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Determination of Nuclear Potential Radii and Its Parameter from Finite – Size Nuclear Model

Adamu A.*, Ngadda Y. H.

Department of Physics, University of Maiduguri, Maiduguri, Nigeria

Abstract The atomic nucleus is not a point source. Thus, the assumption of a finite size for a nucleus leads to a departure from Coulomb potential between electron and nucleus. In this work, we endeavor to determine the nuclear potential charge radius by virtue of the modified finite size nuclear potential. It has been found that an electron moves within a small volume of the nuclear potential charge. We found that the volume of the nuclear potential charge exceeded the nuclear radius by factor √3. Due to the extension of the nuclear potential charge, a new and simple $Z^{1/3}$ – dependent formula for calculating the radii of the extension of nuclear potential charge is proposed. The proposed formula gives effective results for potential charge radius. This work offers us a simple way to predict the nuclear charge radius from the assumption of nuclear finite sized model.

Keywords Nuclear volume, Electric quadrupole moment, Nuclear Shapes, Modified nuclear potential, Potential charge radius

1. Introduction

Nuclear extension in space, often characterized by charge radius, is one of the most important static properties of atomic nuclei [1-4]. In the march towards the new era of nuclear physics, the knowledge of nuclear sizes plays a very important role in understanding complex atomic nuclei. It also plays a key role in studying the characters of nucleus, testing theoretical models of nuclei as well as in studying astrophysics and atomic physics [1]. The developments in the measurement techniques for charge radii of nuclei provide more accurate experimental results which can be used to improve model parameters. Because of this, experimental and theoretical nuclear charge radii studies are one of the important topics in nuclear physics. The radius of atomic nucleus can be determined from its charge density distribution [2]. Both the radius of a nucleus and density distributions are important bulk properties of nuclei that determine the nuclear potential, single-particle orbitals, and wave function. Based on charge distributions, the nuclear size has been studied by electron scattering and muonic atoms [5].

Nucleon distributions have been studied by several nuclear reactions with strong interacting probes. Among those, proton elastic scattering provides the best information [5]. In nuclear density functional theories based on the mean-field approach such as the Hartree-Fock-Bogoliubov (HFB) model and the relativistic mean-field (RMF) theory, nuclear charge radii are calculated in a self-consistent way by folding the charge density distribution. Besides, recent work attempts to deduce charge radii based on the α decay, cluster and proton emission data [1]. The volume or radius of the nucleus is naturally proportional to the nuclear mass number. However, the conventional A – dependent formula, $R = r_0A^{1/3}$ is not globally valid for all nuclei in which there is a significant difference between proton and neutron numbers. Also, the experimental data indicate that the order of magnitude of the range of nuclear forces r_0 is not constant [6]. It is seen from the developed formula that the Z – dependent formula describe nuclei much better [2].

Like many systems governed by the laws of quantum mechanics, the nucleus is an object full of mysteries whose properties are much more difficult to characterize than those of macroscopic objects. Rather than build an exact replica of the nuclear system, nuclear physicists in reality have selected a different approach, using a relatively small number of measurable properties of quantum systems to specify the overall characteristic of the entire nucleus [1].

In this work, we attempt to propose a set of new difference equations of nuclear potential radius that is different from the above approaches. Since the size of a nucleus depends mainly on its charge (proton) distribution, the assumption that atomic nucleus has a finite size charge distribution has been made to determine the radius of nuclear charge that depends mainly on the proton charge distribution.

* Corresponding author:
aliyuadamu703@gmail.com (Adamu A.)
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2. Methodology

The size of a nucleus is characterized by, R_{rms} or by the radius R of the uniform sphere [7]. Both the quantities are related. The mean squared radii of neutron, proton, charge and mass distribution can be defined as follows:

$$\langle r_c^2 \rangle = \frac{\int_0^\infty r^2 4\pi r^2 \rho(r) dr}{\int_0^\infty 4\pi r^2 \rho(r) dr},$$

where $\rho(r)$ is the nuclear charge density [8]. For a uniformly charged sphere [$\rho(r)$ = constant] of radius R. For $r > R$, this gives

$$\langle r^2 \rangle = \frac{\int_0^R r^4 dr}{\int_0^R r^2 dr} = \frac{3}{5} R^2.$$

