

The Generalized Planetary Equations Based Upon Riemannian Geometry and the Golden Metric Tensor

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Abstract In this paper we introduced super general geodesic equation and golden metric tensors. We derived linear velocity vector and linear acceleration vector using golden metric tensor in spherical polar coordinates. Coefficients of affine connection were evaluated based upon golden metric tensor. Based on the evaluated linear velocity vector and acceleration vector, we obtained super general geodesic planetary equation in terms of r, θ, ϕ and x^0 to obtain Riemannian acceleration due to gravity in terms of r, θ, ϕ and x^0 known as gravitational scalar potential that played an important role in dealing with planetary phenomenon. Also in this paper we derived generalized planetary equations based upon Riemannian geometry and the golden metric tensor. The planetary equation obtain in this paper contained Newton's planetary equation, Einstein's planetary equation and the added or contribution term to the existing planetary equations known as post Newton's planetary equation and post Einstein's planetary equation. Whose solution of these equations will be consider in the next edition of this paper.

Keywords Golden metric tensor, Geodesic equation, Coefficient of affine connection, Newton's planetary equation, Einstein's planetary equation

1. Introduction

The properties of geodesics differ from those of straight lines. For example, on a plane, parallel lines never meet, but this is not so far geodesics on the surface of the earth. For example, lines of longitude are parallel at the equator, but intersect at the poles. Analogously, the world lines of test particles in free fall are space time geodesics, the straightest possible lines in space time. But still there are crucial differences between them and the truly straight lines that can be traced out in the gravity-free space time of special relativity [1]. Einstein's equations are the centre piece of general relativity. They provide a precise formulation of the relationship between space time geometry and the properties of matter, using the language of mathematics. More concretely, they are formulated using the concepts of Riemannian geometry, in which the geometric properties of a space (or a space time) are described by a quantity called a metric. The metric encodes the information needed to compute the fundamental geometric notions of distance and angle in a curved space (or space time) [2]. The metric function and its rate of change from point to point can be

defining a geometrical quantity called the Riemann curvature tensor, which describes exactly how the space (or space time) is curved at each point. In general relativity, the metric and the Riemann curvature tensor are quantities define at each point in space time. It is based on the above argument explanation. Howusu introduced, by postulation, a second natural and satisfactorily generalization or extension of the Schwarzschild's metric tensor from the gravitational fields of all static homogeneous spherical distribution of mass to the gravitational fields of all spherical distributions of mass-named as the golden metric tensor for all gravitational fields in nature [3].

2. Golden Metric Tensor

In this paper we introduced golden metric tensor for all gravitational fields in nature as follows [4]. The covariant form of all golden metric tensor for all gravitational fields in nature as

$$g_{11} = \left\{ 1 + \frac{2}{c^2} f(r, \theta, \phi, x^0) \right\}^{-1} \quad (2.1)$$

$$g_{22} = r^2 \left\{ 1 + \frac{2}{c^2} f(r, \theta, \phi, x^0) \right\}^{-1} \quad (2.2)$$

$$g_{33} = r^2 \sin^2 \theta \left\{ 1 + \frac{2}{c^2} f(r, \theta, \phi, x^0) \right\}^{-1} \quad (2.3)$$

$$g_{00} = - \left\{ 1 + \frac{2}{c^2} f(r, \theta, \phi, x^0) \right\} \quad (2.4)$$

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$$g_{uv} = 0, \text{ otherwise} \tag{2.5}$$

Where f is the gravitational scalar potential of the space time, the golden metric tensor and also contains the following physical effects:

- Gravitational space contraction
- Gravitational time dilation
- Gravitational polar angle contraction
- Gravitational azimuthal angle contraction

While the contravariant form of golden metric tensor given as:

$$g^{11} = \left\{ 1 + \frac{2}{c^2} f(r, \theta, \phi, x^0) \right\} \tag{2.6}$$

$$g^{22} = \frac{1}{r^2} \left\{ 1 + \frac{2}{c^2} f(r, \theta, \phi, x^0) \right\} \tag{2.7}$$

$$g^{33} = \frac{1}{r^2 \sin^2 \theta} \left\{ 1 + \frac{2}{c^2} f(r, \theta, \phi, x^0) \right\} \tag{2.8}$$

$$g^{00} = - \left\{ 1 + \frac{2}{c^2} f(r, \theta, \phi, x^0) \right\}^{-1} \tag{2.9}$$

$$g^{uv} = 0, \text{ otherwise} \tag{2.10}$$

3. Formulation of the General Dynamical Laws of Gravitation

It is well known that all of Newton’s dynamical laws of gravitation are founded on the experimental physical facts available in his day. The instantaneous active mass m_A passive mass m_p and the inertial mass m_I of a particle of non-zero rest mass m_0 are given by:

$$m_A = m_p = m_I = m_0 \tag{3.1}$$

In all proper inertial reference frames and proper times.

This statement may be called Newton’s principle of mass. According to the classic scientific method introduced by G. Galileo (the father of mechanics) and Newton the natural laws of mechanics are determined by experimental physical facts. Therefore today’s experimental revisions of the definitions of inertial, passive and active masses of a particle on non-zero rest mass induce a corresponding revision of Newton’s dynamical laws of gravitation which are now formulated (Howusu, 1991).

Super General Planetary Equation

$$(m_I)_H = (m_p)_H = \left[1 - \frac{u^2}{c^2} \left(1 + \frac{2}{c^2} f \right)^{-1} \right]^{\frac{1}{2}} \cdot \left(1 + \frac{2}{c^2} f \right)^{\frac{1}{2}} m_0 \tag{3.2}$$

as given by equation (3.1) above

\underline{g}_H = Riemannian acceleration due to gravity

\underline{u}_H = Riemannian velocity vector

But force is defined as the rate of change of momentum

$$\frac{d}{d\tau} \left\{ \left[1 - \frac{u^2}{c^2} \left(1 + \frac{2}{c^2} f \right)^{-1} \right]^{\frac{1}{2}} \left(1 + \frac{2}{c^2} f \right)^{\frac{1}{2}} m_0 \underline{u}_H \right\} = \left[1 - \frac{u^2}{c^2} \left(1 + \frac{2}{c^2} f \right)^{-1} \right]^{\frac{1}{2}} \cdot \left(1 + \frac{2}{c^2} f \right)^{\frac{1}{2}} m_0 \underline{g}_H \tag{3.3}$$

Where $\left(1 + \frac{2}{c^2} f \right)$ is Riemannian factor.

