:

$$\begin{aligned}dp_{1parallel} &= dt_0 \cosh \chi_1 + k\bar{u}_2 dt_0 \cos \Phi \sinh \chi_1 \\ dp_{1perpendicular} &= k(\bar{u}_1 - \cos \Phi \bar{u}_2) \sinh \chi_1 dt_0\end{aligned} \quad (3.5)$$

For simplicity reason the scalar part of  $dp_1$  was included in

$dp_{1parallel}$ . The second pseudo-rotation acts only on the parallel component:

$$\begin{aligned}dp_{1parallel} e^{k\bar{\chi}_2} &= dt_0 [(\cosh \chi_1 \cosh \chi_2 + \cos \Phi \sinh \chi_1 \sinh \chi_2) \\ &\quad + k\bar{u}_2 (\cosh \chi_1 \sinh \chi_2 + \cos \Phi \cosh \chi_2 \sinh \chi_1)]\end{aligned}$$

Finally it obtains

$$\begin{aligned}dp_t &= dp_{1parallel} e^{k\bar{\chi}_2} + dp_{1perpendicular} = dt + k d\bar{x} \\ dt &= dt_0 (\cosh \chi_1 \cosh \chi_2 + \cos \Phi \sinh \chi_1 \sinh \chi_2) \\ d\bar{x} &= dt_0 \langle \bar{u}_2 [\cosh \chi_1 \sinh \chi_2 + \cos \Phi \sinh \chi_1 (\cosh \chi_2 - 1)] \\ &\quad + \bar{u}_1 \sinh \chi_1 \rangle\end{aligned}$$

The resulting velocity is:

$$\begin{aligned}\bar{v}_t &= \frac{d\bar{x}}{dt} \\ &= \frac{\bar{u}_1 \sinh \chi_1 + \bar{u}_2 [\cosh \chi_1 \sinh \chi_2 + \cos \Phi \sinh \chi_1 (\cosh \chi_2 - 1)]}{\cosh \chi_1 \cosh \chi_2 + \cos \Phi \sinh \chi_1 \sinh \chi_2}\end{aligned} \quad (3.6)$$

But the expressions of individual velocities can be written as:

$$\bar{v}_1 = \bar{u}_1 \tanh \chi_1 = \bar{u}_1 v_1, \text{ and } \bar{v}_2 = \bar{u}_2 \tanh \chi_2 = \bar{u}_2 v_2.$$

The final expression of the magnitude of the resulting velocity is:

$$|v_t| = \frac{(v_1^2 + v_2^2 + 2v_1 v_2 \cos \Phi - v_1^2 v_2^2 \sin^2 \Phi)^{\frac{1}{2}}}{1 + \cos \Phi v_1 v_2} \quad (3.7)$$

The above expression is symmetrical relative to  $v_1$  and  $v_2$ . The absolute value of the velocity's magnitude is less than or equal to 1 (light speed), as we already have expected.

#### 3.2. Multiplication by a Co-imaginary Rotor. Forces Addition Formula

We start again with a particle at rest. The following mapping will be used:

$$dp = dp_0 e^{j\bar{\beta}} = dt_0 (\cos \beta + j\bar{u} \sin \beta) = dt + j d\bar{y} \quad (3.8)$$

The force acting on the particle is:

$$\bar{F} = \frac{d\bar{y}}{dt} = \bar{u} \tan \beta \quad (3.9)$$

Now we will consider two successive rotations with co-imaginary arguments  $\bar{\beta}_1$  and  $\bar{\beta}_2$ . The mechanism of rotation acting on vector part follows the same rule as in the paragraph above.

$$\begin{aligned}dp_{1parallel} &= dt_0 \cos \beta_1 + j\bar{u}_2 dt_0 \cos \Phi \sin \beta_1 \\ dp_{1perpendicular} &= j(\bar{u}_1 - \cos \Phi \bar{u}_2) \sin \beta_1 dt_0\end{aligned} \quad (3.10)$$

After the second rotation it obtains:

$$\begin{aligned} dp_t &= dt + j d\bar{y} \\ dt &= dt_0 (\cos \beta_1 \cos \beta_2 - \cos \Phi \sin \beta_1 \sin \beta_2) \\ d\bar{y} &= dt_0 \left[ \bar{u}_2 [\cos \beta_1 \sin \beta_2 + \cos \Phi \sin \beta_1 (\cos \beta_2 - 1)] \right. \\ &\quad \left. + \bar{u}_1 \sin \beta_1 \right] \end{aligned} \quad (3.11)$$

And thus we obtain the resulting force:

$$\begin{aligned} \bar{F}_t &= \frac{d\bar{y}}{dt} = \\ &= \frac{\bar{u}_1 \sin \beta_1 + \bar{u}_2 [\cos \beta_1 \sin \beta_2 + \cos \Phi \sin \beta_1 (\cos \beta_2 - 1)]}{\cos \beta_1 \cos \beta_2 - \cos \Phi \sin \beta_1 \sin \beta_2} \end{aligned} \quad (3.12)$$