So that the radius of a sphere

$$R = \sqrt{\frac{5}{3}} \langle r_c^2 \rangle^{1/2} \tag{1}$$

The root-mean-square nuclear matter radii (R_{rms}) and the density distributions contain an important insight on nuclear potentials and nuclear wave functions [9]. If the nucleus is a point charge with the distance of electron from the nucleus, r and $k = (4\pi\varepsilon_0,)^{-1}$ then its potential is given by

$$U(r) = -k \frac{Ze^2}{r} \tag{2}$$

The nuclear potential and electron wavefunction change when the nucleus is described as a finite-size source with a uniform distribution of charges [3] of radius R, then the electron wave function can penetrate to $r \le R$, and thus the electron spends part of its time inside the nuclear charge distribution, there it feels a very different interaction [10]. Therefore the potential appropriate for the perturbed electron is no longer of the pure Coulomb form. This is because the electrostatic potential that appears in (2) is no longer due just to the point charge nucleus of electric charge $|e|Z$ [11].

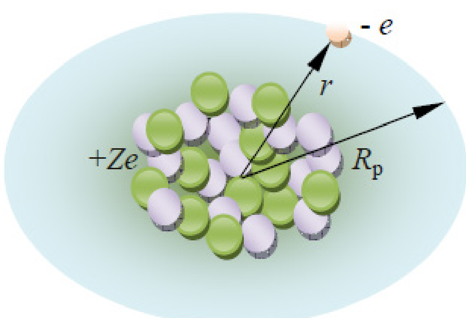

Figure 1. The finite sized nucleus of charge (+*Ze*) orbited by a perturbed electron

The potential inside a spare of radius r due to a point charge $q_{\text{inside}} = e(r/R)^3$, located at the origin is from Coulomb's law:

$$U(r \le R) = -\frac{ke^2}{r} \left(\frac{r}{R}\right)^3 \tag{3}$$

The perturbative potential difference between r and R is defined by:

$$\begin{aligned} U(r \le R) &= -\int_r^R \frac{4\pi e \rho r^2}{r} dr = -\frac{3ke^2}{R^3} \frac{r^2}{2} \Big|_r^R \\ &= -\frac{3ke^2}{2R^3}\left(R^2 - r^2\right) \end{aligned} \tag{4}$$

where $\rho = 3q/4\pi R^3$ is the nuclear charge distribution and in this case it is constant [12]. And

$$R = r_0 A^{1/3} \tag{5}$$

Thus, for $r \le R$ we have the potential:

$$\begin{aligned} U(R) &= -\frac{ke^2}{r}\left(\frac{r}{R}\right)^3 - \frac{3ke^2}{2R^3}(R^2 - r^2) \\ &= -\frac{3ke^2}{2R^3}(3R^2 - r^2) \end{aligned} \tag{6}$$

Equation (6) represents the potential for a finite-size charge nucleus [13]. Now we have seen that due to the finite nuclear size, the electric potentials $U(R)$ and $U(r)$ of the nucleus are different [14]. Therefore, the spherical electrostatic potential function $U(R)$, corresponding to a nuclear charge density distribution, will then be used to replace the common Coulomb potential for a point-like nucleus, (2) [15]. Also compared to a point-like nucleus, the extended nuclear charge distribution also leads to a shift in the energy levels of electron [16, 17].

To understand completely the finite size of nuclei, we here calculate the volume of nucleus to see its deviation from point size. Assuming uniform charge distribution, we have for a nucleus of charge $+Ze$, the volume

$$V = \int_0^{b\sqrt{1-\frac{z^2}{a^2}}} \rho d\rho \int_0^z dz \int_0^{2\pi} d\varphi = \frac{4}{3}\pi a b^2 \tag{7}$$

And hence the density

$$\rho = \frac{3Ze}{4\pi a b^2} \tag{8}$$

The intrinsic quadrupole moment of a symmerty charged distribution is defined by the relation

$$Q_0 = \frac{1}{e}\int \rho(r)\left[3(z)^2 - (r)^2\right] dV \tag{9}$$

The nucleus is assumed to have asymmetry axis along z' and e is the charge on each proton [18]. Using the fact that $r'^2 = x'^2 + y'^2 + z'^2 = \rho'^2 + z'^2$ and $dv = \rho' d\rho' d\varphi' dz'$, we find

$$\begin{aligned} Q_0 &= \frac{3Ze}{4\pi a b^2 e}\iiint \rho(r)\left[3z^2 - z^2 - \rho^2\right] dV \\ &= Z\frac{2}{5}\left(a^2 - b^2\right) \end{aligned} \tag{10}$$

A non-zero quadrupole moment Q_0 indicates that the proton distribution is not spherically symmetric. By convection, the value of Q_0 is taken to be positive (i.e. when $a > b$) if the ellipsoid is prolate and negative (i.e. when $a < b$) if the ellipsoid is oblate and zero (i.e. when $a = b$) if the ellipsoid is a sphere. Figure 2 depicts the possible charge (shape) distribution of nuclei.