Equivalently,

$$\frac{d}{d\tau} \{ (m_I)_H \cdot \underline{u}_H \} = (m_p)_H \underline{g}_H \tag{3.4}$$

$$(m_I)_H \underline{a}_H + \left\{ \frac{d}{d\tau} [(m_I)_H] \right\} \underline{u}_H = (m_p)_H \underline{g}_H \tag{3.5}$$

$$\underline{a}_H + \frac{1}{(m_I)_H} \left\{ \frac{d}{d\tau} [(m_I)_H] \right\} \underline{u}_H = \underline{g}_H \tag{3.6}$$

$\underline{a}_H \equiv$ general acceleration vector.

Equation (3.6) above referred to as super general geodesics equation of motion.

Velocity Tensor

$$\dot{x}^\mu = \{ \dot{x}^1, \dot{x}^2, \dot{x}^3, \dot{x}^0 \} \tag{3.7}$$

$$\begin{aligned}x^1 &= r, \quad x^2 = \theta, \quad x^3 = \phi, \quad x^0 = ict, \\ \dot{x}^\mu &= \{\dot{r}, \dot{\theta}, \dot{\phi}, c\dot{t}\}\end{aligned}\quad (3.8)$$

Velocity vector

$$\underline{u}_H = (\sqrt{g_{11}}\dot{x}^1, \sqrt{g_{22}}\dot{x}^2, \sqrt{g_{33}}\dot{x}^3, \sqrt{g_{00}}\dot{x}^0) \quad (3.9)$$

Based upon golden metric tensor

$$u_r = \sqrt{g_{11}}\dot{r} = \left(1 + \frac{2}{c^2}f\right)^{-\frac{1}{2}}\dot{r} \quad (3.10)$$

$$u_\theta = \sqrt{g_{22}}\dot{\theta} = r\left(1 + \frac{2}{c^2}f\right)^{-\frac{1}{2}}\dot{\theta} \quad (3.11)$$

$$u_\phi = \sqrt{g_{33}}\dot{\phi} = r\sin\theta\left(1 + \frac{2}{c^2}f\right)^{-\frac{1}{2}}\dot{\phi} \quad (3.12)$$

$$u_0 = \sqrt{g_{00}}\dot{x}^0 = \left(1 + \frac{2}{c^2}f\right)^{-\frac{1}{2}}ict \quad (3.13)$$

Where we have make use of Golden metric tensor in equations (2.1)-(2.4).

4. Theoretical Analysis

Acceleration tensors define by geodesics equation as:

$$a^\mu = \ddot{x}^\mu + \Gamma_{\alpha\beta}^\mu \dot{x}^\alpha \dot{x}^\beta \quad (4.1)$$

Where $\Gamma_{\alpha\beta}^\mu$ is defined as coefficient of affine connection gives as:

$$\Gamma_{\alpha\beta}^\mu = \frac{1}{2}g^{\mu\epsilon}(g_{\alpha\epsilon,\beta} + g_{\epsilon\beta,\alpha} - g_{\alpha\beta,\epsilon}) \quad (4.2)$$

Putting $\mu = 1$

$$a^1 = \ddot{r} + \Gamma_{11}^1(\dot{r})^2 + \Gamma_{22}^1(\dot{\theta})^2 + \Gamma_{33}^1(\dot{\phi})^2 \quad (4.3)$$

Putting $\mu = 2$

$$a^2 = \ddot{\theta} + 2\Gamma_{12}^2(\dot{r}\dot{\theta}) + \Gamma_{33}^2(\dot{\phi})^2 \quad (4.4)$$

Putting $\mu = 3$

$$a^3 = \ddot{\phi} + 2\Gamma_{13}^3(\dot{r}\dot{\phi}) + 2\Gamma_{23}^3(\dot{\theta}\dot{\phi}) \quad (4.5)$$

Putting $\mu = 0$

$$a^0 = c\ddot{t} + 2c\Gamma_{01}^0(\dot{t}\dot{r}) \quad (4.6)$$

Using

$$x^0 = ct$$

By employing equation (4.2) above, in our paper (I) coefficients of affine connection are evaluated given in equation [(30) – (58)] [9]

$$\Gamma_{00}^1 = \frac{1}{c^2}\left(1 + \frac{2}{c^2}f\right)f_{,1} \quad (4.7)$$

$$\Gamma_{01}^1 = \Gamma_{10}^1 = -\frac{1}{c^2}\left(1 + \frac{2}{c^2}f\right)^{-1}f_{,0} \quad (4.8)$$

$$\Gamma_{11}^1 = -\frac{1}{c^2}\left(1 + \frac{1}{c^2}f\right)^{-1}f_{,1} \quad (4.9)$$

$$\Gamma_{12}^1 = \Gamma_{21}^1 = -\frac{1}{c^2}\left(1 + \frac{2}{c^2}f\right)^{-1}f_{,2} \quad (4.10)$$

$$\Gamma_{13}^1 = \Gamma_{31}^1 = -\frac{1}{c^2}\left(1 + \frac{2}{c^2}f\right)^{-1}f_{,3} \quad (4.11)$$

$$\Gamma_{22}^1 = -\frac{r^2}{c^2}\left(1 + \frac{2}{c^2}f\right)^{-1}f_{,1} - r \quad (4.12)$$

$$\Gamma_{33}^1 = \frac{r^2\sin^2\theta}{c^2}\left(1 + \frac{2}{c^2}f\right)^{-1}f_{,1} - r\sin^2\theta \quad (4.13)$$

and

$$\Gamma_{00}^2 = \frac{1}{c^2 r^2} \left(1 + \frac{2}{c^2} f\right) f_{,2} \quad (4.14)$$

$$\Gamma_{02}^2 = \Gamma_{20}^2 = \frac{1}{c^2} \left(1 + \frac{2}{c^2} f\right)^{-1} f_{,0} \quad (4.15)$$

$$\Gamma_{11}^2 = \frac{1}{c^2 r^2} \left(1 + \frac{1}{c^2} f\right)^{-1} f_{,2} \quad (4.16)$$

$$\Gamma_{12}^2 = \Gamma_{21}^2 = \frac{1}{r} - \frac{1}{c^2} \left(1 + \frac{2}{c^2} f\right)^{-1} f_{,2} \quad (4.17)$$

