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A Generalization of Newton's Planetary Equation of Motion for Static Homogeneous Spherical Massive Bodies

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Abstract In this article, we applied the generalized Golden (Riemannian) gravitational scalar potential exterior to the body to the well - known Newton's dynamical gravitational equations of motion to obtain generalizations of Newton's dynamical equations of motion. The generalized dynamical gravitational equations of motion are applied to the motion of the planets in the equatorial plane to obtain a generalized planetary equation of motion. The results are that the generalized Newtonian dynamical equations of motion and the generalized planetary equation of motion are augmented with additional correction terms of all orders of c^{-2} which are not found in Newton's dynamical equations of motion or Einstein's geometrical equations of motion.

Keywords Generalized Golden (Riemannian) Gravitational Scalar Potential, Generalized Dynamical equations of Motion, Equatorial plane, Planetary Equation of Motion

1. Introduction

The planetary equations motion of describes the motion of the planets and can be applied to the planets in the solar system or any other system. Newton's dynamical gravitational equations of motion were derived based on Newton's gravitational scalar potential exterior to a static homogeneous spherical body Φ^{+} [1-4] given by

$$f(r) = -\frac{GM}{r} \tag{1}$$

where

G is the universal gravitational constant,

M is the mass of the spherical bodies and

r is the mean distance from the centre of the body

In this article, the generalized golden (Riemannian) gravitational scalar potential exterior to a spherical massive body is applied to the well - known Newton's dynamical gravitational equations of motion to obtain additional post Newton's correction terms of all orders of c^{-2} to Newton's planetary equation of motion.

2. Theoretical Analysis

The generalized golden gravitational scalar potential exterior to the body f^{+} for a static homogeneous spherical massive body [5-8] is given by

$$f^{+}(r) = \frac{-k}{r}\left\{1 - \frac{3k}{5c^2 R}\right\} + \frac{2k^2}{c^2 r^2} \tag{2}$$

where, $K = +\ GM$, r is the mean distance from the sun and c is the speed light in vacuum.

The well - known Newton's dynamical equation of motion [1-3] is given by

$$\underline{a} = -\underline{\nabla} f^{+} \tag{3}$$

where,

f^{+} is Newton's gravitational scalar potential exterior to the body

$\underline{a}$ is the Newtonian acceleration vector

$\underline{\nabla}$ is the gradient operator

By the symmetry of the distribution of mass it follows that the geometrical gravitational field will depend on only the radial coordinate r, therefore, the gradient operator

$$\underline{\nabla} f^{+} = -\hat{r}\frac{\partial f}{\partial r} \tag{4}$$

where, $\hat{r}\ is\ the$ unit vector

Substituting (2) into (4) and differentiating with respect to r we obtain

$$\underline{a} = -\frac{k}{r^2} + \frac{3k^2}{5c^2 R r^2} + \frac{4k^2}{c^2 r^3} \tag{5}$$

Equation (5) is the generalized Golden Newtonian acceleration vector.

Consider a planet or comet of rest mass M (regarded as a particle) in the gravitational field of the sun. Newton's dynamical equation of motion in spherical polar coordinates (r, θ, ϕ) is given by

$$\ddot{r} + r\dot{\theta}^2 + r\dot{\phi}^2 sin^2\,\theta = -\frac{k}{r^2} + \frac{3k^2}{5c^2 R r^2} + \frac{4k^2}{c^2 r^3} \tag{6}$$

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$$\ddot{\theta} + 2\dot{r}\dot{\theta} - r\dot{\phi}^2 sin\theta cos\theta = 0 \tag{7}$$

$$r\ddot{\phi}\, sin\theta + 2\dot{r}\dot{\phi}\, sin\theta + 2r\dot{\theta}\dot{\phi}cos\theta = 0 \tag{8}$$

where, a dot denotes one - time differentiation.

Equations (6 – 8) are the generalized Golden equations of motion of a particle according to the generalization of Newton's dynamical theory of gravitation.