Nuclear deformation has an influence on the nuclear charge radii. The effective deformation parameters (β_{eff}) are deduced from the intrinsic quadrupole moment (Q_0), which is related to the spectroscopic quadrupole moment (Q) via the well-known formula

$$Q = \frac{Q_0 I(2I-1)}{(I+3)(2I+3)} \tag{11}$$

which has been established within the framework of the collective model [18]. The β_{eff} is calculated using

$$Q_0 = \frac{3}{\sqrt{5\pi}} Z R^2 \beta_{eff}\left(1 + \frac{1}{8}\sqrt{\frac{5}{\pi}}\beta_{eff}\right) \tag{12}$$

Thus, the effective deformation parameters can be deduced the quadrupole moments and the charge radii are known. β_{eff} has been deduced for light mirror nuclei [5].

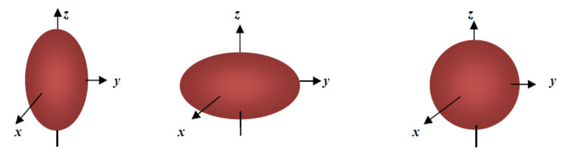

Figure 2. Electric quadrupole moments for different charge distribution

3. Results and Discussion

The potential energies of an electron for a point-like nucleus and for a finite-size nucleus of radius R, are computed for different values of r by using equations (2) and (6) and are presented in Table 1.

Table 1. The values of potential energies for a point-like and finite-size nucleus of hydrogen atom

r (fm)	Potential Energies (MeV)	
	Point – like Nucleus	Finite – size Nucleus
0.0	0.0000	-1.7008
0.5	-2.8800	-1.6129
1.0	-1.4400	-1.3493
1.5	-0.9600	-0.9099
2.0	-0.7200	-0.2948
2.5	-0.5760	0.4961
3.0	-0.4800	1.4627
3.5	-0.4113	2.6050
4.0	-0.3600	3.9232
4.5	-0.3200	5.4170
5.0	-0.2880	7.0866

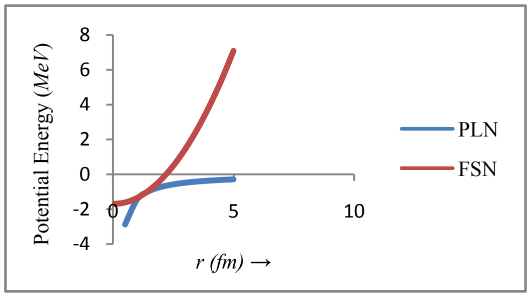

Figure 3. The potential energy curves for a point-like (PLN) and a finite-size nucleus (FSN)

Table 2. The values of finite-size nuclear potential energies of various atomic nuclei