$$\Gamma_{22}^2 = -\frac{1}{c^2} \left(1 + \frac{2}{c^2} f\right)^{-1} f_{,2} \quad (4.18)$$

$$\Gamma_{23}^2 = \Gamma_{32}^2 = -\frac{2}{c^2} \left(1 + \frac{2}{c^2} f\right)^{-1} f_{,3} \quad (4.19)$$

$$\Gamma_{33}^2 = \frac{\sin^2 \theta}{c^2} \left(1 + \frac{2}{c^2} f\right)^{-1} f_{,2} - \sin \theta \cos \theta \quad (4.20)$$

and

$$\Gamma_{00}^3 = \frac{1}{c^2 r^2 \sin^2 \theta} \left(1 + \frac{2}{c^2} f\right) f_{,3} \quad (4.21)$$

$$\Gamma_{03}^3 = \Gamma_{30}^3 = \frac{1}{c^2} \left(1 + \frac{2}{c^2} f\right)^{-1} f_{,3} \quad (4.22)$$

$$\Gamma_{11}^3 = \frac{1}{c^2 r^2 \sin^2 \theta} \left(1 + \frac{2}{c^2} f\right) f_{,3} \quad (4.23)$$

$$\Gamma_{13}^3 = \Gamma_{31}^3 = \frac{1}{r} - \frac{1}{c^2} \left(1 + \frac{2}{c^2} f\right)^{-1} f_{,1} \quad (4.24)$$

$$\Gamma_{22}^3 = \frac{1}{c^2 \sin^2 \theta} \left(1 + \frac{2}{c^2} f\right)^{-1} f_{,3} \quad (4.25)$$

$$\Gamma_{23}^3 = \Gamma_{32}^3 = \cot \theta - \frac{1}{c^2} \left(1 + \frac{2}{c^2} f\right)^{-1} f_{,2} \quad (4.26)$$

$$\Gamma_{33}^3 = -\frac{1}{c^2} \left(1 + \frac{2}{c^2} f\right)^{-1} f_{,3} \quad (4.27)$$

and

$$\Gamma_{00}^0 = \frac{1}{c^2} \left(1 + \frac{2}{c^2} f\right)^{-1} f_{,0} \quad (4.28)$$

$$\Gamma_{01}^0 = \Gamma_{10}^0 = \frac{1}{c^2} \left(1 + \frac{2}{c^2} f\right)^{-1} f_{,1} \quad (4.29)$$

$$\Gamma_{02}^0 = \Gamma_{20}^0 = \frac{1}{c^2} \left(1 + \frac{2}{c^2} f\right)^{-1} f_{,2} \quad (4.30)$$

$$\Gamma_{03}^0 = \Gamma_{30}^0 = \frac{1}{c^2} \left(1 + \frac{2}{c^2} f\right)^{-1} f_{,3} \quad (4.31)$$

$$\Gamma_{01}^0 = -\frac{1}{c^2} \left(1 + \frac{2}{c^2} f\right)^{-3} f_{,0} \quad (4.32)$$

$$\Gamma_{22}^0 = -\frac{r^2}{c^2} \left(1 + \frac{2}{c^2} f\right)^{-3} f_{,0} \quad (4.33)$$

$$\Gamma_{33}^0 = -\frac{r^2 \sin^2 \theta}{c^2} \left(1 + \frac{2}{c^2} f\right)^{-3} f_{,0} \quad (4.34)$$

$$\Gamma_{\alpha\beta}^\mu = 0; \quad \text{otherwise} \quad (4.35)$$

Hence the acceleration vector

$$\underline{a}_H(\sqrt{g_{11}}a^1, \sqrt{g_{22}}a^2, \sqrt{g_{33}}a^3, \sqrt{g_{00}}a^0) \quad (4.36)$$

By putting the results of coefficients of affine connection, covariant form of golden metric tensors into equation (4.15), we obtained acceleration vector equations as:

$$a_r = \left(1 + \frac{2}{c^2} f\right)^{-\frac{1}{2}} \left[\ddot{r} - \frac{1}{c^2} \left(1 + \frac{2}{c^2} f\right)^{-1} f_{,1} (\dot{r})^2 - r \left(1 + \frac{2}{c^2} f\right) (\dot{\theta})^2 - r \sin^2 \theta \left(1 + \frac{2}{c^2} f\right) (\dot{\phi})^2 \right] \quad (4.37)$$

$$a_\theta = r \left(1 + \frac{2}{c^2} f\right)^{-\frac{1}{2}} \left[\ddot{\theta} + \frac{2}{r} (\dot{r} \dot{\theta}) - \sin \theta \cos \theta (\dot{\phi})^2 \right] \quad (4.38)$$

$$a_\phi = r \sin\theta \left(1 + \frac{2}{c^2} f\right)^{-\frac{1}{2}} \left[\ddot{\phi} + \frac{2}{r} (\dot{r}\dot{\theta}) + 2 \cot\theta (\dot{\theta}\dot{\phi}) \right] \quad (4.39)$$

$$a_0 = \left(1 + \frac{2}{c^2} f\right) i \left[c\ddot{t} + \frac{2}{c} \left(1 + \frac{2}{c^2} f\right)^{-1} f_{,1} \right] \quad (4.40)$$

Hence equations (4.37), (4.38), (4.39) and (4.40), are linear acceleration vector equations based upon golden metric tensor.

5. Riemannian Acceleration Due to Gravity

Riemannian acceleration due to gravity in terms of r, θ, ϕ and x^0 is given by [10]

$$a_H + \frac{1}{(m_I)_H} \left\{ \frac{d}{d\tau} [(m_I)_H] \right\} u_H = g_H \quad (5.1)$$

Where a_H = Riemannian acceleration vector [6]

u_H = Riemannian velocity vector

g_H = Riemannian acceleration due to gravity

$(m_I)_H$ = Riemannian inertial mass

Using

$$u_r = \sqrt{g_{11}} \dot{r} = \left(1 + \frac{2}{c^2} f\right)^{-\frac{1}{2}} \dot{r}$$

and

$$a_r = a^1 \left(1 + \frac{2}{c^2} f\right)^{-\frac{1}{2}} \quad (5.2)$$

From equation (4.37) that is

$$a_r = \left(1 + \frac{2}{c^2} f\right)^{-\frac{1}{2}} \left[\ddot{r} - \frac{1}{c^2} \left(1 + \frac{2}{c^2} f\right)^{-1} f_{,1} (\dot{r})^2 - r \left(1 + \frac{2}{c^2} f\right) (\dot{\theta})^2 - r \sin^2\theta \left(1 + \frac{2}{c^2} f\right) (\dot{\phi})^2 \right]$$