3. Motion in the Equatorial Plane (Anomalous Orbital Precession in the Solar System)

Consider the motion of a particle whose motion is confined to the equatorial plane of the sun, such as planets or comets or asteroids, in the solar system

Then,

$$\theta = \frac{\pi}{2}$$

Hence, Newton's dynamical equations of motion (6 – 8) reduces to

$$\ddot{r} - r\dot{\phi}^2 = -\frac{k}{r^2} + \frac{3k^2}{5c^2Rr^2} + \frac{4k^2}{c^2r^3} \tag{9}$$

and

$$r\ddot{\phi} + 2\dot{r}\dot{\phi} = 0 \tag{10}$$

The exact solution of the angular equation (10) is given by

$$\dot{\phi} = \frac{l}{r^2} \tag{11}$$

where, l is the constant of motion corresponding to the angular momentum per unit rest mass. The first integral of the radial equation of motion (9) subject to (11) yields

$$\dot{r}^2(r) = 2k\left(\frac{1}{r} - \frac{1}{r_i}\right) - l^2\left(\frac{1}{r^2} - \frac{1}{r_i^2}\right) - \frac{6k^2}{5c^2R}\left(\frac{1}{r} - \frac{1}{r_i}\right) - \frac{4k^2}{c^2}\left(\frac{1}{r} - \frac{1}{r_i}\right) - \frac{1}{2c^2}\left[2k\left(\frac{1}{r} - \frac{1}{r_i}\right) - l^2\left(\frac{1}{r} - \frac{1}{r_i}\right)\right]^2 \tag{12}$$

where, r_i is any apsidal distance. This is the exact generalized golden Newtonian radial speed of a planet. It follows from (12) subject to (11) that the generalized Golden Newtonian radial acceleration of the planet in terms of radial coordinate is given by

$$\ddot{r} = -\frac{k}{r^2}\left[1 + \frac{2k}{c^2r_1} - \frac{l^2}{c^2{r_1}^2} - \frac{3k}{5c^2R}\right] + \frac{l^2}{r^3}\left[1 + \frac{6k^2}{c^2l^2} + \frac{2k}{c^2r_1} - \frac{l^2}{c^2r_1^2}\right] + \frac{l^2}{r^4}\left[-\frac{3k}{c^2}\right] + \frac{l^2}{r^5}\left[\frac{l^2}{c^2}\right] \tag{13}$$

By transformation that

$$r(\phi) = \frac{1}{u(\phi)} \tag{14}$$

$$\ddot{r} = -l^2u^2\frac{d^2u}{d\phi^2} \tag{15}$$

Substituting (15) into (13) and dividing both sides by u^2l^2 gives

$$\frac{d^2u}{d\phi^2} = \frac{k}{l^2}\left(1 - \frac{3k}{5c^2R}\right) - \left(1 + \frac{6k^2}{c^2l^2}\right)u \tag{16}$$

The generalized Golden Newtonian acceleration vector, equations of motion, radial speed, radial acceleration and radial planetary equation of motion in the equatorial plane are found to be equations (5), (6), (7), (8), (12), (13) and (16) respectively. These results reduces to c^0 to the corresponding pure Newtonian showing that it agrees with the well - known Equivalence Principles in Physics and to the orders of c^2 it contains post Newton and post Einstein correction terms which are uncovered for theoretical development and experimental investigations and applications. It must also be noted that these equations contain additional correction terms not found in [4]. It is most interesting and instructive to note that the post Newtonian correction terms $\left(1 - \frac{3k}{5c^2R}\right) - \left(1 + \frac{6k^2}{c^2l^2}\right)$ in equation (16) can be used to explain the planetary parameters as well as the anomalous orbital precession of the orbit the planets.

4. Remarks and Conclusions

We have in this research work shown how to formulate a generalized Golden Newtonian acceleration vector, equations of motion, radial speed, radial acceleration and radial planetary equation of motion in the equatorial plane using the generalized golden gravitational scalar potential exterior to the body. The immediate consequence of the generalized Newtonian radial planetary equation of motion is that it be used to derive the planetary parameters. The pace is therefore set for the theoretical development and experimental investigations and applications of the post Newton and post Einstein correction terms obtained in this research work.

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