r(fm)	The Finite-Size Nuclear Potential (MeV)							
	$^{1}H_{1}$	$^{7}Li_{3}$	$^{23}Na_{11}$	$^{39}K_{19}$	$^{63}Cu_{29}$	$^{85}Rb_{37}$	$^{107}Ag_{47}$	$^{133}Cs_{55}$
0.0	-1.7008	-2.6674	-6.5786	-9.5291	-12.3956	-14.3126	-16.8378	-18.3257
0.5	-1.6129	-2.6298	-6.5366	-9.4863	-12.3548	-14.2743	-16.7992	-18.2893
1.0	-1.3493	-2.5167	-6.4105	-9.3579	-12.2338	-14.1596	-16.6834	-18.1803
1.5	-0.9099	-2.3284	-6.2003	-9.1438	-12.0316	-13.9683	-16.4904	-17.9987
2.0	-0.2948	-2.0647	-5.9062	-8.8441	-11.7848	-13.7005	-16.2202	-17.7442
2.5	0.4961	-1.7258	-5.5279	-8.4588	-11.3844	-13.3563	-15.8728	-17.4172
3.0	1.4627	-1.3115	-5.0656	-7.9879	-10.9394	-12.9355	-15.4482	-17.0175
3.5	2.6050	-1.3151	-4.5192	-7.4313	-10.4136	-12.4382	-14.9465	-16.5451
4.0	3.9232	-0.2570	-3.8889	-6.7892	-10.2129	-11.8644	-14.3675	-16.0000
4.5	5.4170	0.3833	-3.1744	-6.0614	-9.1192	-11.2141	-13.7113	-15.3823
5.0	7.0866	1.0988	-2.3759	-5.2479	-8.3506	-10.4873	-12.9779	-14.6919
5.5	8.9320	1.8897	-1.4933	-4.3489	-7.5012	-9.6840	-12.1674	-13.9287
6.0	10.9531	2.7559	-0.5267	-3.3642	-6.5708	-8.8042	-11.2796	-13.0930
6.5	13.1499	3.6975	0.5239	-2.2939	-5.5595	-7.8479	-10.3146	-12.1845
7.0	15.5226	4.7143	1.6587	-1.1380	-4.4674	-6.8151	-9.2725	-11.2033
7.5	18.0709	5.8065	2.8775	0.1035	-3.2943	-5.7057	-8.1531	-10.1495
8.0	20.7950	6.9740	4.1803	1.4307	-2.0404	-4.5199	-6.9566	-9.0232
8.5	23.6949	8.2168	5.5672	2.8434	-0.7055	-3.2576	-5.6828	-7.8238
9.0	26.7705	9.5349	7.0382	4.3418	0.7102	-1.9187	-4.3319	-6.5520
9. 5	30.0218	10.9284	8.5932	5.9259	2.2068	-0.5034	-2.9037	-5.2075
10.0	33.4489	12.3972	10.2322	7.5955	3.7844	0.9885	-1.3984	-3.8097
10.5	37.0517	13.9414	11.9554	9.3508	5.4429	2.5568	0.1841	-2.3004
11.0	40.8303	15.5608	13.7625	11.1917	7.1822	4.2017	1.8439	-0.7378
11.5	44.7847	17.2556	15.6537	13.1182	9.0025	5.9231	3.5808	0.8974
12.0	48.9147	19.0256	17.6290	15.1304	10.9036	7.7209	5.3949	2.6053

The potential curve for finite-size nuclear can be seen in Figure 3, represented by the red line, while the point-like potential curve can be seen by the blue line, falling to - ∞ as r approaches 0 and their curves coincide only at the value of r approximately equal to the value of the experimentally measured value of nuclear radius r_0. It can also be observed from the figure that the finite-size potential is finite (with magnitude 1.7008 *MeV*), when $r = 0$. This is evident for a finite size nature of a nucleus.

The finite – size nuclear potential energy U(R) is computed for various nuclei by using equations (6) and are presented in Table 2.

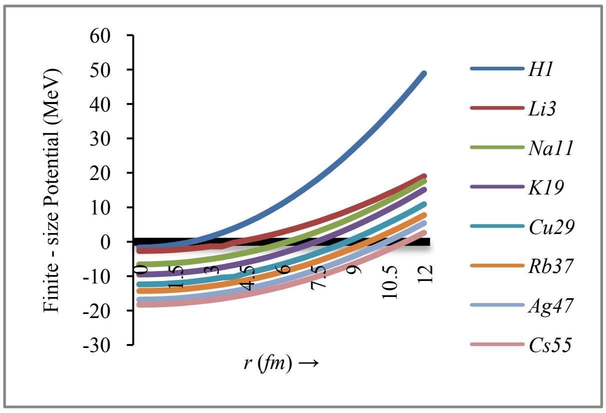

Figure 4. The potential energy curves for various finite-size nuclei (FSN)

Figure 4 gives the information on potential radii of selected nuclei as determined by finite – size nuclear potential model. In the Figure the negative region of the plot of finite size potential is the range over which the potential extend. It is also worth noting that the values of finite size nuclear potential charge change the sign at $U(R) = 0$, the intensity of the potential charge vanishes smoothly from its source and its value is practically zero outside on positive region. It can be observed from the figure that the potential curve for hydrogen atom (a proton nucleus) has different characteristic, there is rapid increase the value of U(R) at large distance, r. The intercept on r axis represent the limit at which the finite sized potential exists. The nuclear charge radius R from equation (5) and the intercept on r axis (the nuclear potential radius, R_p) deducted from Figure 4 are presented in Table 3.