Hence Riemannian radial acceleration due to gravity is given by

$$a_r + \frac{1}{(m_I)_r} \left\{ \frac{d}{d\tau} [(m_I)_r] \right\} u_r = g_r \quad (5.3)$$

Putting (4.37) into equation (5.3) we have

$$\left(1 + \frac{2}{c^2} f\right)^{-\frac{1}{2}} \left[\ddot{r} - \frac{1}{c^2} \left(1 + \frac{2}{c^2} f\right)^{-1} f_{,1} (\dot{r})^2 - r \left(1 + \frac{2}{c^2} f\right) (\dot{\theta})^2 - r \sin^2\theta \left(1 + \frac{2}{c^2} f\right) (\dot{\phi})^2 \right] + \frac{1}{(m_I)_r} \left\{ \frac{d}{d\tau} [(m_I)_r] \right\} \left(1 + \frac{2}{c^2} f\right)^{-\frac{1}{2}} \dot{r} = g_r \quad (5.4)$$

Riemannian θ – acceleration due to gravity is given by

$$a_\theta + \frac{1}{(m_I)_\theta} \left\{ \frac{d}{d\tau} [(m_I)_\theta] \right\} u_\theta = g_\theta \quad (5.5)$$

Putting equation (4.38) into equation (5.5) we get

$$\left(1 + \frac{2}{c^2} f\right)^{-\frac{1}{2}} r \left[\ddot{\theta} + \frac{2}{r} (\dot{r}\dot{\theta}) - \sin\theta \cos\theta (\dot{\phi})^2 \right] + \frac{1}{(m_I)_\theta} \left\{ \frac{d}{d\tau} [(m_I)_\theta] \right\} \left(1 + \frac{2}{c^2} f\right)^{-\frac{1}{2}} \dot{\theta} = g_\theta \quad (5.6)$$

Riemannian ϕ – acceleration due to gravity is given by

$$a_\phi + \frac{1}{(m_I)_\phi} \left\{ \frac{d}{d\tau} [(m_I)_\phi] \right\} u_\phi = g_\phi \quad (5.7)$$

Putting equation (4.39) into equation (5.7) we get

$$\left(1 + \frac{2}{c^2} f\right)^{-\frac{1}{2}} r \sin\theta \left[\ddot{\phi} + \frac{2}{r} (\dot{r}\dot{\theta}) + 2 \cot\theta (\dot{\theta}\dot{\phi}) \right] + \frac{1}{(m_I)_\phi} \left\{ \frac{d}{d\tau} [(m_I)_\phi] \right\} r \sin\theta \left(1 + \frac{2}{c^2} f\right)^{-\frac{1}{2}} \dot{\phi} = g_\phi \quad (5.8)$$

Riemannian x^0 – acceleration due to gravity is given by

$$a_0 + \frac{1}{(m_I)_0} \left\{ \frac{d}{d\tau} [(m_I)_0] \right\} u_0 = g_0 \quad (5.9)$$

Putting equation (4.40) into equation (5.9) we have

$$\left(1 + \frac{2}{c^2}f\right) i \left[c\ddot{t} + \frac{2}{c} \left(1 + \frac{2}{c^2}\right)^{-1} f_{,1} \right] + \frac{1}{(m_l)_0} \left\{ \frac{d}{d\tau} [(m_l)_0] \right\} \left(1 + \frac{2}{c^2}f\right)^{-\frac{1}{2}} ict = g_0 \quad (5.10)$$

Hence equation (5.4), (5.6), (5.8) and (5.10) are super general planetary equations for the components of r, θ, ϕ and x^0 . Using equation (3.2) above we get

$$(m_l)_H = (m_l)_P = \left[1 - \frac{u^2}{c^2} \left(1 + \frac{2}{c^2}f\right)^{-1} \right]^{\frac{1}{2}} \left(1 + \frac{2}{c^2}f\right)^{\frac{1}{2}} m_0 (m_l)_H = \left[1 - \frac{u^2}{c^2} \left(1 + \frac{2}{c^2}f\right)^{-1} \right]^{\frac{1}{2}} \left(1 + \frac{2}{c^2}f\right)^{\frac{1}{2}} m_0 \quad (5.11)$$

Putting equations (5.11) into the equation (5.4), (5.6), (5.8) and (5.10) we get,

$$\begin{aligned} & \left(1 + \frac{2}{c^2}\right)^{-\frac{1}{2}} \left[\ddot{r} - \frac{1}{c^2} \left(1 + \frac{2}{c^2}f\right)^{-1} f_{,1} (\dot{r})^2 - r \left(1 + \frac{2}{c^2}f\right) (\dot{\theta})^2 - r \left(1 + \frac{2}{c^2}f\right) \sin^2\theta \left(1 + \frac{2}{c^2}f\right) (\dot{\phi})^2 \right] \\ & + \left(1 + \frac{2}{c^2}\right)^{-\frac{1}{2}} \left[\frac{1}{\left[1 - \frac{u^2}{c^2} \left(1 + \frac{2}{c^2}f\right)^{-1}\right]^{\frac{1}{2}}} \left\{ \frac{d}{d\tau} \left[1 - \frac{u^2}{c^2} \left(1 + \frac{2}{c^2}f\right)^{-1}\right]^{\frac{1}{2}} \left(1 + \frac{2}{c^2}\right)^{-\frac{1}{2}} \right\} \dot{r} \right] \equiv g_r \end{aligned} \quad (5.12)$$

Equation (5.12) above give rise to super general radial gravitational intensity (or acceleration due to gravity) based upon the golden metric tensor.

Hence equation (5.6) becomes

$$\left(1 + \frac{2}{c^2}\right)^{-\frac{1}{2}} r \left[\ddot{\theta} + \frac{2}{r} (\dot{r}\dot{\theta}) - \sin\theta \cos\theta (\dot{\phi}) \right] + \left(1 + \frac{2}{c^2}\right)^{-\frac{1}{2}} \left[\frac{1}{\left[1 - \frac{u^2}{c^2} \left(1 + \frac{2}{c^2}f\right)^{-1}\right]^{\frac{1}{2}}} \left\{ \frac{d}{d\tau} \left[1 - \frac{u^2}{c^2} \left(1 + \frac{2}{c^2}f\right)^{-1}\right]^{\frac{1}{2}} \left(1 + \frac{2}{c^2}\right)^{-\frac{1}{2}} \right\} r\dot{\theta} \right] \equiv g_\theta \quad (5.13)$$

Equation (5.12) above give rise to general θ – gravitational intensity (or acceleration due to gravity) based upon the golden metric tensor.