Writing the adding forces as:

$\bar{F}_1 = \bar{u}_1 \tan \beta_1$ , and  $\bar{F}_2 = \bar{u}_2 \tan \beta_2$ , it obtains the magnitude of the resulting force:

$$|F_t| = \frac{(F_1^2 + F_2^2 + 2F_1F_2 \cos \Phi - F_1^2 F_2^2 \sin^2 \Phi)^{\frac{1}{2}}}{|1 - \cos \Phi F_1 F_2|} \quad (3.13)$$

If the adding forces are parallel, then the resulting force is:

$$F_t = \tan(\beta_1 + \beta_2) = \frac{F_1 + F_2}{1 - F_1 F_2} \quad (3.14)$$

The conclusion of the analysis presented in this chapter is that forces are not adding linearly, as they use to do in classical mechanics and special relativity. However for practical cases this non-linearity is extremely small as we have mentioned in the previous paper [1] (see paragraph 2.10) published in 2015.

The addition of two or more finite forces can result in an infinite one, because forces are tangent-functions. The reciprocal is also valid, i.e. an infinite force can be split in a number of finite individual forces.

### 3.3. Multiplication by an Imaginary Rotor. Powers Addition Formula

This kind of multiplication will keep the number representing the particle, in the complex plane, but its rest frame suffers a rotation with the angle  $\alpha$ . The corresponding mapping is:

$$dp = dp_0 e^{i\alpha} = dt_0 e^{i\alpha} = dt + idz = dt_0 \cos \alpha + idt_0 \sin \alpha \quad (3.15)$$

The corresponding mass flow rate or power is:

$$P = \frac{dz}{dt} = \tan \alpha \quad (3.16)$$

For  $P=0$  we have the case of the constant mass  $dz=0$ . For  $P>0$  the mass is increasing, i.e. the system receives additional mass (energy) from the exterior. For  $P<0$  the

mass is decreasing, i.e. the system is losing mass (energy) to the environment. If we consider two successive rotations we came to the power addition formula:

$$P_t = \tan(\alpha_1 + \alpha_2) = \frac{P_1 + P_2}{1 - P_1 P_2} \quad (3.17)$$

Because the power is also a tangent-function the addition of powers follows the same rules as the addition of forces. The addition of two or more finite powers can result in an infinite one, and an infinite power can be split in a number of finite individual powers.

### 3.4. Mass of the Universe

Let's play a little around the power's addition rule and try to calculate the mass of the universe. We make the following assumptions:

- The big bang began 13.8 billions years ago, so the age of the universe is  $4.32 \times 10^{17}$  seconds [3].
- The source was a singularity which contained an infinite power and consequently it has no proper time flow.

$$\frac{dz}{dt} = \tan \frac{\pi}{2} = \pm \infty \quad \text{i.e. } dt = 0 \quad (3.18)$$

- At the initial moment the singularity's power made an abrupt transition from  $+\infty$  to  $-\infty$ . Then this negative infinite power split in two finite powers  $P_1=P_2=-1$ , i.e. the total emerging power is 2. The power is expressed in geometrized units.
- Since then the mass flows uniformly in all directions.

Let's calculate the total emerging power (in SI units) using the following table (see [1] Table 2): **The direct and reversed conversion international system of units (SI) to geometrized system of units (GU)**

	GU	SI	SI→GU	GU→SI
Length	m	m	1	1
Time	m	s	c	c <sup>-1</sup>
Velocity	dimensionless	ms <sup>-1</sup>	c <sup>-1</sup>	c
Mass	m	Kg	Gc <sup>-2</sup>	G <sup>-1</sup> c <sup>2</sup>
Momentum	m	Kgms <sup>-1</sup>	Gc <sup>-3</sup>	G <sup>-1</sup> c <sup>3</sup>
Force	dimensionless	N	Gc <sup>-4</sup>	G <sup>-1</sup> c <sup>4</sup>

Using the values [4] of G and c in SI, the emerging power becomes:

$$P_{\text{emerging}} = 2G^{-1}c^3 (\text{Kgs}^{-1}) = 8.09 \times 10^{35} \text{ kg} / \text{s}$$

Age of the universe ( $t_{\text{universe}}$ ) is  $4.32 \times 10^{17}$  seconds.

With all of these, the calculated mass of the universe is:

$$M_{\text{universe}} = P_{\text{emerging}} \times t_{\text{universe}} = 3.49 \times 10^{53} \text{ Kg}$$

It is obvious that this approach is only a hypothesis and the presented calculation a rough estimation. However, the above value belongs to the current estimated range which lies between  $10^{53}$  and  $10^{60}$  Kg, depending on different models and methods of calculation [5].

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