Table 3. The values of nuclear potential radii, R_p deduced from figure 2 and the corresponding value of R using equation (5)

Nuclide: ${}^{A}X_{Z}$	R (*fm*)	R_p (*fm*)	R_p/R
${}^{1}H_{1}$	1.2700	2.1997	1.732
${}^{7}Li_{3}$	2.4294	4.2078	1.732
${}^{23}Na_{11}$	3.6117	6.2556	1.732
${}^{39}K_{19}$	4.3068	7.4596	1.732
${}^{63}Cu_{29}$	5.0534	8.7527	1.732
${}^{85}Rb_{37}$	5.5839	9.6716	1.732
${}^{107}Ag_{47}$	6.0293	10.4431	1.732
${}^{133}Cs_{55}$	6.4827	11.2284	1.732

From Table 3 we can deduce a simple relation:

$$R_p = 1.732\ R = \sqrt{3}R \tag{14}$$

Thus, the nuclear potential radius, R_p estimated by this method is √3 higher than that for the nuclear charge radius, R. This is because the nuclear potential radius, R_p is the measure of the range of the nuclear potential, which is independent of the nature or charge state of the nucleons. Thus a correction due to the finite – size of nuclear potential results in equation (14).

Table 4. The values of nuclear potential radii, R_p deduced from Figure 4 at a point when $U(R) = 0$ and the potential gradients, r_0 obtained from Figure 5

Nuclide: ${}^{A}X_{Z}$	R_p (*fm*)	$Z^{1/3}$	r_p (*fm*)
${}^{7}Li_{3}$	4.2078	1.4422	2.9176
${}^{23}Na_{11}$	6.2556	2.2224	2.8147
${}^{39}K_{19}$	7.4596	2.6684	2.7955
${}^{63}Cu_{29}$	8.7527	3.0723	2.8489
${}^{85}Rb_{37}$	9.6716	3.3322	2.9024
${}^{107}Ag_{47}$	10.4431	3.6088	2.8937
${}^{133}Cs_{55}$	11.2284	3.8029	2.9525

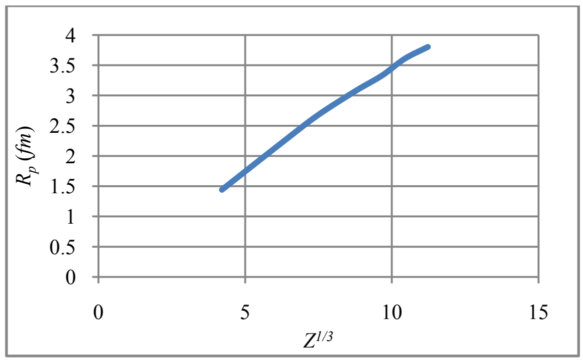

Figure 5. The plots of nuclear potential radius, R_p as a function of a proton number, $Z^{1/3}$

Figure 5 shows the plots of nuclear charge radius, R_p as a function of a proton number, $Z^{1/3}$ and the nuclear charge radius varied directly as the proton number, Z. Plotting R_p against $Z^{1/3}$ (Figure 4) gives us a slope $r_p \approx 2.875$ *fm* for nuclides higher than hydrogen. These results showed that on accounting for the finite sized nuclear charge distribution, significant changes in the R were observed. The results also showed that the charge radii of atomic nuclei, independent of neutron number, follow remarkably very simple relations:

$$R_p \approx 2.875\ Z^{1/3} = r_p\, Z^{1/3} \tag{15}$$

were r_p is the range of the nuclear charge.

The potential curve for hydrogen atom rapidly increases the value of U(R) at large distance, r. This showed that an electron in hydrogen atom does interact with the finite sized potential but only slightly and the relation (15) cannot be applied for hydrogen atom. The order of range of the nuclear potential charge for hydrogen atom is very small approximately 2.2 *fm*. Thus the effects of finite sized of potential charge on electron in hydrogen atom are extremely small.

The difference between finite size and point source nuclear size, being negligible for hydrogen (low atomic number), it grows when Z increases leading to a strong enhancement of the total nuclear size correction for heavy nucleus [19, 20]. In higher nuclei the deviation of the nuclear potential from the Coulomb potential at small distances becomes important at distances r much larger than the radius of the nucleus, R [21].

The theoretical predictions for nuclear charge radius is a challenge for its measurements in the future experiments and thus, for obtaining detailed information on the nuclear potential charge distributions. This is particularly important considering the fact that there are many sources of charge radius data deduced from very different theoretical and experimental techniques. The comparison of the calculated nuclear potential radius, R_p with the future data will be a test of the corresponding theoretical models used for studies of the atomic nuclei structure.

4. Conclusions

Apart from presenting a more physical model for the nuclear charge distribution than the usual point charge model, this work also revealed the important advantage of finite – size nuclear model in determining the nuclear charge radius. Despite the fact that atomic nuclei are complex finite many-body systems governed by the laws of quantum mechanics, these proposed formulas equation (14) and (15) can be used to predict unknown nuclear charge radii. For this reason, the "spherical nuclear" radius R can be replaced by R_p for the distribution of proton charge beyond the radius of the atomic nucleons.

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