Equation (5.8) becomes

$$\begin{aligned} & \left(1 + \frac{2}{c^2}\right)^{-\frac{1}{2}} r \sin\theta \left[\ddot{\phi} + \frac{2}{r} (\dot{r}\dot{\phi}) + 2 \cot\theta (\dot{\theta}\dot{\phi}) \right] \\ & + \left(1 + \frac{2}{c^2}\right)^{-\frac{1}{2}} \left[\frac{1}{\left[1 - \frac{u^2}{c^2} \left(1 + \frac{2}{c^2}f\right)^{-1}\right]^{\frac{1}{2}}} \left\{ \frac{d}{d\tau} \left[1 - \frac{u^2}{c^2} \left(1 + \frac{2}{c^2}f\right)^{-1}\right]^{\frac{1}{2}} \left(1 + \frac{2}{c^2}\right)^{-\frac{1}{2}} \right\} r \sin\theta \dot{\phi} \right] \equiv g_\phi \end{aligned} \quad (5.14)$$

Equation (5.16) above give rise to ϕ – gravitational intensity (or acceleration due to gravity) based upon the golden metric tensor.

Equation (5.10) reduced to

$$\left(1 + \frac{2}{c^2}f\right) i \left[c\ddot{t} + \frac{2}{c} \left(1 + \frac{2}{c^2}\right)^{-1} f_{,1} \right] + \left(1 + \frac{2}{c^2}\right)^{-\frac{1}{2}} \left[\frac{1}{\left[1 - \frac{u^2}{c^2} \left(1 + \frac{2}{c^2}f\right)^{-1}\right]^{\frac{1}{2}}} \left\{ \frac{d}{d\tau} \left[1 - \frac{u^2}{c^2} \left(1 + \frac{2}{c^2}f\right)^{-1}\right]^{\frac{1}{2}} \left(1 + \frac{2}{c^2}\right)^{-\frac{1}{2}} \right\} ict \right] \equiv g_0 \quad (5.15)$$

Equation (5.15) above give rise to x^0 –gravitational intensity (or acceleration due to gravity) based upon the golden metric tensor.

Hence equation (5.12), (5.13), (5.14) and (5.15) are referred to as general gravitational intensity (or acceleration due to gravity) based upon the golden metric tensor in spherical polar coordinates.

6. General Gravitational Intensity or Acceleration due to Gravity Based upon the Golden Metric Tensor in Spherical Polar Coordinates

$$g_r = -(\dot{x}^0)^2 \Gamma_{00}^1 \quad (6.1)$$

$$g_\theta = -(\dot{x}^0)^2 \Gamma_{00}^2 \quad (6.2)$$

$$g_\phi = -(\dot{x}^0)^2 \Gamma_{00}^3 \quad (6.3)$$

$$g_0 = -(\dot{x}^0)^2 \Gamma_{00}^0 \quad (6.4)$$

where $\Gamma_{00}^1, \Gamma_{00}^2, \Gamma_{00}^3, \Gamma_{00}^0$ are coefficient of affine connection evaluated using the formula

$$\Gamma_{\alpha\beta}^{\mu} = \frac{1}{2} g^{\mu\varepsilon} (g_{\alpha\varepsilon,\beta} + g_{\varepsilon\beta,\alpha} + g_{\alpha\beta,\varepsilon}) \quad (6.5)$$

Equations (6.1), (6.2), (6.3) and (6.4) reduce to

$$g_r = -(\dot{x}^0)^2 \frac{1}{c^2} \left(1 + \frac{1}{c^2} f\right) f_{,1} \quad (6.6)$$

$$g_{\theta} = -(\dot{x}^0)^2 \frac{1}{c^2 r^2} \left(1 + \frac{1}{c^2} f\right) f_{,2} \quad (6.7)$$

$$g_{\phi} = -(\dot{x}^0)^2 \frac{1}{c^2 r^2 \sin^2 \theta} \left(1 + \frac{1}{c^2} f\right) f_{,3} \quad (6.8)$$

$$g_0 = -(\dot{x}^0)^2 \frac{1}{c^2} \left(1 + \frac{1}{c^2} f\right)^{-1} f_{,0} \quad (6.9)$$

where

$$\begin{aligned} \dot{x}^0 &= ct \\ (\dot{x}^0)^2 &= c^2 \dot{t}^2 \end{aligned} \quad (6.10)$$

equating equation (5.15) with equation (6.6) we get

$$\begin{aligned} &\left(1 + \frac{2}{c^2}\right)^{-\frac{1}{2}} \left[\dot{r} - \frac{1}{c^2} \left(1 + \frac{2}{c^2} f\right)^{-1} f_{,1} (\dot{r})^2 - r \left(1 + \frac{2}{c^2} f\right) (\dot{\theta})^2 - r \left(1 + \frac{2}{c^2} f\right) \sin^2 \theta \left(1 + \frac{2}{c^2} f\right) (\dot{\phi})^2 \right] \\ &+ \left(1 + \frac{2}{c^2}\right)^{-\frac{1}{2}} \left[1 - \frac{u^2}{c^2} \left(1 + \frac{2}{c^2} f\right)^{-1} \right]^{\frac{1}{2}} \left\{ \frac{d}{d\tau} \left[1 - \frac{u^2}{c^2} \left(1 + \frac{2}{c^2} f\right)^{-1} \right]^{\frac{1}{2}} \left(1 + \frac{2}{c^2}\right)^{-\frac{1}{2}} \right\} \dot{r} \\ &= -\dot{t}^2 \frac{1}{c^2} \left(1 + \frac{2}{c^2} f\right) f_{,1} \end{aligned} \quad (6.11)$$

Equating equation (5.16) with equation (6.7) we get

$$\begin{aligned} &\left(1 + \frac{2}{c^2}\right)^{-\frac{1}{2}} r \left[\ddot{\theta} + \frac{2}{r} (\dot{r}\dot{\theta}) - \sin \theta \cos \theta (\dot{\phi}) \right] \\ &+ \left(1 + \frac{2}{c^2}\right)^{-\frac{1}{2}} \left[1 - \frac{u^2}{c^2} \left(1 + \frac{2}{c^2} f\right)^{-1} \right]^{\frac{1}{2}} \left\{ \frac{d}{d\tau} \left[1 - \frac{u^2}{c^2} \left(1 + \frac{2}{c^2} f\right)^{-1} \right]^{\frac{1}{2}} \left(1 + \frac{2}{c^2}\right)^{-\frac{1}{2}} \right\} r \dot{\theta} \\ &= -\dot{t}^2 \frac{1}{r^2} \left(1 + \frac{2}{c^2} f\right) f_{,2} \end{aligned} \quad (6.12)$$

equating equation (5.17) with equation (6.8) we get

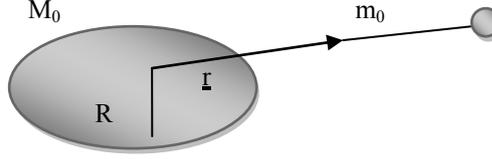
$$\begin{aligned} &\left(1 + \frac{2}{c^2}\right)^{-\frac{1}{2}} r \sin \theta \left[\ddot{\phi} + \frac{2}{r} (\dot{r}\dot{\phi}) + 2 \cot \theta (\dot{\theta}\dot{\phi}) \right] \\ &+ \left(1 + \frac{2}{c^2}\right)^{-\frac{1}{2}} \left[1 - \frac{u^2}{c^2} \left(1 + \frac{2}{c^2} f\right)^{-1} \right]^{\frac{1}{2}} \left\{ \frac{d}{d\tau} \left[1 - \frac{u^2}{c^2} \left(1 + \frac{2}{c^2} f\right)^{-1} \right]^{\frac{1}{2}} \left(1 + \frac{2}{c^2}\right)^{-\frac{1}{2}} \right\} r \sin \theta \dot{\phi} \\ &= -\dot{t}^2 \frac{1}{r^2 \sin^2 \theta} \left(1 + \frac{2}{c^2} f\right) f_{,3} \end{aligned} \quad (6.13)$$

equating equation (5.18) with equation (6.9) we get

$$\begin{aligned} &\left(1 + \frac{2}{c^2} f\right) i \left[c\ddot{t} + \frac{2}{c} \left(1 + \frac{2}{c^2} f\right)^{-1} f_{,1} \right] \\ &+ \left(1 + \frac{2}{c^2}\right)^{-\frac{1}{2}} \left[1 - \frac{u^2}{c^2} \left(1 + \frac{2}{c^2} f\right)^{-1} \right]^{\frac{1}{2}} \left\{ \frac{d}{d\tau} \left[1 - \frac{u^2}{c^2} \left(1 + \frac{2}{c^2} f\right)^{-1} \right]^{\frac{1}{2}} \left(1 + \frac{2}{c^2}\right)^{-\frac{1}{2}} \right\} i c \dot{t} \\ &= -\dot{t}^2 \left(1 + \frac{2}{c^2} f\right)^{-1} f_{,0} \end{aligned} \quad (6.14)$$

7. Application to Planetary Theory

Consider a planet of rest mass m_0 moving in the gravitational field exterior to the sun which is assumed to be a homogeneous spherical body of radius R and rest mass M_0 , shown in the diagram below



In the first place let us assume that the gravitational field is that of Newton:

$$f(r, \theta, \phi, x^0) = -\frac{GM_0}{r} = -\frac{K}{r} \quad (7.1)$$

Where $K = GM_0$

Where G is the universal gravitational constant.

$$f_{,1} = \frac{\partial}{\partial r} f = \frac{K}{r^2} \quad (7.2)$$

$$f_{,2} = 0 \quad (7.3)$$

$$f_{,3} = 0 \quad (7.4)$$

$$f_{,0} = 0 \quad (7.5)$$

$$u^2 = \dot{r}^2 + r^2\dot{\theta}^2 + r^2\sin^2\theta\dot{\phi}^2 \quad (7.6)$$

$$\frac{u^2}{c^2} \left(1 + \frac{2}{c^2} f\right)^{-1} = \frac{1}{c^2} (\dot{r}^2 + r^2\dot{\theta}^2 + r^2\sin^2\theta\dot{\phi}^2) \times \left(1 - \frac{2K}{c^2 r}\right)^{-1} \quad (7.7)$$

$$\dot{t} = \left[1 - \frac{u^2}{c^2} \left(1 + \frac{2}{c^2} f\right)^{-1}\right]^{\frac{1}{2}} \left(1 + \frac{2}{c^2} f\right)^{-\frac{1}{2}} \quad (7.8)$$

Then super general geodesic equation of motion become: hence equation (6.11) becomes

$$\left(1 + \frac{K}{c^2 r}\right) \left[\ddot{r} - \left(\frac{K}{c^2 r^2} + \frac{2K^2}{c^4 r^3}\right) (\dot{r})^2 - r \left(1 - \frac{2K}{c^2 r}\right) (\dot{\theta})^2 - r \left(1 - \frac{2K}{c^2 r}\right) \sin^2\theta (\dot{\phi})^2\right] + \left[\dot{r} \left\{\frac{d}{d\tau} \left[\frac{1}{\dot{t}}\right]\right\}\right] \frac{1}{c^2} = -\dot{t}^2 \left(\frac{K}{r^2} - \frac{2K^2}{c^2 r^3}\right) \quad (7.9)$$

or

$$\left(1 + \frac{K}{c^2 r}\right) \left[\ddot{r} - \left(\frac{K}{c^2 r^2}\right) (\dot{r})^2 - r \left(1 - \frac{2K}{c^2 r}\right) (\dot{\theta})^2 - r \left(1 - \frac{2K}{c^2 r}\right) \sin^2\theta (\dot{\phi})^2\right] - \frac{1}{c^2} [\dot{r} \dot{t}^{-1}] = -\dot{t}^2 \left(\frac{K}{r^2} - \frac{2K^2}{c^2 r^3}\right) \quad (7.10)$$

Equation (6.12) becomes

$$\left(1 - \frac{2K}{c^2 r}\right)^{\frac{1}{2}} r \left[\ddot{\theta} + \frac{2}{r} (\dot{r}\dot{\theta}) - \sin\theta \cos\theta (\dot{\phi})\right] + \left(1 - \frac{2K}{c^2 r}\right)^{\frac{1}{2}} \left[\dot{t} \left(1 - \frac{2K}{c^2 r}\right)^{\frac{1}{2}} \left\{\frac{d}{d\tau} \left[\frac{1}{\dot{t}}\right]\right\} \dot{r}\dot{\theta}\right] = -\dot{t}^2 \frac{1}{r^2} \left(1 - \frac{2K}{c^2 r}\right) f_{,2} \quad (7.11)$$

or

$$\left(1 - \frac{2K}{c^2 r}\right) r \left[\ddot{\theta} + \frac{2}{r} (\dot{r}\dot{\theta}) - \sin\theta \cos\theta (\dot{\phi})\right] - \left(\frac{\dot{r}\dot{\theta}}{\dot{t}}\right) \frac{1}{c^2} \equiv 0 \quad (7.12)$$

Equation (6.13) becomes

$$\left(1 - \frac{2K}{c^2 r}\right)^{\frac{1}{2}} r \sin\theta \left[\ddot{\phi} + \frac{2}{r} (\dot{r}\dot{\phi}) + 2 \cot\theta (\dot{\theta}\dot{\phi})\right] + \left(1 - \frac{2K}{c^2 r}\right)^{\frac{1}{2}} \left[\dot{t} \left(1 - \frac{2K}{c^2 r}\right)^{\frac{1}{2}} \left\{\frac{d}{d\tau} \left[\frac{1}{\dot{t}}\right]\right\} r \sin\theta \dot{\phi}\right] \times \frac{1}{c^2} = -\dot{t}^2 \frac{1}{r^2 \sin^2\theta} \left(1 - \frac{2K}{c^2 r}\right) f_{,2} \quad (7.13)$$

$$\left(1 + \frac{2K}{c^2 r}\right) r \sin\theta \left[\ddot{\phi} + \frac{2}{r} (\dot{r}\dot{\phi}) + 2 \cot\theta (\dot{\theta}\dot{\phi})\right] - \frac{1}{c^2} \left[\frac{\dot{r}\dot{\phi}}{\dot{t}}\right] r \sin\theta = 0 \quad (7.14)$$

Note that planetary equation do not contain t that is time parameter or in the case x^0 parameter, also the same equation that planetary equation contains no polar angle θ , but retained only azimuthal angle ϕ , radial portion of the equation also retained. Based on this argument equation (6.14) and equation (7.11) vanishes only equation (7.10) and equation (7.14) remain for further evaluations. Equation (7.10) reduce to

$$\left(1 + \frac{K}{c^2 r}\right) \left[\ddot{r} - \left(\frac{K}{c^2 r^2}\right) (\dot{r})^2 - r \left(1 - \frac{2K}{c^2 r}\right) (\dot{\theta})^2 - r \left(1 - \frac{2K}{c^2 r}\right) \sin^2\theta (\dot{\phi})^2\right] - \frac{1}{c^2} \left[\left(1 + \frac{u^2}{2c^2} - \frac{K}{c^2 r}\right) (\dot{r})\right] = -\frac{K}{r^2} + \frac{u^2 K}{c^2 r^2} \quad (7.15)$$

Equation (7.14) becomes

$$\left(1 + \frac{K}{c^2 r}\right) [r \sin \theta \ddot{\phi} + 2 \sin \theta (\dot{r} \dot{\phi}) + 2r \cot \theta \sin \theta (\dot{\theta} \dot{\phi})] - \left[\frac{1}{c^2} \left(1 + \frac{u^2}{2c^2} - \frac{K}{c^2 r}\right) r \sin \theta \dot{\phi}\right] = 0 \quad (7.16)$$

For motion along the equatorial plane

$$\begin{aligned} \theta &= \frac{\pi}{2} & \dot{\theta} &= 0 & \ddot{\theta} &= 0 & \sin \frac{\pi}{2} &= 1 \\ \ddot{r} &= -\frac{K}{r^2} + \frac{K^2}{c^2 r^3} + \frac{2K}{c^2 r^2} (\dot{r})^2 + \frac{1}{c^2} + \frac{l^2}{r^3} - \frac{3Kl^2}{c^2 r^4} \\ \dot{r} &= -Ku^2 + \frac{K^2 u^2}{c^2} + \frac{2Ku^2}{c^2} (\dot{r})^2 + \frac{1}{c^2} (\dot{r}) + l^2 u^3 - \frac{3Kl^2 u^2}{c^2} \end{aligned} \quad (7.17)$$

Note that

$$u = \frac{1}{r}$$

For motion along the equatorial plane

$$\theta = \frac{\pi}{2} \quad \dot{\theta} = 0 \quad \ddot{\theta} = 0 \quad \sin \frac{\pi}{2} = 1$$

Equation (7.16) become

$$\ddot{\phi} + \frac{2}{r} (\dot{r} \dot{\phi}) = \frac{1}{c^2} \dot{\phi} \quad (7.18)$$

Solution of equation (7.18) above gives

$$\ddot{\phi} + \frac{2\dot{r}}{r} \dot{\phi} \approx 0$$

$\dot{\phi} = \frac{l}{r^2}$ First approximation

Where l is a constant of the motion. Hence the radial equation becomes.

8. Solution of Radial Equation of Motion

Solution of radial equation of motion, equation (1.17) above becomes

$$\text{let } \dot{\phi} = \frac{l}{r^2} \Rightarrow \dot{\phi}^2 = \frac{l^2}{r^4}$$

$$\ddot{r} = -\frac{K}{r^2} + \frac{K^2}{c^2 r^3} + \frac{2K}{c^2 r^2} (\dot{r})^2 + \frac{1}{c^2} + \frac{l^2}{r^3} - \frac{3Kl^2}{c^2 r^4} \quad (8.1)$$

$$\text{let } u(\phi) = \frac{1}{r(\phi)}$$

$$\begin{aligned} \dot{r} &= \frac{dr}{d\tau} = \frac{dr}{d\phi} \cdot \frac{d\phi}{d\tau} = \dot{\phi} \frac{dr}{d\phi} = \dot{\phi} \frac{dr}{du} \frac{du}{d\phi} = \dot{\phi} \left(-\frac{1}{u^2}\right) \frac{du}{d\phi} \\ \dot{r} &= \frac{l}{r^2} \left(-\frac{1}{u^2}\right) \frac{du}{d\phi} = lu^2 \left(-\frac{1}{u^2}\right) \frac{du}{d\phi} = -l \frac{du}{d\phi} \end{aligned} \quad (8.2)$$

Hence

$$\ddot{r} = -lu^2 \frac{d}{d\phi} \left(l \frac{du}{d\phi}\right) = -l^2 u^2 \frac{d^2 u}{d\phi^2} \quad (8.3)$$

Putting \dot{r} and \ddot{r} into equation (8.1) we get

$$-l^2 u^2 \frac{d^2 u}{d\phi^2} = -Ku^2 + \frac{K^2 u^2}{c^2} + \frac{2Ku^2}{c^2} (\dot{r})^2 + \frac{1}{c^2} (\dot{r}) + l^2 u^2 - \frac{3Kl^2 u^2}{c^2} \quad (8.4)$$

Finally obtain

$$\Rightarrow \frac{d^2 u}{d\phi^2} + u = \frac{K}{l^2} + \frac{3Ku^2}{c^2} - \frac{K^2 u}{c^2 l^2} + \frac{2K}{c^2} \left(\frac{du}{d\phi}\right)^2 + \frac{1}{u^2 c^2 l} \left(\frac{du}{d\phi}\right) \quad (8.5)$$

By neglecting the last two terms or where contribution for the mean time

$$\frac{d^2 u}{d\phi^2} + u = \frac{K}{l^2} \Rightarrow \text{Newton's planetary equation} \quad (8.6)$$

$$\frac{d^2 u}{d\phi^2} + u = \frac{K}{l^2} + \frac{3Ku^2}{c^2} \Rightarrow \text{Einstein's planetary equation} \quad (8.7)$$

The new equation obtained in this work gives two sets of equation

$$\frac{d^2u}{d\phi^2} + \left(1 + \frac{k^2}{c^2l^2}\right)u = \frac{K}{l^2} \tag{8.8}$$

⇒ which give rise to post Newton's planetary equation and

$$\frac{d^2u}{d\phi^2} + \left(1 + \frac{k^2}{c^2l^2}\right)u = \frac{K}{l^2} + \frac{3Ku^2}{c^2} \tag{8.9}$$

⇒ which give rise to post Einstein's planetary equation.

9. Conclusions and Results

In this paper we have succeeded in deriving equations [(3.10)-(3.13)] which referred to as velocity vectors equations based upon the golden metric tensors. We further obtained equations [(4.16)-(4.19)] which referred to as linear acceleration vector equations. Also in this paper concerted effort were made in order to obtained equation (5.15), (5.16), (5.17) and (5.18) are referred to as general gravitational intensity (or acceleration due to gravity) based upon the golden metric tensor in spherical polar coordinates.

The results obtained in this paper are now available for both physics and mathematician alike to apply them in solving planetary problems based upon Riemannian geometry. More effort and time are being devoted in order to offer solution to equations (5.15)-(5.18) obtained in this paper. Also in this paper we offered solution to equation (7.18) which gives rise to azimuthal angle equation along the equatorial plane. Solution of equation (7.17) obtained also in this paper which leads as to equations (8.8) and (8.9). Equation (8.8) is referred to as post Newton's planetary equation while, equation (8.9) referred to as post Einstein's planetary equations. These equations played an important role in planetary phenomena if handle properly. This paved the way for applied mathematician and physics to investigate the implications of those equations. We hope to offer solution of those equations using a powerful method of successive approximation in the next edition of this paper.

Appendix A

Expansion of t , t^2 and t^{-1} to order of c^{-2} .

We get

$$\begin{aligned} t &= \left[1 - \frac{u^2}{c^2} \left(1 + \frac{2}{c^2}f\right)^{-1}\right]^{\frac{1}{2}} \left(1 + \frac{2}{c^2}f\right)^{-\frac{1}{2}} \\ &\approx \left[1 - \frac{u^2}{c^2} \left(1 + \frac{2}{c^2}f\right)^{-1} + \dots\right] \left(1 - \frac{1}{c^2}f \dots\right) \\ &\approx 1 - \frac{1}{c^2}f - \frac{u^2}{2c^2} \end{aligned}$$

But

$$t^2 = \left[1 - \frac{u^2}{c^2} \left(1 + \frac{2}{c^2}f\right)^{-1}\right] \left(1 + \frac{2}{c^2}f\right)^{-1}$$

$$\begin{aligned} &\approx \left[1 - \frac{u^2}{c^2} \left(1 - \frac{2}{c^2}f \dots\right)\right] \left(1 - \frac{2}{c^2}f \dots\right) \\ &\approx \left[\left(1 - \frac{u^2}{c^2} \dots\right) \left(1 - \frac{2}{c^2}f\right)\right] \\ t^2 &\approx \left[1 - \frac{u^2}{c^2} - \frac{2}{c^2}f + \frac{2u^2}{c^4}f \dots\right] \\ &\approx \left[1 - \frac{u^2}{c^2} + \frac{2K}{c^2r} \dots\right] \end{aligned} \tag{2}$$

Next

$$\begin{aligned} \frac{1}{t} &= (t)^{-1} \\ &= \left[1 - \frac{u^2}{c^2} \left(1 + \frac{2}{c^2}f\right)^{-1}\right]^{\frac{1}{2}} \left(1 + \frac{2}{c^2}f\right)^{\frac{1}{2}} \\ &\approx \left[1 + \frac{u^2}{2c^2} \left(1 - \frac{2}{c^2}f \dots\right)\right] \left(1 + \frac{2}{c^2}f \dots\right) \\ &\approx \left[\left(1 + \frac{u^2}{2c^2} \dots\right) \left[1 + \frac{2}{c^2}f \dots\right]\right] \\ (t)^{-1} &\approx 1 + \frac{u^2}{2c^2} + \frac{1}{c^2}f + \dots \\ &\approx 1 + \frac{u^2}{2c^2} + \frac{K}{c^2r} \end{aligned} \tag{3}$$

Appendix B

$$\begin{aligned} \left(1 - \frac{2K}{c^2r}\right)^{-\frac{1}{2}} &\approx \left(1 + \frac{2K}{c^2r}\right) \\ \left(1 - \frac{2K}{c^2r}\right)^{\frac{1}{2}} &\approx \left(1 + \frac{2K}{c^2r}\right) \end{aligned}$$

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